Parameters Série A 2019

Rates for the home and away teams

Model 0

$$\lambda_k = \alpha_i \beta_j \gamma_h$$
$$\mu_k = \alpha_j \beta_i$$

- i: home team index;
- j: away team index;
- α: attack strength parameter;
- $1/\beta$: defense strength parameter;
- γ_h: home advantage parameter.

Model 1

$$\lambda_k(t) = \alpha_i \beta_j \gamma_h$$
$$\mu_k(t) = \alpha_j \beta_i$$

Model 2

$$\lambda_k(t) = \alpha_i \beta_j \gamma_h \tau^{\mathbb{I}\{\text{half} = 2\}}$$
$$\mu_k(t) = \alpha_j \beta_i \tau^{\mathbb{I}\{\text{half} = 2\}}$$

- τ: second half parameter.

Model 3

$$\lambda_k(t) = \alpha_i \beta_j \gamma_h \tau^{\mathbb{I}\{\text{half} = 2\}} \lambda_{xy}$$
$$\mu_k(t) = \alpha_j \beta_i \tau^{\mathbb{I}\{\text{half} = 2\}} \mu_{xy}$$

- $\lambda_{xy} = \begin{cases} 1, \text{ if } x = y; \\ \lambda_{10}, \text{ if } x > y; \\ \lambda_{01}, \text{ if } x < y; \end{cases}$
- $\mu_{xy} = \begin{cases} 1, \text{ if } x = y; \\ \mu_{10}, \text{ if } x > y; \\ \mu_{01}, \text{ if } x < y. \end{cases}$

Stoppage time

For all models except model 0, the stoppage time for the first half, U^1, and the second half, U^2, are modeled as:

$$U^1 \sim \text{Poisson}(\eta_1 + \phi_1 g^1)$$
$$U^2 \sim \text{Poisson}(\eta_2 + \phi_2 g^2 + \kappa c)$$

- g^t is the amount of goals scored in half t until minute 45;
- $c = \begin{cases} 1, & \text{if } |x - y| \leq 1 \text{ at minute 45 of the second half;} \\ 0, & \text{otherwise.} \end{cases}$

Constraint

The constraint used for identifiability in all models was

$$\sum_i^n \log(\alpha_i) = \sum_i^n \log(\beta_i)$$

```r
options(knitr.kable.NA = "-")

library(dplyr)
library(knitr)

load("dados_serie_a_2019.RData")
load("mod_0.RData")
load("mod_1.RData")
load("mod_2.RData")
load("mod_3.RData")
```

```r
alphas = tibble(Time = times$Time,
                mod_0 = exp(mod_0$alpha),
                mod_1 = exp(mod_1$alpha),
                mod_2 = exp(mod_2$alpha),
                mod_3 = exp(mod_3$alpha))
kable(alphas, digits = 4, caption = "Alphas")
```

Table 1: Alphas

Time	mod_0	mod_1	mod_2	mod_3
Athletico-PR	1.1254	0.1134	0.1074	0.1057
Atlético-MG	1.0132	0.1014	0.0960	0.0920
Avaí	0.4108	0.0417	0.0395	0.0362
Bahia	0.9835	0.0991	0.0938	0.0897
Botafogo	0.6940	0.0703	0.0665	0.0638
Ceará	0.8023	0.0805	0.0761	0.0718
Chapecoense	0.6998	0.0709	0.0670	0.0644
Corinthians	0.9286	0.0939	0.0888	0.0867
Cruzeiro	0.6050	0.0615	0.0582	0.0542
CSA	0.5454	0.0554	0.0524	0.0485

Time	mod_0	mod_1	mod_2	mod_3
Flamengo	1.9129	0.1935	0.1833	0.1919
Fluminense	0.8521	0.0857	0.0811	0.0771
Fortaleza	1.1262	0.1139	0.1078	0.1073
Goiás	1.0551	0.1058	0.1002	0.0956
Grêmio	1.4253	0.1448	0.1371	0.1364
Internacional	0.9787	0.0980	0.0928	0.0906
Palmeiras	1.3467	0.1360	0.1287	0.1304
Santos	1.3261	0.1348	0.1276	0.1314
São Paulo	0.8581	0.0861	0.0814	0.0807
Vasco da Gama	0.8735	0.0876	0.0829	0.0810

```r
betas = tibble(Time = times$Time,
                mod_0 = exp(mod_0$beta),
                mod_1 = exp(mod_1$beta),
                mod_2 = exp(mod_2$beta),
                mod_3 = exp(mod_3$beta))
kable(betas, digits = 4, caption = "Betas")
```

