Parameters 2015-2020 model 5

Rates for the home and away goals

$$\ln \lambda_k(t) = \ln \alpha_i + \ln \beta_j + \ln \gamma_h + \mathbb{I}\{\text{half} = 2\}\ln \tau + \ln \lambda_{xy} + \omega_{\text{player}}(y^*(t) - x^*(t))$$

$$\ln \mu_k(t) = \ln \alpha_j + \ln \beta_i + \mathbb{I}\{\text{half} = 2\}\ln \tau + \ln \mu_{xy} + \omega_{\text{player}}(x^*(t) - y^*(t))$$

- i: home team index;

- j: away team index;

- α: attack strength parameter;

- $1/\beta$: defense strength parameter;

- γ_h: home advantage parameter;

- τ: second half parameter;

- $x(t)$: the number of goals of the home team until minute t;

- $y(t)$: the number of goals of the away team until minute t;

- $x^*(t)$: the number of red cards of the home team until minute t;

- $y^*(t)$: the number of red cards of the away team until minute t;

- $\lambda_{xy} = \begin{cases} 1, & \text{for } x = 0, \ y = 0; \\ \lambda_{10}, & \text{for } x = 1, \ y = 0; \\ \lambda_{01}, & \text{for } x = 0, \ y = 1; \\ \lambda_{11}, & \text{for } x = 1, \ y = 1; \\ \lambda_{22}, & \text{for } x - y = 0, \ x, \ y, \geq 2; \\ \lambda_{21}, & \text{for } x - y \geq 1, \ x \geq 2; \\ \lambda_{12}, & \text{for } x - y \leq -1, \ y \geq 2. \end{cases}$

- ω_{player}: parameter that measure the impact of having extra players on the field.

Rates for the home and away red cards

$$\lambda_k^*(t) = A_\lambda \left(t + 45^{\mathbb{I}\{\text{half} = 2\}} \right)$$

$$\mu_k^*(t) = A_\mu \left(t + 45^{\mathbb{I}\{\text{half} = 2\}} \right)$$

Stoppage time

The stoppage time for the first half, U^1, and the second half, U^2, are modeled as:

$$U^1 \sim \text{Poisson}(\eta_1 + \rho_1 r^1)$$

$$U^2 \sim \text{Poisson}(\eta_2 + \rho_2 r^2 + \kappa c)$$

- r^t is the amount of red cards received in half t until minute 45;

- $c = \begin{cases} 1, & \text{if } |x - y| \leq 1 \text{ at minute 45 of the second half;} \\ 0, & \text{otherwise.} \end{cases}$

Constraint

The constraint for identificability is

$$\sum_i^n \log(\alpha_i) = \sum_i^n \log(\beta_i).$$

```
options(knitr.kable.NA = "-")
options(scipen = 999)

library(dplyr)
library(knitr)

load("data/input.RData")
load("data/mod_5.RData")
```

```
alphas_betas = tibble(Team = times$Time,
              alpha = exp(mod_5$alpha),
              beta = exp(mod_5$beta))
kable(alphas_betas, digits = 4, caption = "Alphas and betas",
      col.names = c("Team", "$\\alpha$", "$\\beta$"))
```

Table 1: Alphas and betas

Team	α	β
América-MG	0.0574	0.1031
Athletico-PR	0.0987	0.0721
Atlético-GO	0.0861	0.0941
Atlético-MG	0.1282	0.0900
Avaí	0.0618	0.1103
Bahia	0.0990	0.0917
Botafogo	0.0829	0.0878
Ceará	0.0885	0.0812
Chapecoense	0.0849	0.0965
Corinthians	0.1077	0.0681
Coritiba	0.0788	0.0901
Cruzeiro	0.0876	0.0778
Csa	0.0515	0.1136
Figueirense	0.0728	0.0957

Team	α	β
Flamengo	0.1335	0.0754
Fluminense	0.0956	0.0884
Fortaleza	0.0939	0.0860
Goiás	0.0914	0.1114
Grêmio	0.1151	0.0690
Internacional	0.1019	0.0679
Joinville	0.0575	0.0931
Palmeiras	0.1340	0.0702
Paraná	0.0393	0.1119
Ponte Preta	0.0926	0.0924
Red Bull Bragantino	0.1139	0.0707
Santa Cruz	0.0980	0.1395
Santos	0.1189	0.0728
São Paulo	0.1075	0.0744
Sport	0.0946	0.0988
Vasco da Gama	0.0816	0.0941
Vitória	0.0983	0.1129

```
Parameter = c("$\\gamma_h$", "$\\tau$",
              "$\\lambda_{10}$", "$\\lambda_{01}$",
              "$\\lambda_{11}$", "$\\lambda_{22}$",
              "$\\lambda_{21}$", "$\\lambda_{12}$",
              "$\\mu_{10}$", "$\\mu_{01}$",
              "$\\mu_{11}$", "$\\mu_{22}$",
              "$\\mu_{21}$", "$\\mu_{12}$",
              "$\\omega_{{\\text{player}}}$")
goals = tibble(Parameter,
               Estimative = c(exp(mod_5$gamma), exp(mod_5$tau),
                              exp(mod_5$lambda_xy), exp(mod_5$mu_xy),
                              mod_5$omega))
kable(goals, digits = 4, caption = "Goal rate parameters")
```

Table 2: Goal rate parameters

Parameter	Estimative
γ_h	1.6098
τ	1.2218
λ_{10}	0.8101
λ_{01}	1.0698
λ_{11}	1.0614
λ_{22}	1.0271
λ_{21}	0.8215
λ_{12}	1.1407
μ_{10}	1.1368
μ_{01}	0.9043
μ_{11}	0.9813
μ_{22}	1.1580
μ_{21}	1.1904
μ_{12}	0.9475
ω_{player}	0.3234

```r
Parameter = c("$A_\\lambda$", "$A_\\mu$")
reds = tibble(Parameter, Estimative = exp(mod_5$a))
kable(reds, digits = 8, caption = "Red card rate parameters")
```

Table 3: Red card rate parameters

Parameter	Estimative
A_λ	0.00001973
A_μ	0.00003190

```r
Parameter = c("$\\eta_1$", "$\\eta_2$", "$\\rho_1$", "$\\rho_2$", "$\\kappa$")
st = tibble(Parameter,
            Estimative = c(mod_5$eta, mod_5$rho, mod_5$kappa))
kable(st, digits = 4, caption = "Stoppage time parameters")
```

Table 4: Stoppage time parameters

Parameter	Estimative
η_1	2.4360
η_2	3.7394
ρ_1	1.1400
ρ_2	0.2267
κ	1.0101

```r
mod_5$loglik
```

```
## [1] -25858.02
```