Parameters Série A 2020

Rates for the home and away teams

Model 0

$$\lambda_k = \alpha_i \beta_j \gamma_h$$
$$\mu_k = \alpha_j \beta_i$$

- i: home team index;
- j: away team index;
- α: attack strength parameter;
- $1/\beta$: defense strength parameter;
- γ_h: home advantage parameter.

Model 1

$$\lambda_k(t) = \alpha_i \beta_j \gamma_h$$
$$\mu_k(t) = \alpha_j \beta_i$$

Model 2

$$\lambda_k(t) = \alpha_i \beta_j \gamma_h \tau^{\mathbb{I}\{\text{half} = 2\}}$$
$$\mu_k(t) = \alpha_j \beta_i \tau^{\mathbb{I}\{\text{half} = 2\}}$$

- τ: second half parameter.

Model 3

$$\lambda_k(t) = \alpha_i \beta_j \gamma_h \tau^{\mathbb{I}\{\text{half} = 2\}} \lambda_{xy}$$
$$\mu_k(t) = \alpha_j \beta_i \tau^{\mathbb{I}\{\text{half} = 2\}} \mu_{xy}$$

- $\lambda_{xy} = \begin{cases} 1, \text{ if } x = y; \\ \lambda_{10}, \text{ if } x > y; \\ \lambda_{01}, \text{ if } x < y; \end{cases}$
- $\mu_{xy} = \begin{cases} 1, \text{ if } x = y; \\ \mu_{10}, \text{ if } x > y; \\ \mu_{01}, \text{ if } x < y. \end{cases}$

Stoppage time

For all models except model 0, the stoppage time for the first half, U^1, and the second half, U^2, are modeled as:

$$U^1 \sim \text{Poisson}(\eta_1 + \phi_1 g^1)$$
$$U^2 \sim \text{Poisson}(\eta_2 + \phi_2 g^2 + \kappa c)$$

- g^t is the amount of goals scored in half t until minute 45;
- $c = \begin{cases} 1, & \text{if } |x - y| \leq 1 \text{ at minute 45 of the second half;} \\ 0, & \text{otherwise.} \end{cases}$

Constraint

The constraint for identifiability in all models is

$$\sum_i^n \log(\alpha_i) = \sum_i^n \log(\beta_i)$$

```r
options(knitr.kable.NA = "-")

library(dplyr)
library(knitr)

load("data/input.RData")
load("data/mod_0.RData")
load("data/mod_1.RData")
load("data/mod_2.RData")
load("data/mod_3.RData")
```

```r
alphas = tibble(Time = times$Time,
                mod_0 = exp(mod_0$alpha),
                mod_1 = exp(mod_1$alpha),
                mod_2 = exp(mod_2$alpha),
                mod_3 = exp(mod_3$alpha))
kable(alphas, digits = 4, caption = "Alphas")
```

Table 1: Alphas

Time	mod_0	mod_1	mod_2	mod_3
Athletico-PR	0.8296	0.0838	0.0811	0.0741
Atlético-MG	0.8820	0.0889	0.0860	0.0799
Atlético	1.4131	0.1423	0.1376	0.1278
Bahia	1.0757	0.1082	0.1046	0.0938
Botafogo	0.7187	0.0721	0.0697	0.0619
Ceará	1.1997	0.1210	0.1170	0.1077
Corinthians	0.9925	0.0997	0.0964	0.0901
Coritiba	0.6900	0.0699	0.0676	0.0597
Flamengo	1.5069	0.1518	0.1469	0.1361
CSA	1.2097	0.1221	0.1181	0.1108

Time	mod_0	mod_1	mod_2	mod_3
Fortaleza	0.7486	0.0760	0.0735	0.0669
Goiás	0.9225	0.0921	0.0891	0.0820
Grêmio	1.1630	0.1176	0.1137	0.1083
Internacional	1.3315	0.1337	0.1293	0.1239
Palmeiras	1.1152	0.1130	0.1093	0.1034
Red Bull Bragantino	1.0970	0.1103	0.1067	0.1031
Santos	1.1551	0.1145	0.1107	0.1026
São Paulo	1.2965	0.1303	0.1261	0.1162
Sport	0.6870	0.0695	0.0672	0.0606
Vasco da Gama	0.8257	0.0830	0.0803	0.0732

```
betas = tibble(Time = times$Time,
               mod_0 = exp(mod_0$beta),
               mod_1 = exp(mod_1$beta),
               mod_2 = exp(mod_2$beta),
               mod_3 = exp(mod_3$beta))
kable(betas, digits = 4, caption = "Betas")
```

