Parameters 2015-2019 model 4

Rates for the home and away goals

$$\ln \lambda_k(t) = \ln \alpha_i + \ln \beta_j + \ln \gamma_h + \mathbb{I}\{\text{half} = 2\} \ln \tau + \omega_{\text{goal}}(x(t) - y(t)) + \omega_{\text{goal}^2}(y(t) - x(t))^2 + \omega_{\text{player}}(y^*(t) - x^*(t))$$
$$\ln \mu_k(t) = \ln \alpha_j + \ln \beta_i + \mathbb{I}\{\text{half} = 2\} \ln \tau + \omega_{\text{goal}}(y(t) - x(t)) + \omega_{\text{goal}^2}(x(t) - y(t))^2 + \omega_{\text{player}}(x^*(t) - y^*(t))$$

- i: home team index;
- j: away team index;
- α: attack strength parameter;
- $1/\beta$: defense strength parameter;
- γ_h: home advantage parameter;
- τ: second half parameter;
- $x(t)$: the number of goals of the home team until minute t;
- $y(t)$: the number of goals of the away team until minute t;
- $x^*(t)$: the number of red cards of the home team until minute t;
- $y^*(t)$: the number of red cards of the away team until minute t;
- ω_{goal}: parameter that measure the impact of leading in the score in the rates;
- ω_{player}: parameter that measure the impact of having extra players on the field.

Rates for the home and away red cards

$$\lambda_k^*(t) = A_\lambda \left(t + 45^{\mathbb{I}\{\text{half} = 2\}} \right)$$
$$\mu_k^*(t) = A_\mu \left(t + 45^{\mathbb{I}\{\text{half} = 2\}} \right)$$

Stoppage time

The stoppage time for the first half, U^1, and the second half, U^2, are modeled as:

$$U^1 \sim \text{Poisson}(\eta_1 + \rho_1 r^1)$$
$$U^2 \sim \text{Poisson}(\eta_2 + \rho_2 r^2 + \kappa c)$$

- r^t is the amount of red cards received in half t until minute 45;
- $c = \begin{cases} 1, & \text{if } |x - y| \leq 1 \text{ at minute 45 of the second half;} \\ 0, & \text{otherwise.} \end{cases}$

Constraint

The constraint for identificability is

$$\sum_{i}^{n} \log(\alpha_i) = \sum_{i}^{n} \log(\beta_i).$$

```
options(knitr.kable.NA = "-")
options(scipen = 999)

library(dplyr)
library(knitr)

load("data/input.RData")
load("data/mod_4.RData")
```

```
alphas_betas = tibble(Team = times$Time,
                alpha = exp(mod_4$alpha),
                beta = exp(mod_4$beta))
kable(alphas_betas, digits = 4, caption = "Alphas and betas",
      col.names = c("Team", "$\\alpha$", "$\\beta$"))
```

Table 1: Alphas and betas

Team	α	β
América-MG	0.0576	0.1017
Athletico-PR	0.1022	0.0725
Atlético-GO	0.0826	0.1068
Atlético-MG	0.1257	0.0908
Avaí	0.0617	0.1090
Bahia	0.0977	0.0838
Botafogo	0.0865	0.0805
Ceará	0.0738	0.0745
Chapecoense	0.0853	0.0954
Corinthians	0.1094	0.0646
Coritiba	0.0830	0.0850
Cruzeiro	0.0884	0.0764
Csa	0.0513	0.1130
Figueirense	0.0726	0.0942
Flamengo	0.1302	0.0721
Fluminense	0.0906	0.0900
Fortaleza	0.1121	0.0929
Goiás	0.0931	0.1092
Grêmio	0.1153	0.0672
Internacional	0.0928	0.0690
Joinville	0.0575	0.0917
Palmeiras	0.1385	0.0701
Paraná	0.0394	0.1108
Ponte Preta	0.0930	0.0907
Santa Cruz	0.0981	0.1369
Santos	0.1204	0.0678
São Paulo	0.1028	0.0736
Sport	0.1020	0.0992

Team	α	β
Vasco da Gama	0.0808	0.0918
Vitória	0.0991	0.1119

```
Parameter = c("$\\gamma_h$", "$\\tau$", "$\\omega_{{\\text{goal}}}$",
              "$\\omega_{{\\text{player}}}$", "$\\omega_{{\\text{goal}}^2}$")
goals = tibble(Parameter,
               Estimative = c(exp(mod_4$gamma), exp(mod_4$tau), mod_4$omega))
kable(goals, digits = 4, caption = "Goal rate parameters")
```

Table 2: Goal rate parameters

Parameter	Estimative
γ_h	1.6021
τ	1.2476
ω_{goal}	-0.0951
ω_{player}	0.3108
ω_{goal^2}	-0.0015

```
Parameter = c("$A_\\lambda$", "$A_\\mu$")
reds = tibble(Parameter, Estimative = exp(mod_4$a))
kable(reds, digits = 8, caption = "Red card rate parameters")
```

Table 3: Red card rate parameters

Parameter	Estimative
A_λ	0.00001815
A_μ	0.00003237

```
Parameter = c("$\\eta_1$", "$\\eta_2$", "$\\rho_1$", "$\\rho_2$", "$\\kappa$")
st = tibble(Parameter,
            Estimative = c(mod_4$eta, mod_4$rho, mod_4$kappa))
kable(st, digits = 4, caption = "Stoppage time parameters")
```

Table 4: Stoppage time parameters

Parameter	Estimative
η_1	2.3404
η_2	3.5376
ρ_1	0.8663
ρ_2	0.2537
κ	0.9764

```
mod_4$loglik
```

```
## [1] -11379.88
```