Parameters 2015-2020 model 10

Rates for the home and away goals

$$\ln \lambda_k(t) = \ln \alpha_i + \ln \beta_j + \ln \gamma_h + \mathbb{I}\{\text{half} = 2\} \ln \tau + \ln \lambda_{xy} + \omega_{\text{player}}(y^*(t) - x^*(t)) + \omega_{\text{value}}(v_i - v_j)$$
$$\ln \mu_k(t) = \ln \alpha_j + \ln \beta_i + \mathbb{I}\{\text{half} = 2\} \ln \tau + \ln \mu_{xy} + \omega_{\text{player}}(x^*(t) - y^*(t)) + \omega_{\text{value}}(v_j - v_i)$$

- i: home team index;
- j: away team index;
- α: attack strength parameter;
- $1/\beta$: defense strength parameter;
- γ_h: home advantage parameter;
- τ: second half parameter;
- $x(t)$: the number of goals of the home team until minute t;
- $y(t)$: the number of goals of the away team until minute t;
- $x^*(t)$: the number of red cards of the home team until minute t;
- $y^*(t)$: the number of red cards of the away team until minute t;
- v_i: log of the value of the home team's starting roster in euros;
- v_j: log of the value of the away team's starting roster in euros;
- $\lambda_{xy} = \begin{cases} 1, \text{ for } x = y; \\ \omega_{\text{ahead}}, \text{ for } x - y \geq 1; \\ \omega_{\text{behind}}, \text{ for } x - y \leq -1; \end{cases}$
- $\mu_{xy} = \begin{cases} 1, \text{ for } x = y; \\ \lambda_{\text{ahead}}, \text{ for } y - x \geq 1; \\ \lambda_{\text{behind}}, \text{ for } y - x \leq -1; \end{cases}$
- ω_{player}: parameter that measure the impact of having extra players on the field;
- ω_{value}: parameter that measure the impact of the difference in value of the starting rosters.

Rates for the home and away red cards

$$\lambda_k^*(t) = A_\lambda \left(t + 45^{\mathbb{I}\{\text{half} = 2\}} \right)$$
$$\mu_k^*(t) = A_\mu \left(t + 45^{\mathbb{I}\{\text{half} = 2\}} \right)$$

Stoppage time

The stoppage time for the first half, U^1, and the second half, U^2, are modeled as:

$$U^1 \sim \text{Poisson}(\eta_1 + \rho_1 r^1)$$

$$U^2 \sim \text{Poisson}(\eta_2 + \rho_2 r^2 + \kappa c)$$

- r^t is the amount of red cards received in half t until minute 45;
- $c = \begin{cases} 1, & \text{if } |x - y| \leq 1 \text{ at minute 45 of the second half;} \\ 0, & \text{otherwise.} \end{cases}$

Constraint

The constraint for identificability is

$$\sum_i^n \log(\alpha_i) = \sum_i^n \log(\beta_i).$$

```
options(knitr.kable.NA = "-")
options(scipen = 999)

library(dplyr)
library(knitr)

load("data/input.RData")
load("data/mod_10.RData")
```

```
alphas_betas = tibble(Team = times$Time,
                alpha = exp(mod_10$alpha),
                beta = exp(mod_10$beta))
kable(alphas_betas, digits = 4, caption = "Alphas and betas",
      col.names = c("Team", "$\\alpha$", "$\\beta$"))
```

Table 1: Alphas and betas

Team	α	β
América-MG	0.0635	0.0964
Athletico-PR	0.0972	0.0755
Atlético-GO	0.0951	0.0877
Atlético-MG	0.1165	0.1025
Avaí	0.0709	0.0995
Bahia	0.1026	0.0906
Botafogo	0.0871	0.0865
Ceará	0.0977	0.0755
Chapecoense	0.0893	0.0957
Corinthians	0.0972	0.0776
Coritiba	0.0850	0.0855
Cruzeiro	0.0813	0.0870
Csa	0.0577	0.1035
Figueirense	0.0761	0.0948

Team	α	β
Flamengo	0.1154	0.0905
Fluminense	0.0914	0.0953
Fortaleza	0.1034	0.0799
Goiás	0.1004	0.1047
Grêmio	0.1034	0.0786
Internacional	0.0943	0.0755
Joinville	0.0654	0.0852
Palmeiras	0.1183	0.0822
Paraná	0.0460	0.0986
Ponte Preta	0.0969	0.0919
Red Bull Bragantino	0.1093	0.0738
Santa Cruz	0.1095	0.1302
Santos	0.1079	0.0818
São Paulo	0.0955	0.0865
Sport	0.0956	0.1008
Vasco da Gama	0.0834	0.0950
Vitória	0.1029	0.1131

```
Parameter = c("$\\gamma_h$", "$\\tau$",
              "$\\omega_{{\\text{ahead}}}$",
              "$\\omega_{{\\text{behind}}}$",
              "$\\omega_{{\\text{player}}}$",
              "$\\omega_{\\text{value}}$")
goals = tibble(Parameter,
               Estimative = c(exp(mod_10$gamma),
                              exp(mod_10$tau),
                              exp(mod_10$omega_xy),
                              mod_10$omega))
kable(goals, digits = 4, caption = "Goal rate parameters")
```

Table 2: Goal rate parameters

Parameter	Estimative
γ_h	1.5310
τ	1.2324
ω_{ahead}	0.8333
ω_{behind}	1.1111
ω_{player}	0.3226
ω_{value}	0.1531

```
Parameter = c("$A_\\lambda$", "$A_\\mu$")
reds = tibble(Parameter, Estimative = exp(mod_10$a))
kable(reds, digits = 8, caption = "Red card rate parameters")
```

Table 3: Red card rate parameters

Parameter	Estimative
A_λ	0.00001973

Parameter	Estimative
A_μ	0.00003190

```
Parameter = c("$\\eta_1$", "$\\eta_2$", "$\\rho_1$", "$\\rho_2$", "$\\kappa$")
st = tibble(Parameter,
            Estimative = c(mod_10$eta, mod_10$rho, mod_10$kappa))
kable(st, digits = 4, caption = "Stoppage time parameters")
```

Table 4: Stoppage time parameters

Parameter	Estimative
η_1	2.4359
η_2	3.7394
ρ_1	1.1400
ρ_2	0.2267
κ	1.0101

```
mod_10$loglik
```

```
## [1] -25853.31
```