Parameters Série A 2019 model 1

Rates for the home and away goals

$$\ln \lambda_k(t) = \ln \alpha_i + \ln \beta_j + \ln \gamma_h + \mathbb{I}\{\text{half} = 2\} \ln \tau + \omega_{\lambda x} x(t) + \omega_{\lambda y} y(t) + \omega_{\lambda(y^* - x^*)} x^*(t)$$
$$\ln \mu_k(t) = \ln \alpha_j + \ln \beta_i + \mathbb{I}\{\text{half} = 2\} \ln \tau + \omega_{\mu x} x(t) + \omega_{\mu y} y(t) + \omega_{\mu(x^* - y^*)} y^*(t)$$

- i: home team index;
- j: away team index;
- α: attack strength parameter;
- $1/\beta$: defense strength parameter;
- γ_h: home advantage parameter;
- τ: second half parameter;
- $x(t)$: the number of goals of the home team until minute t;
- $y(t)$: the number of goals of the away team until minute t;
- $x^*(t)$: the number of red cards of the home team until minute t;
- $y^*(t)$: the number of red cards of the away team until minute t;
- $\omega_{\lambda x}$, $\omega_{\lambda y}$, $\omega_{\mu x}$ and $\omega_{\mu y}$: parameters that measure the impact of the scored goals in the rates;
- $\omega_{\lambda(y^* - x^*)}$ and $\omega_{\mu(x^* - y^*)}$: parameters that measure the impact of having extra players on the field.

Rates for the home and away red cards

$$\lambda_k^*(t) = A_\lambda \left(t + 45^{\mathbb{I}\{\text{half} = 2\}} \right)$$
$$\mu_k^*(t) = A_\mu \left(t + 45^{\mathbb{I}\{\text{half} = 2\}} \right)$$

Stoppage time

The stoppage time for the first half, U^1, and the second half, U^2, are modeled as:

$$U^1 \sim \text{Poisson}(\eta_1 + \rho_1 r^1)$$
$$U^2 \sim \text{Poisson}(\eta_2 + \rho_2 r^2 + \kappa c)$$

- r^t is the amount of red cards received in half t until minute 45;
- $c = \begin{cases} 1, & \text{if } |x - y| \leq 1 \text{ at minute 45 of the second half;} \\ 0, & \text{otherwise.} \end{cases}$

Constraint

The constraint for identificability is

$$\sum_{i}^{n} \log(\alpha_i) = \sum_{i}^{n} \log(\beta_i).$$

```
options(knitr.kable.NA = "-")
options(scipen = 999)

library(dplyr)
library(knitr)

load("data/input.RData")
load("data/mod_1.RData")
```

```
alphas_betas = tibble(Team = times$Time,
                alpha = exp(mod_1$alpha),
                beta = exp(mod_1$beta))
kable(alphas_betas, digits = 4, caption = "Alphas and betas",
      col.names = c("Team", "$\\alpha$", "$\\beta$"))
```

Table 1: Alphas and betas

Team	α	β
Athletico-PR	0.1038	0.0631
Atlético-MG	0.0943	0.0946
Avaí	0.0359	0.1308
Bahia	0.0902	0.0858
Botafogo	0.0634	0.0905
Ceará	0.0720	0.0822
Chapecoense	0.0639	0.1046
Corinthians	0.0873	0.0676
Cruzeiro	0.0546	0.0948
Csa	0.0465	0.1232
Flamengo	0.1851	0.0720
Fluminense	0.0781	0.0892
Fortaleza	0.1050	0.0986
Goiás	0.0931	0.1325
Grêmio	0.1364	0.0789
Internacional	0.0903	0.0733
Palmeiras	0.1306	0.0624
Santos	0.1338	0.0610
São Paulo	0.0802	0.0575
Vasco da Gama	0.0792	0.0904

```
Parameter = c("$\\gamma_h$", "$\\tau$", "$\\omega_{\\lambda x}$",
              "$\\omega_{\\lambda y}$", "$\\omega_{\\mu x}$",
              "$\\omega_{\\mu y}$", "$\\omega_{\\lambda (y^*-x^*)}$",
              "$\\omega_{\\mu_(x^*-y^*)}$")
goals = tibble(Parameter,
               Estimative = c(exp(mod_1$gamma), exp(mod_1$tau), mod_1$omega))
```

```
kable(goals, digits = 4, caption = "Goal rate parameters")
```

Table 2: Goal rate parameters

Parameter	Estimative
γ_h	1.6305
τ	1.2199
$\omega_{\lambda x}$	-0.1198
$\omega_{\lambda y}$	0.1016
$\omega_{\mu x}$	0.2114
$\omega_{\mu y}$	-0.1911
$\omega_{\lambda(y^*-x^*)}$	0.4917
$\omega_{\mu_(x^*-y^*)}$	0.3695

```
Parameter = c("$A_\\lambda$", "$A_\\mu$")
reds = tibble(Parameter, Estimative = exp(mod_1$a))
kable(reds, digits = 8, caption = "Red card rate parameters")
```

Table 3: Red card rate parameters

Parameter	Estimative
A_λ	0.00001921
A_μ	0.00003334

```
Parameter = c("$\\eta_1$", "$\\eta_2$", "$\\rho_1$", "$\\rho_2$", "$\\kappa$")
st = tibble(Parameter,
            Estimative = c(mod_1$eta, mod_1$rho, mod_1$kappa))
kable(st, digits = 4, caption = "Stoppage time parameters")
```

Table 4: Stoppage time parameters

Parameter	Estimative
η_1	3.0211
η_2	3.9504
ρ_1	1.1416
ρ_2	0.1989
κ	1.4011

```
mod_1$loglik
```

```
## [1] 1242.245
```