Table 2: Betas

Time	mod_0	mod_1	mod_2	mod_3
Athletico-PR	0.6954	0.0698	0.0660	0.0623
Atlético-MG	1.0584	0.1061	0.1005	0.0977
Avaí	1.2971	0.1322	0.1252	0.1299
Bahia	0.9273	0.0933	0.0883	0.0854
Botafogo	0.9556	0.0968	0.0916	0.0911
Ceará	0.8757	0.0882	0.0834	0.0823
Chapecoense	1.1046	0.1115	0.1055	0.1051
Corinthians	0.7311	0.0738	0.0698	0.0656
Cruzeiro	0.9722	0.0990	0.0937	0.0930
CSA	1.2220	0.1243	0.1177	0.1212
Flamengo	0.8400	0.0847	0.0802	0.0734
Fluminense	0.9851	0.0994	0.0940	0.0919
Fortaleza	1.0649	0.1075	0.1017	0.0977
Goiás	1.3856	0.1391	0.1317	0.1311
Grêmio	0.8616	0.0869	0.0823	0.0796
Internacional	0.8409	0.0847	0.0802	0.0770
Palmeiras	0.7039	0.0712	0.0674	0.0642
Santos	0.7251	0.0736	0.0697	0.0649
São Paulo	0.6426	0.0645	0.0610	0.0578
Vasco da Gama	0.9648	0.0966	0.0914	0.0886

```r
param = tibble(Model = 0:3,
               gamma = c(exp(mod_0$gamma), exp(mod_1$gamma), exp(mod_2$gamma),
                         exp(mod_3$gamma)),
               tau = c(NA, NA, exp(mod_2$tau), exp(mod_3$tau)),
               lambda_10 = c(NA, NA, NA, exp(mod_3$lambda_xy["10"])),
               lambda_01 = c(NA, NA, NA, exp(mod_3$lambda_xy["01"])),
               mu_10 = c(NA, NA, NA, exp(mod_3$mu_xy["10"])),
```

```
                 mu_01 = c(NA, NA, NA, exp(mod_3$mu_xy["01"])),
                 eta_1 = c(NA, mod_1$eta[1], mod_2$eta[1], mod_3$eta[1]),
                 eta_2 = c(NA, mod_1$eta[2], mod_2$eta[2], mod_3$eta[2]),
                 phi_1 = c(NA, mod_1$phi[1], mod_2$phi[1], mod_3$phi[1]),
                 phi_2 = c(NA, mod_1$phi[2], mod_2$phi[2], mod_3$phi[2]),
                 kappa = c(NA, mod_1$kappa, mod_2$kappa, mod_3$kappa))
kable(param, digits = 4, caption = "Other parameters",
      col.names = c("Model", "$\\gamma_h$", "$\\tau$", "$\\lambda_{10}$",
                    "$\\lambda_{01}$", "$\\mu_{10}$", "$\\mu_{01}$", "$\\eta_1$",
                    "$\\eta_2$", "$\\phi_1$", "$\\phi_2$", "$\\kappa$"))
```

Table 3: Other parameters

Model	γ_h	τ	λ_{10}	λ_{01}	μ_{10}	μ_{01}	η_1	η_2	ϕ_1	ϕ_2	κ
0	1.4957	-	-	-	-	-	-	-	-	-	-
1	1.4990	-	-	-	-	-	2.8341	3.6016	0.2261	0.2075	1.5566
2	1.4994	1.2284	-	-	-	-	2.8341	3.6017	0.2261	0.2075	1.5566
3	1.6761	1.2707	0.7697	1.0283	1.2564	0.7582	2.8341	3.6016	0.2261	0.2075	1.5566