Parameters 2019 model 4

Rates for the home and away goals

$$\ln \lambda_k(t) = \ln \alpha_i + \ln \beta_j + \ln \gamma_h + \mathbb{I}\{\text{half} = 2\} \ln \tau + \omega_{\text{goal}}(x(t) - y(t)) + \omega_{\text{goal}^2}(y(t) - x(t))^2 + \omega_{\text{player}}(y^*(t) - x^*(t))$$
$$\ln \mu_k(t) = \ln \alpha_j + \ln \beta_i + \mathbb{I}\{\text{half} = 2\} \ln \tau + \omega_{\text{goal}}(y(t) - x(t)) + \omega_{\text{goal}^2}(x(t) - y(t))^2 + \omega_{\text{player}}(x^*(t) - y^*(t))$$

- i: home team index;
- j: away team index;
- α: attack strength parameter;
- $1/\beta$: defense strength parameter;
- γ_h: home advantage parameter;
- τ: second half parameter;
- $x(t)$: the number of goals of the home team until minute t;
- $y(t)$: the number of goals of the away team until minute t;
- $x^*(t)$: the number of red cards of the home team until minute t;
- $y^*(t)$: the number of red cards of the away team until minute t;
- ω_{goal}: parameter that measure the impact of leading in the score in the rates;
- ω_{player}: parameter that measure the impact of having extra players on the field.

Rates for the home and away red cards

$$\lambda_k^*(t) = A_\lambda \left(t + 45^{\mathbb{I}\{\text{half} = 2\}} \right)$$
$$\mu_k^*(t) = A_\mu \left(t + 45^{\mathbb{I}\{\text{half} = 2\}} \right)$$

Stoppage time

The stoppage time for the first half, U^1, and the second half, U^2, are modeled as:

$$U^1 \sim \text{Poisson}(\eta_1 + \rho_1 r^1)$$
$$U^2 \sim \text{Poisson}(\eta_2 + \rho_2 r^2 + \kappa c)$$

- r^t is the amount of red cards received in half t until minute 45;
- $c = \begin{cases} 1, & \text{if } |x - y| \leq 1 \text{ at minute 45 of the second half;} \\ 0, & \text{otherwise.} \end{cases}$

Constraint

The constraint for identificability is

$$\sum_{i}^{n} \log(\alpha_i) = \sum_{i}^{n} \log(\beta_i).$$

```
options(knitr.kable.NA = "-")
options(scipen = 999)

library(dplyr)
library(knitr)

load("data/input.RData")
load("data/mod_4.RData")
```

```
alphas_betas = tibble(Team = times$Time,
                alpha = exp(mod_4$alpha),
                beta = exp(mod_4$beta))
kable(alphas_betas, digits = 4, caption = "Alphas and betas",
      col.names = c("Team", "$\\alpha$", "$\\beta$"))
```

Table 1: Alphas and betas

Team	α	β
Athletico-PR	0.1048	0.0636
Atlético-MG	0.0944	0.0954
Avaí	0.0356	0.1302
Bahia	0.0909	0.0863
Botafogo	0.0644	0.0918
Ceará	0.0729	0.0833
Chapecoense	0.0637	0.1046
Corinthians	0.0882	0.0683
Cruzeiro	0.0548	0.0954
Csa	0.0466	0.1241
Flamengo	0.1846	0.0712
Fluminense	0.0794	0.0900
Fortaleza	0.1066	0.0996
Goiás	0.0931	0.1323
Grêmio	0.1365	0.0788
Internacional	0.0903	0.0744
Palmeiras	0.1312	0.0627
Santos	0.1344	0.0612
São Paulo	0.0806	0.0576
Vasco da Gama	0.0794	0.0905

```
Parameter = c("$\\gamma_h$", "$\\tau$", "$\\omega_{{\\text{goal}}}$",
              "$\\omega_{{\\text{player}}}$", "$\\omega_{{\\text{goal}}^2}$")
goals = tibble(Parameter,
               Estimative = c(exp(mod_4$gamma), exp(mod_4$tau), mod_4$omega))
kable(goals, digits = 4, caption = "Goal rate parameters")
```

Table 2: Goal rate parameters

Parameter	Estimative
γ_h	1.5980
τ	1.2088
ω_{goal}	-0.1550
ω_{player}	0.4537
ω_{goal^2}	0.0158

```
Parameter = c("$A_\\lambda$", "$A_\\mu$")
reds = tibble(Parameter, Estimative = exp(mod_4$a))
kable(reds, digits = 8, caption = "Red card rate parameters")
```

Table 3: Red card rate parameters

Parameter	Estimative
A_λ	0.00001921
A_μ	0.00003334

```
Parameter = c("$\\eta_1$", "$\\eta_2$", "$\\rho_1$", "$\\rho_2$", "$\\kappa$")
st = tibble(Parameter,
            Estimative = c(mod_4$eta, mod_4$rho, mod_4$kappa))
kable(st, digits = 4, caption = "Stoppage time parameters")
```

Table 4: Stoppage time parameters

Parameter	Estimative
η_1	3.0211
η_2	3.9504
ρ_1	1.1416
ρ_2	0.1989
κ	1.4012

```
mod_4$loglik
```

```
## [1] -1523.916
```