Modelos de Heterocedasticidade Condicional

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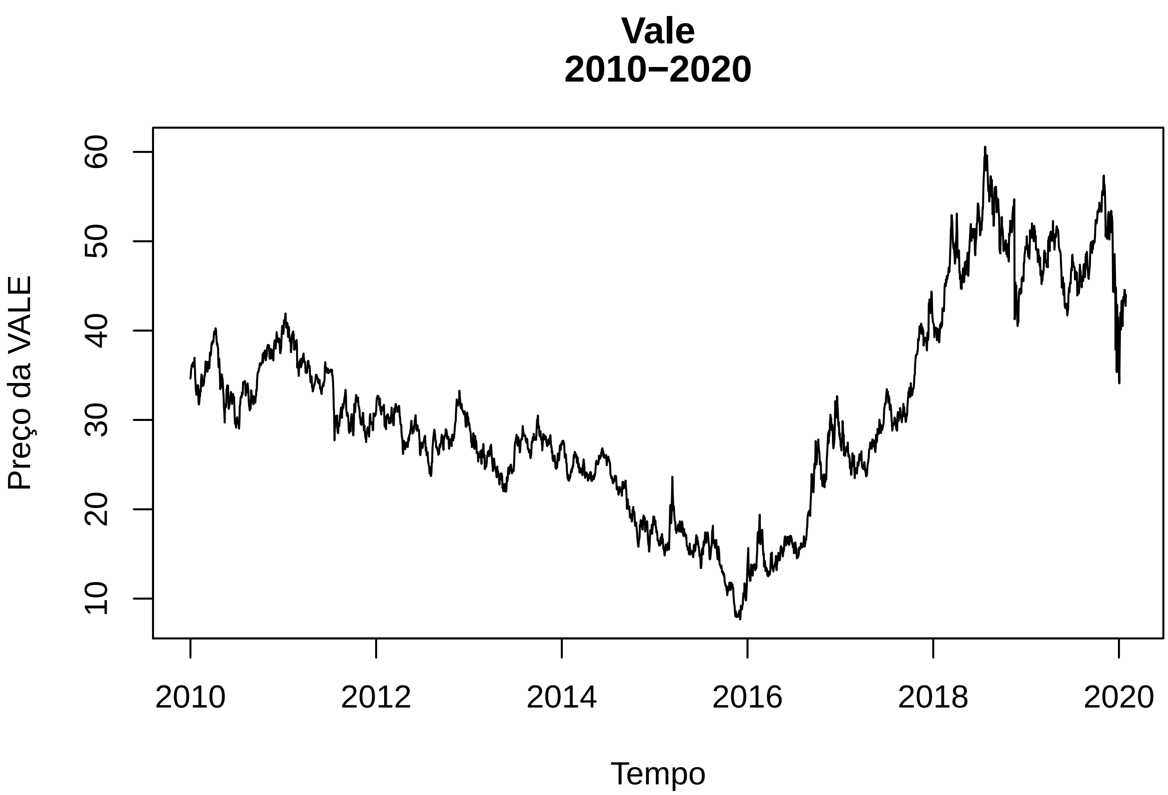
7 Modelos

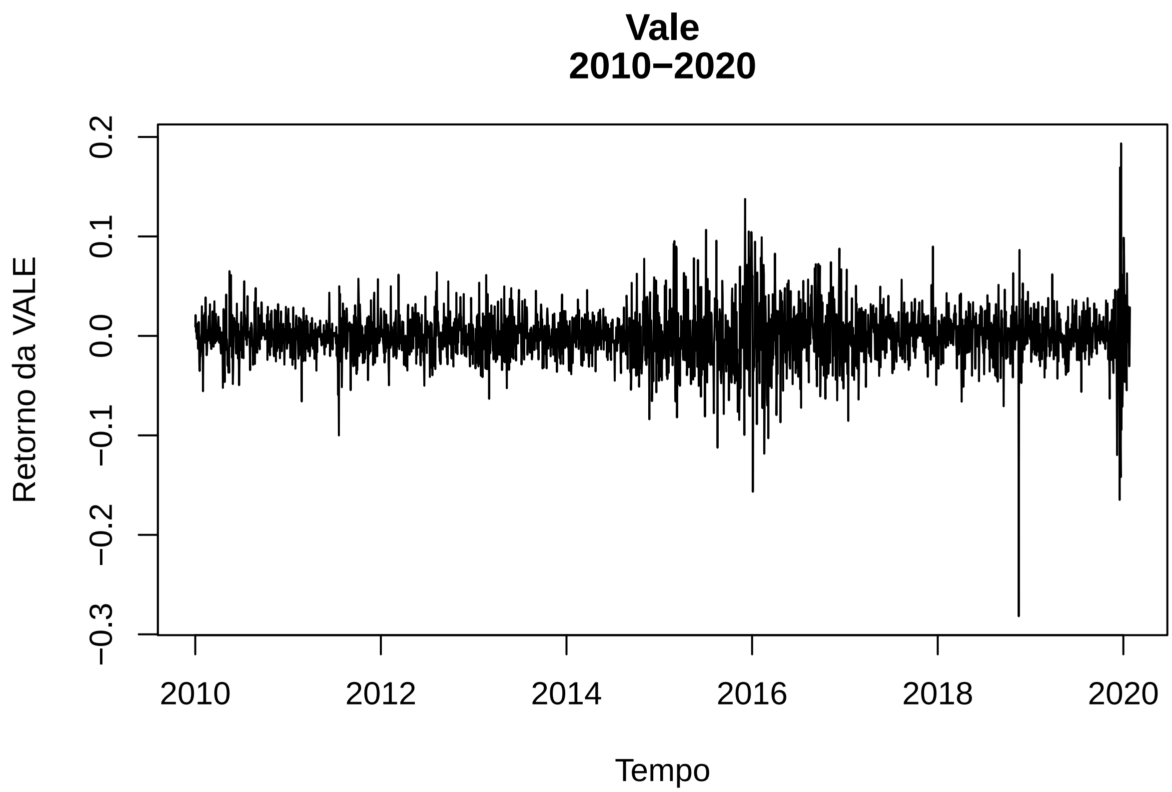
Será apresentado 7 modelos de HC para a série dos retornos da Vale. Primeiramente iremos analisar a série do retorno e buscar o modelo ARMA que melhor se ajusta. E posteriormente, iremos buscar os modelos de HC que melhor se ajuste a série. Os modelos HC são Garch (inovações gaussianas, t e skew t), IGARCH, GARCHM, EGARCH e TGARCH.

Série Historica

```
rtn     = ts(vale, frequency = 252, start = c(2010, 1,1))
par (mfcol = c(1, 1))
plot (rtn, type = 'l', xlab = 'Tempo', ylab = 'Preço da VALE',main= c("Vale","2010-2020"))
```



Log do retorno da Vale



Estatísticas

```
##                     VALE Log do Retorno
## nobs         2540.000000    2539.000000
## NAs             0.000000       0.000000
## Minimum         7.664479      -0.281822
## Maximum        60.594360       0.193574
## 1. Quartile    23.971181      -0.013776
## 3. Quartile    38.080021       0.013365
## Mean           30.919815       0.000094
## Median         29.257683       0.000000
## Sum         78536.330920       0.239289
## SE Mean         0.226132       0.000539
## LCL Mean       30.476392      -0.000962
## UCL Mean       31.363238       0.001150
## Variance      129.885146       0.000737
## Stdev          11.396716       0.027139
## Skewness        0.355837      -0.317886
## Kurtosis       -0.557215       8.729573
```

Série do Retorno

Primeiramente, iremos analisar a série do retorno e verificar se há algum componente que ajude a prever a série.

Teste para a média da série

O test t nos mostra que o retorno da vale foi estatísticamente igual a zero.

```r
t.test (dlvale)
```

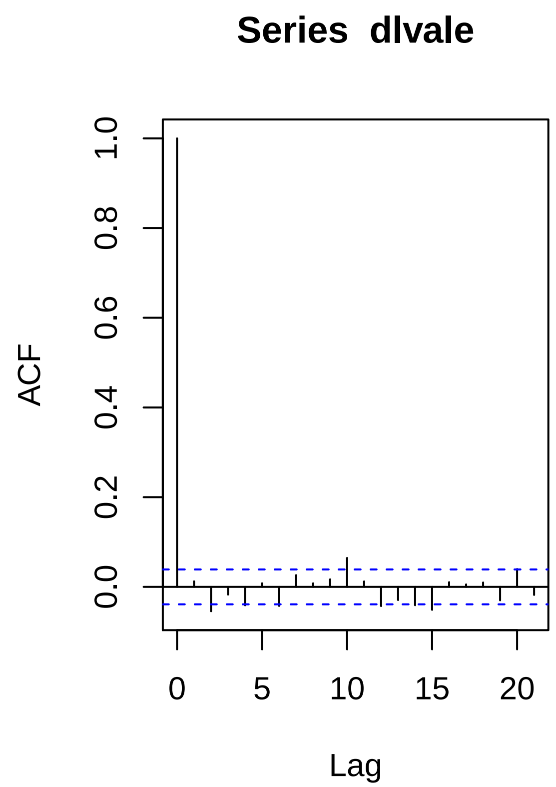
```
##
##  One Sample t-test
##
## data:  dlvale
## t = 0.17498, df = 2538, p-value = 0.8611
## alternative hypothesis: true mean is not equal to 0
## 95 percent confidence interval:
##  -0.0009618838  0.0011503747
## sample estimates:
##    mean of x
## 9.424543e-05
```

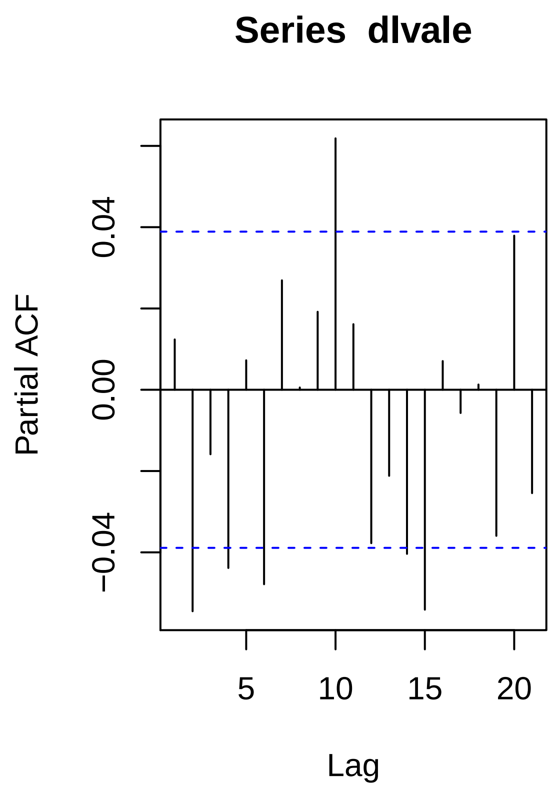
Análise da FAC e FACP da série do retorno

Ao analisarmos a FAC e a FACP conseguimos identificar se o retorno depende de alguma das suas defasagens.Ao analisarmos, podemos concluir que:

Existe algumas defasagens significantes. Portanto, devemos modelar a série do retorno antes de prosseguir com a modlagem da variância.

```r
par(mfcol=c(1,2))
acf(dlvale,lag=21)
pacf(dlvale,lag=21)
```





Modelo proposto para o retorno: ARMA(4,6) com ajuste sazonal no período 15 O modelo incialmente testado foi o ARMA(6,6) já que quase todas as 6 defasagens inicias da FACP e da FAC são significantes.

Após alguns testes, foi concluído que o modelo que melhor se ajusta ao retorno da série é:

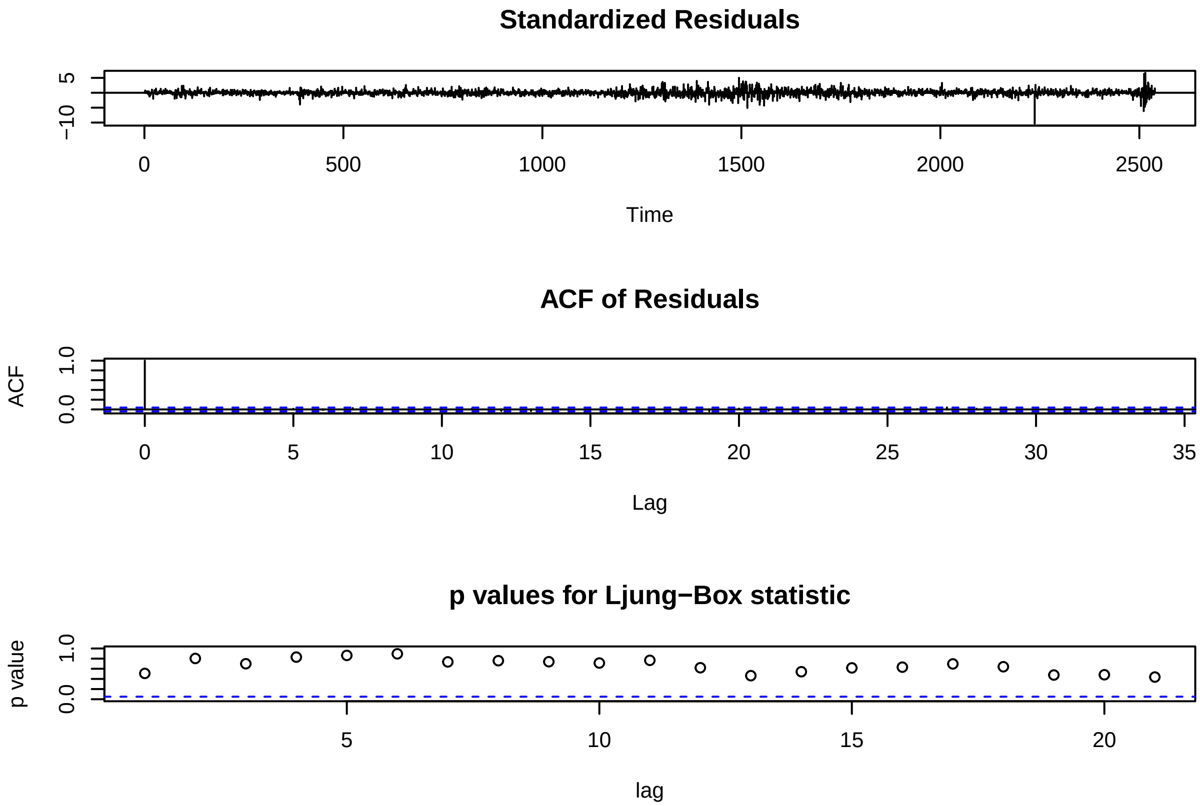
```
c1 <- c(0,NA,0,NA,0,NA,0,NA,0,NA,NA)
m0=arima(dlvale,order=c(4,0,6),fixed=c1,include.mean=F,seasonal = list(order=c(1,0,0),period=15))
coeftest(m0)
```

```
##
## z test of coefficients:
##
##       Estimate Std. Error  z value  Pr(>|z|)
## ar2   0.339851   0.023776  14.2939 < 2.2e-16 ***
## ar4  -0.942207   0.034454 -27.3472 < 2.2e-16 ***
## ma2  -0.401625   0.030937 -12.9821 < 2.2e-16 ***
## ma4   0.932870   0.043460  21.4651 < 2.2e-16 ***
## ma6  -0.074358   0.020524  -3.6229 0.0002913 ***
## sar1 -0.053961   0.019940  -2.7062 0.0068059 **
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
```

