Parameters 2015-2020 model 7

Rates for the home and away goals

$$\ln \lambda_k(t) = \ln \alpha_i + \ln \beta_j + \ln \gamma_h + \mathbb{I}\{\text{half} = 2\} \ln \tau + \ln \lambda_{xy} + \omega_{\text{player}}(y^*(t) - x^*(t))$$
$$\ln \mu_k(t) = \ln \alpha_j + \ln \beta_i + \mathbb{I}\{\text{half} = 2\} \ln \tau + \ln \mu_{xy} + \omega_{\text{player}}(x^*(t) - y^*(t))$$

- i: home team index;
- j: away team index;
- α: attack strength parameter;
- $1/\beta$: defense strength parameter;
- γ_h: home advantage parameter;
- τ: second half parameter;
- $x(t)$: the number of goals of the home team until minute t;
- $y(t)$: the number of goals of the away team until minute t;
- $x^*(t)$: the number of red cards of the home team until minute t;
- $y^*(t)$: the number of red cards of the away team until minute t;
- $\lambda_{xy} = \begin{cases} 1, \text{ for } x = y; \\ \lambda_{10}, \text{ for } x - y >= 1; \\ \lambda_{01}, \text{ for } x - y <= -1; \end{cases}$
- ω_{player}: parameter that measure the impact of having extra players on the field.

Rates for the home and away red cards

$$\lambda_k^*(t) = A_\lambda \left(t + 45^{\mathbb{I}\{\text{half} = 2\}} \right)$$
$$\mu_k^*(t) = A_\mu \left(t + 45^{\mathbb{I}\{\text{half} = 2\}} \right)$$

Stoppage time

The stoppage time for the first half, U^1, and the second half, U^2, are modeled as:

$$U^1 \sim \text{Poisson}(\eta_1 + \rho_1 r^1)$$
$$U^2 \sim \text{Poisson}(\eta_2 + \rho_2 r^2 + \kappa c)$$

- r^t is the amount of red cards received in half t until minute 45;
- $c = \begin{cases} 1, \text{ if } |x - y| \leq 1 \text{ at minute 45 of the second half;} \\ 0, \text{ otherwise.} \end{cases}$

Constraint

The constraint for identificability is

$$\sum_{i}^{n} \log(\alpha_i) = \sum_{i}^{n} \log(\beta_i).$$

```
options(knitr.kable.NA = "-")
options(scipen = 999)

library(dplyr)
library(knitr)

load("data/input.RData")
load("data/mod_7.RData")
```

```
alphas_betas = tibble(Team = times$Time,
                alpha = exp(mod_7$alpha),
                beta = exp(mod_7$beta))
kable(alphas_betas, digits = 4, caption = "Alphas and betas",
      col.names = c("Team", "$\\alpha$", "$\\beta$"))
```

Table 1: Alphas and betas

Team	α	β
América-MG	0.0585	0.1047
Athletico-PR	0.1000	0.0733
Atlético-GO	0.0872	0.0952
Atlético-MG	0.1302	0.0919
Avaí	0.0631	0.1121
Bahia	0.1007	0.0925
Botafogo	0.0845	0.0890
Ceará	0.0895	0.0826
Chapecoense	0.0865	0.0984
Corinthians	0.1092	0.0691
Coritiba	0.0806	0.0905
Cruzeiro	0.0892	0.0792
Csa	0.0526	0.1149
Figueirense	0.0740	0.0973
Flamengo	0.1358	0.0771
Fluminense	0.0970	0.0901
Fortaleza	0.0954	0.0869
Goiás	0.0934	0.1127
Grêmio	0.1167	0.0703
Internacional	0.1028	0.0692
Joinville	0.0586	0.0948
Palmeiras	0.1361	0.0717
Paraná	0.0400	0.1135
Ponte Preta	0.0944	0.0943
Red Bull Bragantino	0.1125	0.0720
Santa Cruz	0.1000	0.1427
Santos	0.1205	0.0741
São Paulo	0.1090	0.0758

Team	α	β
Sport	0.0964	0.1000
Vasco da Gama	0.0830	0.0953
Vitória	0.1005	0.1154

```
Parameter = c("$\\gamma_h$", "$\\tau$",
              "$\\lambda_{10}$", "$\\lambda_{01}$",
              "$\\mu_{10}$", "$\\mu_{01}$",
              "$\\omega_{{\\text{player}}}$")
goals = tibble(Parameter,
               Estimative = c(exp(mod_7$gamma), exp(mod_7$tau),
                              exp(mod_7$lambda_xy), exp(mod_7$mu_xy),
                              mod_7$omega))
kable(goals, digits = 4, caption = "Goal rate parameters")
```

Table 2: Goal rate parameters

Parameter	Estimative
γ_h	1.5358
τ	1.2335
λ_{10}	0.8346
λ_{01}	1.1208
μ_{10}	1.0875
μ_{01}	0.8401
ω_{player}	0.3158

```
Parameter = c("$A_\\lambda$", "$A_\\mu$")
reds = tibble(Parameter, Estimative = exp(mod_7$a))
kable(reds, digits = 8, caption = "Red card rate parameters")
```

Table 3: Red card rate parameters

Parameter	Estimative
A_λ	0.00001973
A_μ	0.00003190

```
Parameter = c("$\\eta_1$", "$\\eta_2$", "$\\rho_1$", "$\\rho_2$", "$\\kappa$")
st = tibble(Parameter,
            Estimative = c(mod_7$eta, mod_7$rho, mod_7$kappa))
kable(st, digits = 4, caption = "Stoppage time parameters")
```

Table 4: Stoppage time parameters

Parameter	Estimative
η_1	2.4359
η_2	3.7394
ρ_1	1.1400

Parameter	Estimative
ρ_2	0.2267
κ	1.0101

```
mod_7$loglik
```

```
## [1] -12912.77
```