

**1 C++**

```
#include <bits/stdc++.h>
using namespace std;

#define fi first
#define se second
#define forn(i, n) for (int i = 0; i < (int)n; ++i)
#define forl(i, n) for (int i = 1; i <= (int)n; ++i)
#define fore(i, l, r) for (int i = (int)l; i <= (int)r; ++i)
#define ford(i, n) for (int i = (int)(n) - 1; i >= 0; --i)
#define fored(i, l, r) for (int i = (int)r; i >= (int)l; --i)
#define pb push_back
#define pf push_front
#define el '\n'
#define d(x) cout << #x << " " << x << el
#define ri(n) scanf("%d", &n)
#define sz(v) int(v.size())
#define all(v) v.begin(), v.end()
#define mset(x, y) memset(x, (y), sizeof(x));

typedef long long ll;
typedef unsigned long long ull;
typedef long double ld;
typedef pair<int, int> pii;
typedef pair<ll, ll> pll;
typedef tuple<int, int, int> iii;
typedef vector<int> vi;
typedef vector<pii> vii;
typedef vector<ll> vll;
typedef vector<ld> vld;

const int INF = 0x3f3f3f3f;
const ull INFL = 0x3f3f3f3f3f3f3f3f;
const int MAX = 1e5 + 200;
const ld PI = acos(-1);
const ld EPS = 1e-9;

int dr[] = {1, -1, 0, 0, 1, -1, -1, 1};
int dc[] = {0, 0, 1, -1, 1, 1, -1, -1};

ostream& operator<<(ostream& os, const pii& pa) {
    return os << "(" << pa.fi << ", " << pa.se << ")";
}

int main() {
    ios_base::sync_with_stdio(false);
    cin.tie(NULL);
    cout.tie(NULL);
    cout << setprecision(20) << fixed;
```

## 1.2 Complexity

## 2 Divide and Conquer

```
// Bisection Method

// Very useful for finding roots of a function
// With 100 repetitions the value already converges.
// This implementation only works if the function in the range [lo, hi] has
    some
// zero.

// F(x)
// ^   F(lo)
// |   *
// |   | *
// |   | *
// |   | * Goal
// |   | *-----> x
// |   -0-
// |   *           |
// |   *           *
// |   *           * F(hi)
// |   * * *
// |

double bisection(double lo, double hi) {
    for (int i = 0; i < 100; i++) {
        double mid = (lo + hi) / 2;
        double F = f(mid); // Declare a function
        if (F > 0)
            lo = mid;
        else
            hi = mid;
    }
    return lo;
}
```

---

```
// Ternary search
// Very useful for finding max/min values between interval
// The function on the interval must be unimodal (only 1 maximum)
```

The diagram shows a coordinate system where the vertical axis represents  $F(x)$ . A horizontal dashed line represents the  $x$ -axis. A point labeled "Goal" is marked at the intersection of the axes, with a value of 0 indicated. Asterisks (\*) represent discrete values or states. Some are located above the  $x$ -axis, forming a curve that peaks near the Goal. Others are below the  $x$ -axis. Labels  $F(l)$  and  $F(r)$  are placed near specific points below the axis.

```
double ternary_search(double l, double r) {
    double eps = 1e-9;
    while (r - l > eps) {
        double m1 = l + (r - l) / 3;
        double m2 = r - (r - l) / 3;
        double f1 = f(m1);
        double f2 = f(m2);
        if (f1 < f2)
            l = m1;
        else
            r = m2;
    }
    return f(l); // Return the maximum of f(x) in [l, r]
}
```

## 2.3 Binary Search

```
// Binary Search
```

```
int binary_search(vector<int> arr, int target) {
    int left = 0;
    int right = arr.size() - 1;
    while (left <= right) {
        int mid = left + (right - left) / 2;
        if (arr[mid] == target) return mid;
        if (arr[mid] < target)
            left = mid + 1;
        else
            right = mid - 1;
    }
    return -1;
}
```

