Parameters 2015-2018 model 4

Rates for the home and away goals

$$\ln \lambda_k(t) = \ln \alpha_i + \ln \beta_j + \ln \gamma_h + \mathbb{I}\{\text{half} = 2\} \ln \tau + \omega_{\text{goal}}(x(t) - y(t)) + \omega_{\text{player}}(y^*(t) - x^*(t)) + \omega_{\text{player}^2}(y^*(t) - x^*(t))^2$$
$$\ln \mu_k(t) = \ln \alpha_j + \ln \beta_i + \mathbb{I}\{\text{half} = 2\} \ln \tau + \omega_{\text{goal}}(y(t) - x(t)) + \omega_{\text{player}}(x^*(t) - y^*(t)) + \omega_{\text{player}^2}(x^*(t) - y^*(t))^2$$

- i: home team index;
- j: away team index;
- α: attack strength parameter;
- $1/\beta$: defense strength parameter;
- γ_h: home advantage parameter;
- τ: second half parameter;
- $x(t)$: the number of goals of the home team until minute t;
- $y(t)$: the number of goals of the away team until minute t;
- $x^*(t)$: the number of red cards of the home team until minute t;
- $y^*(t)$: the number of red cards of the away team until minute t;
- ω_{goal}: parameter that measure the impact of leading in the score in the rates;
- ω_{player}: parameter that measure the impact of having extra players on the field.

Rates for the home and away red cards

$$\lambda_k^*(t) = A_\lambda \left(t + 45^{\mathbb{I}\{\text{half} = 2\}} \right)$$
$$\mu_k^*(t) = A_\mu \left(t + 45^{\mathbb{I}\{\text{half} = 2\}} \right)$$

Stoppage time

The stoppage time for the first half, U^1, and the second half, U^2, are modeled as:

$$U^1 \sim \text{Poisson}(\eta_1 + \rho_1 r^1)$$
$$U^2 \sim \text{Poisson}(\eta_2 + \rho_2 r^2 + \kappa c)$$

- r^t is the amount of red cards received in half t until minute 45;
- $c = \begin{cases} 1, & \text{if } |x - y| \leq 1 \text{ at minute 45 of the second half;} \\ 0, & \text{otherwise.} \end{cases}$

Constraint

The constraint for identificability is

$$\sum_{i}^{n} \log(\alpha_i) = \sum_{i}^{n} \log(\beta_i).$$

```
options(knitr.kable.NA = "-")
options(scipen = 999)

library(dplyr)
library(knitr)

load("data/input.RData")
load("data/mod_4.RData")
```

```
alphas_betas = tibble(Team = times$Time,
                alpha = exp(mod_4$alpha),
                beta = exp(mod_4$beta))
kable(alphas_betas, digits = 4, caption = "Alphas and betas",
      col.names = c("Team", "$\\alpha$", "$\\beta$"))
```

Table 1: Alphas and betas

Team	α	β
América-MG	0.0567	0.1031
Athletico-PR	0.0983	0.0767
Atlético-GO	0.0812	0.1073
Atlético-MG	0.1302	0.0919
Avaí	0.0720	0.1053
Bahia	0.0966	0.0860
Botafogo	0.0908	0.0798
Ceará	0.0684	0.0726
Chapecoense	0.0883	0.0960
Corinthians	0.1115	0.0657
Coritiba	0.0816	0.0858
Cruzeiro	0.0944	0.0747
Figueirense	0.0714	0.0955
Flamengo	0.1124	0.0736
Fluminense	0.0906	0.0925
Goiás	0.0845	0.0964
Grêmio	0.1065	0.0662
Internacional	0.0902	0.0689
Joinville	0.0569	0.0932
Palmeiras	0.1365	0.0737
Paraná	0.0387	0.1125
Ponte Preta	0.0916	0.0919
Santa Cruz	0.0968	0.1384
Santos	0.1136	0.0704
São Paulo	0.1055	0.0796
Sport	0.1004	0.1006
Vasco da Gama	0.0785	0.0956
Vitória	0.0976	0.1131

```
Parameter = c("$\\gamma_h$", "$\\tau$", "$\\omega_{{\\text{goal}}}$",
              "$\\omega_{{\\text{player}}}$", "$\\omega_{{\\text{player}}^2}$")
goals = tibble(Parameter,
               Estimative = c(exp(mod_4$gamma), exp(mod_4$tau), mod_4$omega))
kable(goals, digits = 4, caption = "Goal rate parameters")
```

Table 2: Goal rate parameters

Parameter	Estimative
γ_h	1.6142
τ	1.2655
ω_{goal}	-0.0907
ω_{player}	0.2559
ω_{player^2}	-0.0104

```
Parameter = c("$A_\\lambda$", "$A_\\mu$")
reds = tibble(Parameter, Estimative = exp(mod_4$a))
kable(reds, digits = 8, caption = "Red card rate parameters")
```

Table 3: Red card rate parameters

Parameter	Estimative
A_λ	0.00001788
A_μ	0.00003212

```
Parameter = c("$\\eta_1$", "$\\eta_2$", "$\\rho_1$", "$\\rho_2$", "$\\kappa$")
st = tibble(Parameter,
            Estimative = c(mod_4$eta, mod_4$rho, mod_4$kappa))
kable(st, digits = 4, caption = "Stoppage time parameters")
```

Table 4: Stoppage time parameters

Parameter	Estimative
η_1	2.1709
η_2	3.4403
ρ_1	0.7630
ρ_2	0.2504
κ	0.8641

```
mod_4$loglik
```

```
## [1] -9750.144
```