# Parameters Série A 2019

## Rates for the home and away teams

### Model 0

$$\lambda_k = \alpha_i \beta_j \gamma_h$$
$$\mu_k = \alpha_j \beta_i$$

- $i$: home team index;
- $j$: away team index;
- $\alpha$: attack strength parameter;
- $1/\beta$: defense strength parameter;
- $\gamma_h$: home advantage parameter.

### Model 1

$$\lambda_k(t) = \alpha_i \beta_j \gamma_h$$
$$\mu_k(t) = \alpha_j \beta_i$$

### Model 2

$$\lambda_k(t) = \alpha_i \beta_j \gamma_h \tau^{\mathbb{I}\{\text{half} = 2\}}$$
$$\mu_k(t) = \alpha_j \beta_i \tau^{\mathbb{I}\{\text{half} = 2\}}$$

- $\tau$: second half parameter.

### Model 3

$$\lambda_k(t) = \alpha_i \beta_j \gamma_h \tau^{\mathbb{I}\{\text{half} = 2\}} \lambda_{xy}$$
$$\mu_k(t) = \alpha_j \beta_i \tau^{\mathbb{I}\{\text{half} = 2\}} \mu_{xy}$$

- $\lambda_{xy} = \begin{cases} 1, & \text{if } x = y; \\ \lambda_{10}, & \text{if } x > y; \\ \lambda_{01}, & \text{if } x < y; \end{cases}$
- $\mu_{xy} = \begin{cases} 1, & \text{if } x = y; \\ \mu_{10}, & \text{if } x > y; \\ \mu_{01}, & \text{if } x < y. \end{cases}$

## Stoppage time

For all models except model 0, the stoppage time for the first half, $U^1$, and the second half, $U^2$, are modeled as:

$$U^1 \sim \text{Poisson}(\eta_1 + \phi_1 g^1)$$
$$U^2 \sim \text{Poisson}(\eta_2 + \phi_2 g^2 + \kappa c)$$

- $g^t$ is the amount of goals scored in half $t$ until minute 45;
- $c = \begin{cases} 1, & \text{if } |x - y| \leq 1 \text{ at minute 45 of the second half;} \\ 0, & \text{otherwise.} \end{cases}$

## Constraint

The constraint for identificability in all models is

$$\sum_i^n \log(\alpha_i) = \sum_i^n \log(\beta_i)$$

```
options(knitr.kable.NA = "-")

library(dplyr)
library(knitr)

load("data/input.RData")
load("data/mod_0.RData")
load("data/mod_1.RData")
load("data/mod_2.RData")
load("data/mod_3.RData")
```

```
alphas = tibble(Time = times$Time,
                mod_0 = exp(mod_0$alpha),
                mod_1 = exp(mod_1$alpha),
                mod_2 = exp(mod_2$alpha),
                mod_3 = exp(mod_3$alpha))
kable(alphas, digits = 4, caption = "Alphas")
```

Table 1: Alphas

| Time | mod_0 | mod_1 | mod_2 | mod_3 |
|---|---|---|---|---|
| Athletico-PR | 1.1254 | 0.1134 | 0.1074 | 0.1055 |
| Atlético-MG | 1.0132 | 0.1014 | 0.0960 | 0.0917 |
| Avaí | 0.4108 | 0.0417 | 0.0395 | 0.0360 |
| Bahia | 0.9835 | 0.0991 | 0.0938 | 0.0893 |
| Botafogo | 0.6940 | 0.0703 | 0.0665 | 0.0635 |
| Ceará | 0.8023 | 0.0805 | 0.0761 | 0.0715 |
| Chapecoense | 0.6998 | 0.0709 | 0.0670 | 0.0642 |
| Corinthians | 0.9286 | 0.0939 | 0.0888 | 0.0864 |
| Cruzeiro | 0.6050 | 0.0615 | 0.0582 | 0.0540 |
| CSA | 0.5454 | 0.0554 | 0.0524 | 0.0483 |

| Time | mod_0 | mod_1 | mod_2 | mod_3 |
|---|---|---|---|---|
| Flamengo | 1.9129 | 0.1935 | 0.1833 | 0.1916 |
| Fluminense | 0.8521 | 0.0857 | 0.0811 | 0.0768 |
| Fortaleza | 1.1262 | 0.1139 | 0.1078 | 0.1070 |
| Goiás | 1.0551 | 0.1058 | 0.1002 | 0.0953 |
| Grêmio | 1.4253 | 0.1448 | 0.1371 | 0.1360 |
| Internacional | 0.9787 | 0.0980 | 0.0928 | 0.0903 |
| Palmeiras | 1.3467 | 0.1360 | 0.1287 | 0.1301 |
| Santos | 1.3261 | 0.1348 | 0.1276 | 0.1312 |
| São Paulo | 0.8581 | 0.0861 | 0.0814 | 0.0805 |
| Vasco da Gama | 0.8735 | 0.0876 | 0.0829 | 0.0807 |

```
betas = tibble(Time = times$Time,
               mod_0 = exp(mod_0$beta),
               mod_1 = exp(mod_1$beta),
               mod_2 = exp(mod_2$beta),
               mod_3 = exp(mod_3$beta))
kable(betas, digits = 4, caption = "Betas")
```

