Parameters Série A 2018 model 1

Rates for the home and away goals

$$\ln \lambda_k(t) = \ln \alpha_i + \ln \beta_j + \ln \gamma_h + \mathbb{I}\{\text{half} = 2\} \ln \tau + \omega_{\lambda x} x(t) + \omega_{\lambda y} y(t) + \omega_{\lambda(y^* - x^*)} x^*(t)$$
$$\ln \mu_k(t) = \ln \alpha_j + \ln \beta_i + \mathbb{I}\{\text{half} = 2\} \ln \tau + \omega_{\mu x} x(t) + \omega_{\mu y} y(t) + \omega_{\mu(x^* - y^*)} y^*(t)$$

- i: home team index;
- j: away team index;
- α: attack strength parameter;
- $1/\beta$: defense strength parameter;
- γ_h: home advantage parameter;
- τ: second half parameter;
- $x(t)$: the number of goals of the home team until minute t;
- $y(t)$: the number of goals of the away team until minute t;
- $x^*(t)$: the number of red cards of the home team until minute t;
- $y^*(t)$: the number of red cards of the away team until minute t;
- $\omega_{\lambda x}$, $\omega_{\lambda y}$, $\omega_{\mu x}$ and $\omega_{\mu y}$: parameters that measure the impact of the scored goals in the rates;
- $\omega_{\lambda(y^* - x^*)}$ and $\omega_{\mu(x^* - y^*)}$: parameters that measure the impact of having extra players on the field.

Rates for the home and away red cards

$$\lambda_k^*(t) = A_\lambda \left(t + 45^{\mathbb{I}\{\text{half} = 2\}} \right)$$
$$\mu_k^*(t) = A_\mu \left(t + 45^{\mathbb{I}\{\text{half} = 2\}} \right)$$

Stoppage time

The stoppage time for the first half, U^1, and the second half, U^2, are modeled as:

$$U^1 \sim \text{Poisson}(\eta_1 + \rho_1 r^1)$$
$$U^2 \sim \text{Poisson}(\eta_2 + \rho_2 r^2 + \kappa c)$$

- r^t is the amount of red cards received in half t until minute 45;
- $c = \begin{cases} 1, & \text{if } |x - y| \leq 1 \text{ at minute 45 of the second half;} \\ 0, & \text{otherwise.} \end{cases}$

Constraint

The constraint for identificability is

$$\sum_{i}^{n} \log(\alpha_i) = \sum_{i}^{n} \log(\beta_i).$$

```
options(knitr.kable.NA = "-")
options(scipen = 999)

library(dplyr)
library(knitr)

load("data/input.RData")
load("data/mod_1.RData")
```

```
alphas_betas = tibble(Team = times$Time,
                alpha = exp(mod_1$alpha),
                beta = exp(mod_1$beta))
kable(alphas_betas, digits = 4, caption = "Alphas and betas",
      col.names = c("Team", "$\\alpha$", "$\\beta$"))
```

Table 1: Alphas and betas

Team	α	β
América-MG	0.0592	0.0955
Athletico-PR	0.1170	0.0726
Atlético-MG	0.1134	0.0832
Bahia	0.0757	0.0820
Botafogo	0.0767	0.0903
Ceará	0.0612	0.0737
Chapecoense	0.0645	0.1008
Corinthians	0.0673	0.0678
Cruzeiro	0.0670	0.0681
Flamengo	0.1266	0.0540
Fluminense	0.0647	0.0936
Grêmio	0.0946	0.0518
Internacional	0.1022	0.0544
Palmeiras	0.1365	0.0464
Paraná	0.0329	0.1237
Santos	0.0939	0.0791
São Paulo	0.0956	0.0634
Sport	0.0675	0.1146
Vasco da Gama	0.0755	0.0950
Vitória	0.0675	0.1358

```
Parameter = c("$\\gamma_h$", "$\\tau$", "$\\omega_{\\lambda x}$",
              "$\\omega_{\\lambda y}$", "$\\omega_{\\mu x}$",
              "$\\omega_{\\mu y}$", "$\\omega_{\\lambda (y^*-x^*)}$",
              "$\\omega_{\\mu_(x^*-y^*)}$")
goals = tibble(Parameter,
               Estimative = c(exp(mod_1$gamma), exp(mod_1$tau), mod_1$omega))
```

```
kable(goals, digits = 4, caption = "Goal rate parameters")
```

Table 2: Goal rate parameters

Parameter	Estimative
γ_h	1.8018
τ	1.3757
$\omega_{\lambda x}$	-0.2378
$\omega_{\lambda y}$	0.3064
$\omega_{\mu x}$	0.0974
$\omega_{\mu y}$	-0.1270
$\omega_{\lambda(y^*-x^*)}$	0.3886
$\omega_{\mu_(x^*-y^*)}$	-0.0357

```
Parameter = c("$A_\\lambda$", "$A_\\mu$")
reds = tibble(Parameter, Estimative = exp(mod_1$a))
kable(reds, digits = 8, caption = "Red card rate parameters")
```

Table 3: Red card rate parameters

Parameter	Estimative
A_λ	0.00001960
A_μ	0.00003227

```
Parameter = c("$\\eta_1$", "$\\eta_2$", "$\\rho_1$", "$\\rho_2$", "$\\kappa$")
st = tibble(Parameter,
            Estimative = c(mod_1$eta, mod_1$rho, mod_1$kappa))
kable(st, digits = 4, caption = "Stoppage time parameters")
```

Table 4: Stoppage time parameters

Parameter	Estimative
η_1	2.3879
η_2	3.5898
ρ_1	0.9702
ρ_2	0.2034
κ	1.0271

```
mod_1$loglik
```

```
## [1] -21.18098
```