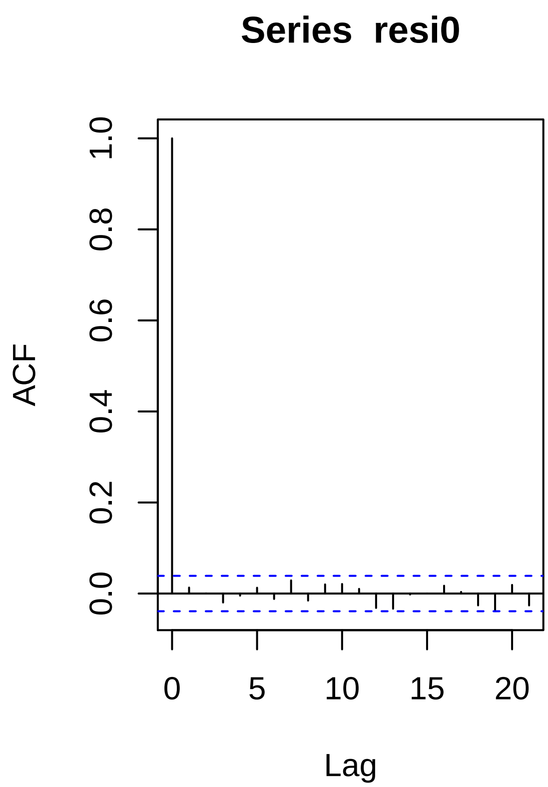
Análise dos resíduos de M0

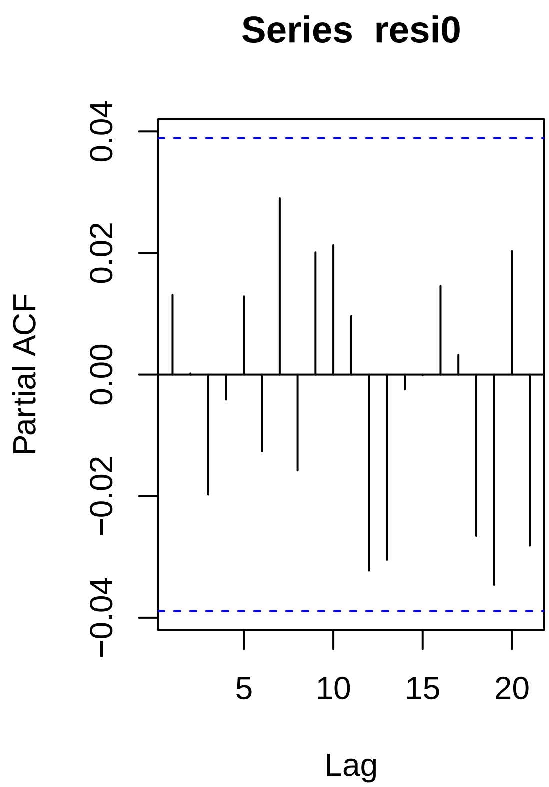
O teste de Ljung-Box mostra que os resíduos são não autocorrelacionados com suas defasagens. Ou seja, todas as informações que estava contida na série historica foram extraidas.

```r
tsdiag(m0,gof=21)
```




```r
resi0 = residuals (m0, standardize = T)
par (mfcol = c(1, 2))
acf (resi0, lag = 21)
pacf (resi0, lag = 21)
```





Análise dos resíduos quadraticos de M0

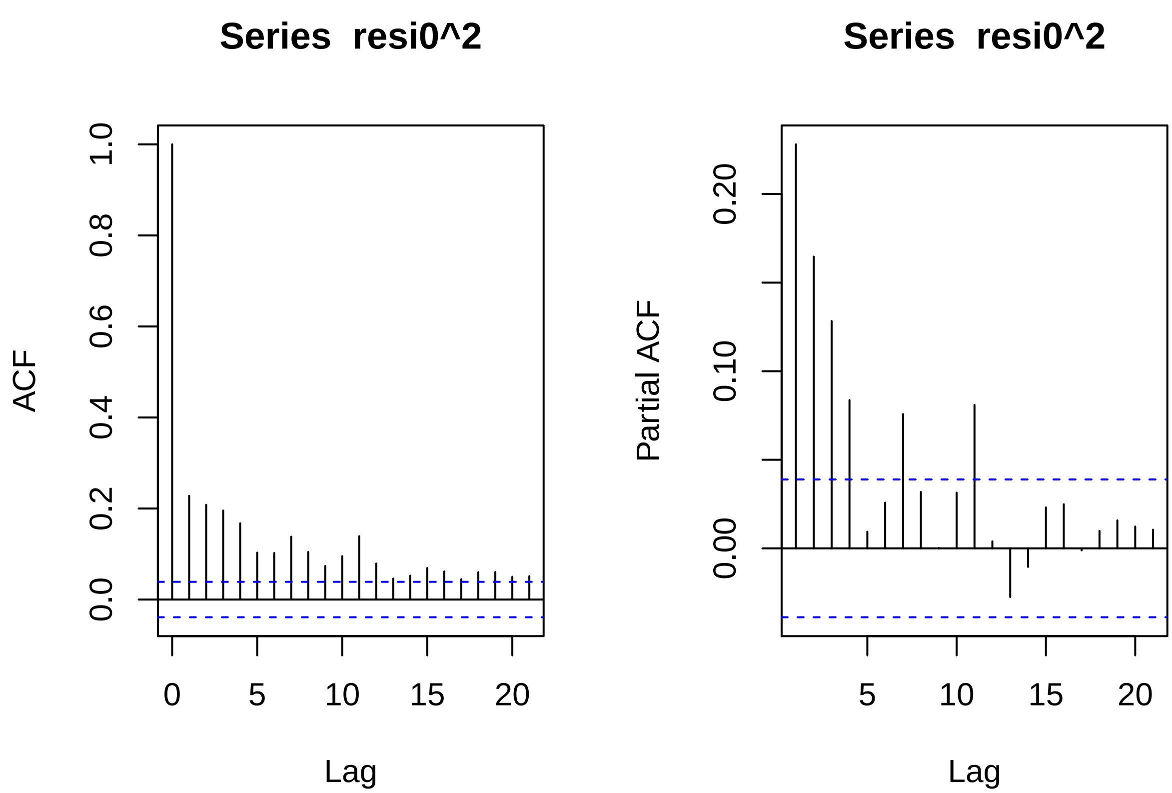
O teste arch e as defasagens dos resíduos quadraticos nos mostram que há dependência entre os resíduos. Portanto, a variância é heterocedastica e pode ser modelada.

```r
archTest (resi0, 21)
```

```
##
## Call:
## lm(formula = atsq ~ x)
##
## Residuals:
##       Min        1Q    Median        3Q       Max
## -0.011901 -0.000489 -0.000318  0.000058  0.076843
##
## Coefficients:
##               Estimate Std. Error t value Pr(>|t|)
## (Intercept)  2.712e-04  5.487e-05   4.942 8.23e-07 ***
## x1           1.491e-01  2.002e-02   7.448 1.30e-13 ***
## x2           1.266e-01  2.024e-02   6.254 4.68e-10 ***
## x3           1.011e-01  2.039e-02   4.955 7.70e-07 ***
## x4           5.970e-02  2.049e-02   2.913  0.00361 **
## x5          -1.012e-02  2.053e-02  -0.493  0.62195
## x6           1.032e-02  2.052e-02   0.503  0.61526
## x7           6.265e-02  2.052e-02   3.053  0.00229 **
## x8           1.616e-02  2.056e-02   0.786  0.43208
```

```
## x9          -1.649e-02  2.056e-02  -0.802  0.42279
## x10          2.092e-02  2.057e-02   1.017  0.30909
## x11          8.278e-02  2.050e-02   4.037 5.58e-05 ***
## x12          4.364e-03  2.057e-02   0.212  0.83198
## x13         -3.252e-02  2.056e-02  -1.581  0.11395
## x14         -1.826e-02  2.057e-02  -0.888  0.37481
## x15          1.757e-02  2.054e-02   0.855  0.39236
## x16          2.155e-02  2.054e-02   1.049  0.29440
## x17         -6.721e-03  2.056e-02  -0.327  0.74384
## x18          4.883e-03  2.057e-02   0.237  0.81238
## x19          1.307e-02  2.048e-02   0.638  0.52340
## x20          1.106e-02  2.032e-02   0.544  0.58620
## x21          1.049e-02  2.010e-02   0.522  0.60193
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
##
## Residual standard error: 0.002195 on 2496 degrees of freedom
## Multiple R-squared:  0.1149, Adjusted R-squared:  0.1075
## F-statistic: 15.43 on 21 and 2496 DF,  p-value: < 2.2e-16
```

```r
par (mfcol = c(1, 2))
acf (resi0^2, lag = 21)
pacf (resi0^2, lag = 21)
```



Modelos de Volatilidade

Modelos GARCH

1. Modelo GARCH com inovações gaussianas

O modelo GARCH é escrito da seguinte forma:

$$\sigma_t^2 = \alpha_0 + \alpha_1 a_{t-1}^2 + \beta_1 \sigma_{t-1}^2; \quad \epsilon_t \sim N(0,1)$$

Todos os coefs do GARCH(1,1) com inovações gaussianas foram significantes. Mas é possível verificar que o Teste de Jarque-Bera para os resíduos padronizados não foi aceito.

```
m1 <- garchFit(~garch(1,1),data=resi0, trace= F,include.mean = F)
summary(m1)
```

```
##
## Title:
##  GARCH Modelling
##
## Call:
##  garchFit(formula = ~garch(1, 1), data = resi0, include.mean = F,
##     trace = F)
##
## Mean and Variance Equation:
##  data ~ garch(1, 1)
## <environment: 0x000000001e799480>
##  [data = resi0]
##
## Conditional Distribution:
##  norm
##
## Coefficient(s):
##      omega      alpha1       beta1
## 8.2365e-06  5.4093e-02  9.3624e-01
##
## Std. Errors:
##  based on Hessian
##
## Error Analysis:
##         Estimate  Std. Error  t value Pr(>|t|)
## omega  8.237e-06   2.348e-06    3.508 0.000452 ***
## alpha1 5.409e-02   6.989e-03    7.739 9.99e-15 ***
## beta1  9.362e-01   8.481e-03  110.391  < 2e-16 ***
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
##
## Log Likelihood:
##  5860.139    normalized:  2.30805
##
## Description:
##  Fri May 01 18:02:46 2020 by user: R
##
```

```
##
## Standardised Residuals Tests:
##                                 Statistic p-Value
##  Jarque-Bera Test   R    Chi^2  16190.04  0
##  Shapiro-Wilk Test  R    W      0.9548268 0
##  Ljung-Box Test     R    Q(10)  6.559389  0.7662831
##  Ljung-Box Test     R    Q(15)  10.82604  0.7648256
##  Ljung-Box Test     R    Q(20)  13.61407  0.8495122
##  Ljung-Box Test     R^2  Q(10)  0.8185557 0.9999318
##  Ljung-Box Test     R^2  Q(15)  3.243863  0.9993494
##  Ljung-Box Test     R^2  Q(20)  3.659116  0.9999778
##  LM Arch Test       R    TR^2   0.9909464 0.9999865
##
## Information Criterion Statistics:
##       AIC       BIC       SIC      HQIC
## -4.613737 -4.606837 -4.613740 -4.611234
```

$$\sigma_t^2 = 0.0000 + 0.0539a_{t-1}^2 + 0.9374\sigma_{t-1}^2$$

Análise dos resíduos quadraticos de M1:

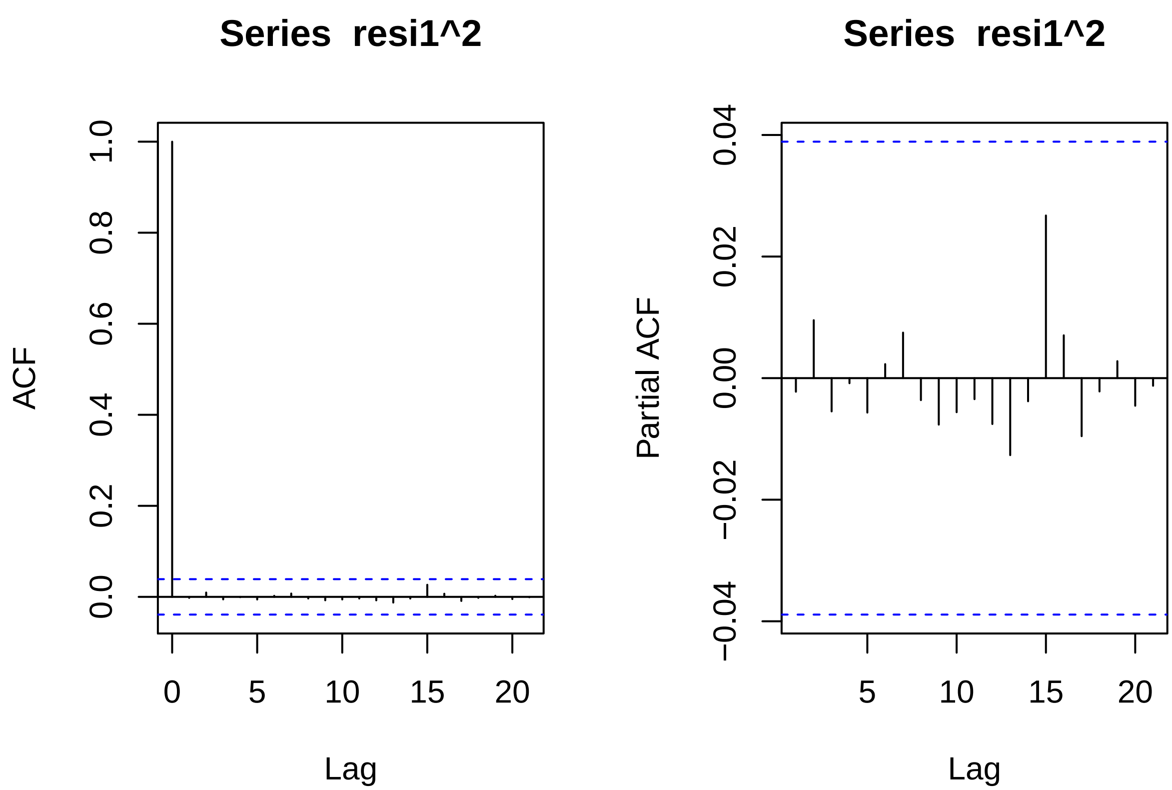
O arch teste mostrou que Aceitamos H0, ou seja, os resíduos não são autocorrelacionados.Portanto, todas as informações contidas na série foram extraidas.

```r
resi1 = residuals (m1, standardize = T)
archTest (resi1, 21)
```

```
##
## Call:
## lm(formula = atsq ~ x)
##
## Residuals:
##     Min      1Q  Median      3Q     Max
##  -5.337  -0.918  -0.668   0.102 169.348
##
## Coefficients:
##              Estimate Std. Error t value Pr(>|t|)
## (Intercept)  1.022441   0.119769   8.537   <2e-16 ***
## x1          -0.002660   0.020016  -0.133    0.894
## x2           0.009958   0.020016   0.497    0.619
## x3          -0.005411   0.020017  -0.270    0.787
## x4          -0.001110   0.020017  -0.055    0.956
## x5          -0.005624   0.020016  -0.281    0.779
## x6           0.002553   0.020016   0.128    0.899
## x7           0.007540   0.020009   0.377    0.706
## x8          -0.003956   0.020010  -0.198    0.843
## x9          -0.007919   0.020008  -0.396    0.692
## x10         -0.005497   0.020008  -0.275    0.784
## x11         -0.003367   0.020008  -0.168    0.866
## x12         -0.007567   0.020008  -0.378    0.705
## x13         -0.012999   0.020008  -0.650    0.516
## x14         -0.003926   0.020009  -0.196    0.844
## x15          0.026735   0.020009   1.336    0.182
```

```
## x16          0.006973   0.020016   0.348    0.728
## x17         -0.009677   0.020016  -0.483    0.629
## x18         -0.002170   0.020017  -0.108    0.914
## x19          0.002753   0.020016   0.138    0.891
## x20         -0.004577   0.020015  -0.229    0.819
## x21         -0.001264   0.020015  -0.063    0.950
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
##
## Residual standard error: 3.79 on 2496 degrees of freedom
## Multiple R-squared:  0.001459,   Adjusted R-squared:  -0.006942
## F-statistic: 0.1737 on 21 and 2496 DF,  p-value: 1
```

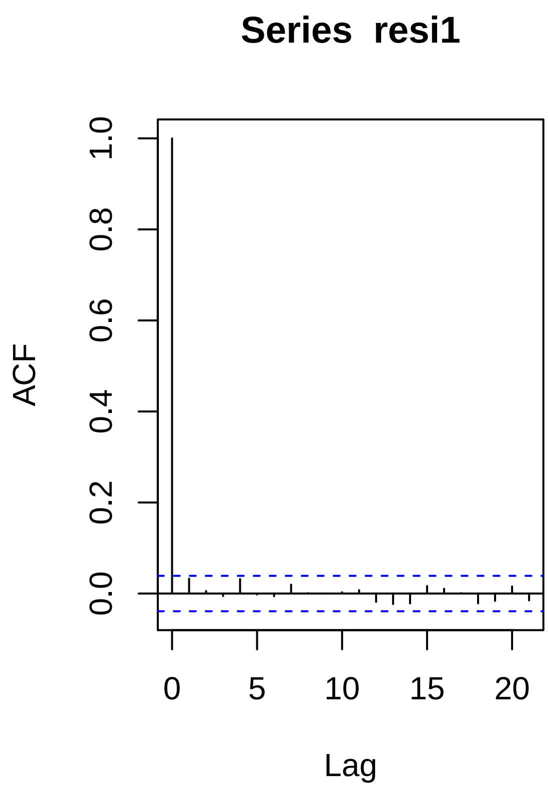
```r
par (mfcol = c(1, 2))
acf (resi1^2, lag = 21)
pacf (resi1^2, lag = 21)
```