Table 2: Betas

| Time | mod_0 | mod_1 | mod_2 | mod_3 |
|---|---|---|---|---|
| Athletico-PR | 0.6954 | 0.0698 | 0.0660 | 0.0620 |
| Atlético-MG | 1.0584 | 0.1061 | 0.1005 | 0.0973 |
| Avaí | 1.2971 | 0.1322 | 0.1252 | 0.1297 |
| Bahia | 0.9273 | 0.0933 | 0.0883 | 0.0852 |
| Botafogo | 0.9556 | 0.0968 | 0.0916 | 0.0909 |
| Ceará | 0.8757 | 0.0882 | 0.0834 | 0.0821 |
| Chapecoense | 1.1046 | 0.1115 | 0.1055 | 0.1048 |
| Corinthians | 0.7311 | 0.0738 | 0.0698 | 0.0654 |
| Cruzeiro | 0.9722 | 0.0990 | 0.0937 | 0.0928 |
| CSA | 1.2220 | 0.1243 | 0.1177 | 0.1210 |
| Flamengo | 0.8400 | 0.0847 | 0.0802 | 0.0730 |
| Fluminense | 0.9851 | 0.0994 | 0.0940 | 0.0917 |
| Fortaleza | 1.0649 | 0.1075 | 0.1017 | 0.0974 |
| Goiás | 1.3856 | 0.1391 | 0.1317 | 0.1307 |
| Grêmio | 0.8616 | 0.0869 | 0.0823 | 0.0793 |
| Internacional | 0.8409 | 0.0847 | 0.0802 | 0.0768 |
| Palmeiras | 0.7039 | 0.0712 | 0.0674 | 0.0639 |
| Santos | 0.7251 | 0.0736 | 0.0697 | 0.0646 |
| São Paulo | 0.6426 | 0.0645 | 0.0610 | 0.0576 |
| Vasco da Gama | 0.9648 | 0.0966 | 0.0914 | 0.0884 |

```
param = tibble(Model = 0:3,
              gamma = c(exp(mod_0$gamma), exp(mod_1$gamma), exp(mod_2$gamma),
                        exp(mod_3$gamma)),
              tau = c(NA, NA, exp(mod_2$tau), exp(mod_3$tau)),
              lambda_10 = c(NA, NA, NA, exp(mod_3$lambda_xy["10"])),
              lambda_01 = c(NA, NA, NA, exp(mod_3$lambda_xy["01"])),
              mu_10 = c(NA, NA, NA, exp(mod_3$mu_xy["10"])),
```

```
                 mu_01 = c(NA, NA, NA, exp(mod_3$mu_xy["01"])),
                 eta_1 = c(NA, mod_1$eta[1], mod_2$eta[1], mod_3$eta[1]),
                 eta_2 = c(NA, mod_1$eta[2], mod_2$eta[2], mod_3$eta[2]),
                 phi_1 = c(NA, mod_1$phi[1], mod_2$phi[1], mod_3$phi[1]),
                 phi_2 = c(NA, mod_1$phi[2], mod_2$phi[2], mod_3$phi[2]),
                 kappa = c(NA, mod_1$kappa, mod_2$kappa, mod_3$kappa))
kable(param, digits = 4, caption = "Other parameters",
      col.names = c("Model", "$\\gamma_h$", "$\\tau$", "$\\lambda_{10}$",
                    "$\\lambda_{01}$", "$\\mu_{10}$", "$\\mu_{01}$", "$\\eta_1$",
                    "$\\eta_2$", "$\\phi_1$", "$\\phi_2$", "$\\kappa$"))
```

Table 3: Other parameters

| Model | $\gamma_h$ | $\tau$ | $\lambda_{10}$ | $\lambda_{01}$ | $\mu_{10}$ | $\mu_{01}$ | $\eta_1$ | $\eta_2$ | $\phi_1$ | $\phi_2$ | $\kappa$ |
|---|---|---|---|---|---|---|---|---|---|---|---|
| 0 | 1.4957 | – | – | – | – | – | – | – | – | – | – |
| 1 | 1.4990 | – | – | – | – | – | 2.8342 | 3.6017 | 0.2261 | 0.2075 | 1.5565 |
| 2 | 1.4994 | 1.2284 | – | – | – | – | 2.8341 | 3.6017 | 0.2261 | 0.2075 | 1.5566 |
| 3 | 1.6930 | 1.2709 | 0.7617 | 1.0247 | 1.2748 | 0.7623 | 2.8341 | 3.6016 | 0.2261 | 0.2075 | 1.5566 |