

Parameters 2015-2020 model 6

Rates for the home and away goals

$$\ln \lambda_k(t) = \ln \alpha_i + \ln \beta_j + \ln \gamma_h + \mathbb{I}\{\text{half} = 2\} \ln \tau + \ln \lambda_{xy} + \omega_{\text{player}}(y^*(t) - x^*(t))$$

$$\ln \mu_k(t) = \ln \alpha_j + \ln \beta_i + \mathbb{I}\{\text{half} = 2\} \ln \tau + \ln \mu_{xy} + \omega_{\text{player}}(x^*(t) - y^*(t))$$

- i : home team index;
- j : away team index;
- α : attack strength parameter;
- $1/\beta$: defense strength parameter;
- γ_h : home advantage parameter;
- τ : second half parameter;
- $x(t)$: the number of goals of the home team until minute t ;
- $y(t)$: the number of goals of the away team until minute t ;
- $x^*(t)$: the number of red cards of the home team until minute t ;
- $y^*(t)$: the number of red cards of the away team until minute t ;

$$\lambda_{xy} = \begin{cases} 1, & \text{for } x = y; \\ \lambda_{10}, & \text{for } x - y = 1; \\ \lambda_{20}, & \text{for } x - y \geq 2; \\ \lambda_{01}, & \text{for } x - y = -1; \\ \lambda_{02}, & \text{for } x - y \leq -2. \end{cases}$$

- ω_{player} : parameter that measure the impact of having extra players on the field.

Rates for the home and away red cards

$$\lambda_k^*(t) = A_\lambda \left(t + 45^{\mathbb{I}\{\text{half} = 2\}} \right)$$

$$\mu_k^*(t) = A_\mu \left(t + 45^{\mathbb{I}\{\text{half} = 2\}} \right)$$

Stoppage time

The stoppage time for the first half, U^1 , and the second half, U^2 , are modeled as:

$$U^1 \sim \text{Poisson}(\eta_1 + \rho_1 r^1)$$

$$U^2 \sim \text{Poisson}(\eta_2 + \rho_2 r^2 + \kappa c)$$

- r^t is the amount of red cards received in half t until minute 45;
- $c = \begin{cases} 1, & \text{if } |x - y| \leq 1 \text{ at minute 45 of the second half;} \\ 0, & \text{otherwise.} \end{cases}$

Constraint

The constraint for identificability is

$$\sum_i^n \log(\alpha_i) = \sum_i^n \log(\beta_i).$$

```
options(knitr.kable.NA = "-")
options(scipen = 999)
```

```
library(dplyr)
library(knitr)
```

```
load("data/input.RData")
load("data/mod_6.RData")
```

```
alphas_betas = tibble(Team = times$Time,
                      alpha = exp(mod_6$alpha),
                      beta = exp(mod_6$beta))
kable(alphas_betas, digits = 4, caption = "Alphas and betas",
      col.names = c("Team", "$\\alpha$", "$\\beta$"))
```

Table 1: Alphas and betas

Team	α	β
América-MG	0.0584	0.1047
Athletico-PR	0.1000	0.0732
Atlético-GO	0.0872	0.0953
Atlético-MG	0.1304	0.0919
Avaí	0.0631	0.1124
Bahia	0.1008	0.0925
Botafogo	0.0847	0.0890
Ceará	0.0896	0.0827
Chapecoense	0.0865	0.0984
Corinthians	0.1094	0.0691
Coritiba	0.0806	0.0905
Cruzeiro	0.0893	0.0792
Csa	0.0525	0.1151
Figueirense	0.0741	0.0975

Team	α	β
Flamengo	0.1359	0.0770
Fluminense	0.0971	0.0902
Fortaleza	0.0955	0.0870
Goiás	0.0932	0.1126
Grêmio	0.1169	0.0702
Internacional	0.1030	0.0693
Joinville	0.0585	0.0948
Palmeiras	0.1362	0.0717
Paraná	0.0400	0.1138
Ponte Preta	0.0944	0.0942
Red Bull Bragantino	0.1127	0.0720
Santa Cruz	0.0999	0.1427
Santos	0.1205	0.0739
São Paulo	0.1091	0.0758
Sport	0.0964	0.1001
Vasco da Gama	0.0831	0.0956
Vitória	0.1004	0.1152

```

Parameter = c("$\\gamma_h$", "$\\tau$",
              "$\\lambda_{10}$", "$\\lambda_{20}$",
              "$\\lambda_{01}$", "$\\lambda_{02}$",
              "$\\mu_{10}$", "$\\mu_{20}$",
              "$\\mu_{01}$", "$\\mu_{02}$",
              "$\\omega_{\\{\\text{player}\\}}$")
goals = tibble(Parameter,
               Estimative = c(exp(mod_6$gamma), exp(mod_6$tau),
                             exp(mod_6$lambda_xy), exp(mod_6$mu_xy),
                             mod_6$omega))
kable(goals, digits = 4, caption = "Goal rate parameters")

```

Table 2: Goal rate parameters

Parameter	Estimative
γ_h	1.5359
τ	1.2309
λ_{10}	0.8261
λ_{20}	0.8544
λ_{01}	1.1197
λ_{02}	1.1298
μ_{10}	1.0616
μ_{20}	1.1598
μ_{01}	0.8577
μ_{02}	0.7775
ω_{player}	0.3163

```

Parameter = c("$A_\\lambda$", "$A_\\mu$")
reds = tibble(Parameter, Estimative = exp(mod_6$a))
kable(reds, digits = 8, caption = "Red card rate parameters")

```

Table 3: Red card rate parameters

Parameter	Estimative
A_λ	0.00001973
A_μ	0.00003190

```

Parameter = c("$\\eta_1$", "$\\eta_2$", "$\\rho_1$", "$\\rho_2$", "$\\kappa$")
st = tibble(Parameter,
              Estimative = c(mod_6$eta, mod_6$rho, mod_6$kappa))
kable(st, digits = 4, caption = "Stoppage time parameters")

```

Table 4: Stoppage time parameters

Parameter	Estimative
η_1	2.4360
η_2	3.7394
ρ_1	1.1400
ρ_2	0.2267
κ	1.0101

```
mod_6$loglik
```

```
## [1] -12911.92
```