Parameters 2019 model 2

Rates for the home and away goals

$$\ln \lambda_k(t) = \ln \alpha_i + \ln \beta_j + \ln \gamma_h + \mathbb{I}\{\text{half} = 2\} \ln \tau + \omega_{\lambda(x-y)}(x(t) - y(t)) + \omega_{\lambda(y^*-x^*)}(y^*(t) - x^*(t))$$
$$\ln \mu_k(t) = \ln \alpha_j + \ln \beta_i + \mathbb{I}\{\text{half} = 2\} \ln \tau + \omega_{\mu(y-x)}(y(t) - x(t)) + \omega_{\mu(x^*-y^*)}(x^*(t) - y^*(t))$$

- i: home team index;
- j: away team index;
- α: attack strength parameter;
- $1/\beta$: defense strength parameter;
- γ_h: home advantage parameter;
- τ: second half parameter;
- $x(t)$: the number of goals of the home team until minute t;
- $y(t)$: the number of goals of the away team until minute t;
- $x^*(t)$: the number of red cards of the home team until minute t;
- $y^*(t)$: the number of red cards of the away team until minute t;
- $\omega_{\lambda(x-y)}$, $\omega_{\mu(y-x)}$: parameters that measure the impact of leading in the score in the rates;
- $\omega_{\lambda(y^*-x^*)}$ and $\omega_{\mu(x^*-y^*)}$: parameters that measure the impact of having extra players on the field.

Rates for the home and away red cards

$$\lambda_k^*(t) = A_\lambda\left(t + 45^{\mathbb{I}\{\text{half} = 2\}}\right)$$
$$\mu_k^*(t) = A_\mu\left(t + 45^{\mathbb{I}\{\text{half} = 2\}}\right)$$

Stoppage time

The stoppage time for the first half, U^1, and the second half, U^2, are modeled as:

$$U^1 \sim \text{Poisson}(\eta_1 + \rho_1 r^1)$$
$$U^2 \sim \text{Poisson}(\eta_2 + \rho_2 r^2 + \kappa c)$$

- r^t is the amount of red cards received in half t until minute 45;
- $c = \begin{cases} 1, & \text{if } |x - y| \leq 1 \text{ at minute 45 of the second half;} \\ 0, & \text{otherwise.} \end{cases}$

Constraint

The constraint for identificability is

$$\sum_{i}^{n} \log(\alpha_i) = \sum_{i}^{n} \log(\beta_i).$$

```
options(knitr.kable.NA = "-")
options(scipen = 999)

library(dplyr)
library(knitr)

load("data/input.RData")
load("data/mod_2.RData")
```

```
alphas_betas = tibble(Team = times$Time,
                alpha = exp(mod_2$alpha),
                beta = exp(mod_2$beta))
kable(alphas_betas, digits = 4, caption = "Alphas and betas",
      col.names = c("Team", "$\\alpha$", "$\\beta$"))
```

Table 1: Alphas and betas

Team	α	β
Athletico-PR	0.1045	0.0634
Atlético-MG	0.0946	0.0953
Avaí	0.0361	0.1317
Bahia	0.0908	0.0863
Botafogo	0.0638	0.0910
Ceará	0.0724	0.0828
Chapecoense	0.0642	0.1052
Corinthians	0.0878	0.0680
Cruzeiro	0.0550	0.0953
Csa	0.0467	0.1241
Flamengo	0.1858	0.0724
Fluminense	0.0788	0.0895
Fortaleza	0.1059	0.0990
Goiás	0.0936	0.1333
Grêmio	0.1372	0.0793
Internacional	0.0908	0.0738
Palmeiras	0.1313	0.0628
Santos	0.1344	0.0615
São Paulo	0.0807	0.0578
Vasco da Gama	0.0796	0.0909

```
Parameter = c("$\\gamma_h$", "$\\tau$", "$\\omega_{\\lambda (x-y)}$",
              "$\\omega_{\\mu (y-x)}$", "$\\omega_{\\lambda (y^*-x^*)}$",
              "$\\omega_{\\mu_(x^*-y^*)}$")
goals = tibble(Parameter,
               Estimative = c(exp(mod_2$gamma), exp(mod_2$tau), mod_2$omega))
kable(goals, digits = 4, caption = "Goal rate parameters")
```

Table 2: Goal rate parameters

Parameter	Estimative
γ_h	1.5980
τ	1.2181
$\omega_{\lambda(x-y)}$	-0.1143
$\omega_{\mu(y-x)}$	-0.2057
$\omega_{\lambda(y^*-x^*)}$	0.4899
$\omega_{\mu_(x^*-y^*)}$	0.3714

```
Parameter = c("$A_\\lambda$", "$A_\\mu$")
reds = tibble(Parameter, Estimative = exp(mod_2$a))
kable(reds, digits = 8, caption = "Red card rate parameters")
```

Table 3: Red card rate parameters

Parameter	Estimative
A_λ	0.00001921
A_μ	0.00003334

```
Parameter = c("$\\eta_1$", "$\\eta_2$", "$\\rho_1$", "$\\rho_2$", "$\\kappa$")
st = tibble(Parameter,
            Estimative = c(mod_2$eta, mod_2$rho, mod_2$kappa))
kable(st, digits = 4, caption = "Stoppage time parameters")
```

Table 4: Stoppage time parameters

Parameter	Estimative
η_1	3.0211
η_2	3.9504
ρ_1	1.1416
ρ_2	0.1989
κ	1.4012

```
mod_2$loglik
```

```
## [1] -1523.425
```