Parameters 2015-2019 model 1

Rates for the home and away goals

$$\ln \lambda_k(t) = \ln \alpha_i + \ln \beta_j + \ln \gamma_h + \mathbb{I}\{\text{half} = 2\} \ln \tau + \omega_{\lambda x} x(t) + \omega_{\lambda y} y(t) + \omega_{\lambda x^*} x^*(t) + \omega_{\lambda y^*} y^*(t)$$
$$\ln \mu_k(t) = \ln \alpha_j + \ln \beta_i + \mathbb{I}\{\text{half} = 2\} \ln \tau + \omega_{\mu x} x(t) + \omega_{\mu y} y(t) + \omega_{\mu x^*} x^*(t) + \omega_{\mu y^*} y^*(t)$$

- i: home team index;
- j: away team index;
- α: attack strength parameter;
- $1/\beta$: defense strength parameter;
- γ_h: home advantage parameter;
- τ: second half parameter;
- $x(t)$: the number of goals of the home team until minute t;
- $y(t)$: the number of goals of the away team until minute t;
- $x^*(t)$: the number of red cards of the home team until minute t;
- $y^*(t)$: the number of red cards of the away team until minute t;
- $\omega_{\lambda x}$, $\omega_{\lambda y}$, $\omega_{\mu x}$ and $\omega_{\mu y}$: parameters that measure the impact of the scored goals in the rates;
- $\omega_{\lambda x^*}$, $\omega_{\lambda y^*}$, $\omega_{\mu x^*}$ and $\omega_{\mu y^*}$: parameters that measure the impact of red cards in the rates.

Rates for the home and away red cards

$$\lambda_k^*(t) = A_\lambda \left(t + 45^{\mathbb{I}\{\text{half} = 2\}} \right)$$
$$\mu_k^*(t) = A_\mu \left(t + 45^{\mathbb{I}\{\text{half} = 2\}} \right)$$

Stoppage time

The stoppage time for the first half, U^1, and the second half, U^2, are modeled as:

$$U^1 \sim \text{Poisson}(\eta_1 + \rho_1 r^1)$$
$$U^2 \sim \text{Poisson}(\eta_2 + \rho_2 r^2 + \kappa c)$$

- r^t is the amount of red cards received in half t until minute 45;
- $c = \begin{cases} 1, & \text{if } |x - y| \leq 1 \text{ at minute 45 of the second half;} \\ 0, & \text{otherwise.} \end{cases}$

Constraint

The constraint for identificability is

$$\sum_{i}^{n} \log(\alpha_i) = \sum_{i}^{n} \log(\beta_i).$$

```
options(knitr.kable.NA = "-")
options(scipen = 999)

library(dplyr)
library(knitr)

load("data/input.RData")
load("data/mod_1.RData")
```

```
alphas_betas = tibble(Team = times$Time,
                alpha = exp(mod_1$alpha),
                beta = exp(mod_1$beta))
kable(alphas_betas, digits = 4, caption = "Alphas and betas",
      col.names = c("Team", "$\\alpha$", "$\\beta$"))
```

Table 1: Alphas and betas

Team	α	β
América-MG	0.0575	0.1015
Athletico-PR	0.1019	0.0724
Atlético-GO	0.0818	0.1056
Atlético-MG	0.1249	0.0902
Avaí	0.0615	0.1087
Bahia	0.0980	0.0840
Botafogo	0.0862	0.0804
Ceará	0.0737	0.0745
Chapecoense	0.0849	0.0952
Corinthians	0.1091	0.0644
Coritiba	0.0829	0.0847
Cruzeiro	0.0882	0.0764
Csa	0.0514	0.1131
Figueirense	0.0727	0.0943
Flamengo	0.1295	0.0719
Fluminense	0.0903	0.0897
Fortaleza	0.1117	0.0920
Goiás	0.0929	0.1090
Grêmio	0.1149	0.0669
Internacional	0.0930	0.0692
Joinville	0.0575	0.0919
Palmeiras	0.1374	0.0694
Paraná	0.0398	0.1112
Ponte Preta	0.0929	0.0907
Santa Cruz	0.0975	0.1359
Santos	0.1199	0.0676
São Paulo	0.1027	0.0733
Sport	0.1016	0.0988

Team	α	β
Vasco da Gama	0.0808	0.0918
Vitória	0.0985	0.1112

```
Parameter = c("$\\gamma_h$", "$\\tau$", "$\\omega_{\\lambda x}$",
              "$\\omega_{\\lambda y}$", "$\\omega_{\\mu x}$",
              "$\\omega_{\\mu y}$", "$\\omega_{\\lambda x^*}$",
              "$\\omega_{\\lambda y^*}$", "$\\omega_{\\mu x^*}$",
              "$\\omega_{\\mu y^*}$")
goals = tibble(Parameter,
               Estimative = c(exp(mod_1$gamma), exp(mod_1$tau), mod_1$omega))
kable(goals, digits = 4, caption = "Goal rate parameters")
```

Table 2: Goal rate parameters

Parameter	Estimative
γ_h	1.5933
τ	1.2259
$\omega_{\lambda x}$	-0.0793
$\omega_{\lambda y}$	0.1274
$\omega_{\mu x}$	0.1031
$\omega_{\mu y}$	-0.0702
$\omega_{\lambda x^*}$	-0.4782
$\omega_{\lambda y^*}$	0.2356
$\omega_{\mu x^*}$	0.2964
$\omega_{\mu y^*}$	-0.3965

```
Parameter = c("$A_\\lambda$", "$A_\\mu$")
reds = tibble(Parameter, Estimative = exp(mod_1$a))
kable(reds, digits = 8, caption = "Red card rate parameters")
```

Table 3: Red card rate parameters

Parameter	Estimative
A_λ	0.00001815
A_μ	0.00003237

```
Parameter = c("$\\eta_1$", "$\\eta_2$", "$\\rho_1$", "$\\rho_2$", "$\\kappa$")
st = tibble(Parameter,
            Estimative = c(mod_1$eta, mod_1$rho, mod_1$kappa))
kable(st, digits = 4, caption = "Stoppage time parameters")
```

Table 4: Stoppage time parameters

Parameter	Estimative
η_1	2.3404
η_2	3.5377

Parameter	Estimative
ρ_1	0.8663
ρ_2	0.2537
κ	0.9764

```
mod_1$loglik
```

```
## [1] -11377.56
```