Parameters 2015-2020 model 11

Rates for the home and away goals

$$\ln \lambda_k(t) = \ln \alpha_i + \ln \beta_j + \ln \gamma_h + \mathbb{I}\{\text{half} = 2\} \ln \tau + \omega_{\text{goal}}(x(t) - y(t)) + \omega_{\text{player}}(y^*(t) - x^*(t)) + \omega_{\text{value}}(v_i - v_j)$$

$$\ln \mu_k(t) = \ln \alpha_j + \ln \beta_i + \mathbb{I}\{\text{half} = 2\} \ln \tau + \omega_{\text{goal}}(y(t) - x(t)) + \omega_{\text{player}}(x^*(t) - y^*(t)) + \omega_{\text{value}}(v_j - v_i)$$

- i: home team index;

- j: away team index;

- α: attack strength parameter;

- $1/\beta$: defense strength parameter;

- γ_h: home advantage parameter;

- τ: second half parameter;

- $x(t)$: the number of goals of the home team until minute t;

- $y(t)$: the number of goals of the away team until minute t;

- $x^*(t)$: the number of red cards of the home team until minute t;

- $y^*(t)$: the number of red cards of the away team until minute t;

- v_i: the value of the home team's starting roster in millions of euros;

- v_j: the value of the away team's starting roster in millions of euros;

- ω_{goal}: parameter that measure the impact of leading in the score in the rates;

- ω_{player}: parameter that measure the impact of having extra players on the field;

- ω_{value}: parameter that measure the impact of the difference in value of the starting rosters.

Rates for the home and away red cards

$$\lambda_k^*(t) = A_\lambda \left(t + 45^{\mathbb{I}\{\text{half} = 2\}} \right)$$

$$\mu_k^*(t) = A_\mu \left(t + 45^{\mathbb{I}\{\text{half} = 2\}} \right)$$

Stoppage time

The stoppage time for the first half, U^1, and the second half, U^2, are modeled as:

$$U^1 \sim \text{Poisson}(\eta_1 + \rho_1 r^1)$$

$$U^2 \sim \text{Poisson}(\eta_2 + \rho_2 r^2 + \kappa c)$$

- r^t is the amount of red cards received in half t until minute 45;

- $c = \begin{cases} 1, & \text{if } |x - y| \leq 1 \text{ at minute 45 of the second half;} \\ 0, & \text{otherwise.} \end{cases}$

Constraint

The constraint for identificability is

$$\sum_i^n \log(\alpha_i) = \sum_i^n \log(\beta_i).$$

```
options(knitr.kable.NA = "-")
options(scipen = 999)

library(dplyr)
library(knitr)

load("data/input.RData")
load("data/mod_11.RData")
```

```
alphas_betas = tibble(Team = times$Time,
                alpha = exp(mod_11$alpha),
                beta = exp(mod_11$beta))
kable(alphas_betas, digits = 4, caption = "Alphas and betas",
      col.names = c("Team", "$\\alpha$", "$\\beta$"))
```

Table 1: Alphas and betas

Team	α	β
América-MG	0.0607	0.0982
Athletico-PR	0.0994	0.0727
Atlético-GO	0.0914	0.0903
Atlético-MG	0.1203	0.0975
Avaí	0.0662	0.1040
Bahia	0.1030	0.0888
Botafogo	0.0862	0.0859
Ceará	0.0953	0.0764
Chapecoense	0.0883	0.0946
Corinthians	0.0991	0.0748
Coritiba	0.0833	0.0865
Cruzeiro	0.0834	0.0829
Csa	0.0555	0.1038
Figueirense	0.0750	0.0942

Team	α	β
Flamengo	0.1130	0.0896
Fluminense	0.0934	0.0917
Fortaleza	0.1009	0.0814
Goiás	0.0976	0.1064
Grêmio	0.1037	0.0770
Internacional	0.0979	0.0711
Joinville	0.0618	0.0881
Palmeiras	0.1203	0.0789
Paraná	0.0420	0.1052
Ponte Preta	0.0961	0.0906
Red Bull Bragantino	0.1158	0.0709
Santa Cruz	0.1031	0.1341
Santos	0.1112	0.0780
São Paulo	0.0973	0.0833
Sport	0.0959	0.0991
Vasco da Gama	0.0835	0.0935
Vitória	0.1016	0.1121

```
Parameter = c("$\\gamma_h$", "$\\tau$",
              "$\\omega_{{\\text{goal}}}$",
              "$\\omega_{{\\text{player}}}$",
              "$\\omega_{\\text{value}}$")
goals = tibble(Parameter,
               Estimative = c(exp(mod_11$gamma),
                              exp(mod_11$tau),
                              mod_11$omega))
kable(goals, digits = 4, caption = "Goal rate parameters")
```

Table 2: Goal rate parameters

Parameter	Estimative
γ_h	1.5439
τ	1.2250
ω_{goal}	-0.0989
ω_{player}	0.3271
ω_{value}	0.0065

```
Parameter = c("$A_\\lambda$", "$A_\\mu$")
reds = tibble(Parameter, Estimative = exp(mod_11$a))
kable(reds, digits = 8, caption = "Red card rate parameters")
```

Table 3: Red card rate parameters

Parameter	Estimative
A_λ	0.00001973
A_μ	0.00003190

```
Parameter = c("$\\eta_1$", "$\\eta_2$", "$\\rho_1$", "$\\rho_2$", "$\\kappa$")
st = tibble(Parameter,
            Estimative = c(mod_11$eta, mod_11$rho, mod_11$kappa))
kable(st, digits = 4, caption = "Stoppage time parameters")
```

Table 4: Stoppage time parameters

Parameter	Estimative
η_1	2.4360
η_2	3.7394
ρ_1	1.1400
ρ_2	0.2267
κ	1.0101

```
mod_11$loglik
```

```
## [1] -25848.87
```