Parameters 2015-2020 model 2

Rates for the home and away goals

$$\ln \lambda_k(t) = \ln \alpha_i + \ln \beta_j + \ln \gamma_h + \mathbb{I}\{\text{half} = 2\} \ln \tau + \omega_{\lambda(x-y)}(x(t) - y(t)) + \omega_{\lambda(y^*-x^*)}(y^*(t) - x^*(t))$$

$$\ln \mu_k(t) = \ln \alpha_j + \ln \beta_i + \mathbb{I}\{\text{half} = 2\} \ln \tau + \omega_{\mu(y-x)}(y(t) - x(t)) + \omega_{\mu(x^*-y^*)}(x^*(t) - y^*(t))$$

- i: home team index;
- j: away team index;
- α: attack strength parameter;
- $1/\beta$: defense strength parameter;
- γ_h: home advantage parameter;
- τ: second half parameter;
- $x(t)$: the number of goals of the home team until minute t;
- $y(t)$: the number of goals of the away team until minute t;
- $x^*(t)$: the number of red cards of the home team until minute t;
- $y^*(t)$: the number of red cards of the away team until minute t;
- $\omega_{\lambda(x-y)}$, $\omega_{\mu(y-x)}$: parameters that measure the impact of leading in the score in the rates;
- $\omega_{\lambda(y^*-x^*)}$ and $\omega_{\mu(x^*-y^*)}$: parameters that measure the impact of having extra players on the field.

Rates for the home and away red cards

$$\lambda_k^*(t) = A_\lambda \left(t + 45^{\mathbb{I}\{\text{half} = 2\}} \right)$$

$$\mu_k^*(t) = A_\mu \left(t + 45^{\mathbb{I}\{\text{half} = 2\}} \right)$$

Stoppage time

The stoppage time for the first half, U^1, and the second half, U^2, are modeled as:

$$U^1 \sim \text{Poisson}(\eta_1 + \rho_1 r^1)$$

$$U^2 \sim \text{Poisson}(\eta_2 + \rho_2 r^2 + \kappa c)$$

- r^t is the amount of red cards received in half t until minute 45;
- $c = \begin{cases} 1, & \text{if } |x - y| \leq 1 \text{ at minute 45 of the second half;} \\ 0, & \text{otherwise.} \end{cases}$

Constraint

The constraint for identificability is

$$\sum_i^n \log(\alpha_i) = \sum_i^n \log(\beta_i).$$

```r
options(knitr.kable.NA = "-")
options(scipen = 999)

library(dplyr)
library(knitr)

load("data/input.RData")
load("data/mod_2.RData")
```

```r
alphas_betas = tibble(Team = times$Time,
                alpha = exp(mod_2$alpha),
                beta = exp(mod_2$beta))
kable(alphas_betas, digits = 4, caption = "Alphas and betas",
      col.names = c("Team", "$\\alpha$", "$\\beta$"))
```

Table 1: Alphas and betas

Team	α	β
América-MG	0.0577	0.1034
Athletico-PR	0.0994	0.0725
Atlético-GO	0.0864	0.0946
Atlético-MG	0.1295	0.0906
Avaí	0.0619	0.1112
Bahia	0.0998	0.0919
Botafogo	0.0833	0.0886
Ceará	0.0892	0.0815
Chapecoense	0.0854	0.0973
Corinthians	0.1082	0.0684
Coritiba	0.0796	0.0901
Cruzeiro	0.0885	0.0779
Csa	0.0514	0.1147
Figueirense	0.0730	0.0963
Flamengo	0.1346	0.0759
Fluminense	0.0963	0.0892
Fortaleza	0.0945	0.0870
Goiás	0.0918	0.1128
Grêmio	0.1162	0.0692
Internacional	0.1023	0.0681
Joinville	0.0579	0.0936
Palmeiras	0.1351	0.0705
Paraná	0.0393	0.1115
Ponte Preta	0.0934	0.0929
Red Bull Bragantino	0.1149	0.0714
Santa Cruz	0.0981	0.1398
Santos	0.1198	0.0731
São Paulo	0.1080	0.0749

Team	α	β
Sport	0.0951	0.0994
Vasco da Gama	0.0815	0.0953
Vitória	0.0992	0.1139

```
Parameter = c("$\\gamma_h$", "$\\tau$", "$\\omega_{\\lambda (x-y)}$",
              "$\\omega_{\\mu (y-x)}$", "$\\omega_{\\lambda (y^*-x^*)}$",
              "$\\omega_{\\mu_(x^*-y^*)}$")
goals = tibble(Parameter,
               Estimative = c(exp(mod_2$gamma), exp(mod_2$tau), mod_2$omega))
kable(goals, digits = 4, caption = "Goal rate parameters")
```

Table 2: Goal rate parameters

Parameter	Estimative
γ_h	1.5553
τ	1.2274
$\omega_{\lambda(x-y)}$	-0.0999
$\omega_{\mu(y-x)}$	-0.0842
$\omega_{\lambda(y^*-x^*)}$	0.2985
$\omega_{\mu(x^*-y^*)}$	0.3634

```
Parameter = c("$A_\\lambda$", "$A_\\mu$")
reds = tibble(Parameter, Estimative = exp(mod_2$a))
kable(reds, digits = 8, caption = "Red card rate parameters")
```

Table 3: Red card rate parameters

Parameter	Estimative
A_λ	0.00001973
A_μ	0.00003190

```
Parameter = c("$\\eta_1$", "$\\eta_2$", "$\\rho_1$", "$\\rho_2$", "$\\kappa$")
st = tibble(Parameter,
            Estimative = c(mod_2$eta, mod_2$rho, mod_2$kappa))
kable(st, digits = 4, caption = "Stoppage time parameters")
```

Table 4: Stoppage time parameters

Parameter	Estimative
η_1	2.4360
η_2	3.7394
ρ_1	1.1400
ρ_2	0.2266
κ	1.0101

```
mod_2$loglik
```

```
## [1] -12910.92
```