

Parameters 2018 model 3

Rates for the home and away goals

$$\begin{aligned}\ln \lambda_k(t) &= \ln \alpha_i + \ln \beta_j + \ln \gamma_h + \mathbb{I}\{\text{half} = 2\} \ln \tau + \omega_{\text{goal}}(x(t) - y(t)) + \omega_{\text{player}}(y^*(t) - x^*(t)) \\ \ln \mu_k(t) &= \ln \alpha_j + \ln \beta_i + \mathbb{I}\{\text{half} = 2\} \ln \tau + \omega_{\text{goal}}(y(t) - x(t)) + \omega_{\text{player}}(x^*(t) - y^*(t))\end{aligned}$$

- i : home team index;
- j : away team index;
- α : attack strength parameter;
- $1/\beta$: defense strength parameter;
- γ_h : home advantage parameter;
- τ : second half parameter;
- $x(t)$: the number of goals of the home team until minute t ;
- $y(t)$: the number of goals of the away team until minute t ;
- $x^*(t)$: the number of red cards of the home team until minute t ;
- $y^*(t)$: the number of red cards of the away team until minute t ;
- ω_{goal} : parameter that measure the impact of leading in the score in the rates;
- ω_{player} : parameter that measure the impact of having extra players on the field.

Rates for the home and away red cards

$$\begin{aligned}\lambda_k^*(t) &= A_\lambda \left(t + 45^{\mathbb{I}\{\text{half} = 2\}} \right) \\ \mu_k^*(t) &= A_\mu \left(t + 45^{\mathbb{I}\{\text{half} = 2\}} \right)\end{aligned}$$

Stoppage time

The stoppage time for the first half, U^1 , and the second half, U^2 , are modeled as:

$$\begin{aligned}U^1 &\sim \text{Poisson}(\eta_1 + \rho_1 r^1) \\ U^2 &\sim \text{Poisson}(\eta_2 + \rho_2 r^2 + \kappa c)\end{aligned}$$

- r^t is the amount of red cards received in half t until minute 45;
- $c = \begin{cases} 1, & \text{if } |x - y| \leq 1 \text{ at minute 45 of the second half;} \\ 0, & \text{otherwise.} \end{cases}$

Constraint

The constraint for identificability is

$$\sum_i^n \log(\alpha_i) = \sum_i^n \log(\beta_i).$$

```
options(knitr.kable.NA = "-")
options(scipen = 999)
```

```
library(dplyr)
library(knitr)
```

```
load("data/input.RData")
load("data/mod_3.RData")
```

```
alphas_betas = tibble(Team = times$Time,
                      alpha = exp(mod_3$alpha),
                      beta = exp(mod_3$beta))
kable(alphas_betas, digits = 4, caption = "Alphas and betas",
      col.names = c("Team", "$\\alpha$", "$\\beta$"))
```

Table 1: Alphas and betas

Team	α	β
América-MG	0.0581	0.0926
Athletico-PR	0.1134	0.0686
Atlético-MG	0.1112	0.0825
Bahia	0.0742	0.0809
Botafogo	0.0744	0.0888
Ceará	0.0605	0.0727
Chapecoense	0.0633	0.0996
Corinthians	0.0659	0.0664
Cruzeiro	0.0660	0.0665
Flamengo	0.1236	0.0537
Fluminense	0.0622	0.0903
Grêmio	0.0943	0.0501
Internacional	0.1002	0.0535
Palmeiras	0.1342	0.0459
Paraná	0.0321	0.1189
Santos	0.0919	0.0771
São Paulo	0.0930	0.0627
Sport	0.0655	0.1143
Vasco da Gama	0.0759	0.0932
Vitória	0.0660	0.1342

```
Parameter = c("$\\gamma_h$", "$\\tau$", "$\\omega_{\\{\\text{goal}\\}}$",
              "$\\omega_{\\{\\text{player}\\}}$")
goals = tibble(Parameter,
               Estimative = c(exp(mod_3$gamma), exp(mod_3$tau), mod_3$omega))
kable(goals, digits = 4, caption = "Goal rate parameters")
```

Table 2: Goal rate parameters

Parameter	Estimative
γ_h	1.9282
τ	1.3827
ω_{goal}	-0.2057
ω_{player}	0.2387

```
Parameter = c("$A_\\lambda$", "$A_\\mu$")
reds = tibble(Parameter, Estimative = exp(mod_3$a))
kable(reds, digits = 8, caption = "Red card rate parameters")
```

Table 3: Red card rate parameters

Parameter	Estimative
A_λ	0.00001960
A_μ	0.00003227

```
Parameter = c("$\\eta_1$", "$\\eta_2$", "$\\rho_1$", "$\\rho_2$", "$\\kappa$")
st = tibble(Parameter,
             Estimative = c(mod_3$eta, mod_3$rho, mod_3$kappa))
kable(st, digits = 4, caption = "Stoppage time parameters")
```

Table 4: Stoppage time parameters

Parameter	Estimative
η_1	2.3879
η_2	3.5898
ρ_1	0.9702
ρ_2	0.2034
κ	1.0271

```
mod_3$loglik
```

```
## [1] -2052.097
```