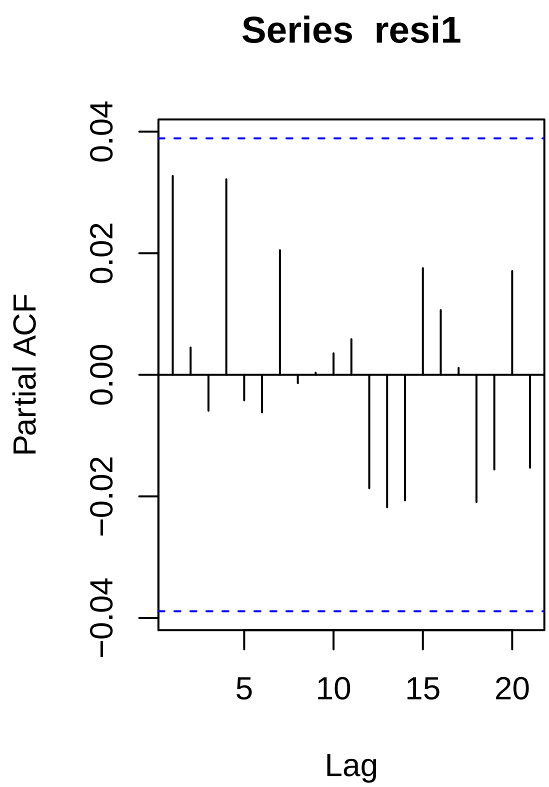


Análise das inovações:

Podemos verificar que as inovações se comportam como um ruído branco. Já que não há defasagens significantes.

```r
par (mfcol = c(1, 2))
acf (resi1, lag = 21)
pacf (resi1, lag = 21)
```





2. Modelo GARCH com inovações t

O modelo GARCH, onde as inovações se distribuem como um t, é escrito da seguinte forma:

$$\sigma_t^2 = \alpha_0 + \alpha_1 a_{t-1}^2 + \beta_1 \sigma_{t-1}^2, \quad \epsilon_t \sim t_{gl}^*$$

Todos os coefs do GARCH(1,1) com inovações com distribuição T de Student foram significantes.

```
m2 <- garchFit(~garch(1,1),data=resi0, trace= F, include.mean = F, cond.dist = "std")
summary(m2)
```

```
##
## Title:
##  GARCH Modelling
##
## Call:
##  garchFit(formula = ~garch(1, 1), data = resi0, cond.dist = "std",
##     include.mean = F, trace = F)
##
## Mean and Variance Equation:
##  data ~ garch(1, 1)
## <environment: 0x00000000222a9cf0>
##  [data = resi0]
##
## Conditional Distribution:
##  std
```

```
##
## Coefficient(s):
##      omega      alpha1       beta1       shape
## 6.8910e-06  5.6933e-02  9.3389e-01  6.7007e+00
##
## Std. Errors:
##  based on Hessian
##
## Error Analysis:
##         Estimate  Std. Error  t value Pr(>|t|)
## omega  6.891e-06   2.714e-06    2.539   0.0111 *
## alpha1 5.693e-02   1.006e-02    5.659 1.52e-08 ***
## beta1  9.339e-01   1.211e-02   77.125  < 2e-16 ***
## shape  6.701e+00   8.299e-01    8.074 6.66e-16 ***
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
##
## Log Likelihood:
##  5970.998    normalized:  2.351713
##
## Description:
##  Fri May 01 18:02:47 2020 by user: R
##
##
## Standardised Residuals Tests:
##                                 Statistic p-Value
##  Jarque-Bera Test   R    Chi^2  16917.41  0
##  Shapiro-Wilk Test  R    W      0.954282  0
##  Ljung-Box Test     R    Q(10)  6.805846  0.7436384
##  Ljung-Box Test     R    Q(15)  11.10531  0.7450937
##  Ljung-Box Test     R    Q(20)  13.82161  0.8394242
##  Ljung-Box Test     R^2  Q(10)  0.8477533 0.9999198
##  Ljung-Box Test     R^2  Q(15)  3.306741  0.9992688
##  Ljung-Box Test     R^2  Q(20)  3.731885  0.9999738
##  LM Arch Test       R    TR^2   1.083829  0.9999778
##
## Information Criterion Statistics:
##       AIC       BIC       SIC      HQIC
## -4.700274 -4.691075 -4.700279 -4.696937
```

O coefeciente *shape* indica que a distribuição t possui 6.72 gls. Quanto mais próximo de 1, mais pesada será a cauda da distribuição. Portanto, a estimativa do coeficiente está de acordo com o esperado. Já que a série apresenta excesso de curtose. Ou seja, cauda mais pesada do que a distribuição Normal.

$$\sigma_t^2 = 0.0000 + 0.0561 a_{t-1}^2 + 0.9359 \sigma_{t-1}^2, \quad \epsilon_t \sim t^*_{6.7217}$$

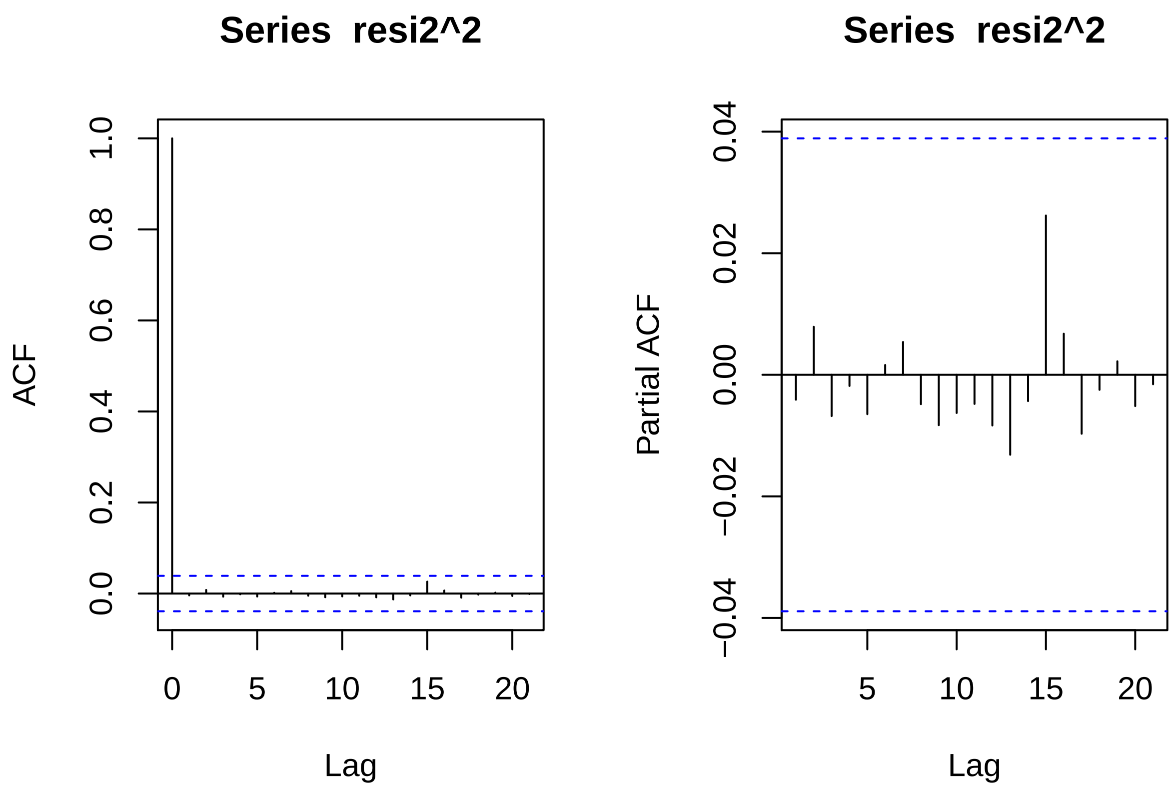
Análise dos resíduos quadraticos de M2:

É possível verificar que os resíduos quadraticos não são autocorrelacionados. Portanto, todas as informações que poderiam ser extraidas da variância já foram extraídas.

```
resi2 = residuals (m2, standardize = T)
archTest (resi2, 20)
```

```
##
## Call:
## lm(formula = atsq ~ x)
##
## Residuals:
##     Min      1Q  Median      3Q     Max
##  -5.553  -0.958  -0.698   0.116 178.715
##
## Coefficients:
##              Estimate Std. Error t value Pr(>|t|)
## (Intercept)  1.082321   0.124164   8.717   <2e-16 ***
## x1          -0.004538   0.020008  -0.227    0.821
## x2           0.008198   0.020008   0.410    0.682
## x3          -0.006756   0.020008  -0.338    0.736
## x4          -0.002088   0.020008  -0.104    0.917
## x5          -0.006448   0.020008  -0.322    0.747
## x6           0.001760   0.020001   0.088    0.930
## x7           0.005399   0.020002   0.270    0.787
## x8          -0.005212   0.020000  -0.261    0.794
## x9          -0.008603   0.019999  -0.430    0.667
## x10         -0.006211   0.020000  -0.311    0.756
## x11         -0.004710   0.020000  -0.235    0.814
## x12         -0.008327   0.020000  -0.416    0.677
## x13         -0.013420   0.020000  -0.671    0.502
## x14         -0.004402   0.020001  -0.220    0.826
## x15          0.026195   0.020001   1.310    0.190
## x16          0.006691   0.020008   0.334    0.738
## x17         -0.009811   0.020008  -0.490    0.624
## x18         -0.002418   0.020008  -0.121    0.904
## x19          0.002170   0.020008   0.108    0.914
## x20         -0.005161   0.020007  -0.258    0.796
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
##
## Residual standard error: 3.986 on 2498 degrees of freedom
## Multiple R-squared:  0.001485,   Adjusted R-squared:  -0.006509
## F-statistic: 0.1858 on 20 and 2498 DF,  p-value: 1
```

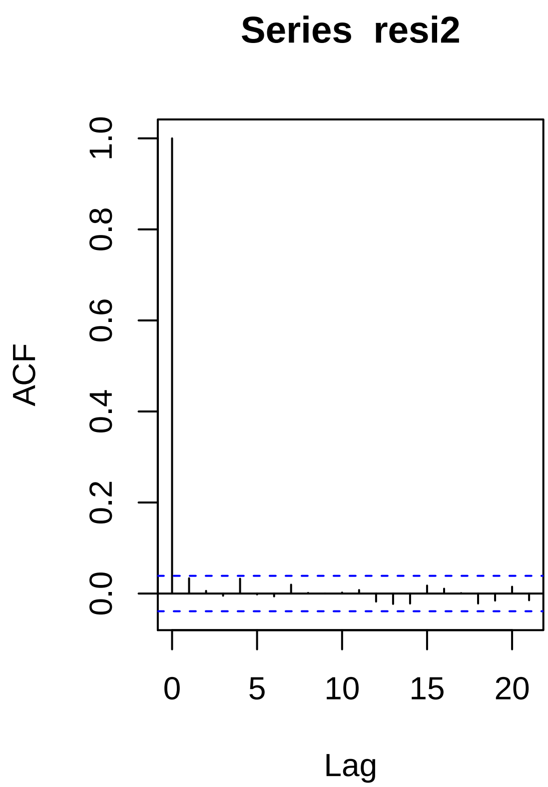
```r
par (mfcol = c(1, 2))
acf (resi2^2, lag = 21)
pacf (resi2^2, lag = 21)
```

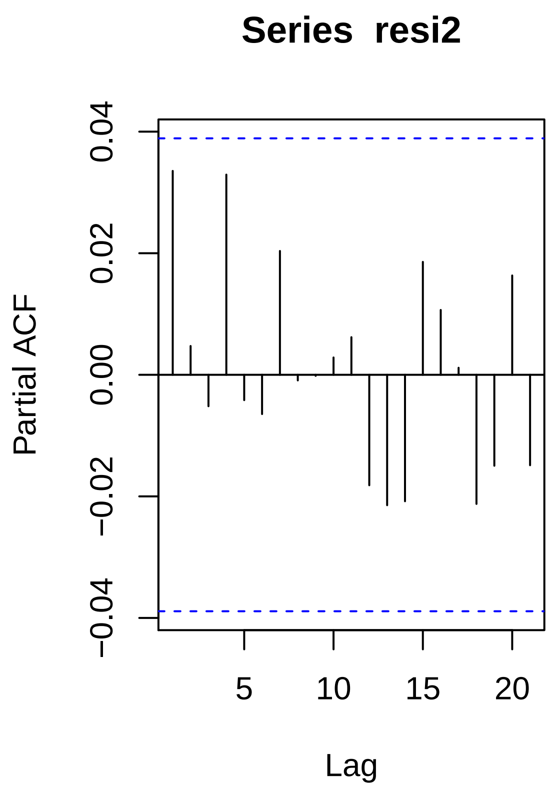


Análise das inovações:

Podemos verificar que as inovações se comportam como um ruído branco.

```
par (mfcol = c(1, 2))
acf (resi2, lag = 21)
pacf (resi2, lag = 21)
```





3. Modelo GARCH com inovações t e com assimetria

O modelo GARCH, com distribuição t e skew, é escrito da seguinte forma:

$$\sigma_t^2 = \alpha_0 + \alpha_1 a_{t-1}^2 + \beta_1 \sigma_{t-1}^2, \quad \epsilon_t \sim t^*_{sk,gl}$$

Todos os coefs do modelo GARCH(1,1) foram significantes.

```
m3 <- garchFit(~garch(1,1),data=resi0, trace= F, include.mean = F, cond.dist = "sstd")
summary(m3)
```

```
##
## Title:
##  GARCH Modelling
##
## Call:
##  garchFit(formula = ~garch(1, 1), data = resi0, cond.dist = "sstd",
##     include.mean = F, trace = F)
##
## Mean and Variance Equation:
##  data ~ garch(1, 1)
## <environment: 0x0000000024a3dd58>
##  [data = resi0]
##
## Conditional Distribution:
##  sstd
```

```
##
## Coefficient(s):
##      omega      alpha1       beta1        skew       shape
## 6.8473e-06  5.6895e-02  9.3403e-01  9.8866e-01  6.6951e+00
##
## Std. Errors:
##  based on Hessian
##
## Error Analysis:
##         Estimate  Std. Error  t value Pr(>|t|)
## omega  6.847e-06   2.709e-06    2.528   0.0115 *
## alpha1 5.689e-02   1.005e-02    5.659 1.53e-08 ***
## beta1  9.340e-01   1.209e-02   77.238  < 2e-16 ***
## skew   9.887e-01   2.565e-02   38.547  < 2e-16 ***
## shape  6.695e+00   8.301e-01    8.066 6.66e-16 ***
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
##
## Log Likelihood:
##  5971.095    normalized:  2.351751
##
## Description:
##  Fri May 01 18:02:49 2020 by user: R
##
##
## Standardised Residuals Tests:
##                                 Statistic p-Value
##  Jarque-Bera Test   R    Chi^2  16903.76  0
##  Shapiro-Wilk Test  R    W      0.9542845 0
##  Ljung-Box Test     R    Q(10)  6.814094  0.7428714
##  Ljung-Box Test     R    Q(15)  11.11513  0.7443913
##  Ljung-Box Test     R    Q(20)  13.83158  0.8389317
##  Ljung-Box Test     R^2  Q(10)  0.8480658 0.9999196
##  Ljung-Box Test     R^2  Q(15)  3.307835  0.9992673
##  Ljung-Box Test     R^2  Q(20)  3.732729  0.9999738
##  LM Arch Test       R    TR^2   1.084534  0.9999778
##
## Information Criterion Statistics:
##       AIC       BIC       SIC      HQIC
## -4.699563 -4.688063 -4.699571 -4.695391
```

$$\sigma_t^2 = 0.0000 + 0.0561a_{t-1}^2 + 0.9360\sigma_{t-1}^2, \quad \epsilon_t \sim t^*_{0.9886,6.7162}$$

Análise dos resíduos quadraticos de M3:

