Parameters 2020 model 3

Rates for the home and away goals

$$\ln \lambda_k(t) = \ln \alpha_i + \ln \beta_j + \ln \gamma_h + \mathbb{I}\{\text{half} = 2\} \ln \tau + \omega_{\text{goal}}(x(t) - y(t)) + \omega_{\text{player}}(y^*(t) - x^*(t))$$
$$\ln \mu_k(t) = \ln \alpha_j + \ln \beta_i + \mathbb{I}\{\text{half} = 2\} \ln \tau + \omega_{\text{goal}}(y(t) - x(t)) + \omega_{\text{player}}(x^*(t) - y^*(t))$$

- i: home team index;
- j: away team index;
- α: attack strength parameter;
- $1/\beta$: defense strength parameter;
- γ_h: home advantage parameter;
- τ: second half parameter;
- $x(t)$: the number of goals of the home team until minute t;
- $y(t)$: the number of goals of the away team until minute t;
- $x^*(t)$: the number of red cards of the home team until minute t;
- $y^*(t)$: the number of red cards of the away team until minute t;
- ω_{goal}: parameter that measure the impact of leading in the score in the rates;
- ω_{player}: parameter that measure the impact of having extra players on the field.

Rates for the home and away red cards

$$\lambda_k^*(t) = A_\lambda \left(t + 45^{\mathbb{I}\{\text{half} = 2\}} \right)$$
$$\mu_k^*(t) = A_\mu \left(t + 45^{\mathbb{I}\{\text{half} = 2\}} \right)$$

Stoppage time

The stoppage time for the first half, U^1, and the second half, U^2, are modeled as:

$$U^1 \sim \text{Poisson}(\eta_1 + \rho_1 r^1)$$
$$U^2 \sim \text{Poisson}(\eta_2 + \rho_2 r^2 + \kappa c)$$

- r^t is the amount of red cards received in half t until minute 45;
- $c = \begin{cases} 1, & \text{if } |x - y| \leq 1 \text{ at minute 45 of the second half;} \\ 0, & \text{otherwise.} \end{cases}$

Constraint

The constraint for identificability is

$$\sum_{i}^{n} \log(\alpha_i) = \sum_{i}^{n} \log(\beta_i).$$

```
options(knitr.kable.NA = "-")
options(scipen = 999)

library(dplyr)
library(knitr)

load("data/input.RData")
load("data/mod_3.RData")
```

```
alphas_betas = tibble(Team = times$Time,
                alpha = exp(mod_3$alpha),
                beta = exp(mod_3$beta))
kable(alphas_betas, digits = 4, caption = "Alphas and betas",
      col.names = c("Team", "$\\alpha$", "$\\beta$"))
```

Table 1: Alphas and betas

Team	α	β
Athletico-PR	0.0793	0.0749
Atlético-GO	0.0847	0.0941
Atlético-MG	0.1398	0.0918
Bahia	0.0984	0.1301
Botafogo	0.0650	0.1323
Ceará	0.1144	0.1070
Corinthians	0.0950	0.0914
Coritiba	0.0638	0.1152
Flamengo	0.1485	0.0995
Fluminense	0.1172	0.0875
Fortaleza	0.0720	0.0917
Goiás	0.0823	0.1337
Grêmio	0.1147	0.0826
Internacional	0.1357	0.0685
Palmeiras	0.1106	0.0757
Red Bull Bragantino	0.1092	0.0814
Santos	0.1090	0.1054
São Paulo	0.1250	0.0857
Sport	0.0624	0.1076
Vasco da Gama	0.0784	0.1184

```
Parameter = c("$\\gamma_h$", "$\\tau$", "$\\omega_{{\\text{goal}}}$",
              "$\\omega_{{\\text{player}}}$")
goals = tibble(Parameter,
               Estimative = c(exp(mod_3$gamma), exp(mod_3$tau), mod_3$omega))
kable(goals, digits = 4, caption = "Goal rate parameters")
```

Table 2: Goal rate parameters

Parameter	Estimative
γ_h	1.3806
τ	1.1358
ω_{goal}	-0.1256
ω_{player}	0.3878

```
Parameter = c("$A_\\lambda$", "$A_\\mu$")
reds = tibble(Parameter, Estimative = exp(mod_3$a))
kable(reds, digits = 8, caption = "Red card rate parameters")
```

Table 3: Red card rate parameters

Parameter	Estimative
A_λ	0.00002736
A_μ	0.00002960

```
Parameter = c("$\\eta_1$", "$\\eta_2$", "$\\rho_1$", "$\\rho_2$", "$\\kappa$")
st = tibble(Parameter,
            Estimative = c(mod_3$eta, mod_3$rho, mod_3$kappa))
kable(st, digits = 4, caption = "Stoppage time parameters")
```

Table 4: Stoppage time parameters

Parameter	Estimative
η_1	2.9222
η_2	4.7355
ρ_1	1.8709
ρ_2	0.1346
κ	1.1870

```
mod_3$loglik
```

```
## [1] -1410.89
```