Parameters Série A 2020

Rates for the home and away teams

Model 0

$$\lambda_k = \alpha_i \beta_j \gamma_h$$
$$\mu_k = \alpha_j \beta_i$$

- i: home team index;
- j: away team index;
- α: attack strength parameter;
- $1/\beta$: defense strength parameter;
- γ_h: home advantage parameter.

Model 1

$$\lambda_k(t) = \alpha_i \beta_j \gamma_h$$
$$\mu_k(t) = \alpha_j \beta_i$$

Model 2

$$\lambda_k(t) = \alpha_i \beta_j \gamma_h \tau^{\mathbb{I}\{\text{half} = 2\}}$$
$$\mu_k(t) = \alpha_j \beta_i \tau^{\mathbb{I}\{\text{half} = 2\}}$$

- τ: second half parameter.

Model 3

$$\lambda_k(t) = \alpha_i \beta_j \gamma_h \tau^{\mathbb{I}\{\text{half} = 2\}} \lambda_{xy}$$
$$\mu_k(t) = \alpha_j \beta_i \tau^{\mathbb{I}\{\text{half} = 2\}} \mu_{xy}$$

- $\lambda_{xy} = \begin{cases} 1, \text{ if } x = y; \\ \lambda_{10}, \text{ if } x > y; \\ \lambda_{01}, \text{ if } x < y; \end{cases}$
- $\mu_{xy} = \begin{cases} 1, \text{ if } x = y; \\ \mu_{10}, \text{ if } x > y; \\ \mu_{01}, \text{ if } x < y. \end{cases}$

Model 4

$$\lambda_k(t) = \alpha_i \beta_j \gamma_h \tau^{\mathbb{I}\{\text{half } = \, 2\}} e^{x(t)\omega_{\lambda x}} e^{y(t)\omega_{\lambda y}}$$
$$\mu_k(t) = \alpha_j \beta_i \tau^{\mathbb{I}\{\text{half } = \, 2\}} e^{x(t)\omega_{\mu x}} e^{y(t)\omega_{\mu y}}$$

- $x(t)$ is the number of goals of the home team until minute t;
- $y(t)$ is the number of goals of the away team until minute t;
- $\omega_{\lambda x}$, $\omega_{\lambda y}$, $\omega_{\mu x}$ and $\omega_{\mu y}$: parameters that measure the impact of the scored goals in the rates.

Stoppage time

For all models except model 0, the stoppage time for the first half, U^1, and the second half, U^2, are modeled as:

$$U^1 \sim \text{Poisson}(\eta_1 + \phi_1 g^1)$$
$$U^2 \sim \text{Poisson}(\eta_2 + \phi_2 g^2 + \kappa c)$$

- g^t is the amount of goals scored in half t until minute 45;
- $c = \begin{cases} 1, & \text{if } |x - y| \leq 1 \text{ at minute 45 of the second half;} \\ 0, & \text{otherwise.} \end{cases}$

Constraint

The constraint for identificability in all models is

$$\sum_i^n \log(\alpha_i) = \sum_i^n \log(\beta_i)$$

```
options(knitr.kable.NA = "-")

library(dplyr)
library(knitr)

load("data/input.RData")
load("data/mod_0.RData")
load("data/mod_1.RData")
load("data/mod_2.RData")
load("data/mod_3.RData")
load("data/mod_4.RData")
```

```
alphas = tibble(Time = times$Time,
                mod_0 = exp(mod_0$alpha),
                mod_1 = exp(mod_1$alpha),
                mod_2 = exp(mod_2$alpha),
                mod_3 = exp(mod_3$alpha),
                mod_4 = exp(mod_4$alpha))
kable(alphas, digits = 4, caption = "Alphas")
```

Table 1: Alphas

Time	mod_0	mod_1	mod_2	mod_3	mod_4
Athletico-PR	0.8296	0.0838	0.0811	0.0741	0.0759
Atlético-MG	0.8820	0.0889	0.0860	0.0799	0.0821
Atlético	1.4131	0.1423	0.1376	0.1278	0.1359
Bahia	1.0757	0.1082	0.1046	0.0938	0.0963
Botafogo	0.7187	0.0721	0.0697	0.0619	0.0628
Ceará	1.1997	0.1210	0.1170	0.1077	0.1094
Corinthians	0.9925	0.0997	0.0964	0.0901	0.0929
Coritiba	0.6900	0.0699	0.0676	0.0597	0.0609
Flamengo	1.5069	0.1518	0.1469	0.1361	0.1423
CSA	1.2097	0.1221	0.1181	0.1108	0.1136
Fortaleza	0.7486	0.0760	0.0735	0.0669	0.0681
Goiás	0.9225	0.0921	0.0891	0.0820	0.0818
Grêmio	1.1630	0.1176	0.1137	0.1083	0.1120
Internacional	1.3315	0.1337	0.1293	0.1239	0.1287
Palmeiras	1.1152	0.1130	0.1093	0.1034	0.1084
Red Bull Bragantino	1.0970	0.1103	0.1067	0.1031	0.1051
Santos	1.1551	0.1145	0.1107	0.1026	0.1080
São Paulo	1.2965	0.1303	0.1261	0.1162	0.1191
Sport	0.6870	0.0695	0.0672	0.0606	0.0618
Vasco da Gama	0.8257	0.0830	0.0803	0.0732	0.0740

```
betas = tibble(Time = times$Time,
               mod_0 = exp(mod_0$beta),
               mod_1 = exp(mod_1$beta),
               mod_2 = exp(mod_2$beta),
               mod_3 = exp(mod_3$beta),
               mod_4 = exp(mod_4$beta))
kable(betas, digits = 4, caption = "Betas")
```

