Parameters 2015-2018 model 2

Rates for the home and away goals

$$\ln \lambda_k(t) = \ln \alpha_i + \ln \beta_j + \ln \gamma_h + \mathbb{I}\{\text{half} = 2\}\ln \tau + \omega_{\lambda(x-y)}(x(t) - y(t)) + \omega_{\lambda(y^*-x^*)}(y^*(t) - x^*(t))$$

$$\ln \mu_k(t) = \ln \alpha_j + \ln \beta_i + \mathbb{I}\{\text{half} = 2\}\ln \tau + \omega_{\mu(y-x)}(y(t) - x(t)) + \omega_{\mu(x^*-y^*)}(x^*(t) - y^*(t))$$

- i: home team index;

- j: away team index;

- α: attack strength parameter;

- $1/\beta$: defense strength parameter;

- γ_h: home advantage parameter;

- τ: second half parameter;

- $x(t)$: the number of goals of the home team until minute t;

- $y(t)$: the number of goals of the away team until minute t;

- $x^*(t)$: the number of red cards of the home team until minute t;

- $y^*(t)$: the number of red cards of the away team until minute t;

- $\omega_{\lambda(x-y)}$, $\omega_{\mu(y-x)}$: parameters that measure the impact of leading in the score in the rates;

- $\omega_{\lambda(y^*-x^*)}$ and $\omega_{\mu(x^*-y^*)}$: parameters that measure the impact of having extra players on the field.

Rates for the home and away red cards

$$\lambda_k^*(t) = A_\lambda \left(t + 45^{\mathbb{I}\{\text{half} = 2\}}\right)$$

$$\mu_k^*(t) = A_\mu \left(t + 45^{\mathbb{I}\{\text{half} = 2\}}\right)$$

Stoppage time

The stoppage time for the first half, U^1, and the second half, U^2, are modeled as:

$$U^1 \sim \text{Poisson}(\eta_1 + \rho_1 r^1)$$

$$U^2 \sim \text{Poisson}(\eta_2 + \rho_2 r^2 + \kappa c)$$

- r^t is the amount of red cards received in half t until minute 45;

- $c = \begin{cases} 1, & \text{if } |x - y| \leq 1 \text{ at minute 45 of the second half;} \\ 0, & \text{otherwise.} \end{cases}$

Constraint

The constraint for identifiability is

$$\sum_i^n \log(\alpha_i) = \sum_i^n \log(\beta_i).$$

```
options(knitr.kable.NA = "-")
options(scipen = 999)

library(dplyr)
library(knitr)

load("data/input.RData")
load("data/mod_2.RData")
```

```
alphas_betas = tibble(Team = times$Time,
                alpha = exp(mod_2$alpha),
                beta = exp(mod_2$beta))
kable(alphas_betas, digits = 4, caption = "Alphas and betas",
      col.names = c("Team", "$\\alpha$", "$\\beta$"))
```

Table 1: Alphas and betas

Team	α	β
América-MG	0.0566	0.1029
Athletico-PR	0.0984	0.0768
Atlético-GO	0.0806	0.1065
Atlético-MG	0.1299	0.0917
Avaí	0.0718	0.1051
Bahia	0.0967	0.0860
Botafogo	0.0908	0.0799
Ceará	0.0683	0.0725
Chapecoense	0.0882	0.0959
Corinthians	0.1115	0.0656
Coritiba	0.0815	0.0856
Cruzeiro	0.0942	0.0745
Figueirense	0.0716	0.0958
Flamengo	0.1123	0.0736
Fluminense	0.0906	0.0925
Goiás	0.0846	0.0964
Grêmio	0.1063	0.0660
Internacional	0.0902	0.0688
Joinville	0.0568	0.0932
Palmeiras	0.1361	0.0733
Paraná	0.0386	0.1119
Ponte Preta	0.0916	0.0919
Santa Cruz	0.0960	0.1375
Santos	0.1137	0.0705
São Paulo	0.1054	0.0795
Sport	0.1003	0.1005
Vasco da Gama	0.0782	0.0952
Vitória	0.0974	0.1125

```r
Parameter = c("$\\gamma_h$", "$\\tau$", "$\\omega_{\\lambda (x-y)}$",
              "$\\omega_{\\mu (y-x)}$", "$\\omega_{\\lambda (y^*-x^*)}$",
              "$\\omega_{\\mu_(x^*-y^*)}$")
goals = tibble(Parameter,
               Estimative = c(exp(mod_2$gamma), exp(mod_2$tau), mod_2$omega))
kable(goals, digits = 4, caption = "Goal rate parameters")
```

Table 2: Goal rate parameters

Parameter	Estimative
γ_h	1.6126
τ	1.2584
$\omega_{\lambda(x-y)}$	-0.1068
$\omega_{\mu(y-x)}$	-0.0674
$\omega_{\lambda(y^*-x^*)}$	0.2156
$\omega_{\mu(x^*-y^*)}$	0.3233

```r
Parameter = c("$A_\\lambda$", "$A_\\mu$")
reds = tibble(Parameter, Estimative = exp(mod_2$a))
kable(reds, digits = 8, caption = "Red card rate parameters")
```

Table 3: Red card rate parameters

Parameter	Estimative
A_λ	0.00001788
A_μ	0.00003212

```r
Parameter = c("$\\eta_1$", "$\\eta_2$", "$\\rho_1$", "$\\rho_2$", "$\\kappa$")
st = tibble(Parameter,
            Estimative = c(mod_2$eta, mod_2$rho, mod_2$kappa))
kable(st, digits = 4, caption = "Stoppage time parameters")
```

Table 4: Stoppage time parameters

Parameter	Estimative
η_1	2.1709
η_2	3.4403
ρ_1	0.7630
ρ_2	0.2504
κ	0.8641

```r
mod_2$loglik
```

```
## [1] -9749.841
```