### 3 Sorting

### 3.1 Merge Sort (Inversion Counter)

```
// Merge sort
// The provided implementation enhances this basic approach by counting the
// number of inversions in the array.
// Time complexity:  $O(n \log n)$ 
```

```
// Space complexity:  $O(n)$ 
```

```
int merge(vi arr, vi aux, int lo, int hi, int mid) {
    int inv = 0;
    for (int k = lo; k <= hi; k++) aux[k] = arr[k];
    int i = lo;
    int j = mid + 1;
    for (int k = lo; k <= hi; k++) {
        if (i > mid)
            arr[k] = aux[j++];
        else if (j > hi)
            arr[k] = aux[i++];
        else if (aux[j] < aux[i]) {
            arr[k] = aux[j++];
            inv += mid + 1 - i;
        } else
            arr[k] = aux[i++];
    }
    return inv;
}

int merge_sort(vi arr, vi aux, int lo, int hi) {
    int inv = 0;
    if (lo >= hi) return inv;
    int mid = lo + (hi - lo) / 2;
    inv += merge_sort(arr, aux, lo, mid);
    inv += merge_sort(arr, aux, mid + 1, hi);
    inv += merge(arr, aux, lo, hi, mid);
    return inv;
}
```

## 4 Graph Algorithms

## 4.1 DFS

```
// Depth first search
```

```
int V;
vector<vi> adj;
bool vis[VMAX];
vi topsort; // Topological Sort.
            // Only works in directed acyclic graph.
```

```
void dfs(int s) {
    vis[s] = true;
    for (auto a : adj[s]) {
        if (!vis[a]) {
            dfs(a);
        }
    }
    topsort.push_back(s); // Only works in DAG.
}
```

## 4.2 Bellman Ford

```
// Bellman-Ford
```

```

typedef tuple<int, int, int> iii;

int V;
vector<iii> edges; // <u, v, w>
int dist[VMAX];

int bellman_ford(int s) {
    memset(dist, INF, sizeof(dist));
    dist[s] = 0;
    for (int i = 0; i < V - 1; i++) {
        for (auto uv : edges) {
            auto [u, v, w] = uv;
            if (dist[u] + w < dist[v]) {
                dist[v] = dist[u] + w;
            }
        }
    }
    for (auto uv : edges) {
        auto [u, v, w] = uv;
        if (dist[u] + w < dist[v]) {
            return true; // Negative cycle
        }
    }
    return false;
}

```

### 4.3 Strongly Connected Components

```

// Tarjan's Algorithm
// Finding strongly connected components (Directed Graph)

int V;
vector<vi> adj;
vi dfslow, dfsnum;
bool vis[VMAX];
int SCC, TIME;
stack<int> aux;

void tarjan_dfs(int s) {
    dfslow[s] = dfsnum[s] = ++TIME;
    aux.push(s);
    vis[s] = true;
    for (auto a : adj[s]) {
        if (!dfsnum[a]) tarjan_dfs(a);
        if (vis[a]) dfslow[s] = min(dfslow[s], dfslow[a]);
    }
    if (dfslow[s] == dfsnum[s]) {
        SCC += 1;
        while (1) {
            int v = aux.top();
            aux.pop();
            vis[v] = 0;
            if (s == v) break;
        }
    }
}

void scc() {
    aux = stack<int>();
}

```

```

dfslow = vi(V, 0);
dfsnum = vi(V, 0);
memset(vis, false, sizeof(vis));
TIME = SCC = 0;
for (int i = 0; i < V; i++) {
    if (!dfsnum[i]) tarjan_dfs(i);
}
}

```

### 4.4 Edmonds-Karp

```

// Edmonds-Karp Algorithm
// Min-Cut/Max-Flow problem

int V;
vector<vi> adj;
vector<vi> capacity;

int bfs(int s, int t, vi& parent) {
    fill(parent.begin(), parent.end(), -1);
    parent[s] = -2;
    queue<pair<int, int>> q;
    q.push({s, INF});
    while (!q.empty()) {
        int cur = q.front().first;
        int flow = q.front().second;
        q.pop();

        for (int next : adj[cur]) {
            if (parent[next] == -1 && capacity[cur][next]) {
                parent[next] = cur;
                int new_flow = min(flow, capacity[cur][next]);
                if (next == t) return new_flow;
                q.push({next, new_flow});
            }
        }
    }
    return 0;
}

int maxflow(int s, int t) {
    int flow = 0;
    vi parent(V);
    int new_flow;
    while ((new_flow = bfs(s, t, parent))) {
        flow += new_flow;
        int cur = t;
        while (cur != s) {
            int prev = parent[cur];
            capacity[prev][cur] -= new_flow;
            capacity[cur][prev] += new_flow;
            cur = prev;
        }
    }
    return flow;
}

```