

Parameters 2020 model 5

Rates for the home and away goals

$$\ln \lambda_k(t) = \ln \alpha_i + \ln \beta_j + \ln \gamma_h + \mathbb{I}\{\text{half} = 2\} \ln \tau + \ln \lambda_{xy} + \omega_{\text{player}}(y^*(t) - x^*(t))$$

$$\ln \mu_k(t) = \ln \alpha_j + \ln \beta_i + \mathbb{I}\{\text{half} = 2\} \ln \tau + \ln \mu_{xy} + \omega_{\text{player}}(x^*(t) - y^*(t))$$

- i : home team index;
- j : away team index;
- α : attack strength parameter;
- $1/\beta$: defense strength parameter;
- γ_h : home advantage parameter;
- τ : second half parameter;
- $x(t)$: the number of goals of the home team until minute t ;
- $y(t)$: the number of goals of the away team until minute t ;
- $x^*(t)$: the number of red cards of the home team until minute t ;
- $y^*(t)$: the number of red cards of the away team until minute t ;

$$\lambda_{xy} = \begin{cases} 1, & \text{for } x = 0, y = 0; \\ \lambda_{10}, & \text{for } x = 1, y = 0; \\ \lambda_{01}, & \text{for } x = 0, y = 1; \\ \lambda_{11}, & \text{for } x = 1, y = 1; \\ \lambda_{22}, & \text{for } x - y = 0, x, y \geq 2; \\ \lambda_{21}, & \text{for } x - y \geq 1, x \geq 2; \\ \lambda_{12}, & \text{for } x - y \leq -1, y \geq 2. \end{cases}$$

- ω_{player} : parameter that measure the impact of having extra players on the field.

Rates for the home and away red cards

$$\lambda_k^*(t) = A_\lambda \left(t + 45^{\mathbb{I}\{\text{half} = 2\}} \right)$$

$$\mu_k^*(t) = A_\mu \left(t + 45^{\mathbb{I}\{\text{half} = 2\}} \right)$$

Stoppage time

The stoppage time for the first half, U^1 , and the second half, U^2 , are modeled as:

$$U^1 \sim \text{Poisson}(\eta_1 + \rho_1 r^1)$$

$$U^2 \sim \text{Poisson}(\eta_2 + \rho_2 r^2 + \kappa c)$$

- r^t is the amount of red cards received in half t until minute 45;
- $c = \begin{cases} 1, & \text{if } |x - y| \leq 1 \text{ at minute 45 of the second half;} \\ 0, & \text{otherwise.} \end{cases}$

Constraint

The constraint for identificability is

$$\sum_i^n \log(\alpha_i) = \sum_i^n \log(\beta_i).$$

```
options(knitr.kable.NA = "-")
options(scipen = 999)
```

```
library(dplyr)
library(knitr)
```

```
load("data/input.RData")
load("data/mod_5.RData")
```

```
alphas_betas = tibble(Team = times$Time,
                      alpha = exp(mod_5$alpha),
                      beta = exp(mod_5$beta))
kable(alphas_betas, digits = 4, caption = "Alphas and betas",
      col.names = c("Team", "$\\alpha$", "$\\beta$"))
```

Table 1: Alphas and betas

| Team | α | β |
|---------------|----------|---------|
| Athletico-PR | 0.0731 | 0.0725 |
| Atlético-GO | 0.0805 | 0.0882 |
| Atlético-MG | 0.1322 | 0.0889 |
| Bahia | 0.0923 | 0.1219 |
| Botafogo | 0.0611 | 0.1234 |
| Ceará | 0.1087 | 0.1031 |
| Corinthians | 0.0927 | 0.0852 |
| Coritiba | 0.0587 | 0.1087 |
| Flamengo | 0.1412 | 0.0979 |
| Fluminense | 0.1110 | 0.0836 |
| Fortaleza | 0.0676 | 0.0877 |
| Goiás | 0.0792 | 0.1254 |
| Grêmio | 0.1095 | 0.0772 |
| Internacional | 0.1275 | 0.0648 |

| Team | α | β |
|---------------------|----------|---------|
| Palmeiras | 0.1058 | 0.0717 |
| Red Bull Bragantino | 0.1070 | 0.0774 |
| Santos | 0.1035 | 0.0998 |
| São Paulo | 0.1182 | 0.0825 |
| Sport | 0.0577 | 0.0970 |
| Vasco da Gama | 0.0747 | 0.1113 |

```

Parameter = c("$\\gamma_h$", "$\\tau$",
              "$\\lambda_{10}$", "$\\lambda_{01}$",
              "$\\lambda_{11}$", "$\\lambda_{22}$",
              "$\\lambda_{21}$", "$\\lambda_{12}$",
              "$\\mu_{10}$", "$\\mu_{01}$",
              "$\\mu_{11}$", "$\\mu_{22}$",
              "$\\mu_{21}$", "$\\mu_{12}$",
              "$\\omega_{\\text{player}}$")
goals = tibble(Parameter,
                Estimative = c(exp(mod_5$gamma), exp(mod_5$tau),
                               exp(mod_5$lambda_xy), exp(mod_5$mu_xy),
                               mod_5$omega))
kable(goals, digits = 4, caption = "Goal rate parameters")

```

Table 2: Goal rate parameters

| Parameter | Estimative |
|--------------------------|------------|
| γ_h | 1.7180 |
| τ | 1.1884 |
| λ_{10} | 0.7767 |
| λ_{01} | 0.8905 |
| λ_{11} | 0.7593 |
| λ_{22} | 1.0159 |
| λ_{21} | 0.5935 |
| λ_{12} | 1.0214 |
| μ_{10} | 1.4619 |
| μ_{01} | 0.9683 |
| μ_{11} | 0.7734 |
| μ_{22} | 1.2509 |
| μ_{21} | 1.3393 |
| μ_{12} | 1.1844 |
| ω_{player} | 0.3840 |

```

Parameter = c("$A_\\lambda$", "$A_\\mu$")
reds = tibble(Parameter, Estimative = exp(mod_5$a))
kable(reds, digits = 8, caption = "Red card rate parameters")

```

Table 3: Red card rate parameters

| Parameter | Estimative |
|-------------|------------|
| A_λ | 0.00002736 |

| Parameter | Estimative |
|-----------|------------|
| A_μ | 0.00002960 |

```
Parameter = c("$\\eta_1$", "$\\eta_2$", "$\\rho_1$", "$\\rho_2$", "$\\kappa$")
st = tibble(Parameter,
             Estimative = c(mod_5$eta, mod_5$rho, mod_5$kappa))
kable(st, digits = 4, caption = "Stoppage time parameters")
```

Table 4: Stoppage time parameters

| Parameter | Estimative |
|-----------|------------|
| η_1 | 2.9223 |
| η_2 | 4.7355 |
| ρ_1 | 1.8709 |
| ρ_2 | 0.1346 |
| κ | 1.1870 |

```
mod_5$loglik
```

```
## [1] -1402.943
```