

## Parameters 2015-2019 model 2

### Rates for the home and away goals

$$\begin{aligned}\ln \lambda_k(t) &= \ln \alpha_i + \ln \beta_j + \ln \gamma_h + \mathbb{I}\{\text{half} = 2\} \ln \tau + \omega_{\lambda(x-y)}(x(t) - y(t)) + \omega_{\lambda(y^*-x^*)}(y^*(t) - x^*(t)) \\ \ln \mu_k(t) &= \ln \alpha_j + \ln \beta_i + \mathbb{I}\{\text{half} = 2\} \ln \tau + \omega_{\mu(y-x)}(y(t) - x(t)) + \omega_{\mu(x^*-y^*)}(x^*(t) - y^*(t))\end{aligned}$$

- $i$ : home team index;
- $j$ : away team index;
- $\alpha$ : attack strength parameter;
- $1/\beta$ : defense strength parameter;
- $\gamma_h$ : home advantage parameter;
- $\tau$ : second half parameter;
- $x(t)$ : the number of goals of the home team until minute  $t$ ;
- $y(t)$ : the number of goals of the away team until minute  $t$ ;
- $x^*(t)$ : the number of red cards of the home team until minute  $t$ ;
- $y^*(t)$ : the number of red cards of the away team until minute  $t$ ;
- $\omega_{\lambda(x-y)}, \omega_{\mu(y-x)}$ : parameters that measure the impact of leading in the score in the rates;
- $\omega_{\lambda(y^*-x^*)}$  and  $\omega_{\mu(x^*-y^*)}$ : parameters that measure the impact of having extra players on the field.

### Rates for the home and away red cards

$$\begin{aligned}\lambda_k^*(t) &= A_\lambda \left( t + 45^{\mathbb{I}\{\text{half} = 2\}} \right) \\ \mu_k^*(t) &= A_\mu \left( t + 45^{\mathbb{I}\{\text{half} = 2\}} \right)\end{aligned}$$

### Stoppage time

The stoppage time for the first half,  $U^1$ , and the second half,  $U^2$ , are modeled as:

$$\begin{aligned}U^1 &\sim \text{Poisson}(\eta_1 + \rho_1 r^1) \\ U^2 &\sim \text{Poisson}(\eta_2 + \rho_2 r^2 + \kappa c)\end{aligned}$$

- $r^t$  is the amount of red cards received in half  $t$  until minute 45;
- $c = \begin{cases} 1, & \text{if } |x - y| \leq 1 \text{ at minute 45 of the second half;} \\ 0, & \text{otherwise.} \end{cases}$

## Constraint

The constraint for identificability is

$$\sum_i^n \log(\alpha_i) = \sum_i^n \log(\beta_i).$$

```
options(knitr.kable.NA = "-")
options(scipen = 999)

library(dplyr)
library(knitr)

load("data/input.RData")
load("data/mod_2.RData")

alphas_betas = tibble(Team = times$Time,
                      alpha = exp(mod_2$alpha),
                      beta = exp(mod_2$beta))
kable(alphas_betas, digits = 4, caption = "Alphas and betas",
      col.names = c("Team", "$\\alpha$", "$\\beta$"))
```

Table 1: Alphas and betas

Team	$\alpha$	$\beta$
América-MG	0.0576	0.1017
Athletico-PR	0.1022	0.0726
Atlético-GO	0.0824	0.1066
Atlético-MG	0.1257	0.0907
Avaí	0.0616	0.1090
Bahia	0.0977	0.0837
Botafogo	0.0865	0.0805
Ceará	0.0738	0.0745
Chapecoense	0.0853	0.0954
Corinthians	0.1093	0.0646
Coritiba	0.0829	0.0849
Cruzeiro	0.0884	0.0764
Csa	0.0513	0.1130
Figueirense	0.0726	0.0943
Flamengo	0.1302	0.0721
Fluminense	0.0906	0.0901
Fortaleza	0.1122	0.0929
Goiás	0.0931	0.1092
Grêmio	0.1152	0.0671
Internacional	0.0927	0.0690
Joinville	0.0575	0.0917
Palmeiras	0.1384	0.0700
Paraná	0.0394	0.1105
Ponte Preta	0.0930	0.0907
Santa Cruz	0.0979	0.1366
Santos	0.1204	0.0678
São Paulo	0.1028	0.0735
Sport	0.1020	0.0992

Team	$\alpha$	$\beta$
Vasco da Gama	0.0808	0.0917
Vitória	0.0991	0.1118

```

Parameter = c("$\\gamma_h$", "$\\tau$", "$\\omega_{\\lambda (x-y)}$",
              "$\\omega_{\\mu (y-x)}$", "$\\omega_{\\lambda (y^*-x^*)}$",
              "$\\omega_{\\mu (x^*-y^*)}$")
goals = tibble(Parameter,
                Estimative = c(exp(mod_2$gamma), exp(mod_2$tau), mod_2$omega))
kable(goals, digits = 4, caption = "Goal rate parameters")

```

Table 2: Goal rate parameters

Parameter	Estimative
$\gamma_h$	1.6027
$\tau$	1.2474
$\omega_{\lambda(x-y)}$	-0.0982
$\omega_{\mu(y-x)}$	-0.0907
$\omega_{\lambda(y^*-x^*)}$	0.2895
$\omega_{\mu(x^*-y^*)}$	0.3471

```

Parameter = c("$A_{\\lambda}$", "$A_{\\mu}$")
reds = tibble(Parameter, Estimative = exp(mod_2$a))
kable(reds, digits = 8, caption = "Red card rate parameters")

```

Table 3: Red card rate parameters

Parameter	Estimative
$A_{\lambda}$	0.00001815
$A_{\mu}$	0.00003237

```

Parameter = c("$\\eta_1$", "$\\eta_2$", "$\\rho_1$", "$\\rho_2$", "$\\kappa$")
st = tibble(Parameter,
             Estimative = c(mod_2$eta, mod_2$rho, mod_2$kappa))
kable(st, digits = 4, caption = "Stoppage time parameters")

```

Table 4: Stoppage time parameters

Parameter	Estimative
$\eta_1$	2.3404
$\eta_2$	3.5376
$\rho_1$	0.8663
$\rho_2$	0.2537
$\kappa$	0.9764

```
mod_2$loglik
```

```
## [1] -11379.75
```