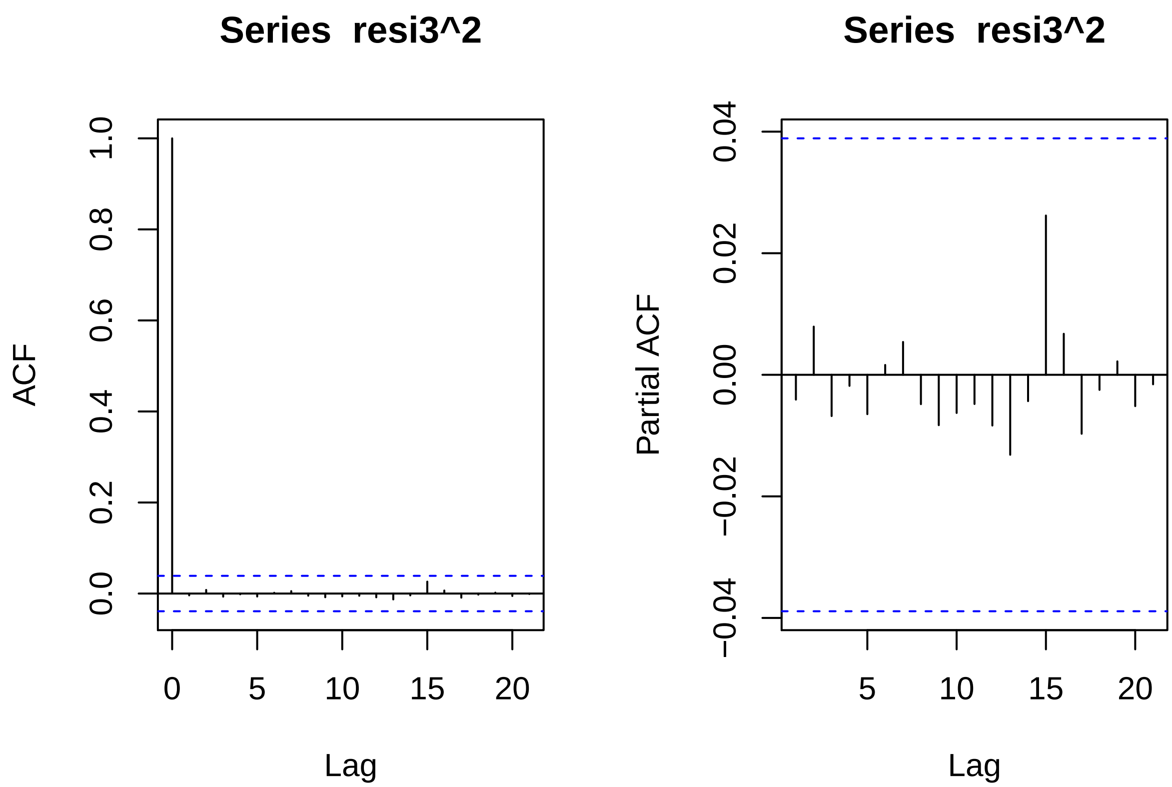
Aceitamos H0 do arch teste, ou seja, não há correlação entre os resíduos.

```
resi3 = residuals (m3, standardize = T)
archTest (resi3, 20)
```

```
##
## Call:
## lm(formula = atsq ~ x)
```

```
##
## Residuals:
##     Min      1Q  Median      3Q     Max
##  -5.551  -0.958  -0.698   0.116 178.630
##
## Coefficients:
##              Estimate Std. Error t value Pr(>|t|)
## (Intercept)  1.082079   0.124125   8.718   <2e-16 ***
## x1          -0.004523   0.020008  -0.226    0.821
## x2           0.008229   0.020008   0.411    0.681
## x3          -0.006749   0.020008  -0.337    0.736
## x4          -0.002070   0.020008  -0.103    0.918
## x5          -0.006443   0.020008  -0.322    0.747
## x6           0.001762   0.020001   0.088    0.930
## x7           0.005400   0.020002   0.270    0.787
## x8          -0.005209   0.020000  -0.260    0.795
## x9          -0.008603   0.019999  -0.430    0.667
## x10         -0.006208   0.020000  -0.310    0.756
## x11         -0.004720   0.020000  -0.236    0.813
## x12         -0.008332   0.020000  -0.417    0.677
## x13         -0.013427   0.020000  -0.671    0.502
## x14         -0.004405   0.020001  -0.220    0.826
## x15          0.026194   0.020001   1.310    0.190
## x16          0.006679   0.020008   0.334    0.739
## x17         -0.009815   0.020008  -0.491    0.624
## x18         -0.002426   0.020008  -0.121    0.903
## x19          0.002157   0.020008   0.108    0.914
## x20         -0.005164   0.020007  -0.258    0.796
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
##
## Residual standard error: 3.984 on 2498 degrees of freedom
## Multiple R-squared:  0.001485,   Adjusted R-squared:  -0.006509
## F-statistic: 0.1858 on 20 and 2498 DF,  p-value: 1
```

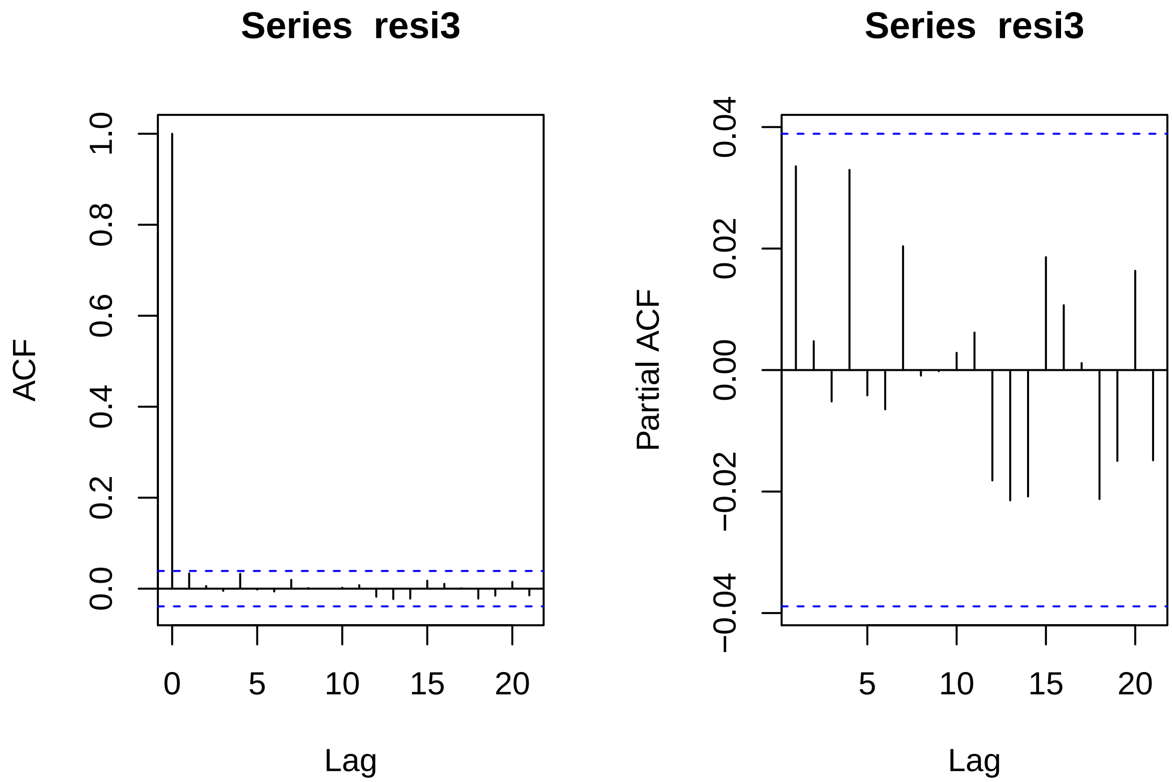
```r
par (mfcol = c(1, 2))
acf (resi3^2, lag = 21)
pacf (resi3^2, lag = 21)
```



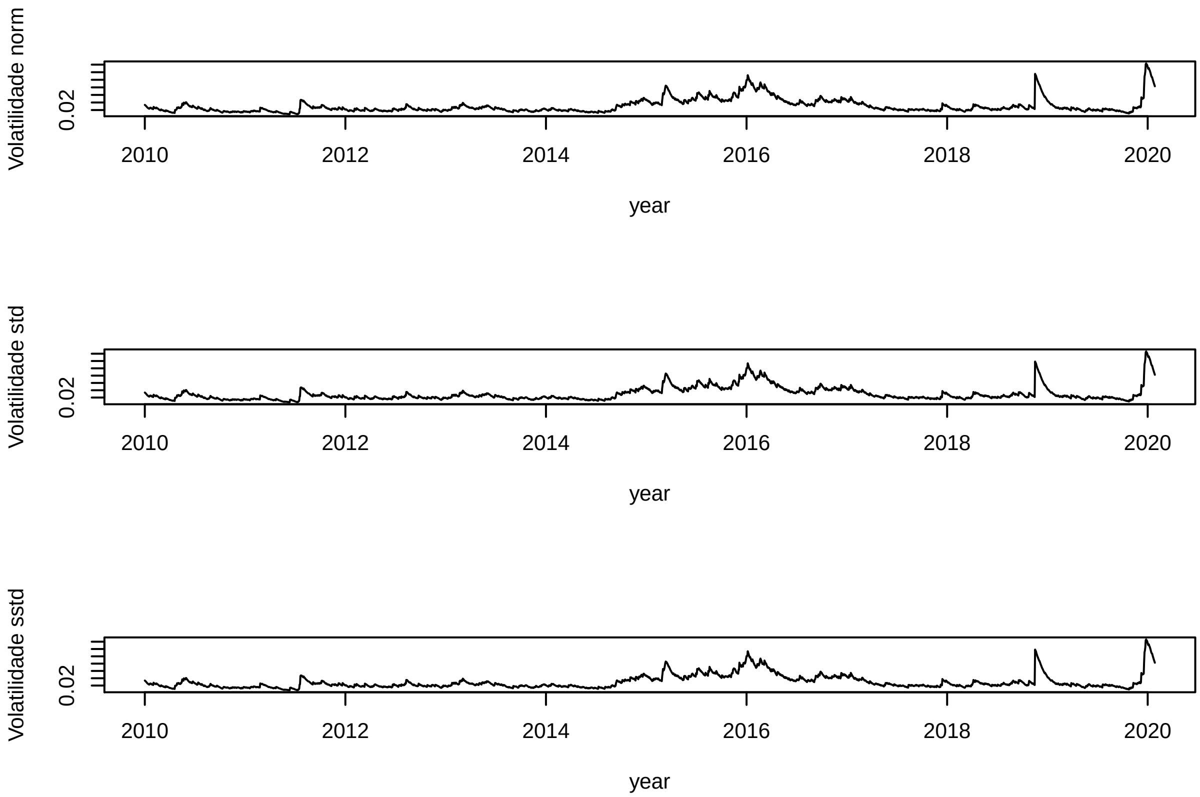
Análise das inovações:

Podemos verificar que as inovações se comportam como um ruído branco.

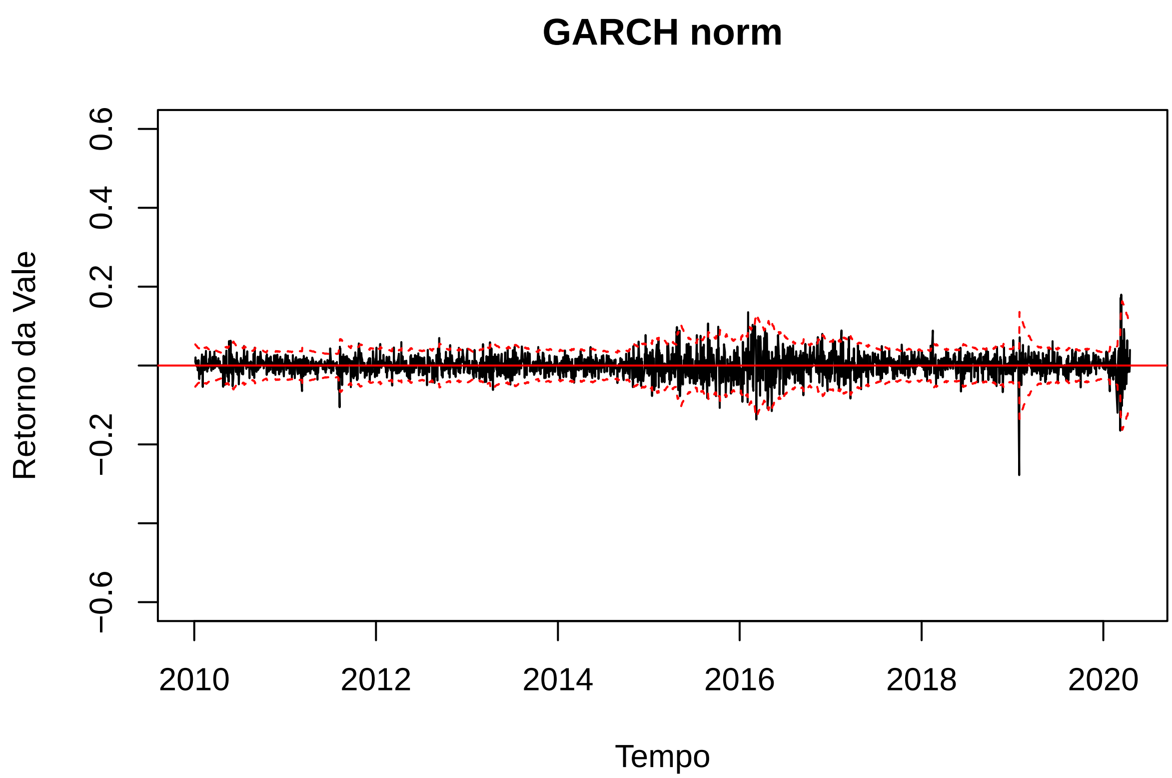
```
par (mfcol = c(1, 2))
acf (resi3, lag = 21)
pacf (resi3, lag = 21)
```