Table 2: Betas

Time	mod_0	mod_1	mod_2	mod_3	mod_4
Athletico-PR	0.7768	0.0785	0.0760	0.0719	0.0724
Atlético-MG	0.9735	0.0983	0.0950	0.0878	0.0894
Atlético	1.0001	0.1008	0.0975	0.0887	0.0901
Bahia	1.2889	0.1297	0.1254	0.1194	0.1248
Botafogo	1.3304	0.1333	0.1289	0.1229	0.1264
Ceará	1.1212	0.1128	0.1091	0.1008	0.1035
Corinthians	0.9789	0.0980	0.0948	0.0861	0.0887
Coritiba	1.1571	0.1172	0.1134	0.1088	0.1128
Flamengo	1.0720	0.1077	0.1042	0.0953	0.0975
CSA	0.9238	0.0930	0.0900	0.0819	0.0844
Fortaleza	0.9456	0.0962	0.0930	0.0878	0.0900
Goiás	1.3657	0.1365	0.1321	0.1227	0.1301
Grêmio	0.8777	0.0887	0.0858	0.0774	0.0783
Internacional	0.7746	0.0780	0.0754	0.0666	0.0683
Palmeiras	0.8099	0.0820	0.0794	0.0719	0.0722
Red Bull Bragantino	0.8748	0.0879	0.0850	0.0762	0.0777
Santos	1.1186	0.1109	0.1072	0.0986	0.1011

Time	mod_0	mod_1	mod_2	mod_3	mod_4
São Paulo	0.9058	0.0911	0.0881	0.0810	0.0842
Sport	1.0712	0.1083	0.1047	0.0974	0.1009
Vasco da Gama	1.2081	0.1219	0.1179	0.1107	0.1163

```
rate = tibble(Model = 0:4,
              gamma = c(exp(mod_0$gamma), exp(mod_1$gamma), exp(mod_2$gamma),
                        exp(mod_3$gamma), exp(mod_4$gamma)),
              tau = c(NA, NA, exp(mod_2$tau), exp(mod_3$tau), exp(mod_4$tau)),
              lambda_10 = c(NA, NA, NA, exp(mod_3$lambda_xy["10"]),
                            mod_4$omega["lambda_x"]),
              lambda_01 = c(NA, NA, NA, exp(mod_3$lambda_xy["01"]),
                            mod_4$omega["lambda_y"]),
              mu_10 = c(NA, NA, NA, exp(mod_3$mu_xy["10"]), mod_4$omega["mu_x"]),
              mu_01 = c(NA, NA, NA, exp(mod_3$mu_xy["01"]), mod_4$omega["mu_y"]))
kable(rate, digits = 4, caption = "Other parameters",
      col.names = c("Model", "$\\gamma_h$", "$\\tau$",
                    "$\\lambda_{10} \\text{ or } \\omega_{\\lambda x}$",
                    "$\\lambda_{01} \\text{ or } \\omega_{\\lambda y}$",
                    "$\\mu_{10} \\text{ or } \\omega_{\\mu x}$",
                    "$\\mu_{01} \\text{ or } \\omega_{\\mu y}$"))
```

Table 3: Other parameters

Model	γ_h	τ	λ_{10} or $\omega_{\lambda x}$	λ_{01} or $\omega_{\lambda y}$	μ_{10} or $\omega_{\mu x}$	μ_{01} or $\omega_{\mu y}$
0	1.3137	–	–	–	–	–
1	1.3146	–	–	–	–	–
2	1.3146	1.1346	–	–	–	–
3	1.6784	1.1267	0.7592	0.9981	1.4699	1.1435
4	1.6019	1.1832	-0.2079	0.0400	0.1222	-0.0311

```
st = tibble(Model = 0:4,
            eta_1 = c(NA, mod_1$eta[1], mod_2$eta[1], mod_3$eta[1], mod_4$eta[1]),
            eta_2 = c(NA, mod_1$eta[2], mod_2$eta[2], mod_3$eta[2], mod_4$eta[2]),
            phi_1 = c(NA, mod_1$phi[1], mod_2$phi[1], mod_3$phi[1], mod_4$phi[1]),
            phi_2 = c(NA, mod_1$phi[2], mod_2$phi[2], mod_3$phi[2], mod_4$phi[2]),
            kappa = c(NA, mod_1$kappa, mod_2$kappa, mod_3$kappa, mod_4$kappa))
kable(st, digits = 4, caption = "Stoppage time parameters",
      col.names = c("Model", "$\\eta_1$", "$\\eta_2$", "$\\phi_1$", "$\\phi_2$",
                    "$\\kappa$"))
```

Table 4: Stoppage time parameters

Model	η_1	η_2	ϕ_1	ϕ_2	κ
0	–	–	–	–	–
1	2.9580	4.6987	0.0556	0.0354	1.208
2	2.9581	4.6988	0.0556	0.0354	1.208
3	2.9580	4.6987	0.0556	0.0354	1.208
4	2.9580	4.6987	0.0556	0.0354	1.208

```
log_lik = tibble(Model = 1:4,
                 `log-likelihood` = c(mod_1$loglik, mod_2$loglik, mod_3$loglik, mod_4$loglik))
kable(log_lik, digits = 4, caption = "Log-likelihoods")
```

Table 5: Log-likelihoods

Model	log-likelihood
1	2197.634
2	2199.505
3	2209.217
4	2209.288