Table 2: Betas

Time	mod_0	mod_1	mod_2	mod_3
Athletico-PR	0.7768	0.0785	0.0760	0.0719
Atlético-MG	0.9735	0.0983	0.0950	0.0878
Atlético	1.0001	0.1008	0.0975	0.0888
Bahia	1.2889	0.1297	0.1254	0.1194
Botafogo	1.3304	0.1333	0.1289	0.1229
Ceará	1.1212	0.1128	0.1091	0.1008
Corinthians	0.9789	0.0980	0.0948	0.0861
Coritiba	1.1571	0.1172	0.1134	0.1088
Flamengo	1.0720	0.1077	0.1042	0.0953
CSA	0.9238	0.0930	0.0900	0.0819
Fortaleza	0.9456	0.0962	0.0930	0.0878
Goiás	1.3657	0.1365	0.1321	0.1227
Grêmio	0.8777	0.0887	0.0858	0.0774
Internacional	0.7746	0.0780	0.0754	0.0666
Palmeiras	0.8099	0.0820	0.0794	0.0719
Red Bull Bragantino	0.8748	0.0879	0.0850	0.0762
Santos	1.1186	0.1109	0.1072	0.0986
São Paulo	0.9058	0.0911	0.0881	0.0810
Sport	1.0712	0.1083	0.1047	0.0974
Vasco da Gama	1.2081	0.1219	0.1179	0.1107

```
param = tibble(Model = 0:3,
              gamma = c(exp(mod_0$gamma), exp(mod_1$gamma), exp(mod_2$gamma),
                        exp(mod_3$gamma)),
              tau = c(NA, NA, exp(mod_2$tau), exp(mod_3$tau)),
              lambda_10 = c(NA, NA, NA, exp(mod_3$lambda_xy["10"])),
              lambda_01 = c(NA, NA, NA, exp(mod_3$lambda_xy["01"])),
              mu_10 = c(NA, NA, NA, exp(mod_3$mu_xy["10"])),
```

```
                mu_01 = c(NA, NA, NA, exp(mod_3$mu_xy["01"])),
                eta_1 = c(NA, mod_1$eta[1], mod_2$eta[1], mod_3$eta[1]),
                eta_2 = c(NA, mod_1$eta[2], mod_2$eta[2], mod_3$eta[2]),
                phi_1 = c(NA, mod_1$phi[1], mod_2$phi[1], mod_3$phi[1]),
                phi_2 = c(NA, mod_1$phi[2], mod_2$phi[2], mod_3$phi[2]),
                kappa = c(NA, mod_1$kappa, mod_2$kappa, mod_3$kappa))
kable(param, digits = 4, caption = "Other parameters",
      col.names = c("Model", "$\\gamma_h$", "$\\tau$", "$\\lambda_{10}$",
                    "$\\lambda_{01}$", "$\\mu_{10}$", "$\\mu_{01}$", "$\\eta_1$",
                    "$\\eta_2$", "$\\phi_1$", "$\\phi_2$", "$\\kappa$"))
```

Table 3: Other parameters

Model	γ_h	τ	λ_{10}	λ_{01}	μ_{10}	μ_{01}	η_1	η_2	ϕ_1	ϕ_2	κ
0	1.3137	–	–	–	–	–	–	–	–	–	–
1	1.3146	–	–	–	–	–	2.8358	4.5385	0.1591	0.1236	1.2549
2	1.3146	1.1346	–	–	–	–	2.8358	4.5385	0.1591	0.1236	1.2549
3	1.6784	1.1267	0.7592	0.9981	1.4699	1.1434	2.8358	4.5385	0.1591	0.1236	1.2549