

# Parameters Série A 2020 model 1

## Rates for the home and away goals

$$\begin{aligned}\ln \lambda_k(t) &= \ln \alpha_i + \ln \beta_j + \ln \gamma_h + \mathbb{I}\{\text{half} = 2\} \ln \tau + \omega_{\lambda x} x(t) + \omega_{\lambda y} y(t) + \omega_{\lambda(y^* - x^*)} x^*(t) \\ \ln \mu_k(t) &= \ln \alpha_j + \ln \beta_i + \mathbb{I}\{\text{half} = 2\} \ln \tau + \omega_{\mu x} x(t) + \omega_{\mu y} y(t) + \omega_{\mu(x^* - y^*)} y^*(t)\end{aligned}$$

- $i$ : home team index;
- $j$ : away team index;
- $\alpha$ : attack strength parameter;
- $1/\beta$ : defense strength parameter;
- $\gamma_h$ : home advantage parameter;
- $\tau$ : second half parameter;
- $x(t)$ : the number of goals of the home team until minute  $t$ ;
- $y(t)$ : the number of goals of the away team until minute  $t$ ;
- $x^*(t)$ : the number of red cards of the home team until minute  $t$ ;
- $y^*(t)$ : the number of red cards of the away team until minute  $t$ ;
- $\omega_{\lambda x}$ ,  $\omega_{\lambda y}$ ,  $\omega_{\mu x}$  and  $\omega_{\mu y}$ : parameters that measure the impact of the scored goals in the rates;
- $\omega_{\lambda(y^* - x^*)}$  and  $\omega_{\mu(x^* - y^*)}$ : parameters that measure the impact of having extra players on the field.

## Rates for the home and away red cards

$$\begin{aligned}\lambda_k^*(t) &= A_\lambda \left( t + 45^{\mathbb{I}\{\text{half} = 2\}} \right) \\ \mu_k^*(t) &= A_\mu \left( t + 45^{\mathbb{I}\{\text{half} = 2\}} \right)\end{aligned}$$

## Stoppage time

The stoppage time for the first half,  $U^1$ , and the second half,  $U^2$ , are modeled as:

$$\begin{aligned}U^1 &\sim \text{Poisson}(\eta_1 + \rho_1 r^1) \\ U^2 &\sim \text{Poisson}(\eta_2 + \rho_2 r^2 + \kappa c)\end{aligned}$$

- $r^t$  is the amount of red cards received in half  $t$  until minute 45;
- $c = \begin{cases} 1, & \text{if } |x - y| \leq 1 \text{ at minute 45 of the second half;} \\ 0, & \text{otherwise.} \end{cases}$

## Constraint

The constraint for identificability is

$$\sum_i^n \log(\alpha_i) = \sum_i^n \log(\beta_i).$$

```
options(knitr.kable.NA = "-")
options(scipen = 999)
```

```
library(dplyr)
library(knitr)
```

```
load("data/input.RData")
load("data/mod_1.RData")
```

```
alphas_betas = tibble(Team = times$Time,
                      alpha = exp(mod_1$alpha),
                      beta = exp(mod_1$beta))
kable(alphas_betas, digits = 4, caption = "Alphas and betas",
      col.names = c("Team", "$\\alpha$", "$\\beta$"))
```

Table 1: Alphas and betas

Team	$\alpha$	$\beta$
Athletico-PR	0.0747	0.0725
Atlético-GO	0.0817	0.0890
Atlético-MG	0.1362	0.0887
Bahia	0.0943	0.1259
Botafogo	0.0621	0.1269
Ceará	0.1083	0.1035
Corinthians	0.0928	0.0865
Coritiba	0.0604	0.1125
Flamengo	0.1423	0.0967
Fluminense	0.1120	0.0843
Fortaleza	0.0684	0.0885
Goiás	0.0788	0.1303
Grêmio	0.1115	0.0780
Internacional	0.1307	0.0656
Palmeiras	0.1085	0.0718
Red Bull Bragantino	0.1065	0.0770
Santos	0.1072	0.1011
São Paulo	0.1189	0.0842
Sport	0.0599	0.1024
Vasco da Gama	0.0751	0.1150

```
Parameter = c("$\\gamma_h$", "$\\tau$", "$\\omega_{\\lambda x}$",
              "$\\omega_{\\lambda y}$", "$\\omega_{\\mu x}$",
              "$\\omega_{\\mu y}$", "$\\omega_{\\lambda (y^*-x^*)}$",
              "$\\omega_{\\mu (x^*-y^*)}$")
goals = tibble(Parameter,
               Estimative = c(exp(mod_1$gamma), exp(mod_1$tau), mod_1$omega))
```

```
kable(goals, digits = 4, caption = "Goal rate parameters")
```

Table 2: Goal rate parameters

Parameter	Estimative
$\gamma_h$	1.6215
$\tau$	1.1780
$\omega_{\lambda x}$	-0.2221
$\omega_{\lambda y}$	0.0538
$\omega_{\mu x}$	0.1401
$\omega_{\mu y}$	-0.0422
$\omega_{\lambda(y^*-x^*)}$	0.3615
$\omega_{\mu(x^*-y^*)}$	0.4370

```
Parameter = c("$A_\\lambda$", "$A_\\mu$")
reds = tibble(Parameter, Estimative = exp(mod_1$a))
kable(reds, digits = 8, caption = "Red card rate parameters")
```

Table 3: Red card rate parameters

Parameter	Estimative
$A_\lambda$	0.00002736
$A_\mu$	0.00002960

```
Parameter = c("$\\eta_1$", "$\\eta_2$", "$\\rho_1$", "$\\rho_2$", "$\\kappa$")
st = tibble(Parameter,
             Estimative = c(mod_1$eta, mod_1$rho, mod_1$kappa))
kable(st, digits = 4, caption = "Stoppage time parameters")
```

Table 4: Stoppage time parameters

Parameter	Estimative
$\eta_1$	2.9222
$\eta_2$	4.7355
$\rho_1$	1.8709
$\rho_2$	0.1346
$\kappa$	1.1871

```
mod_1$loglik
```

```
## [1] 1724.679
```