Parameters Série A 2020 model 1

Rates for the home and away goals

$$\ln \lambda_k(t) = \ln \alpha_i + \ln \beta_j + \ln \gamma_h + \mathbb{I}\{\text{half} = 2\}\ln \tau + x\omega_{\lambda x} + y\omega_{\lambda y} + x^*\omega_{\lambda x^*} + y^*\omega_{\lambda y^*}$$

$$\ln \mu_k(t) = \ln \alpha_j + \ln \beta_i + \mathbb{I}\{\text{half} = 2\}\ln \tau + x\omega_{\mu x} + y\omega_{\mu y} + x^*\omega_{\mu x^*} + y^*\omega_{\mu y^*}$$

- i: home team index;

- j: away team index;

- α: attack strength parameter;

- $1/\beta$: defense strength parameter;

- γ_h: home advantage parameter;

- τ: second half parameter;

- x: the number of goals of the home team until minute t;

- y: the number of goals of the away team until minute t;

- x^*: the number of red cards of the home team until minute t;

- y^*: the number of red cards of the away team until minute t;

- $\omega_{\lambda x}$, $\omega_{\lambda y}$, $\omega_{\mu x}$ and $\omega_{\mu y}$: parameters that measure the impact of the scored goals in the rates;

- $\omega_{\lambda x^*}$, $\omega_{\lambda y^*}$, $\omega_{\mu x^*}$ and $\omega_{\mu y^*}$: parameters that measure the impact of the received red cards in the rates.

Rates for the home and away red cards

$$\lambda_k^*(t) = A_\lambda t$$

where

$$A_\lambda = \begin{cases} A_\lambda^1, & \text{if half} = 1; \\ A_\lambda^2, & \text{if half} = 2. \end{cases}$$

$$\mu_k^*(t) = A_\mu t$$

where

$$A_\mu = \begin{cases} A_\mu^1, & \text{if half} = 1; \\ A_\mu^2, & \text{if half} = 2. \end{cases}$$

Stoppage time

The stoppage time for the first half, U^1, and the second half, U^2, are modeled as:

$$U^1 \sim \text{Poisson}(\eta_1 + \rho_1 r^1)$$

$$U^2 \sim \text{Poisson}(\eta_2 + \rho_2 r^2 + \kappa c)$$

- r^t is the amount of red cards received in half t until minute 45;

- $c = \begin{cases} 1, & \text{if } |x - y| \leq 1 \text{ at minute 45 of the second half;} \\ 0, & \text{otherwise.} \end{cases}$

Constraint

The constraint for identificability is

$$\sum_{i}^{n} \log(\alpha_i) = \sum_{i}^{n} \log(\beta_i).$$

```
options(knitr.kable.NA = "-")
options(scipen = 999)

library(dplyr)
library(knitr)

load("data/input.RData")
load("data/mod_1.RData")
```

```
alphas_betas = tibble(Team = times$Time,
                alpha = exp(mod_1$alpha),
                beta = exp(mod_1$beta))
kable(alphas_betas, digits = 4, caption = "Alphas and betas",
      col.names = c("Team", "$\\alpha$", "$\\beta$"))
```

Table 1: Alphas and betas

Team	α	β
Athletico-PR	0.0747	0.0725
Athletico-GO	0.0817	0.0889
Atlético-MG	0.1360	0.0888
Bahia	0.0942	0.1259
Botafogo	0.0621	0.1268
Ceará	0.1084	0.1034
Corinthians	0.0928	0.0864
Coritiba	0.0602	0.1125
Flamengo	0.1424	0.0966
Fluminense	0.1121	0.0843
Fortaleza	0.0685	0.0883
Goiás	0.0789	0.1299
Grêmio	0.1114	0.0780
Internacional	0.1305	0.0656

Team	α	β
Palmeiras	0.1084	0.0718
Red Bull Bragantino	0.1065	0.0770
Santos	0.1070	0.1011
São Paulo	0.1188	0.0842
Sport	0.0598	0.1025
Vasco da Gama	0.0752	0.1150

```
Parameter = c("$\\gamma_h$", "$\\tau$", "$\\omega_{\\lambda x}$",
              "$\\omega_{\\lambda y}$", "$\\omega_{\\mu x}$",
              "$\\omega_{\\mu y}$", "$\\omega_{\\lambda x^*}$",
              "$\\omega_{\\lambda y^*}$", "$\\omega_{\\mu x^*}$",
              "$\\omega_{\\mu y^*}$")
goals = tibble(Parameter,
               Estimative = c(exp(mod_1$gamma), exp(mod_1$tau), mod_1$omega))
kable(goals, digits = 4, caption = "Goal rate parameters")
```

Table 2: Goal rate parameters

Parameter	Estimative
γ_h	1.6237
τ	1.1768
$\omega_{\lambda x}$	-0.2215
$\omega_{\lambda y}$	0.0544
$\omega_{\mu x}$	0.1389
$\omega_{\mu y}$	-0.0433
$\omega_{\lambda x^*}$	-0.3693
$\omega_{\lambda y^*}$	0.3559
$\omega_{\mu x^*}$	0.4602
$\omega_{\mu y^*}$	-0.3879

```
Parameter = c("$A_\\lambda^1$", "$A_\\lambda^2$", "$A_\\mu^1$", "$A_\\mu^2$")
reds = tibble(Parameter, Estimative = exp(c(mod_1$a_lambda, mod_1$a_mu)))
kable(reds, digits = 8, caption = "Red card rate parameters")
```

Table 3: Red card rate parameters

Parameter	Estimative
A_λ^1	0.00002508
A_λ^2	0.00007803
A_μ^1	0.00002736
A_μ^2	0.00008419

```
Parameter = c("$\\eta_1$", "$\\eta_2$", "$\\rho_1$", "$\\rho_2$", "$\\kappa$")
st = tibble(Parameter,
            Estimative = c(mod_1$eta, mod_1$rho, mod_1$kappa))
kable(st, digits = 4, caption = "Stoppage time parameters")
```

Table 4: Stoppage time parameters

Parameter	Estimative
η_1	2.9223
η_2	4.7355
ρ_1	1.8709
ρ_2	0.1346
κ	1.1870

```
mod_1$loglik
```

```
## [1] 1693.657
```