

Parameters 2015-2018 model 1

Rates for the home and away goals

$$\begin{aligned}\ln \lambda_k(t) &= \ln \alpha_i + \ln \beta_j + \ln \gamma_h + \mathbb{I}\{\text{half} = 2\} \ln \tau + \omega_{\lambda x} x(t) + \omega_{\lambda y} y(t) + \omega_{\lambda x^*} x^*(t) + \omega_{\lambda y^*} y^*(t) \\ \ln \mu_k(t) &= \ln \alpha_j + \ln \beta_i + \mathbb{I}\{\text{half} = 2\} \ln \tau + \omega_{\mu x} x(t) + \omega_{\mu y} y(t) + \omega_{\mu x^*} x^*(t) + \omega_{\mu y^*} y^*(t)\end{aligned}$$

- i : home team index;
- j : away team index;
- α : attack strength parameter;
- $1/\beta$: defense strength parameter;
- γ_h : home advantage parameter;
- τ : second half parameter;
- $x(t)$: the number of goals of the home team until minute t ;
- $y(t)$: the number of goals of the away team until minute t ;
- $x^*(t)$: the number of red cards of the home team until minute t ;
- $y^*(t)$: the number of red cards of the away team until minute t ;
- $\omega_{\lambda x}$, $\omega_{\lambda y}$, $\omega_{\mu x}$ and $\omega_{\mu y}$: parameters that measure the impact of the scored goals in the rates;
- $\omega_{\lambda x^*}$, $\omega_{\lambda y^*}$, $\omega_{\mu x^*}$ and $\omega_{\mu y^*}$: parameters that measure the impact of red cards in the rates.

Rates for the home and away red cards

$$\begin{aligned}\lambda_k^*(t) &= A_\lambda \left(t + 45^{\mathbb{I}\{\text{half} = 2\}} \right) \\ \mu_k^*(t) &= A_\mu \left(t + 45^{\mathbb{I}\{\text{half} = 2\}} \right)\end{aligned}$$

Stoppage time

The stoppage time for the first half, U^1 , and the second half, U^2 , are modeled as:

$$\begin{aligned}U^1 &\sim \text{Poisson}(\eta_1 + \rho_1 r^1) \\ U^2 &\sim \text{Poisson}(\eta_2 + \rho_2 r^2 + \kappa c)\end{aligned}$$

- r^t is the amount of red cards received in half t until minute 45;
- $c = \begin{cases} 1, & \text{if } |x - y| \leq 1 \text{ at minute 45 of the second half;} \\ 0, & \text{otherwise.} \end{cases}$

Constraint

The constraint for identificability is

$$\sum_i^n \log(\alpha_i) = \sum_i^n \log(\beta_i).$$

```
options(knitr.kable.NA = "-")
options(scipen = 999)

library(dplyr)
library(knitr)

load("data/input.RData")
load("data/mod_1.RData")

alphas_betas = tibble(Team = times$Time,
                      alpha = exp(mod_1$alpha),
                      beta = exp(mod_1$beta))
kable(alphas_betas, digits = 4, caption = "Alphas and betas",
      col.names = c("Team", "$\\alpha$", "$\\beta$"))
```

Table 1: Alphas and betas

Team	α	β
América-MG	0.0566	0.1029
Athletico-PR	0.0982	0.0768
Atlético-GO	0.0802	0.1058
Atlético-MG	0.1297	0.0914
Avaí	0.0715	0.1048
Bahia	0.0970	0.0865
Botafogo	0.0905	0.0799
Ceará	0.0681	0.0726
Chapecoense	0.0880	0.0958
Corinthians	0.1115	0.0655
Coritiba	0.0816	0.0854
Cruzeiro	0.0942	0.0747
Figueirense	0.0719	0.0960
Flamengo	0.1122	0.0737
Fluminense	0.0905	0.0924
Goiás	0.0850	0.0965
Grêmio	0.1066	0.0660
Internacional	0.0907	0.0691
Joinville	0.0569	0.0935
Palmeiras	0.1354	0.0729
Paraná	0.0391	0.1131
Ponte Preta	0.0918	0.0922
Santa Cruz	0.0960	0.1373
Santos	0.1137	0.0704
São Paulo	0.1054	0.0793
Sport	0.1002	0.1004
Vasco da Gama	0.0784	0.0956
Vitória	0.0972	0.1124

```

Parameter = c("$\\gamma_h$", "$\\tau$", "$\\omega_{\\lambda x}$",
              "$\\omega_{\\lambda y}$", "$\\omega_{\\mu x}$",
              "$\\omega_{\\mu y}$", "$\\omega_{\\lambda x^*}$",
              "$\\omega_{\\lambda y^*}$", "$\\omega_{\\mu x^*}$",
              "$\\omega_{\\mu y^*}$")
goals = tibble(Parameter,
               Estimative = c(exp(mod_1$gamma), exp(mod_1$tau), mod_1$omega))
kable(goals, digits = 4, caption = "Goal rate parameters")

```

Table 2: Goal rate parameters

Parameter	Estimative
γ_h	1.5946
τ	1.2424
$\omega_{\lambda x}$	-0.0885
$\omega_{\lambda y}$	0.1340
$\omega_{\mu x}$	0.0768
$\omega_{\mu y}$	-0.0535
$\omega_{\lambda x^*}$	-0.4147
$\omega_{\lambda y^*}$	0.1489
$\omega_{\mu x^*}$	0.2362
$\omega_{\mu y^*}$	-0.4103

```

Parameter = c("$A_{\\lambda}$", "$A_{\\mu}$")
reds = tibble(Parameter, Estimative = exp(mod_1$a))
kable(reds, digits = 8, caption = "Red card rate parameters")

```

Table 3: Red card rate parameters

Parameter	Estimative
A_{λ}	0.00001788
A_{μ}	0.00003212

```

Parameter = c("$\\eta_1$", "$\\eta_2$", "$\\rho_1$", "$\\rho_2$", "$\\kappa$")
st = tibble(Parameter,
            Estimative = c(mod_1$eta, mod_1$rho, mod_1$kappa))
kable(st, digits = 4, caption = "Stoppage time parameters")

```

Table 4: Stoppage time parameters

Parameter	Estimative
η_1	2.1709
η_2	3.4403
ρ_1	0.7630
ρ_2	0.2504
κ	0.8641

```
mod_1$loglik
```

```
## [1] -9747.911
```