Among applications which need to solve non-linear systems equations there is the Bairstow's Method, which enable to find two roots of a polynomial.
The problem is defined by this way,
we have $P(x) = \sum_{k=0}^{n} a_k \cdot X^k$
we put $P(X) = (X^2 + B \cdot X + C) \cdot Q(X) + R(B,C) \cdot X + S(B,C)$
and

$$f : \begin{bmatrix} B \\ C \end{bmatrix} \rightarrow \begin{bmatrix} R(B,C) \\ S(B,C) \end{bmatrix}$$

We have $f(B,C) = \begin{bmatrix} 0 \\ 0 \end{bmatrix} \Leftrightarrow P(X) = Q(X) \cdot (X^2 + B \cdot X + C)$
We start by choose B and C, and Suppose
$f(B + \delta B, C + \delta C) = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$
In neighborhood of a root, a first-order Taylor series approximates the variation of R and S with respect the small changes in B, C, we get a Newton-Raphson form $f(U) + H(U) \cdot V$
which is

$$\begin{cases} R(B + \delta B, C + \delta C) = \frac{\delta R}{\delta B} \cdot \delta B + \frac{\delta R}{\delta C} \cdot \delta C \\ S(B + \delta B, C + \delta C) = \frac{\delta S}{\delta B} \cdot \delta B + \frac{\delta S}{\delta C} \cdot \delta C \end{cases}$$

Finally, we get a Newton-Raphson Form ($f(U) + J(U) \cdot V$)

with
$f(U) = \begin{bmatrix} R(B,C) \\ S(B,C) \end{bmatrix}$

$V = \begin{bmatrix} \delta B \\ \delta C \end{bmatrix}$, and

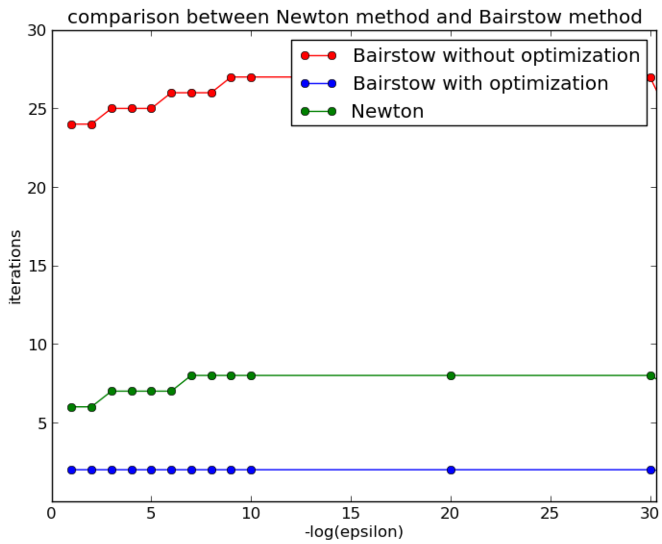
$$J(U) = \begin{bmatrix} \frac{\delta R}{\delta B} \cdot \delta B & \frac{\delta R}{\delta C} \cdot \delta C \\ \frac{\delta S}{\delta B} \cdot \delta B & \frac{\delta S}{\delta C} \cdot \delta C \end{bmatrix}$$

To get the coefficients of J(U), we must put a second polynomial division
we put $Q(X) = G(X) \cdot (X^2 + B \cdot X + C) + R_1 \cdot X + S_1$

afterwards we get, $\begin{cases} \frac{\delta R}{\delta B} = B \cdot R_1 - S_1 \\ \frac{\delta R}{\delta C} = -R_1 \\ \frac{\delta S}{\delta B} = C \cdot R_1 \\ \frac{\delta S}{\delta C} = -S_1 \end{cases}$

After every iteration we get a new B and a new C by adding δB and δC.
Finally, we get Two roots of P thanks to B and C
Indeed, $-B = X_+ + X_-$ and $C = X_+ \cdot X_-$
actually, we get X_+ and X_- by solving $(X^2 + B \cdot X + C)$
Then, we have compared this method with a solving by newton Raphson in dimension 1
As well, we can optimize the algorithm by taking $B = \frac{a_{n-1}}{a_n}$ and $C = \frac{a_{n-2}}{a_n}$

We have test those methods, and we have depended the average iteration of every algorithm.



To Conclude, Bairstow's method optimized is the most efficient and it is more automatic than the others. Moreover it find two roots unlike the newton raphson in 1 dimension which find a root.