

Parameters 2015-2020 model 1

Rates for the home and away goals

$$\begin{aligned}\ln \lambda_k(t) &= \ln \alpha_i + \ln \beta_j + \ln \gamma_h + \mathbb{I}\{\text{half} = 2\} \ln \tau + \omega_{\lambda x} x(t) + \omega_{\lambda y} y(t) + \omega_{\lambda x^*} x^*(t) + \omega_{\lambda y^*} y^*(t) \\ \ln \mu_k(t) &= \ln \alpha_j + \ln \beta_i + \mathbb{I}\{\text{half} = 2\} \ln \tau + \omega_{\mu x} x(t) + \omega_{\mu y} y(t) + \omega_{\mu x^*} x^*(t) + \omega_{\mu y^*} y^*(t)\end{aligned}$$

- i : home team index;
- j : away team index;
- α : attack strength parameter;
- $1/\beta$: defense strength parameter;
- γ_h : home advantage parameter;
- τ : second half parameter;
- $x(t)$: the number of goals of the home team until minute t ;
- $y(t)$: the number of goals of the away team until minute t ;
- $x^*(t)$: the number of red cards of the home team until minute t ;
- $y^*(t)$: the number of red cards of the away team until minute t ;
- $\omega_{\lambda x}, \omega_{\lambda y}, \omega_{\mu x}$ and $\omega_{\mu y}$: parameters that measure the impact of the scored goals in the rates;
- $\omega_{\lambda x^*}, \omega_{\lambda y^*}, \omega_{\mu x^*}$ and $\omega_{\mu y^*}$: parameters that measure the impact of red cards in the rates.

Rates for the home and away red cards

$$\begin{aligned}\lambda_k^*(t) &= A_\lambda \left(t + 45^{\mathbb{I}\{\text{half} = 2\}} \right) \\ \mu_k^*(t) &= A_\mu \left(t + 45^{\mathbb{I}\{\text{half} = 2\}} \right)\end{aligned}$$

Stoppage time

The stoppage time for the first half, U^1 , and the second half, U^2 , are modeled as:

$$\begin{aligned}U^1 &\sim \text{Poisson}(\eta_1 + \rho_1 r^1) \\ U^2 &\sim \text{Poisson}(\eta_2 + \rho_2 r^2 + \kappa c)\end{aligned}$$

- r^t is the amount of red cards received in half t until minute 45;
- $c = \begin{cases} 1, & \text{if } |x - y| \leq 1 \text{ at minute 45 of the second half;} \\ 0, & \text{otherwise.} \end{cases}$

Constraint

The constraint for identificability is

$$\sum_i^n \log(\alpha_i) = \sum_i^n \log(\beta_i).$$

```
options(knitr.kable.NA = "-")
options(scipen = 999)
```

```
library(dplyr)
library(knitr)
```

```
load("data/input.RData")
load("data/mod_1.RData")
```

```
alphas_betas = tibble(Team = times$Time,
                      alpha = exp(mod_1$alpha),
                      beta = exp(mod_1$beta))
kable(alphas_betas, digits = 4, caption = "Alphas and betas",
      col.names = c("Team", "$\\alpha$", "$\\beta$"))
```

Table 1: Alphas and betas

Team	α	β
América-MG	0.0571	0.1023
Athletico-PR	0.0982	0.0717
Atlético-GO	0.0855	0.0935
Atlético-MG	0.1277	0.0892
Avaí	0.0613	0.1098
Bahia	0.0989	0.0912
Botafogo	0.0824	0.0876
Ceará	0.0882	0.0806
Chapecoense	0.0844	0.0962
Corinthians	0.1070	0.0677
Coritiba	0.0788	0.0893
Cruzeiro	0.0876	0.0772
Csa	0.0510	0.1138
Figueirense	0.0724	0.0955
Flamengo	0.1327	0.0749
Fluminense	0.0951	0.0882
Fortaleza	0.0935	0.0860
Goiás	0.0908	0.1115
Grêmio	0.1149	0.0684
Internacional	0.1015	0.0676
Joinville	0.0573	0.0930
Palmeiras	0.1331	0.0695
Paraná	0.0393	0.1109
Ponte Preta	0.0925	0.0920
Red Bull Bragantino	0.1137	0.0703
Santa Cruz	0.0967	0.1378
Santos	0.1183	0.0723
São Paulo	0.1066	0.0741

Team	α	β
Sport	0.0939	0.0983
Vasco da Gama	0.0806	0.0944
Vitória	0.0979	0.1123

```

Parameter = c("$\\gamma_h$", "$\\tau$", "$\\omega_{\\lambda x}$",
              "$\\omega_{\\lambda y}$", "$\\omega_{\\mu x}$",
              "$\\omega_{\\mu y}$", "$\\omega_{\\lambda x^*}$",
              "$\\omega_{\\lambda y^*}$", "$\\omega_{\\mu x^*}$",
              "$\\omega_{\\mu y^*}$")
goals = tibble(Parameter,
               Estimative = c(exp(mod_1$gamma), exp(mod_1$tau), mod_1$omega))
kable(goals, digits = 4, caption = "Goal rate parameters")

```

Table 2: Goal rate parameters

Parameter	Estimative
γ_h	1.5954
τ	1.2092
$\omega_{\lambda x}$	-0.0915
$\omega_{\lambda y}$	0.1114
$\omega_{\mu x}$	0.1061
$\omega_{\mu y}$	-0.0465
$\omega_{\lambda x^*}$	-0.4505
$\omega_{\lambda y^*}$	0.2495
$\omega_{\mu x^*}$	0.3342
$\omega_{\mu y^*}$	-0.3928

```

Parameter = c("$A_{\\lambda}$", "$A_{\\mu}$")
reds = tibble(Parameter, Estimative = exp(mod_1$a))
kable(reds, digits = 8, caption = "Red card rate parameters")

```

Table 3: Red card rate parameters

Parameter	Estimative
A_{λ}	0.00001973
A_{μ}	0.00003190

```

Parameter = c("$\\eta_1$", "$\\eta_2$", "$\\rho_1$", "$\\rho_2$", "$\\kappa$")
st = tibble(Parameter,
             Estimative = c(mod_1$eta, mod_1$rho, mod_1$kappa))
kable(st, digits = 4, caption = "Stoppage time parameters")

```

Table 4: Stoppage time parameters

Parameter	Estimative
η_1	2.4359
η_2	3.7394
ρ_1	1.1400
ρ_2	0.2267
κ	1.0101

```
mod_1$loglik
```

```
## [1] -12908.56
```