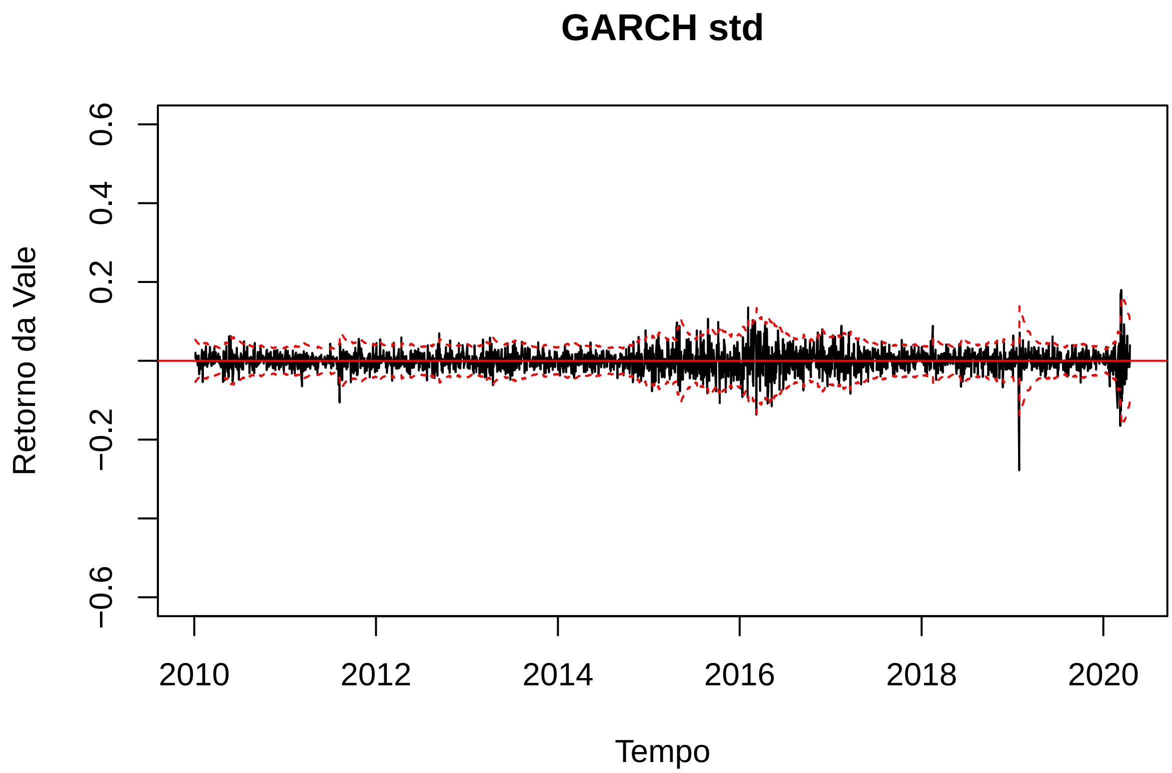


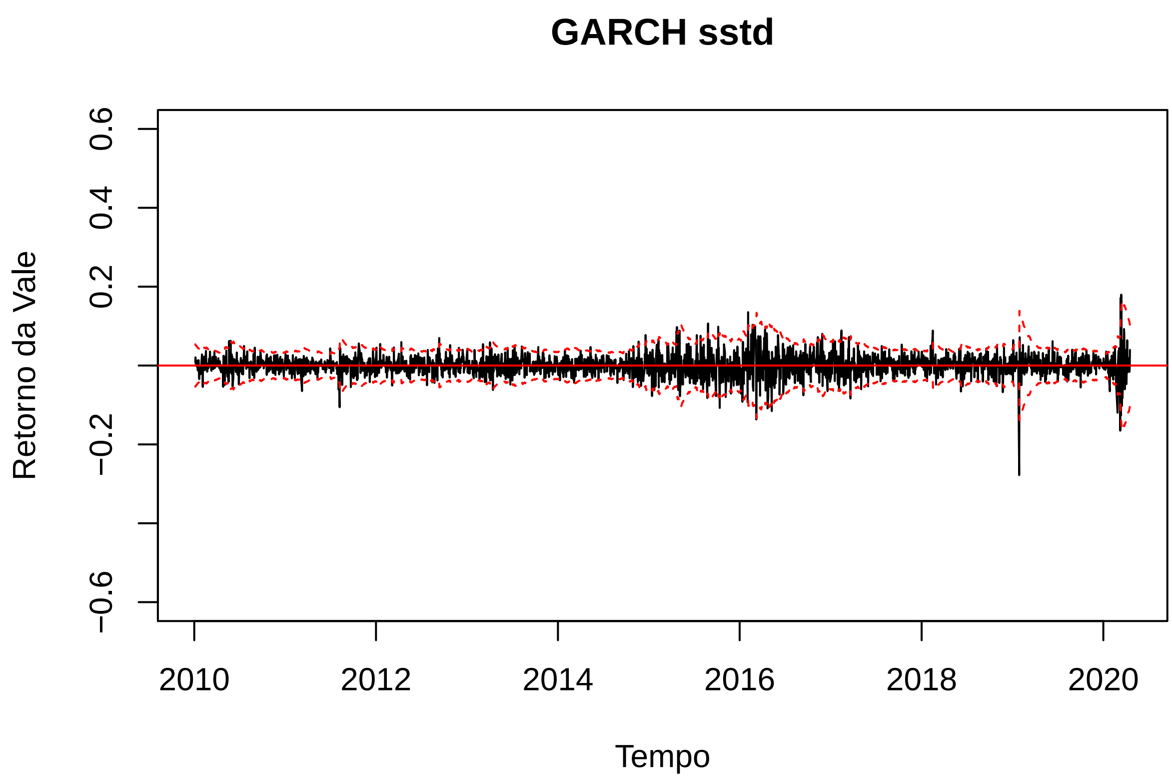
Volatilidade dos modelos



Intervalo de Confiança


GARCH norm
Retorno da Vale
0.6
0.4
0.2
−0.2
−0.6
2010
2012
2014
2016
2018
2020
Tempo


GARCH std
Retorno da Vale
0.6
0.4
0.2
−0.2
−0.6
2010
2012
2014
2016
2018
2020
Tempo



Modelo IGARCH

No modelo IGARCH, partimos do pressuposto de que $\alpha_1 + \beta 1 = 1$. Portanto, sugere que a série apresenta raíz unitária. Não é estacionaria.

$$\sigma_t^2 = \alpha_0 + (1 - \beta_1) a_{t-1}^2 + \beta_1 \sigma_{t-1}^2, \quad \epsilon_t \sim N(0,1)$$

IGARCH(1,1) com inovações Gaussianas. O coef Beta é estatisticamente significante.

```
m5 = Igarch (resi0)
```

```
## Estimates:  0.9580592
## Maximized log-likehood:  -5835.991
##
## Coefficient(s):
##        Estimate  Std. Error  t value   Pr(>|t|)
## beta 0.95805925  0.00431906  221.821 < 2.22e-16 ***
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
```

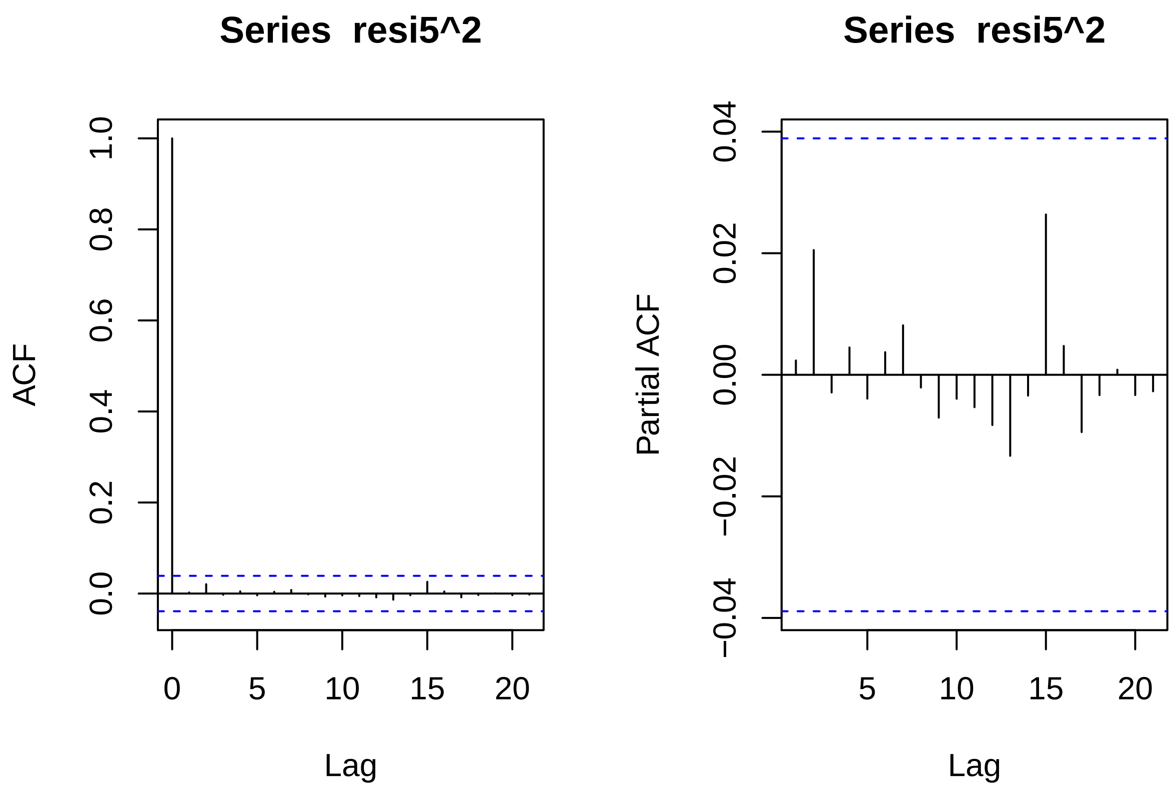
```
resi5 = resi0 / m5$volatility
```

$$\sigma_t^2 = 0.0421 a_{t-1}^2 + 0.9579 \sigma_{t-1}^2$$

Análise dos resíduos quadraticos de M5:

De acordo o arch teste e a análise da FAC e FACP não há autocorrelação entre as defasagens dos resíduos quadraticos.

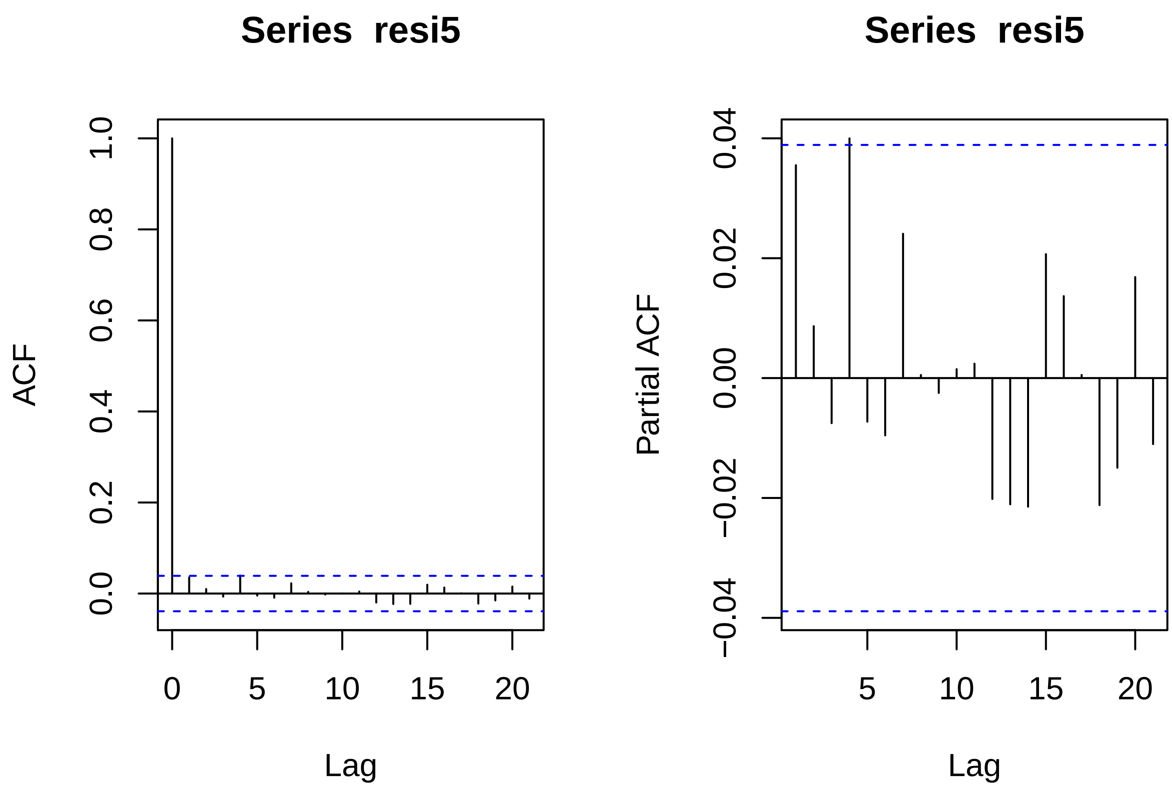
```
##
## Call:
## lm(formula = atsq ~ x)
##
## Residuals:
##     Min      1Q  Median      3Q     Max
##  -5.729  -1.043  -0.760   0.105 181.240
##
## Coefficients:
##               Estimate Std. Error t value Pr(>|t|)
## (Intercept)  1.1369983  0.1324516   8.584   <2e-16 ***
## x1           0.0018567  0.0200157   0.093    0.926
## x2           0.0207712  0.0200157   1.038    0.299
## x3          -0.0028663  0.0200202  -0.143    0.886
## x4           0.0042427  0.0200201   0.212    0.832
## x5          -0.0040294  0.0200193  -0.201    0.840
## x6           0.0039513  0.0200194   0.197    0.844
## x7           0.0082235  0.0200127   0.411    0.681
## x8          -0.0024327  0.0200135  -0.122    0.903
## x9          -0.0072116  0.0200114  -0.360    0.719
## x10         -0.0038203  0.0200112  -0.191    0.849
## x11         -0.0052024  0.0200112  -0.260    0.795
## x12         -0.0082448  0.0200115  -0.412    0.680
## x13         -0.0139129  0.0200117  -0.695    0.487
## x14         -0.0036880  0.0200134  -0.184    0.854
## x15          0.0264671  0.0200127   1.323    0.186
## x16          0.0047412  0.0200195   0.237    0.813
## x17         -0.0095114  0.0200193  -0.475    0.635
## x18         -0.0032934  0.0200199  -0.165    0.869
## x19          0.0008598  0.0200200   0.043    0.966
## x20         -0.0033686  0.0200158  -0.168    0.866
## x21         -0.0027125  0.0200157  -0.136    0.892
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
##
## Residual standard error: 4.166 on 2496 degrees of freedom
## Multiple R-squared:  0.001739,   Adjusted R-squared:  -0.00666
## F-statistic: 0.207 on 21 and 2496 DF,  p-value: 1
```



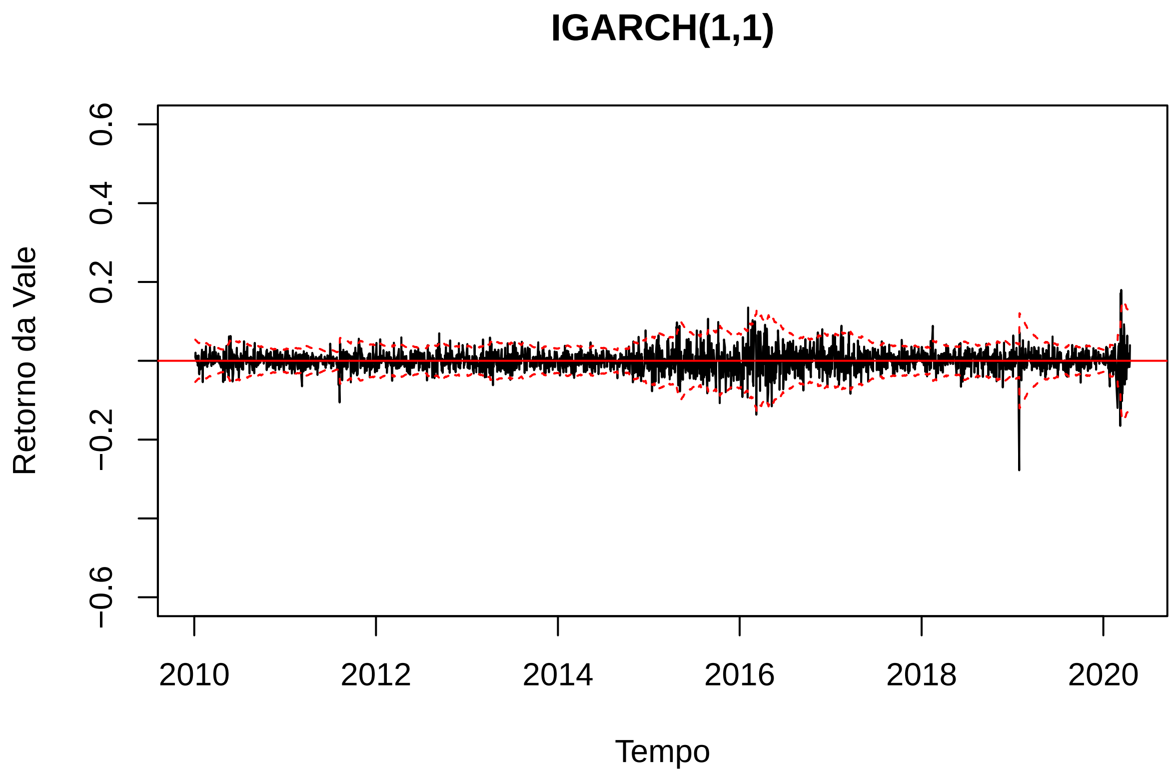
Análise das inovações:

A FAC e a FACP nos mostram que apenas a defasagem 4 é significante. Talvez ela seja um outlier

```
par (mfcol = c(1, 2))
acf (resi5, lag = 21)
pacf (resi5, lag = 21)
```



Intervalo de COnfiança



Modelo GARCH-M

No GARCH-M, além de estimar a variância da série estimamos também a estrutura do retorno.

$$r_t = \mu + c\sigma_t^2 + a_t$$

O termo c é conhecido como prêmio de risco. E sugere que uma maior volatilidade impacta positivamente o retorno. Remete a ideia de que uma maior volatilidade é compensada por um maior retorno.

$$\sigma_t^2 = \alpha_0 + \alpha_1 a_{t-1}^2 + \beta_1 \sigma_{t-1}^2, \quad \epsilon_t \sim N(0,1)$$

GARCH-M(1,1) com inovações Gaussianas

```
m6 = garchM (dlvale, type = 2)
```

```
## Maximized log-likehood:  5848.268
##
## Coefficient(s):
##            Estimate   Std. Error   t value   Pr(>|t|)
## mu    -4.71192e-04  1.71330e-03  -0.27502 0.78330116
## gamma  2.46197e-02  7.61953e-02   0.32311 0.74661001
## omega  7.60446e-06  2.25022e-06   3.37943 0.00072637 ***
## alpha  5.19025e-02  6.69724e-03   7.74983 9.1038e-15 ***
```

```
## beta   9.39536e-01  8.11310e-03 115.80475 < 2.22e-16 ***
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
```

$$r_t = \mu + a_t$$

Os coeficientes estimados sugerem que o parâmetro c não é significante. Ou seja, o modelo é semelhante ao GARCH(1,1).

$$\sigma_t^2 = 0.0000 + 0.0518a_{t-1}^2 + 0.9404\sigma_{t-1}^2$$

Análise dos resíduos quadraticos de M6:

