Parameters 2015-2020 model 9

Rates for the home and away goals

$$\ln \lambda_k(t) = \ln \alpha_i + \ln \beta_j + \ln \gamma_h + \mathbb{I}\{\text{half} = 2\} \ln \tau + \ln \lambda_{xy} + \omega_{\text{player}}(y^*(t) - x^*(t)) + \omega_{\text{value}}(v_i - v_j)$$

$$\ln \mu_k(t) = \ln \alpha_j + \ln \beta_i + \mathbb{I}\{\text{half} = 2\} \ln \tau + \ln \mu_{xy} + \omega_{\text{player}}(x^*(t) - y^*(t)) + \omega_{\text{value}}(v_j - v_i)$$

- i: home team index;

- j: away team index;

- α: attack strength parameter;

- $1/\beta$: defense strength parameter;

- γ_h: home advantage parameter;

- τ: second half parameter;

- $x(t)$: the number of goals of the home team until minute t;

- $y(t)$: the number of goals of the away team until minute t;

- $x^*(t)$: the number of red cards of the home team until minute t;

- $y^*(t)$: the number of red cards of the away team until minute t;

- v_i: the value of the home team's starting roster in millions of (?);

- v_j: the value of the away team's starting roster in millions of (?);

- $\lambda_{xy} = \begin{cases} 1, & \text{for } x = y; \\ \omega_{\text{ahead}}, & \text{for } x - y \geq 1; \\ \omega_{\text{behind}}, & \text{for } x - y \leq -1; \end{cases}$

- $\mu_{xy} = \begin{cases} 1, & \text{for } x = y; \\ \lambda_{\text{ahead}}, & \text{for } y - x \geq 1; \\ \lambda_{\text{behind}}, & \text{for } y - x \leq -1; \end{cases}$

- ω_{player}: parameter that measure the impact of having extra players on the field;

- ω_{value}: parameter that measure the impact of the difference in value of the starting rosters.

Rates for the home and away red cards

$$\lambda_k^*(t) = A_\lambda \left(t + 45^{\mathbb{I}\{\text{half} = 2\}} \right)$$

$$\mu_k^*(t) = A_\mu \left(t + 45^{\mathbb{I}\{\text{half} = 2\}} \right)$$

Stoppage time

The stoppage time for the first half, U^1, and the second half, U^2, are modeled as:

$$U^1 \sim \text{Poisson}(\eta_1 + \rho_1 r^1)$$

$$U^2 \sim \text{Poisson}(\eta_2 + \rho_2 r^2 + \kappa c)$$

- r^t is the amount of red cards received in half t until minute 45;

- $c = \begin{cases} 1, & \text{if } |x - y| \leq 1 \text{ at minute 45 of the second half;} \\ 0, & \text{otherwise.} \end{cases}$

Constraint

The constraint for identificability is

$$\sum_i^n \log(\alpha_i) = \sum_i^n \log(\beta_i).$$

```
options(knitr.kable.NA = "-")
options(scipen = 999)

library(dplyr)
library(knitr)

load("data/input.RData")
load("data/mod_9.RData")
```

```
alphas_betas = tibble(Team = times$Time,
              alpha = exp(mod_9$alpha),
              beta = exp(mod_9$beta))
kable(alphas_betas, digits = 4, caption = "Alphas and betas",
      col.names = c("Team", "$\\alpha$", "$\\beta$"))
```

Table 1: Alphas and betas

Team	α	β
América-MG	0.0614	0.0996
Athletico-PR	0.0999	0.0734
Atlético-GO	0.0919	0.0907
Atlético-MG	0.1211	0.0987
Avaí	0.0673	0.1050
Bahia	0.1037	0.0894
Botafogo	0.0873	0.0864
Ceará	0.0951	0.0775
Chapecoense	0.0893	0.0957
Corinthians	0.1001	0.0751
Coritiba	0.0840	0.0869
Cruzeiro	0.0841	0.0840
Csa	0.0567	0.1044
Figueirense	0.0759	0.0951

Team	α	β
Flamengo	0.1145	0.0904
Fluminense	0.0942	0.0925
Fortaleza	0.1015	0.0815
Goiás	0.0989	0.1065
Grêmio	0.1044	0.0778
Internacional	0.0983	0.0720
Joinville	0.0624	0.0893
Palmeiras	0.1215	0.0799
Paraná	0.0425	0.1068
Ponte Preta	0.0969	0.0920
Red Bull Bragantino	0.1130	0.0714
Santa Cruz	0.1047	0.1366
Santos	0.1120	0.0787
São Paulo	0.0985	0.0838
Sport	0.0971	0.0996
Vasco da Gama	0.0849	0.0934
Vitória	0.1028	0.1134

```
Parameter = c("$\\gamma_h$", "$\\tau$",
              "$\\omega_{{\\text{ahead}}}$",
              "$\\omega_{{\\text{behind}}}$",
              "$\\omega_{{\\text{player}}}$",
              "$\\omega_{\\text{value}}$")
goals = tibble(Parameter,
                Estimative = c(exp(mod_9$gamma),
                               exp(mod_9$tau),
                               exp(mod_9$omega_xy),
                               mod_9$omega))
kable(goals, digits = 4, caption = "Goal rate parameters")
```

Table 2: Goal rate parameters

Parameter	Estimative
γ_h	1.5323
τ	1.2329
ω_{ahead}	0.8319
ω_{behind}	1.1114
ω_{player}	0.3183
ω_{value}	0.0062

```
Parameter = c("$A_\\lambda$", "$A_\\mu$")
reds = tibble(Parameter, Estimative = exp(mod_9$a))
kable(reds, digits = 8, caption = "Red card rate parameters")
```

Table 3: Red card rate parameters

Parameter	Estimative
A_λ	0.00001973

Parameter	Estimative
A_μ	0.00003190

```
Parameter = c("$\\eta_1$", "$\\eta_2$", "$\\rho_1$", "$\\rho_2$", "$\\kappa$")
st = tibble(Parameter,
            Estimative = c(mod_9$eta, mod_9$rho, mod_9$kappa))
kable(st, digits = 4, caption = "Stoppage time parameters")
```

Table 4: Stoppage time parameters

Parameter	Estimative
η_1	2.4359
η_2	3.7394
ρ_1	1.1400
ρ_2	0.2267
κ	1.0101

```
mod_9$loglik
```

```
## [1] -12899.89
```