

## Parameters 2020 model 4

### Rates for the home and away goals

$$\ln \lambda_k(t) = \ln \alpha_i + \ln \beta_j + \ln \gamma_h + \mathbb{I}\{\text{half} = 2\} \ln \tau + \omega_{\text{goal}}(x(t) - y(t)) + \omega_{\text{player}}(y^*(t) - x^*(t)) + \omega_{\text{player}^2}(y^*(t) - x^*(t))^2$$

$$\ln \mu_k(t) = \ln \alpha_j + \ln \beta_i + \mathbb{I}\{\text{half} = 2\} \ln \tau + \omega_{\text{goal}}(y(t) - x(t)) + \omega_{\text{player}}(x^*(t) - y^*(t)) + \omega_{\text{player}^2}(x^*(t) - y^*(t))^2$$

- $i$ : home team index;
- $j$ : away team index;
- $\alpha$ : attack strength parameter;
- $1/\beta$ : defense strength parameter;
- $\gamma_h$ : home advantage parameter;
- $\tau$ : second half parameter;
- $x(t)$ : the number of goals of the home team until minute  $t$ ;
- $y(t)$ : the number of goals of the away team until minute  $t$ ;
- $x^*(t)$ : the number of red cards of the home team until minute  $t$ ;
- $y^*(t)$ : the number of red cards of the away team until minute  $t$ ;
- $\omega_{\text{goal}}$ : parameter that measure the impact of leading in the score in the rates;
- $\omega_{\text{player}}$ : parameter that measure the impact of having extra players on the field.

### Rates for the home and away red cards

$$\lambda_k^*(t) = A_\lambda \left( t + 45 \mathbb{I}\{\text{half} = 2\} \right)$$

$$\mu_k^*(t) = A_\mu \left( t + 45 \mathbb{I}\{\text{half} = 2\} \right)$$

### Stoppage time

The stoppage time for the first half,  $U^1$ , and the second half,  $U^2$ , are modeled as:

$$U^1 \sim \text{Poisson}(\eta_1 + \rho_1 r^1)$$

$$U^2 \sim \text{Poisson}(\eta_2 + \rho_2 r^2 + \kappa c)$$

- $r^t$  is the amount of red cards received in half  $t$  until minute 45;
- $c = \begin{cases} 1, & \text{if } |x - y| \leq 1 \text{ at minute 45 of the second half;} \\ 0, & \text{otherwise.} \end{cases}$

## Constraint

The constraint for identificability is

$$\sum_i^n \log(\alpha_i) = \sum_i^n \log(\beta_i).$$

```
options(knitr.kable.NA = "-")
options(scipen = 999)
```

```
library(dplyr)
library(knitr)
```

```
load("data/input.RData")
load("data/mod_4.RData")
```

```
alphas_betas = tibble(Team = times$Time,
                      alpha = exp(mod_4$alpha),
                      beta = exp(mod_4$beta))
kable(alphas_betas, digits = 4, caption = "Alphas and betas",
      col.names = c("Team", "$\\alpha$", "$\\beta$"))
```

Table 1: Alphas and betas

| Team                | $\alpha$ | $\beta$ |
|---------------------|----------|---------|
| Athletico-PR        | 0.0793   | 0.0749  |
| Atlético-GO         | 0.0847   | 0.0941  |
| Atlético-MG         | 0.1403   | 0.0921  |
| Bahia               | 0.0988   | 0.1305  |
| Botafogo            | 0.0650   | 0.1323  |
| Ceará               | 0.1145   | 0.1071  |
| Corinthians         | 0.0951   | 0.0915  |
| Coritiba            | 0.0638   | 0.1154  |
| Flamengo            | 0.1490   | 0.1000  |
| Fluminense          | 0.1174   | 0.0876  |
| Fortaleza           | 0.0721   | 0.0918  |
| Goiás               | 0.0825   | 0.1338  |
| Grêmio              | 0.1148   | 0.0827  |
| Internacional       | 0.1357   | 0.0686  |
| Palmeiras           | 0.1108   | 0.0758  |
| Red Bull Bragantino | 0.1093   | 0.0815  |
| Santos              | 0.1090   | 0.1055  |
| São Paulo           | 0.1252   | 0.0859  |
| Sport               | 0.0624   | 0.1076  |
| Vasco da Gama       | 0.0787   | 0.1188  |

```
Parameter = c("$\\gamma_h$", "$\\tau$", "$\\omega_{\\text{goal}}$",
              "$\\omega_{\\text{player}}$", "$\\omega_{\\text{player}}^2$")
goals = tibble(Parameter,
               Estimative = c(exp(mod_4$gamma), exp(mod_4$tau), mod_4$omega))
kable(goals, digits = 4, caption = "Goal rate parameters")
```

Table 2: Goal rate parameters

| Parameter                  | Estimative |
|----------------------------|------------|
| $\gamma_h$                 | 1.3803     |
| $\tau$                     | 1.1433     |
| $\omega_{\text{goal}}$     | -0.1260    |
| $\omega_{\text{player}}$   | 0.3883     |
| $\omega_{\text{player}^2}$ | -0.0055    |

```
Parameter = c("$A_\\lambda$", "$A_\\mu$")
reds = tibble(Parameter, Estimative = exp(mod_4$a))
kable(reds, digits = 8, caption = "Red card rate parameters")
```

Table 3: Red card rate parameters

| Parameter   | Estimative |
|-------------|------------|
| $A_\lambda$ | 0.00002736 |
| $A_\mu$     | 0.00002960 |

```
Parameter = c("$\\eta_1$", "$\\eta_2$", "$\\rho_1$", "$\\rho_2$", "$\\kappa$")
st = tibble(Parameter,
  Estimative = c(mod_4$eta, mod_4$rho, mod_4$kappa))
kable(st, digits = 4, caption = "Stoppage time parameters")
```

Table 4: Stoppage time parameters

| Parameter | Estimative |
|-----------|------------|
| $\eta_1$  | 2.9222     |
| $\eta_2$  | 4.7355     |
| $\rho_1$  | 1.8709     |
| $\rho_2$  | 0.1346     |
| $\kappa$  | 1.1871     |

```
mod_4$loglik
```

```
## [1] -1410.828
```