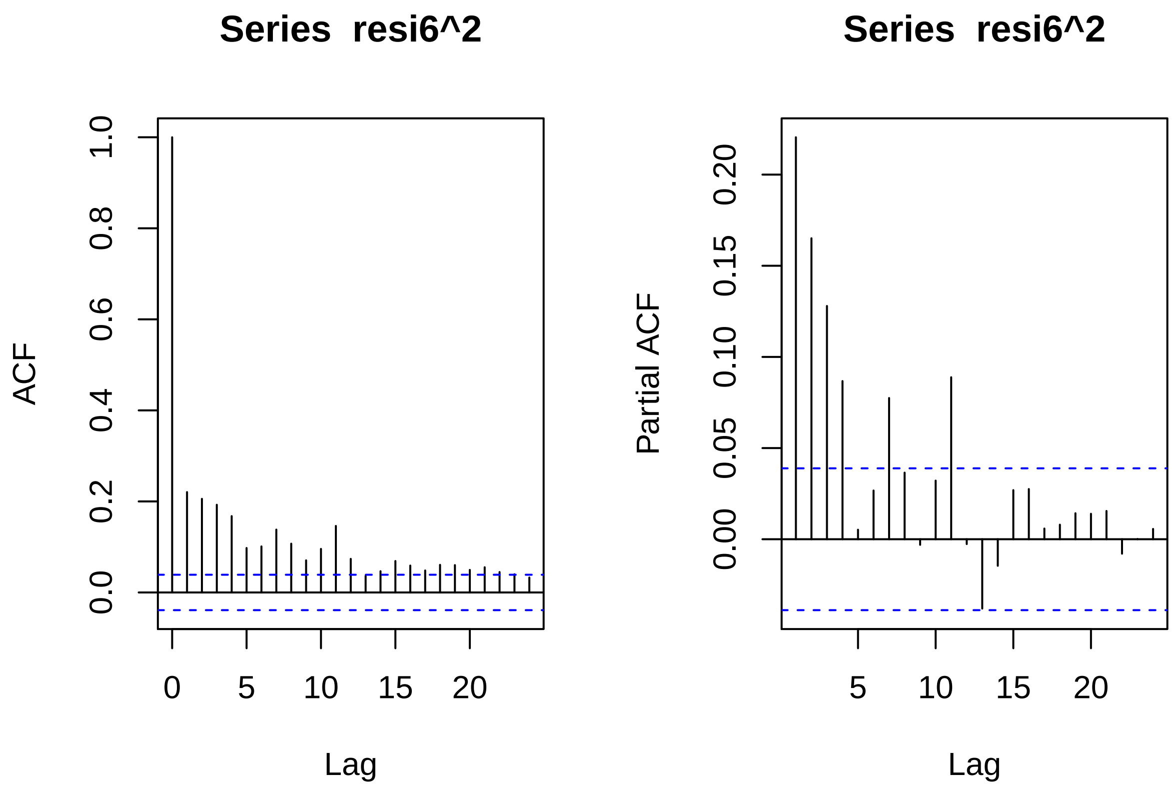
Rejeitamos H0 do arch teste e a FAC e FACP nos mostra que há autocorrelação entre os resíduos quadraticos.

```
resi6 = m6$residuals
archTest (resi6, 20)
```

```
##
## Call:
## lm(formula = atsq ~ x)
##
## Residuals:
##       Min        1Q    Median        3Q       Max
## -0.011706 -0.000507 -0.000328  0.000040  0.078950
##
## Coefficients:
##              Estimate Std. Error t value Pr(>|t|)
## (Intercept)  0.0002802  0.0000565   4.959 7.57e-07 ***
## x1           0.1423614  0.0200058   7.116 1.45e-12 ***
## x2           0.1298697  0.0202067   6.427 1.55e-10 ***
## x3           0.1013055  0.0203728   4.973 7.05e-07 ***
## x4           0.0615828  0.0204732   3.008  0.00266 **
## x5          -0.0145866  0.0205049  -0.711  0.47692
## x6           0.0124642  0.0205045   0.608  0.54333
## x7           0.0634857  0.0205040   3.096  0.00198 **
## x8           0.0198988  0.0205243   0.970  0.33238
## x9          -0.0201611  0.0205374  -0.982  0.32635
## x10          0.0235026  0.0204549   1.149  0.25067
## x11          0.0937726  0.0204563   4.584 4.79e-06 ***
## x12         -0.0008109  0.0205389  -0.039  0.96851
## x13         -0.0437789  0.0205359  -2.132  0.03312 *
## x14         -0.0233019  0.0205151  -1.136  0.25613
## x15          0.0206898  0.0205186   1.008  0.31339
## x16          0.0235158  0.0205240   1.146  0.25200
## x17          0.0014069  0.0205083   0.069  0.94531
## x18          0.0040626  0.0204595   0.199  0.84262
## x19          0.0126875  0.0202874   0.625  0.53177
## x20          0.0142581  0.0200953   0.710  0.47807
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
##
```

```
## Residual standard error: 0.002285 on 2498 degrees of freedom
## Multiple R-squared:  0.115,  Adjusted R-squared:  0.1079
## F-statistic: 16.23 on 20 and 2498 DF,  p-value: < 2.2e-16
```

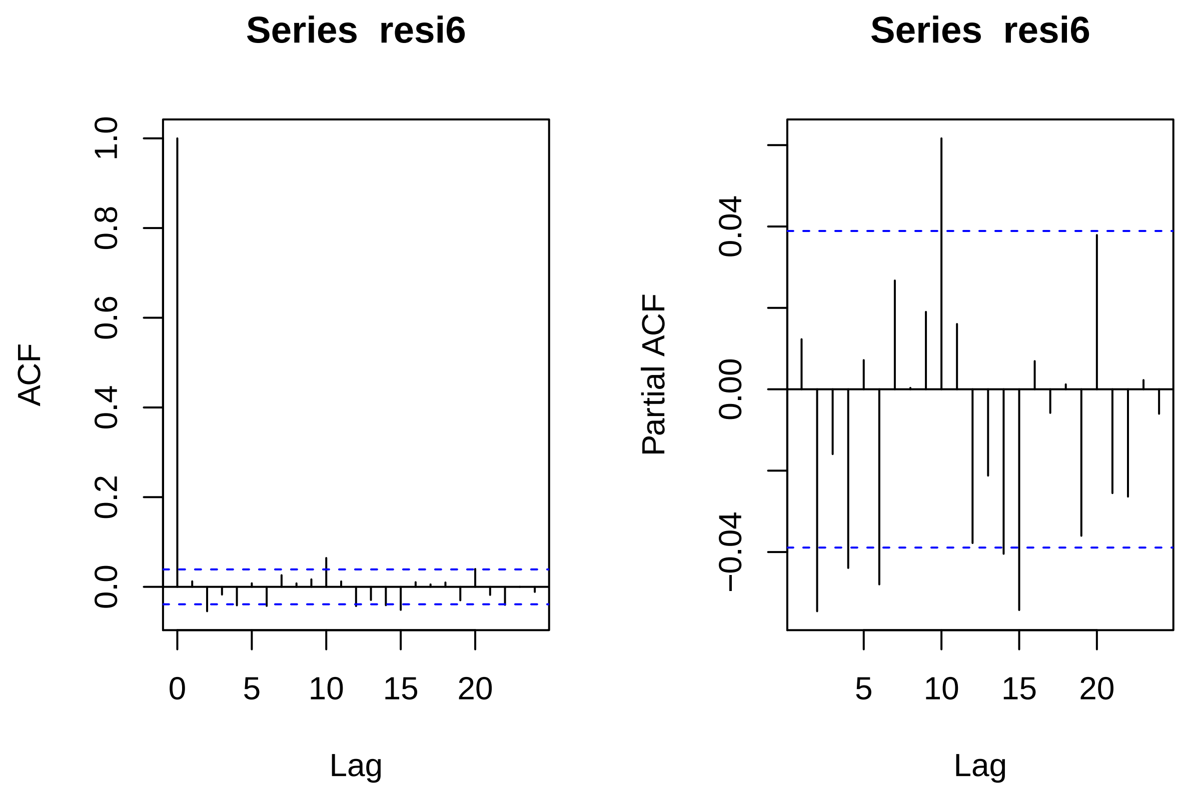
```
par (mfcol = c(1, 2))
acf (resi6^2, lag = 24)
pacf (resi6^2, lag = 24)
```



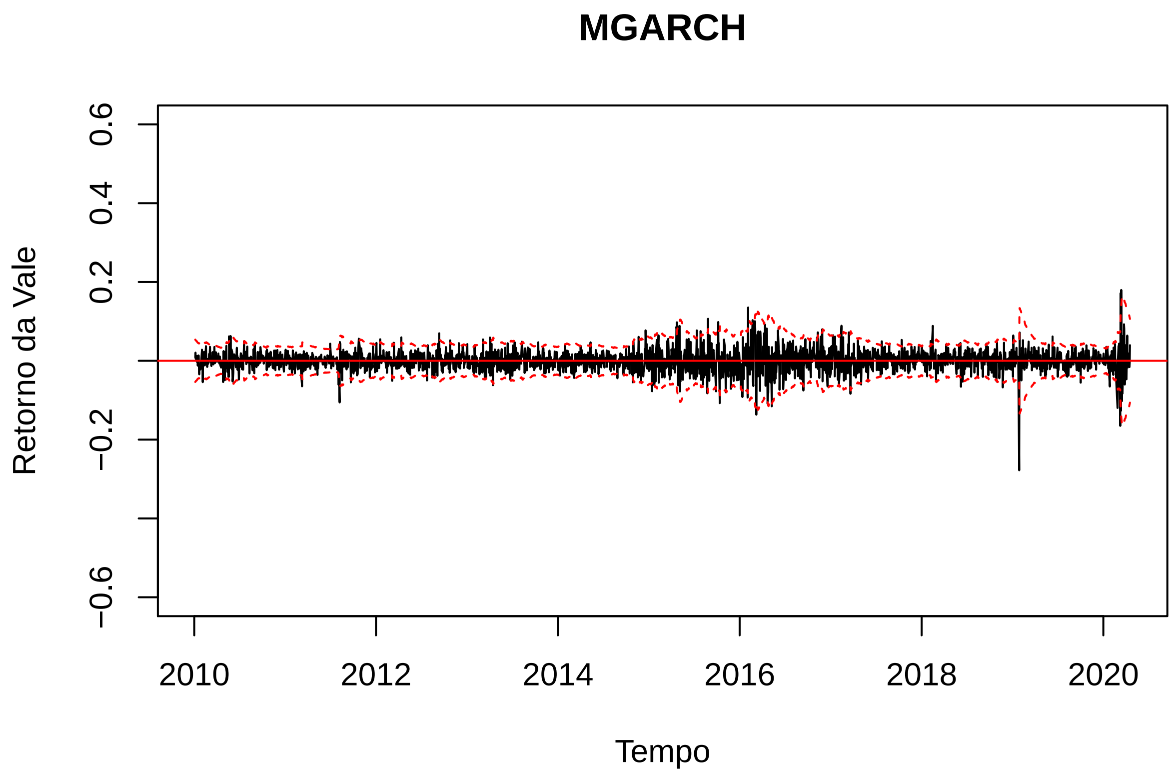
Análise das inovações:

As inovações não se comportam como um ruído branco.

```
par (mfcol = c(1, 2))
acf (resi6, lag = 24)
pacf (resi6, lag = 24)
```



Intervalo de Confiança



Modelo EGARCH

$$(1-\alpha\beta)ln(\sigma_t^2) = (1-\alpha)\alpha_0 + g(\epsilon_{t-1}), \quad \epsilon_t \sim N(0,1)$$

O modelo EGARCH tenta capturar o *Leverage Effect*. Ou seja, tenta capturar a assimentria na volatilidade. Já que os fatos estilizados sugerem que a volatilidade quando há choques negativos é maior do que a volatilidade quando os choques são positivos.

$$(1-\alpha\beta)ln(\sigma_t^2) = \begin{cases} \alpha_* + (\gamma+\theta)\epsilon_{t-1} & \epsilon_{t-1} \geq 0 \\ \alpha_* + (\gamma-\theta)(-\epsilon_{t-1}) & \epsilon_{t-1} < 0 \end{cases}$$

EGARCH(1,1) com inovações gaussianas

```
m7 = Egarch(resi0)
```

```
##
## Estimation results of EGARCH(1,1) model:
## estimates:  -0.0004562515 -1.129569 0.2698486 0 0.8742895
## std.errors:  0.0004627206 NaN NaN 0.06006264 NaN
## t-ratio:  -0.9860195 NaN NaN 0 NaN
```

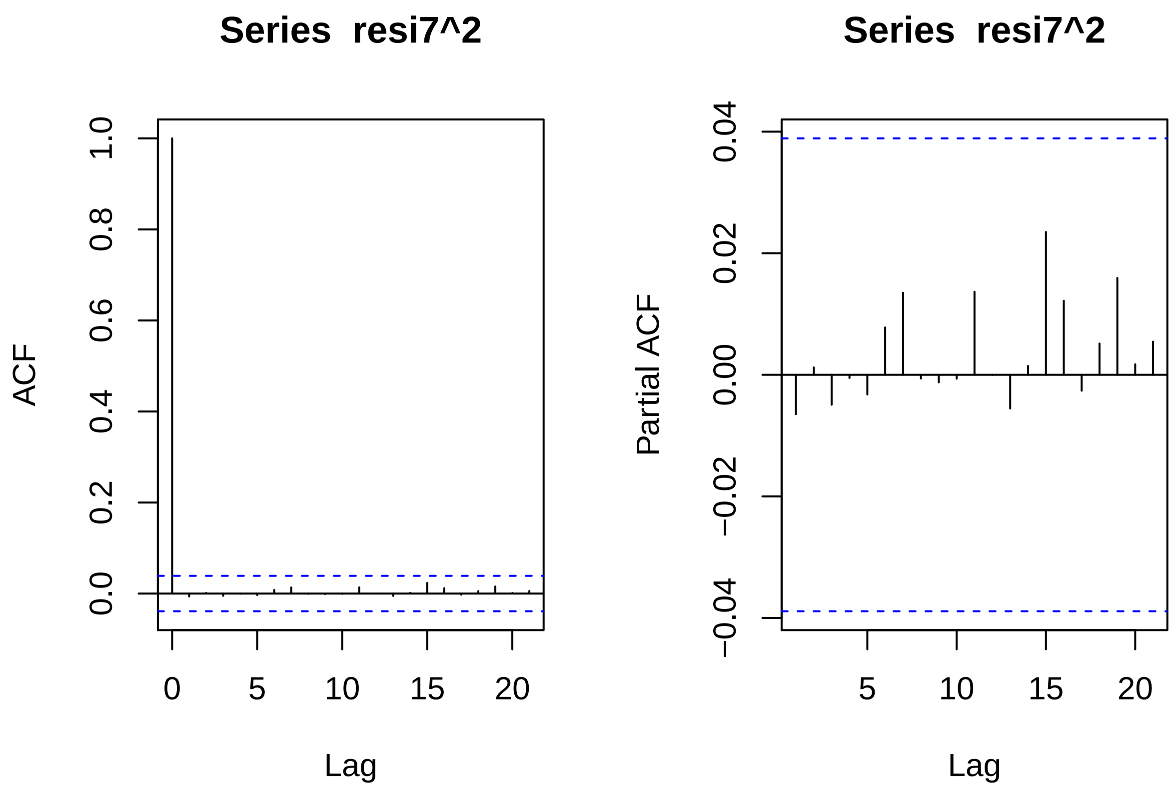
Análise dos resíduos quadraticos de M7:

Aceitamos H0 do arch teste e as defasagens da FAC e da FACP não são significantes. Tudo indica que os as defasagens dos resíduos quadraticos não são significnates.

```r
resi7 = m7$residuals / m7$volatility
archTest (resi7, 21)
```

```
##
## Call:
## lm(formula = atsq ~ x)
##
## Residuals:
##     Min      1Q  Median      3Q     Max
##  -4.794  -0.911  -0.668   0.067 194.341
##
## Coefficients:
##               Estimate Std. Error t value Pr(>|t|)
## (Intercept)  0.9305208  0.1234510   7.538 6.66e-14 ***
## x1          -0.0071335  0.0200165  -0.356    0.722
## x2           0.0012503  0.0200170   0.062    0.950
## x3          -0.0050334  0.0200145  -0.251    0.801
## x4          -0.0015111  0.0200144  -0.076    0.940
## x5          -0.0036450  0.0200142  -0.182    0.856
## x6           0.0076529  0.0200129   0.382    0.702
## x7           0.0138912  0.0200081   0.694    0.488
## x8          -0.0007229  0.0200102  -0.036    0.971
## x9          -0.0016479  0.0200110  -0.082    0.934
## x10         -0.0002629  0.0200111  -0.013    0.990
## x11          0.0134336  0.0200094   0.671    0.502
## x12         -0.0005527  0.0200114  -0.028    0.978
## x13         -0.0058412  0.0200114  -0.292    0.770
## x14          0.0016062  0.0200115   0.080    0.936
## x15          0.0233942  0.0200096   1.169    0.242
## x16          0.0123110  0.0200143   0.615    0.539
## x17         -0.0027793  0.0200162  -0.139    0.890
## x18          0.0050833  0.0200188   0.254    0.800
## x19          0.0158774  0.0200187   0.793    0.428
## x20          0.0017008  0.0200212   0.085    0.932
## x21          0.0052428  0.0200207   0.262    0.793
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
##
## Residual standard error: 4.224 on 2496 degrees of freedom
## Multiple R-squared:  0.001552,   Adjusted R-squared:  -0.006849
## F-statistic: 0.1847 on 21 and 2496 DF,  p-value: 1
```

