Parameters 2019 model 5

Rates for the home and away goals

$$\ln \lambda_k(t) = \ln \alpha_i + \ln \beta_j + \ln \gamma_h + \mathbb{I}\{\text{half} = 2\} \ln \tau + \ln \lambda_{xy} + \omega_{\text{player}}(y^*(t) - x^*(t))$$
$$\ln \mu_k(t) = \ln \alpha_j + \ln \beta_i + \mathbb{I}\{\text{half} = 2\} \ln \tau + \ln \mu_{xy} + \omega_{\text{player}}(x^*(t) - y^*(t))$$

- i: home team index;
- j: away team index;
- α: attack strength parameter;
- $1/\beta$: defense strength parameter;
- γ_h: home advantage parameter;
- τ: second half parameter;
- $x(t)$: the number of goals of the home team until minute t;
- $y(t)$: the number of goals of the away team until minute t;
- $x^*(t)$: the number of red cards of the home team until minute t;
- $y^*(t)$: the number of red cards of the away team until minute t;
- $\lambda_{xy} = \begin{cases} 1, \text{ for } x = 0,\ y = 0; \\ \lambda_{10}, \text{ for } x = 1,\ y = 0; \\ \lambda_{01}, \text{ for } x = 0,\ y = 1; \\ \lambda_{11}, \text{ for } x = 1,\ y = 1; \\ \lambda_{22}, \text{ for } x - y = 0,\ x,\ y, \geq 2; \\ \lambda_{21}, \text{ for } x - y \geq 1,\ x \geq 2; \\ \lambda_{12}, \text{ for } x - y \leq -1,\ y \geq 2. \end{cases}$
- ω_{player}: parameter that measure the impact of having extra players on the field.

Rates for the home and away red cards

$$\lambda_k^*(t) = A_\lambda\Big(t + 45^{\mathbb{I}\{\text{half} = 2\}}\Big)$$
$$\mu_k^*(t) = A_\mu\Big(t + 45^{\mathbb{I}\{\text{half} = 2\}}\Big)$$

Stoppage time

The stoppage time for the first half, U^1, and the second half, U^2, are modeled as:

$$U^1 \sim \text{Poisson}(\eta_1 + \rho_1 r^1)$$

$$U^2 \sim \text{Poisson}(\eta_2 + \rho_2 r^2 + \kappa c)$$

- r^t is the amount of red cards received in half t until minute 45;
- $c = \begin{cases} 1, & \text{if } |x - y| \leq 1 \text{ at minute 45 of the second half;} \\ 0, & \text{otherwise.} \end{cases}$

Constraint

The constraint for identificability is

$$\sum_i^n \log(\alpha_i) = \sum_i^n \log(\beta_i).$$

```
options(knitr.kable.NA = "-")
options(scipen = 999)

library(dplyr)
library(knitr)

load("data/input.RData")
load("data/mod_5.RData")
```

```
alphas_betas = tibble(Team = times$Time,
                      alpha = exp(mod_5$alpha),
                      beta = exp(mod_5$beta))
kable(alphas_betas, digits = 4, caption = "Alphas and betas",
      col.names = c("Team", "$\\alpha$", "$\\beta$"))
```

Table 1: Alphas and betas

Team	α	β
Athletico-PR	0.1028	0.0622
Atlético-MG	0.0932	0.0937
Avaí	0.0364	0.1289
Bahia	0.0887	0.0859
Botafogo	0.0632	0.0908
Ceará	0.0701	0.0822
Chapecoense	0.0639	0.1032
Corinthians	0.0864	0.0650
Cruzeiro	0.0529	0.0935
Csa	0.0474	0.1222
Flamengo	0.1854	0.0734
Fluminense	0.0772	0.0893
Fortaleza	0.1055	0.0974
Goiás	0.0951	0.1299

Team	α	β
Grêmio	0.1351	0.0802
Internacional	0.0885	0.0734
Palmeiras	0.1299	0.0622
Santos	0.1341	0.0639
São Paulo	0.0803	0.0563
Vasco da Gama	0.0786	0.0885

```
Parameter = c("$\\gamma_h$", "$\\tau$",
              "$\\lambda_{10}$", "$\\lambda_{01}$",
              "$\\lambda_{11}$", "$\\lambda_{22}$",
              "$\\lambda_{21}$", "$\\lambda_{12}$",
              "$\\mu_{10}$", "$\\mu_{01}$",
              "$\\mu_{11}$", "$\\mu_{22}$",
              "$\\mu_{21}$", "$\\mu_{12}$",
              "$\\omega_{{\\text{player}}}$")
goals = tibble(Parameter,
               Estimative = c(exp(mod_5$gamma), exp(mod_5$tau),
                              exp(mod_5$lambda_xy), exp(mod_5$mu_xy),
                              mod_5$omega))
kable(goals, digits = 4, caption = "Goal rate parameters")
```

Table 2: Goal rate parameters

Parameter	Estimative
γ_h	1.6932
τ	1.2437
λ_{10}	0.7527
λ_{01}	1.0702
λ_{11}	1.1343
λ_{22}	0.8118
λ_{21}	0.7479
λ_{12}	1.0426
μ_{10}	1.3970
μ_{01}	0.7985
μ_{11}	1.0650
μ_{22}	1.4137
μ_{21}	1.2809
μ_{12}	0.7306
ω_{player}	0.4574

```
Parameter = c("$A_\\lambda$", "$A_\\mu$")
reds = tibble(Parameter, Estimative = exp(mod_5$a))
kable(reds, digits = 8, caption = "Red card rate parameters")
```

Table 3: Red card rate parameters

Parameter	Estimative
A_λ	0.00001921

Parameter	Estimative
A_μ	0.00003334

```
Parameter = c("$\\eta_1$", "$\\eta_2$", "$\\rho_1$", "$\\rho_2$", "$\\kappa$")
st = tibble(Parameter,
            Estimative = c(mod_5$eta, mod_5$rho, mod_5$kappa))
kable(st, digits = 4, caption = "Stoppage time parameters")
```

Table 4: Stoppage time parameters

Parameter	Estimative
η_1	3.0211
η_2	3.9504
ρ_1	1.1416
ρ_2	0.1988
κ	1.4011

```
mod_5$loglik
```

```
## [1] -1522.751
```