

Parameters 2015-2020 model 9

Rates for the home and away goals

$$\begin{aligned}\ln \lambda_k(t) &= \ln \alpha_i + \ln \beta_j + \ln \gamma_h + \mathbb{I}\{\text{half} = 2\} \ln \tau + \ln \lambda_{xy} + \omega_{\text{player}}(y^*(t) - x^*(t)) + \omega_{\text{value}}(v_i - v_j) \\ \ln \mu_k(t) &= \ln \alpha_j + \ln \beta_i + \mathbb{I}\{\text{half} = 2\} \ln \tau + \ln \mu_{xy} + \omega_{\text{player}}(x^*(t) - y^*(t)) + \omega_{\text{value}}(v_j - v_i)\end{aligned}$$

- i : home team index;
- j : away team index;
- α : attack strength parameter;
- $1/\beta$: defense strength parameter;
- γ_h : home advantage parameter;
- τ : second half parameter;
- $x(t)$: the number of goals of the home team until minute t ;
- $y(t)$: the number of goals of the away team until minute t ;
- $x^*(t)$: the number of red cards of the home team until minute t ;
- $y^*(t)$: the number of red cards of the away team until minute t ;
- v_i : the value of the home team's starting roster in millions of euros;
- v_j : the value of the away team's starting roster in millions of euros;
- $\lambda_{xy} = \begin{cases} 1, & \text{for } x = y; \\ \omega_{\text{ahead}}, & \text{for } x - y \geq 1; \\ \omega_{\text{behind}}, & \text{for } x - y \leq -1; \end{cases}$
- $\mu_{xy} = \begin{cases} 1, & \text{for } x = y; \\ \lambda_{\text{ahead}}, & \text{for } y - x \geq 1; \\ \lambda_{\text{behind}}, & \text{for } y - x \leq -1; \end{cases}$
- ω_{player} : parameter that measure the impact of having extra players on the field;
- ω_{value} : parameter that measure the impact of the difference in value of the starting rosters.

Rates for the home and away red cards

$$\begin{aligned}\lambda_k^*(t) &= A_\lambda \left(t + 45^{\mathbb{I}\{\text{half} = 2\}} \right) \\ \mu_k^*(t) &= A_\mu \left(t + 45^{\mathbb{I}\{\text{half} = 2\}} \right)\end{aligned}$$

Stoppage time

The stoppage time for the first half, U^1 , and the second half, U^2 , are modeled as:

$$U^1 \sim \text{Poisson}(\eta_1 + \rho_1 r^1)$$

$$U^2 \sim \text{Poisson}(\eta_2 + \rho_2 r^2 + \kappa c)$$

- r^t is the amount of red cards received in half t until minute 45;
- $c = \begin{cases} 1, & \text{if } |x - y| \leq 1 \text{ at minute 45 of the second half;} \\ 0, & \text{otherwise.} \end{cases}$

Constraint

The constraint for identificability is

$$\sum_i^n \log(\alpha_i) = \sum_i^n \log(\beta_i).$$

```
options(knitr.kable.NA = "-")
options(scipen = 999)
```

```
library(dplyr)
library(knitr)
```

```
load("data/input.RData")
load("data/mod_9.RData")
```

```
alphas_betas = tibble(Team = times$Time,
                      alpha = exp(mod_9$alpha),
                      beta = exp(mod_9$beta))
kable(alphas_betas, digits = 4, caption = "Alphas and betas",
      col.names = c("Team", "$\\alpha$", "$\\beta$"))
```

Table 1: Alphas and betas

| Team | α | β |
|--------------|----------|---------|
| América-MG | 0.0614 | 0.0996 |
| Athletico-PR | 0.0999 | 0.0734 |
| Atlético-GO | 0.0919 | 0.0907 |
| Atlético-MG | 0.1211 | 0.0987 |
| Avaí | 0.0673 | 0.1050 |
| Bahia | 0.1037 | 0.0894 |
| Botafogo | 0.0873 | 0.0864 |
| Ceará | 0.0951 | 0.0775 |
| Chapecoense | 0.0893 | 0.0957 |
| Corinthians | 0.1001 | 0.0751 |
| Coritiba | 0.0840 | 0.0869 |
| Cruzeiro | 0.0841 | 0.0840 |
| Csa | 0.0567 | 0.1044 |
| Figueirense | 0.0759 | 0.0951 |

| Team | α | β |
|---------------------|----------|---------|
| Flamengo | 0.1145 | 0.0904 |
| Fluminense | 0.0942 | 0.0925 |
| Fortaleza | 0.1015 | 0.0815 |
| Goiás | 0.0989 | 0.1065 |
| Grêmio | 0.1044 | 0.0778 |
| Internacional | 0.0983 | 0.0720 |
| Joinville | 0.0624 | 0.0893 |
| Palmeiras | 0.1215 | 0.0799 |
| Paraná | 0.0425 | 0.1068 |
| Ponte Preta | 0.0969 | 0.0920 |
| Red Bull Bragantino | 0.1130 | 0.0714 |
| Santa Cruz | 0.1047 | 0.1366 |
| Santos | 0.1120 | 0.0787 |
| São Paulo | 0.0985 | 0.0838 |
| Sport | 0.0971 | 0.0996 |
| Vasco da Gama | 0.0849 | 0.0934 |
| Vitória | 0.1028 | 0.1134 |

```

Parameter = c("$\\gamma_h$", "$\\tau$",
              "$\\omega_{\\text{ahead}}$",
              "$\\omega_{\\text{behind}}$",
              "$\\omega_{\\text{player}}$",
              "$\\omega_{\\text{value}}$")
goals = tibble(Parameter,
               Estimative = c(exp(mod_9$gamma),
                             exp(mod_9$tau),
                             exp(mod_9$omega_xy),
                             mod_9$omega))
kable(goals, digits = 4, caption = "Goal rate parameters")

```

Table 2: Goal rate parameters

| Parameter | Estimative |
|--------------------------|------------|
| γ_h | 1.5323 |
| τ | 1.2329 |
| ω_{ahead} | 0.8319 |
| ω_{behind} | 1.1114 |
| ω_{player} | 0.3183 |
| ω_{value} | 0.0062 |

```

Parameter = c("$A_\\lambda$", "$A_\\mu$")
reds = tibble(Parameter, Estimative = exp(mod_9$a))
kable(reds, digits = 8, caption = "Red card rate parameters")

```

Table 3: Red card rate parameters

| Parameter | Estimative |
|-------------|------------|
| A_λ | 0.00001973 |

| Parameter | Estimative |
|-----------|------------|
| A_μ | 0.00003190 |

```
Parameter = c("$\\eta_1$", "$\\eta_2$", "$\\rho_1$", "$\\rho_2$", "$\\kappa$")
st = tibble(Parameter,
             Estimative = c(mod_9$eta, mod_9$rho, mod_9$kappa))
kable(st, digits = 4, caption = "Stoppage time parameters")
```

Table 4: Stoppage time parameters

| Parameter | Estimative |
|-----------|------------|
| η_1 | 2.4359 |
| η_2 | 3.7394 |
| ρ_1 | 1.1400 |
| ρ_2 | 0.2267 |
| κ | 1.0101 |

```
mod_9$loglik
```

```
## [1] -25851.51
```