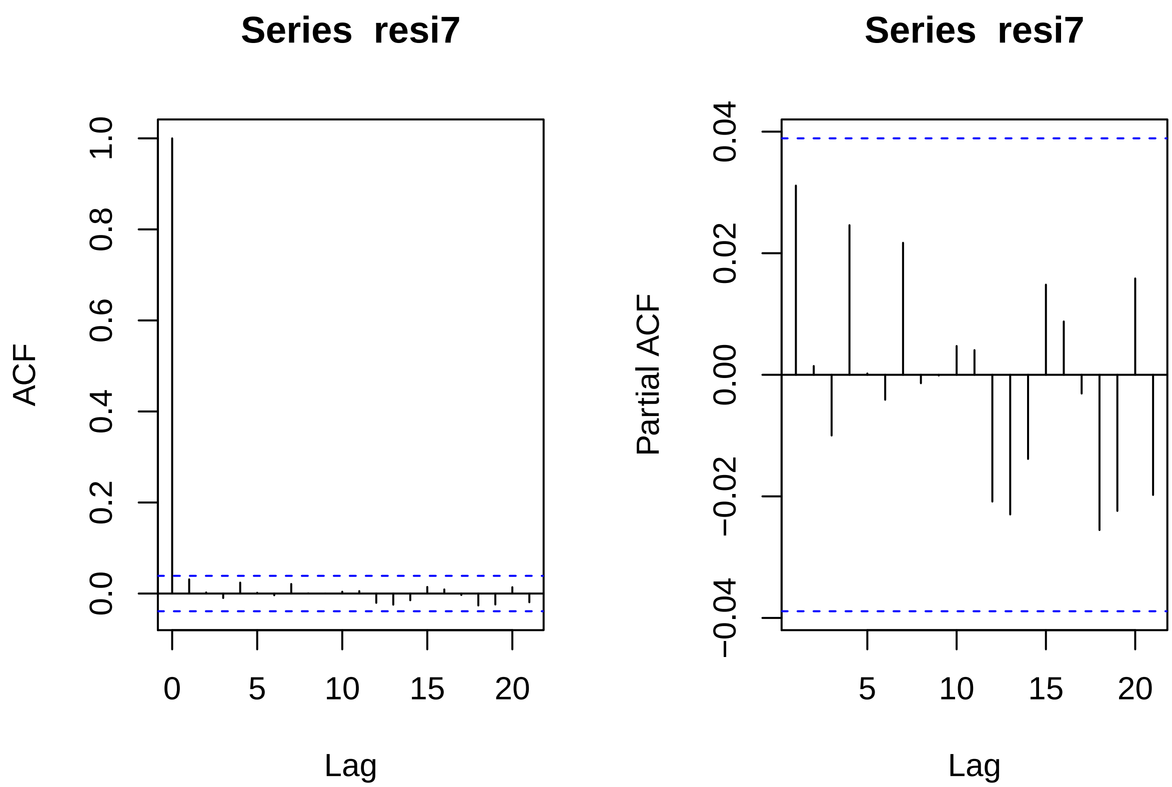
```r
par (mfcol = c(1, 2))
acf (resi7^2, lag = 21)
pacf (resi7^2, lag = 21)
```



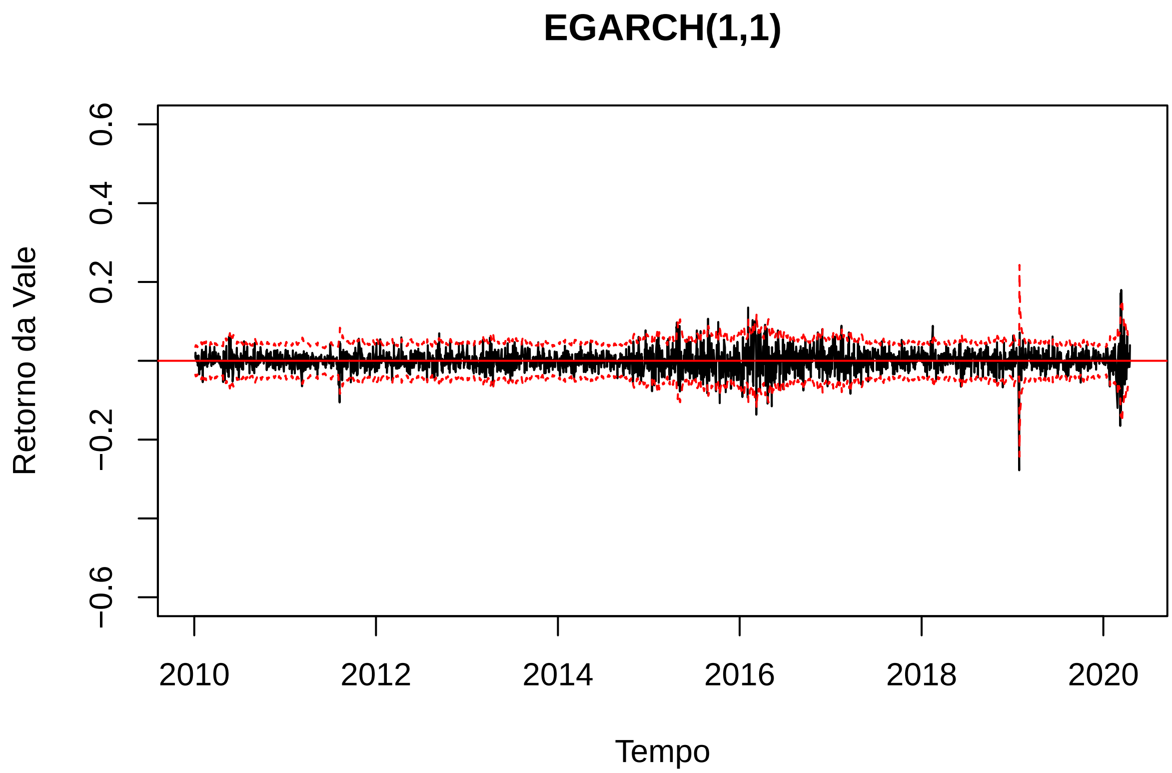
Análise das inovações:

As inovações se comportam como um ruído branco.

```
par (mfcol = c(1, 2))
acf (resi7, lag = 21)
pacf (resi7, lag = 21)
```



Intervalo de Confiança



Modelo TGARCH

$$\sigma_t^2 = \alpha_0 + (\alpha_1 + \gamma_1 N_{t-1})a_{t-1}^2 + \beta_1 \sigma_{t-1}^2, \quad \epsilon_t \sim N(0,1)$$

O TGARCH também tenta a cpturar o *Leverage Effect*.

$$N_{t-1} = \begin{cases} 1 & se \quad a_{t-1} < 0 \\ 0 & se \quad a_{t-1} \geq 0 \end{cases}$$

TGARCH(1,1) com inovações gaussianas

```
m8 = Tgarch11 (resi0)
```

```
## Log likelihood at MLEs:
## [1] 5830.004
```

```
## Warning in sqrt(diag(solve(Hessian))): NaNs produzidos
```

```
##
## Coefficient(s):
##          Estimate  Std. Error  t value Pr(>|t|)
## mu    1.11314e-04 4.51805e-04  0.24638  0.80539
## omega 5.11669e-05          NA       NA       NA
## alpha 1.00000e-01 9.48893e-03 10.53860  < 2e-16 ***
```

```
## gam1  2.00000e-02 2.04666e-02  0.97720  0.32847
## beta  8.10000e-01          NA       NA       NA
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
```

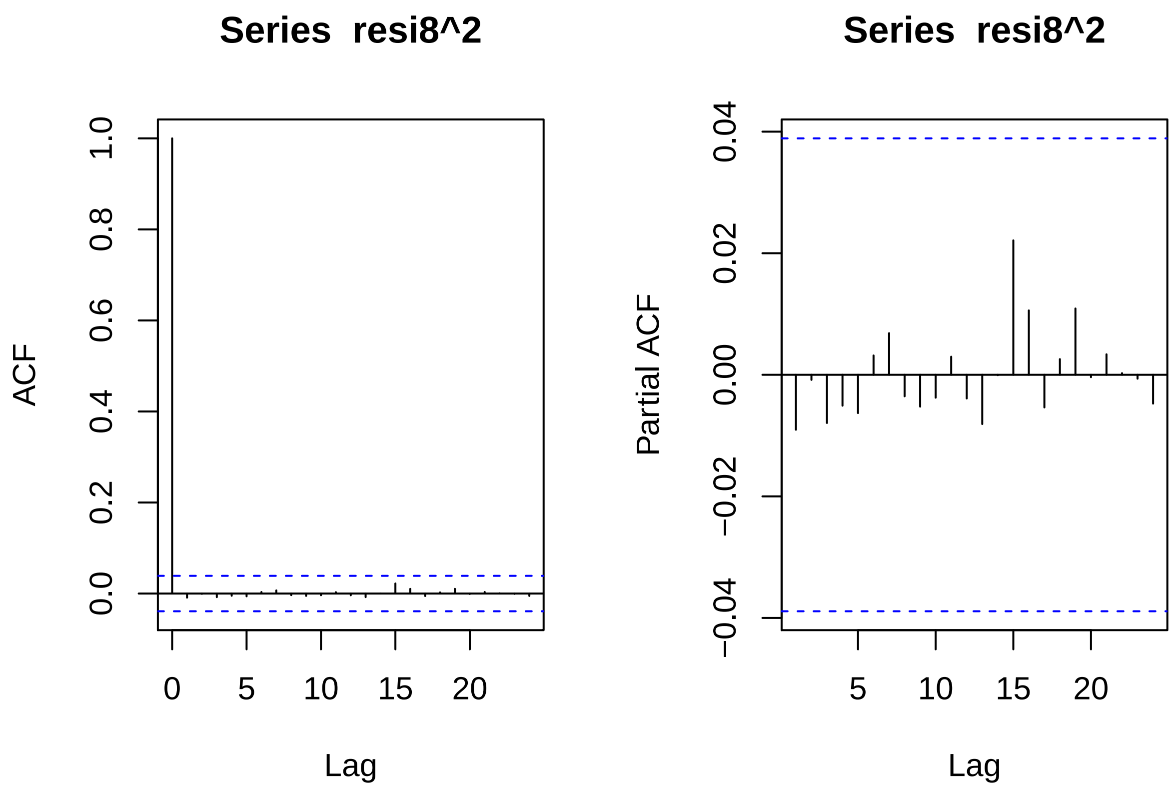
Análise dos resíduos quadraticos de M8:

Não há autocorrelação entres os resíduos quadraticos e aceitamos H0 do arch teste.

```
resi8 = m8$residuals / m8$volatility
archTest (resi8, 20)
```

```
##
## Call:
## lm(formula = atsq ~ x)
##
## Residuals:
##     Min      1Q  Median      3Q     Max
##  -5.078  -0.943  -0.686   0.087 201.906
##
## Coefficients:
##              Estimate Std. Error t value Pr(>|t|)
## (Intercept)  1.0283577  0.1280637   8.030 1.49e-15 ***
## x1          -0.0094749  0.0200081  -0.474    0.636
## x2          -0.0006087  0.0200079  -0.030    0.976
## x3          -0.0080383  0.0200078  -0.402    0.688
## x4          -0.0055108  0.0200081  -0.275    0.783
## x5          -0.0062684  0.0200073  -0.313    0.754
## x6           0.0035608  0.0200029   0.178    0.859
## x7           0.0068813  0.0200032   0.344    0.731
## x8          -0.0038417  0.0200027  -0.192    0.848
## x9          -0.0054763  0.0200028  -0.274    0.784
## x10         -0.0036895  0.0200029  -0.184    0.854
## x11          0.0031579  0.0200031   0.158    0.875
## x12         -0.0038551  0.0200030  -0.193    0.847
## x13         -0.0080916  0.0200029  -0.405    0.686
## x14          0.0001103  0.0200030   0.006    0.996
## x15          0.0222730  0.0200026   1.114    0.266
## x16          0.0105781  0.0200071   0.529    0.597
## x17         -0.0053630  0.0200078  -0.268    0.789
## x18          0.0027249  0.0200075   0.136    0.892
## x19          0.0108998  0.0200077   0.545    0.586
## x20         -0.0003882  0.0200079  -0.019    0.985
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
##
## Residual standard error: 4.369 on 2498 degrees of freedom
## Multiple R-squared:  0.001171,   Adjusted R-squared:  -0.006826
## F-statistic: 0.1464 on 20 and 2498 DF,  p-value: 1
```

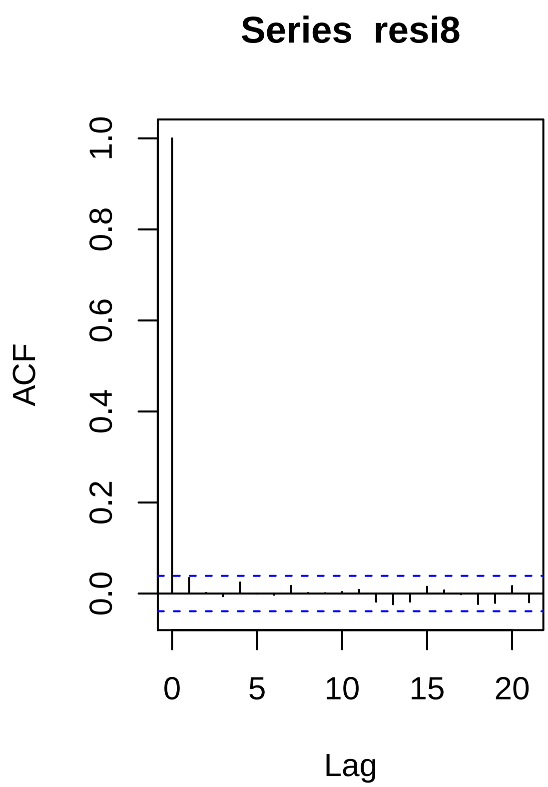
```
par (mfcol = c(1, 2))
acf (resi8^2, lag = 24)
pacf (resi8^2, lag = 24)
```

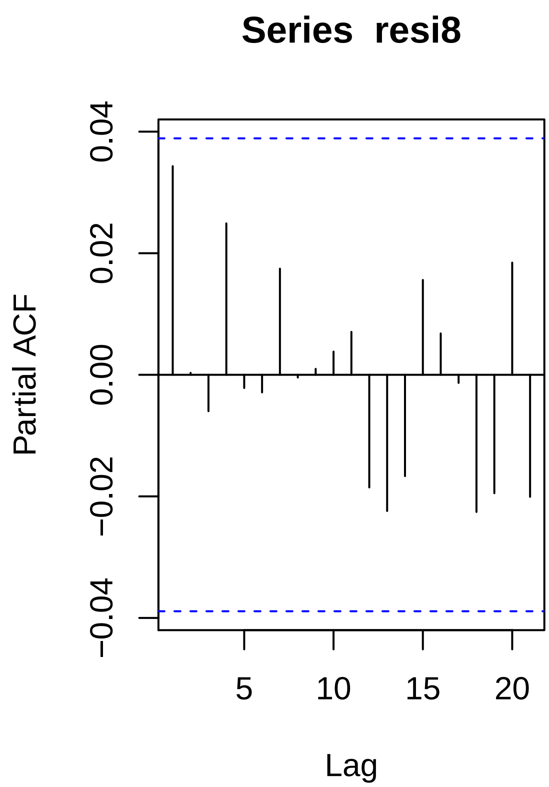


Análise das inovações:

