Parameters 2018 model 2

Rates for the home and away goals

$$\ln \lambda_k(t) = \ln \alpha_i + \ln \beta_j + \ln \gamma_h + \mathbb{I}\{\text{half} = 2\} \ln \tau + \omega_{\lambda(x-y)}(x(t) - y(t)) + \omega_{\lambda(y^*-x^*)}(y^*(t) - x^*(t))$$
$$\ln \mu_k(t) = \ln \alpha_j + \ln \beta_i + \mathbb{I}\{\text{half} = 2\} \ln \tau + \omega_{\mu(y-x)}(y(t) - x(t)) + \omega_{\mu(x^*-y^*)}(x^*(t) - y^*(t))$$

- i: home team index;
- j: away team index;
- α: attack strength parameter;
- $1/\beta$: defense strength parameter;
- γ_h: home advantage parameter;
- τ: second half parameter;
- $x(t)$: the number of goals of the home team until minute t;
- $y(t)$: the number of goals of the away team until minute t;
- $x^*(t)$: the number of red cards of the home team until minute t;
- $y^*(t)$: the number of red cards of the away team until minute t;
- $\omega_{\lambda(x-y)}$, $\omega_{\mu(y-x)}$: parameters that measure the impact of leading in the score in the rates;
- $\omega_{\lambda(y^*-x^*)}$ and $\omega_{\mu(x^*-y^*)}$: parameters that measure the impact of having extra players on the field.

Rates for the home and away red cards

$$\lambda_k^*(t) = A_\lambda\left(t + 45^{\mathbb{I}\{\text{half} = 2\}}\right)$$
$$\mu_k^*(t) = A_\mu\left(t + 45^{\mathbb{I}\{\text{half} = 2\}}\right)$$

Stoppage time

The stoppage time for the first half, U^1, and the second half, U^2, are modeled as:

$$U^1 \sim \text{Poisson}(\eta_1 + \rho_1 r^1)$$
$$U^2 \sim \text{Poisson}(\eta_2 + \rho_2 r^2 + \kappa c)$$

- r^t is the amount of red cards received in half t until minute 45;
- $c = \begin{cases} 1, & \text{if } |x - y| \leq 1 \text{ at minute 45 of the second half;} \\ 0, & \text{otherwise.} \end{cases}$

Constraint

The constraint for identificability is

$$\sum_{i}^{n} \log(\alpha_i) = \sum_{i}^{n} \log(\beta_i).$$

```
options(knitr.kable.NA = "-")
options(scipen = 999)

library(dplyr)
library(knitr)

load("data/input.RData")
load("data/mod_2.RData")
```

```
alphas_betas = tibble(Team = times$Time,
                alpha = exp(mod_2$alpha),
                beta = exp(mod_2$beta))
kable(alphas_betas, digits = 4, caption = "Alphas and betas",
      col.names = c("Team", "$\\alpha$", "$\\beta$"))
```

Table 1: Alphas and betas

Team	α	β
América-MG	0.0585	0.0938
Athletico-PR	0.1164	0.0710
Atlético-MG	0.1123	0.0829
Bahia	0.0744	0.0808
Botafogo	0.0756	0.0897
Ceará	0.0605	0.0726
Chapecoense	0.0636	0.0995
Corinthians	0.0660	0.0667
Cruzeiro	0.0663	0.0668
Flamengo	0.1246	0.0537
Fluminense	0.0635	0.0927
Grêmio	0.0945	0.0508
Internacional	0.1004	0.0538
Palmeiras	0.1350	0.0460
Paraná	0.0326	0.1214
Santos	0.0926	0.0782
São Paulo	0.0940	0.0629
Sport	0.0662	0.1144
Vasco da Gama	0.0751	0.0935
Vitória	0.0669	0.1344

```
Parameter = c("$\\gamma_h$", "$\\tau$", "$\\omega_{\\lambda (x-y)}$",
              "$\\omega_{\\mu (y-x)}$", "$\\omega_{\\lambda (y^*-x^*)}$",
              "$\\omega_{\\mu_(x^*-y^*)}$")
goals = tibble(Parameter,
               Estimative = c(exp(mod_2$gamma), exp(mod_2$tau), mod_2$omega))
kable(goals, digits = 4, caption = "Goal rate parameters")
```

Table 2: Goal rate parameters

Parameter	Estimative
γ_h	1.8940
τ	1.4007
$\omega_{\lambda(x-y)}$	-0.2652
$\omega_{\mu(y-x)}$	-0.1066
$\omega_{\lambda(y^*-x^*)}$	0.4124
$\omega_{\mu_(x^*-y^*)}$	-0.0287

```
Parameter = c("$A_\\lambda$", "$A_\\mu$")
reds = tibble(Parameter, Estimative = exp(mod_2$a))
kable(reds, digits = 8, caption = "Red card rate parameters")
```

Table 3: Red card rate parameters

Parameter	Estimative
A_λ	0.00001960
A_μ	0.00003227

```
Parameter = c("$\\eta_1$", "$\\eta_2$", "$\\rho_1$", "$\\rho_2$", "$\\kappa$")
st = tibble(Parameter,
            Estimative = c(mod_2$eta, mod_2$rho, mod_2$kappa))
kable(st, digits = 4, caption = "Stoppage time parameters")
```

Table 4: Stoppage time parameters

Parameter	Estimative
η_1	2.3879
η_2	3.5898
ρ_1	0.9702
ρ_2	0.2034
κ	1.0271

```
mod_2$loglik
```

```
## [1] -2048.924
```