Parameters 2020 model 2

Rates for the home and away goals

$$\ln \lambda_k(t) = \ln \alpha_i + \ln \beta_j + \ln \gamma_h + \mathbb{I}\{\text{half} = 2\} \ln \tau + \omega_{\lambda(x-y)}(x(t) - y(t)) + \omega_{\lambda(y^*-x^*)}(y^*(t) - x^*(t))$$
$$\ln \mu_k(t) = \ln \alpha_j + \ln \beta_i + \mathbb{I}\{\text{half} = 2\} \ln \tau + \omega_{\mu(y-x)}(y(t) - x(t)) + \omega_{\mu(x^*-y^*)}(x^*(t) - y^*(t))$$

- i: home team index;
- j: away team index;
- α: attack strength parameter;
- $1/\beta$: defense strength parameter;
- γ_h: home advantage parameter;
- τ: second half parameter;
- $x(t)$: the number of goals of the home team until minute t;
- $y(t)$: the number of goals of the away team until minute t;
- $x^*(t)$: the number of red cards of the home team until minute t;
- $y^*(t)$: the number of red cards of the away team until minute t;
- $\omega_{\lambda(x-y)}$, $\omega_{\mu(y-x)}$: parameters that measure the impact of leading in the score in the rates;
- $\omega_{\lambda(y^*-x^*)}$ and $\omega_{\mu(x^*-y^*)}$: parameters that measure the impact of having extra players on the field.

Rates for the home and away red cards

$$\lambda_k^*(t) = A_\lambda\left(t + 45^{\mathbb{I}\{\text{half} = 2\}}\right)$$
$$\mu_k^*(t) = A_\mu\left(t + 45^{\mathbb{I}\{\text{half} = 2\}}\right)$$

Stoppage time

The stoppage time for the first half, U^1, and the second half, U^2, are modeled as:

$$U^1 \sim \text{Poisson}(\eta_1 + \rho_1 r^1)$$
$$U^2 \sim \text{Poisson}(\eta_2 + \rho_2 r^2 + \kappa c)$$

- r^t is the amount of red cards received in half t until minute 45;
- $c = \begin{cases} 1, & \text{if } |x - y| \leq 1 \text{ at minute 45 of the second half;} \\ 0, & \text{otherwise.} \end{cases}$

Constraint

The constraint for identificability is

$$\sum_{i}^{n} \log(\alpha_i) = \sum_{i}^{n} \log(\beta_i).$$

```
options(knitr.kable.NA = "-")
options(scipen = 999)

library(dplyr)
library(knitr)

load("data/input.RData")
load("data/mod_2.RData")
```

```
alphas_betas = tibble(Team = times$Time,
                alpha = exp(mod_2$alpha),
                beta = exp(mod_2$beta))
kable(alphas_betas, digits = 4, caption = "Alphas and betas",
      col.names = c("Team", "$\\alpha$", "$\\beta$"))
```

Table 1: Alphas and betas

Team	α	β
Athletico-PR	0.0794	0.0750
Atlético-GO	0.0848	0.0942
Atlético-MG	0.1412	0.0930
Bahia	0.0984	0.1301
Botafogo	0.0650	0.1325
Ceará	0.1142	0.1066
Corinthians	0.0951	0.0913
Coritiba	0.0637	0.1153
Flamengo	0.1480	0.0991
Fluminense	0.1176	0.0876
Fortaleza	0.0721	0.0917
Goiás	0.0823	0.1333
Grêmio	0.1146	0.0827
Internacional	0.1352	0.0682
Palmeiras	0.1111	0.0762
Red Bull Bragantino	0.1096	0.0818
Santos	0.1097	0.1062
São Paulo	0.1253	0.0857
Sport	0.0623	0.1076
Vasco da Gama	0.0788	0.1190

```
Parameter = c("$\\gamma_h$", "$\\tau$", "$\\omega_{\\lambda (x-y)}$",
              "$\\omega_{\\mu (y-x)}$", "$\\omega_{\\lambda (y^*-x^*)}$",
              "$\\omega_{\\mu_(x^*-y^*)}$")
goals = tibble(Parameter,
               Estimative = c(exp(mod_2$gamma), exp(mod_2$tau), mod_2$omega))
kable(goals, digits = 4, caption = "Goal rate parameters")
```

Table 2: Goal rate parameters

Parameter	Estimative
γ_h	1.3797
τ	1.1396
$\omega_{\lambda(x-y)}$	-0.1459
$\omega_{\mu(y-x)}$	-0.0991
$\omega_{\lambda(y^*-x^*)}$	0.3456
$\omega_{\mu_(x^*-y^*)}$	0.4402

```
Parameter = c("$A_\\lambda$", "$A_\\mu$")
reds = tibble(Parameter, Estimative = exp(mod_2$a))
kable(reds, digits = 8, caption = "Red card rate parameters")
```

Table 3: Red card rate parameters

Parameter	Estimative
A_λ	0.00002736
A_μ	0.00002960

```
Parameter = c("$\\eta_1$", "$\\eta_2$", "$\\rho_1$", "$\\rho_2$", "$\\kappa$")
st = tibble(Parameter,
            Estimative = c(mod_2$eta, mod_2$rho, mod_2$kappa))
kable(st, digits = 4, caption = "Stoppage time parameters")
```

Table 4: Stoppage time parameters

Parameter	Estimative
η_1	2.9222
η_2	4.7355
ρ_1	1.8709
ρ_2	0.1346
κ	1.1871

```
mod_2$loglik
```

```
## [1] -1410.491
```