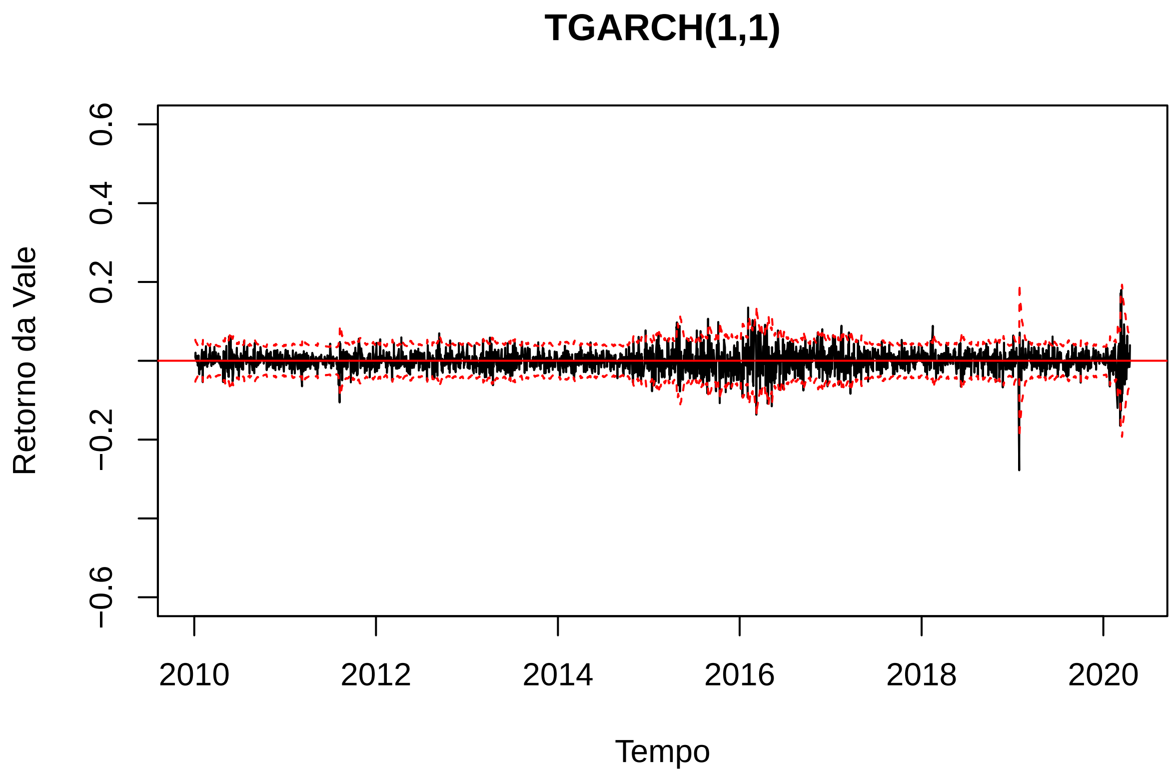
Inovações se comportam como um ruído branco.

```r
par (mfcol = c(1, 2))
acf (resi8, lag = 21)
pacf (resi8, lag = 21)
```



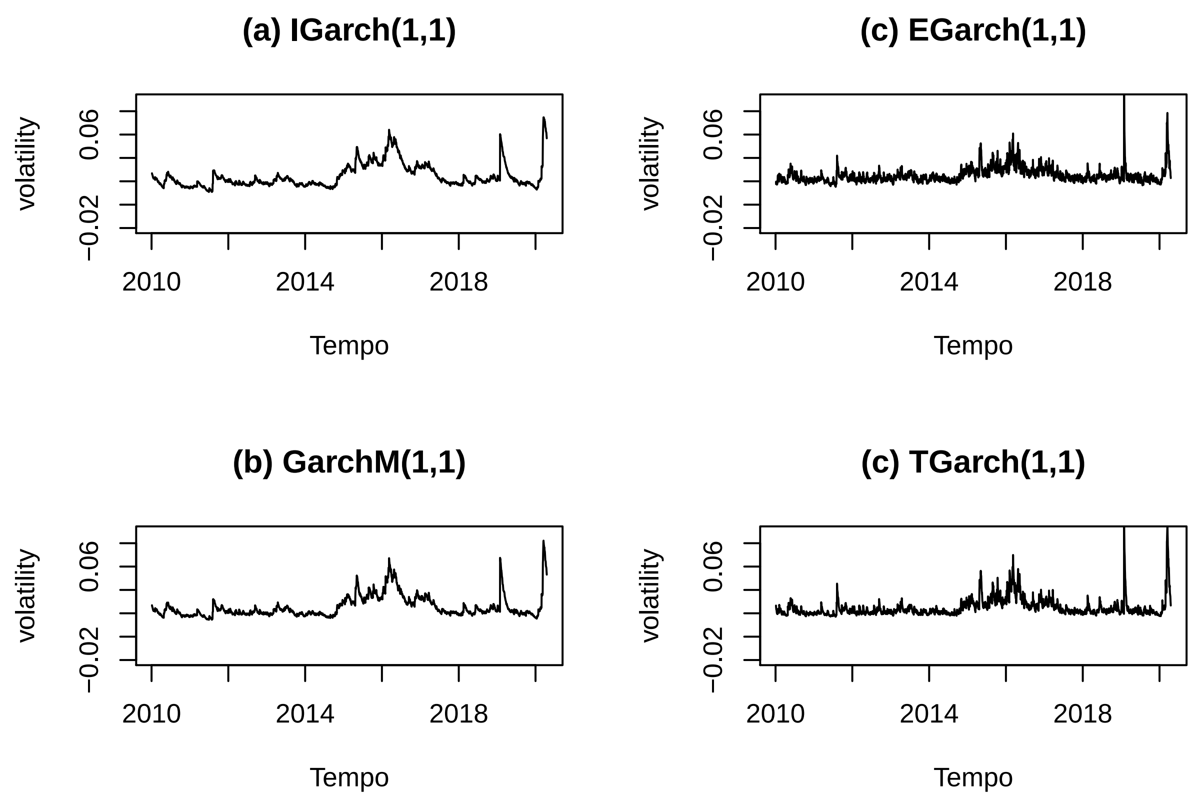


Intervalo de Confiança



TGARCH(1,1)
Retorno da Vale
0.6
0.4
0.2
−0.2
−0.6
2010
2012
2014
2016
2018
2020
Tempo

Comparação dos Modelos



Criterio de Informação (AIC)

Podemos comparar os três primeiros modelos utilizando criterio de informação. O críterio escolhido foi o AIC e o menor valor do AIC é obtido no modelo GARCH onde as inovações se distribuem como uma t de student.

```
## [1] "GARCH (norm): -4.61374"
```

```
## [1] "GARCH (std): -4.70027"
```

```
## [1] "GARCH (sstd): -4.69956"
```

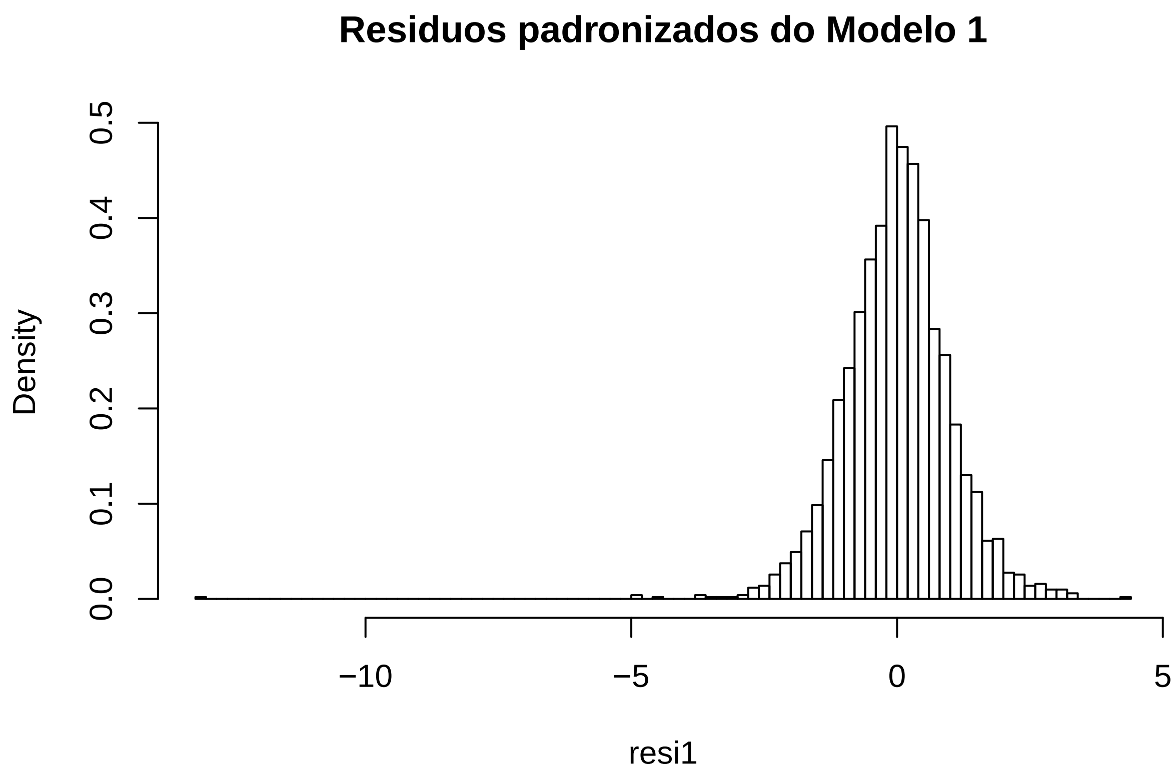
Correlação

```
##              GARCH (norm) GARCH (std) GARCH (sstd)    IGARCH    GARCHM
## GARCH (norm)    1.0000000   0.9999233    0.9999301 0.9879006 0.9981631
## GARCH (std)     0.9999233   1.0000000    0.9999997 0.9865397 0.9977821
## GARCH (sstd)    0.9999301   0.9999997    1.0000000 0.9866591 0.9978086
## IGARCH          0.9879006   0.9865397    0.9866591 1.0000000 0.9902150
## GARCHM          0.9981631   0.9977821    0.9978086 0.9902150 1.0000000
## EGARCH          0.8429523   0.8474986    0.8472518 0.7921749 0.8326570
## TGARCH          0.9140042   0.9176416    0.9173867 0.8572338 0.9035887
##                    EGARCH    TGARCH
```

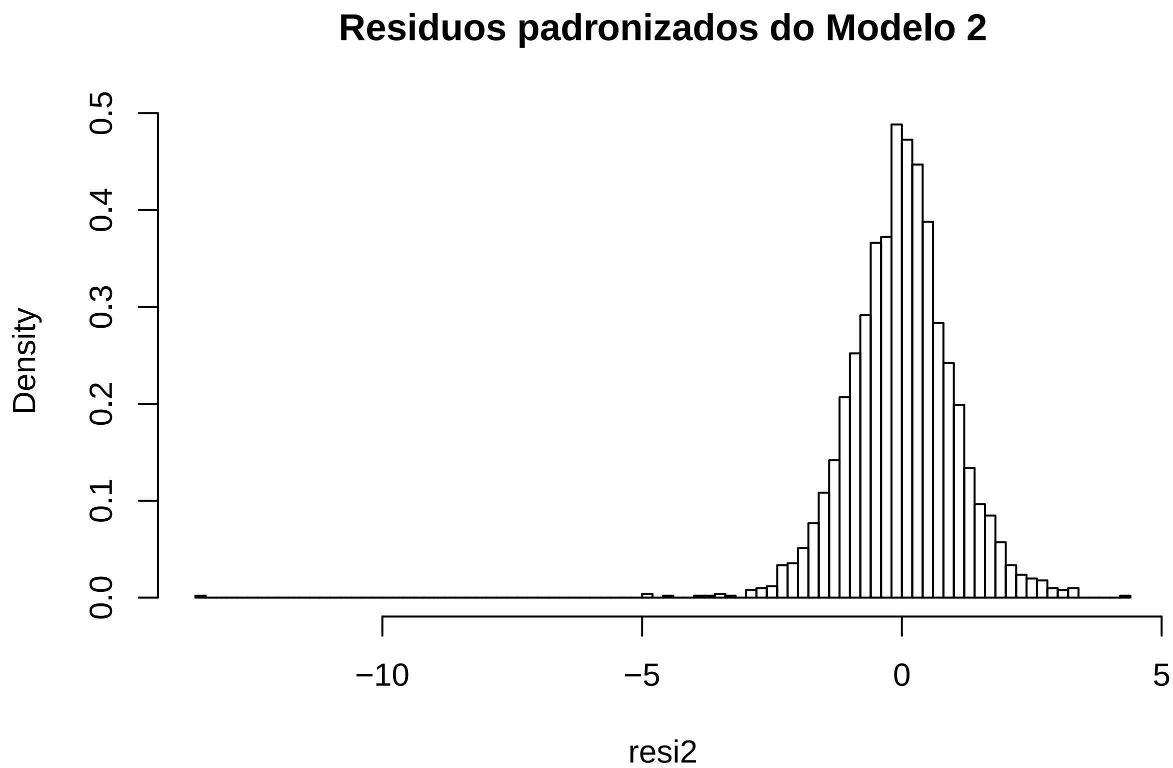
```
## GARCH (norm) 0.8429523 0.9140042
## GARCH (std)  0.8474986 0.9176416
## GARCH (sstd) 0.8472518 0.9173867
## IGARCH       0.7921749 0.8572338
## GARCHM       0.8326570 0.9035887
## EGARCH       1.0000000 0.9606895
## TGARCH       0.9606895 1.0000000
```

Histogramas dos resíduos padronizados dos Modelos.

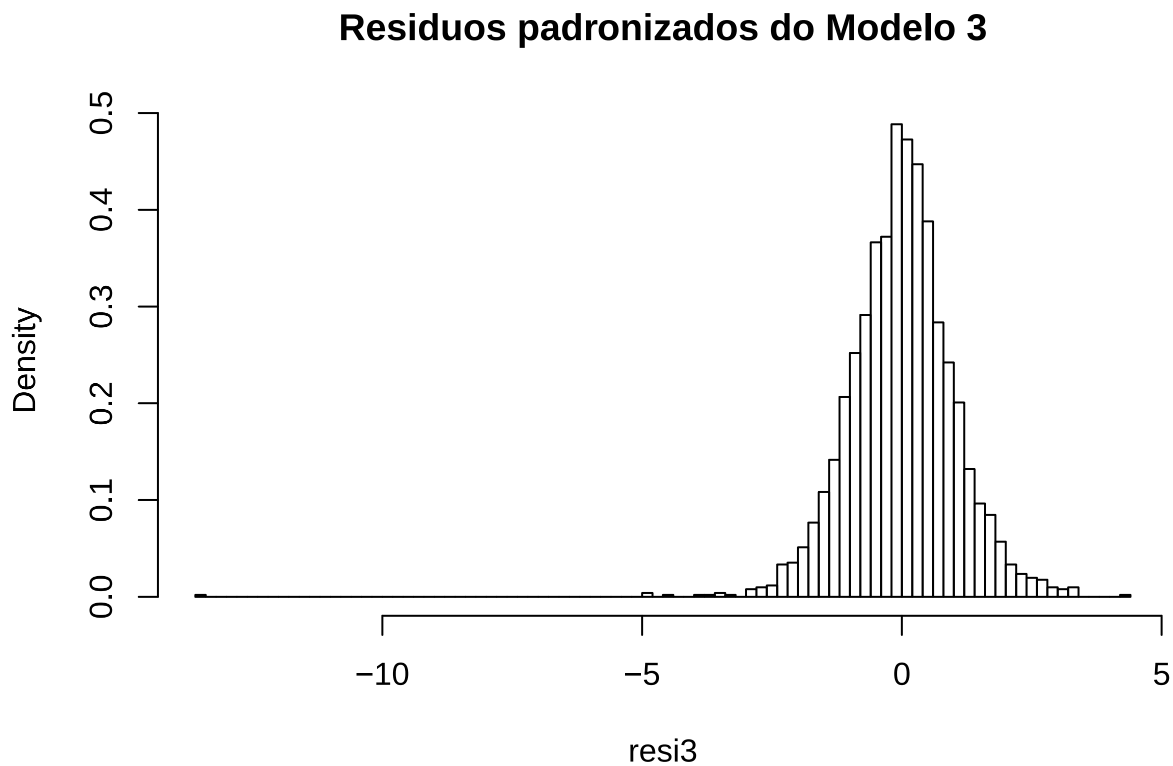
```r
hist.FD(resi1,main='Residuos padronizados do Modelo 1')
```




```r
hist.FD(resi2,main='Residuos padronizados do Modelo 2')
```




```
hist.FD(resi3,main='Residuos padronizados do Modelo 3')
```



Residuos padronizados do Modelo 3
Density
0.0
0.1
0.2
0.3
0.4
0.5
−10
−5
0
5
resi3