

# Parameters 2018 model 1

## Rates for the home and away goals

$$\begin{aligned}\ln \lambda_k(t) &= \ln \alpha_i + \ln \beta_j + \ln \gamma_h + \mathbb{I}\{\text{half} = 2\} \ln \tau + \omega_{\lambda x} x(t) + \omega_{\lambda y} y(t) + \omega_{\lambda x^*} x^*(t) + \omega_{\lambda y^*} y^*(t) \\ \ln \mu_k(t) &= \ln \alpha_j + \ln \beta_i + \mathbb{I}\{\text{half} = 2\} \ln \tau + \omega_{\mu x} x(t) + \omega_{\mu y} y(t) + \omega_{\mu x^*} x^*(t) + \omega_{\mu y^*} y^*(t)\end{aligned}$$

- $i$ : home team index;
- $j$ : away team index;
- $\alpha$ : attack strength parameter;
- $1/\beta$ : defense strength parameter;
- $\gamma_h$ : home advantage parameter;
- $\tau$ : second half parameter;
- $x(t)$ : the number of goals of the home team until minute  $t$ ;
- $y(t)$ : the number of goals of the away team until minute  $t$ ;
- $x^*(t)$ : the number of red cards of the home team until minute  $t$ ;
- $y^*(t)$ : the number of red cards of the away team until minute  $t$ ;
- $\omega_{\lambda x}$ ,  $\omega_{\lambda y}$ ,  $\omega_{\mu x}$  and  $\omega_{\mu y}$ : parameters that measure the impact of the scored goals in the rates;
- $\omega_{\lambda x^*}$ ,  $\omega_{\lambda y^*}$ ,  $\omega_{\mu x^*}$  and  $\omega_{\mu y^*}$ : parameters that measure the impact of red cards in the rates.

## Rates for the home and away red cards

$$\begin{aligned}\lambda_k^*(t) &= A_\lambda \left( t + 45^{\mathbb{I}\{\text{half} = 2\}} \right) \\ \mu_k^*(t) &= A_\mu \left( t + 45^{\mathbb{I}\{\text{half} = 2\}} \right)\end{aligned}$$

## Stoppage time

The stoppage time for the first half,  $U^1$ , and the second half,  $U^2$ , are modeled as:

$$\begin{aligned}U^1 &\sim \text{Poisson}(\eta_1 + \rho_1 r^1) \\ U^2 &\sim \text{Poisson}(\eta_2 + \rho_2 r^2 + \kappa c)\end{aligned}$$

- $r^t$  is the amount of red cards received in half  $t$  until minute 45;
- $c = \begin{cases} 1, & \text{if } |x - y| \leq 1 \text{ at minute 45 of the second half;} \\ 0, & \text{otherwise.} \end{cases}$

## Constraint

The constraint for identificability is

$$\sum_i^n \log(\alpha_i) = \sum_i^n \log(\beta_i).$$

```
options(knitr.kable.NA = "-")
options(scipen = 999)
```

```
library(dplyr)
library(knitr)
```

```
load("data/input.RData")
load("data/mod_1.RData")
```

```
alphas_betas = tibble(Team = times$Time,
                      alpha = exp(mod_1$alpha),
                      beta = exp(mod_1$beta))
kable(alphas_betas, digits = 4, caption = "Alphas and betas",
      col.names = c("Team", "$\\alpha$", "$\\beta$"))
```

Table 1: Alphas and betas

| Team          | $\alpha$ | $\beta$ |
|---------------|----------|---------|
| América-MG    | 0.0594   | 0.0955  |
| Athletico-PR  | 0.1163   | 0.0725  |
| Atlético-MG   | 0.1142   | 0.0834  |
| Bahia         | 0.0758   | 0.0824  |
| Botafogo      | 0.0767   | 0.0904  |
| Ceará         | 0.0609   | 0.0737  |
| Chapecoense   | 0.0643   | 0.1006  |
| Corinthians   | 0.0672   | 0.0677  |
| Cruzeiro      | 0.0670   | 0.0682  |
| Flamengo      | 0.1269   | 0.0541  |
| Fluminense    | 0.0648   | 0.0940  |
| Grêmio        | 0.0945   | 0.0517  |
| Internacional | 0.1020   | 0.0545  |
| Palmeiras     | 0.1362   | 0.0462  |
| Paraná        | 0.0334   | 0.1246  |
| Santos        | 0.0939   | 0.0790  |
| São Paulo     | 0.0960   | 0.0634  |
| Sport         | 0.0672   | 0.1143  |
| Vasco da Gama | 0.0757   | 0.0954  |
| Vitória       | 0.0677   | 0.1361  |

```
Parameter = c("$\\gamma_h$", "$\\tau$", "$\\omega_{\\lambda x}$",
              "$\\omega_{\\lambda y}$", "$\\omega_{\\mu x}$",
              "$\\omega_{\\mu y}$", "$\\omega_{\\lambda x^*}$",
              "$\\omega_{\\lambda y^*}$", "$\\omega_{\\mu x^*}$",
              "$\\omega_{\\mu y^*}$")
goals = tibble(Parameter,
```

```
Estimative = c(exp(mod_1$gamma), exp(mod_1$tau), mod_1$omega))
kable(goals, digits = 4, caption = "Goal rate parameters")
```

Table 2: Goal rate parameters

| Parameter              | Estimative |
|------------------------|------------|
| $\gamma_h$             | 1.7998     |
| $\tau$                 | 1.3853     |
| $\omega_{\lambda x}$   | -0.2350    |
| $\omega_{\lambda y}$   | 0.3093     |
| $\omega_{\mu x}$       | 0.0996     |
| $\omega_{\mu y}$       | -0.1241    |
| $\omega_{\lambda x^*}$ | -0.6620    |
| $\omega_{\lambda y^*}$ | 0.3196     |
| $\omega_{\mu x^*}$     | -0.1808    |
| $\omega_{\mu y^*}$     | -0.0581    |

```
Parameter = c("$A_\\lambda$", "$A_\\mu$")
reds = tibble(Parameter, Estimative = exp(mod_1$a))
kable(reds, digits = 8, caption = "Red card rate parameters")
```

Table 3: Red card rate parameters

| Parameter   | Estimative |
|-------------|------------|
| $A_\lambda$ | 0.00001959 |
| $A_\mu$     | 0.00003227 |

```
Parameter = c("$\\eta_1$", "$\\eta_2$", "$\\rho_1$", "$\\rho_2$", "$\\kappa$")
st = tibble(Parameter,
  Estimative = c(mod_1$eta, mod_1$rho, mod_1$kappa))
kable(st, digits = 4, caption = "Stoppage time parameters")
```

Table 4: Stoppage time parameters

| Parameter | Estimative |
|-----------|------------|
| $\eta_1$  | 2.3879     |
| $\eta_2$  | 3.5898     |
| $\rho_1$  | 0.9702     |
| $\rho_2$  | 0.2034     |
| $\kappa$  | 1.0271     |

```
mod_1$loglik
```

```
## [1] -2047.963
```