

## Parameters 2020 model 4

### Rates for the home and away goals

$$\ln \lambda_k(t) = \ln \alpha_i + \ln \beta_j + \ln \gamma_h + \mathbb{I}\{\text{half} = 2\} \ln \tau + \omega_{\text{goal}}(x(t) - y(t)) + \omega_{\text{goal}^2}(y(t) - x(t))^2 + \omega_{\text{player}}(y^*(t) - x^*(t))$$

$$\ln \mu_k(t) = \ln \alpha_j + \ln \beta_i + \mathbb{I}\{\text{half} = 2\} \ln \tau + \omega_{\text{goal}}(y(t) - x(t)) + \omega_{\text{goal}^2}(x(t) - y(t))^2 + \omega_{\text{player}}(x^*(t) - y^*(t))$$

- $i$ : home team index;
- $j$ : away team index;
- $\alpha$ : attack strength parameter;
- $1/\beta$ : defense strength parameter;
- $\gamma_h$ : home advantage parameter;
- $\tau$ : second half parameter;
- $x(t)$ : the number of goals of the home team until minute  $t$ ;
- $y(t)$ : the number of goals of the away team until minute  $t$ ;
- $x^*(t)$ : the number of red cards of the home team until minute  $t$ ;
- $y^*(t)$ : the number of red cards of the away team until minute  $t$ ;
- $\omega_{\text{goal}}$ : parameter that measure the impact of leading in the score in the rates;
- $\omega_{\text{player}}$ : parameter that measure the impact of having extra players on the field.

### Rates for the home and away red cards

$$\lambda_k^*(t) = A_\lambda \left( t + 45^{\mathbb{I}\{\text{half} = 2\}} \right)$$

$$\mu_k^*(t) = A_\mu \left( t + 45^{\mathbb{I}\{\text{half} = 2\}} \right)$$

### Stoppage time

The stoppage time for the first half,  $U^1$ , and the second half,  $U^2$ , are modeled as:

$$U^1 \sim \text{Poisson}(\eta_1 + \rho_1 r^1)$$

$$U^2 \sim \text{Poisson}(\eta_2 + \rho_2 r^2 + \kappa c)$$

- $r^t$  is the amount of red cards received in half  $t$  until minute 45;
- $c = \begin{cases} 1, & \text{if } |x - y| \leq 1 \text{ at minute 45 of the second half;} \\ 0, & \text{otherwise.} \end{cases}$

## Constraint

The constraint for identificability is

$$\sum_i^n \log(\alpha_i) = \sum_i^n \log(\beta_i).$$

```
options(knitr.kable.NA = "-")
options(scipen = 999)
```

```
library(dplyr)
library(knitr)
```

```
load("data/input.RData")
load("data/mod_4.RData")
```

```
alphas_betas = tibble(Team = times$Time,
                      alpha = exp(mod_4$alpha),
                      beta = exp(mod_4$beta))
kable(alphas_betas, digits = 4, caption = "Alphas and betas",
      col.names = c("Team", "$\\alpha$", "$\\beta$"))
```

Table 1: Alphas and betas

Team	$\alpha$	$\beta$
Athletico-PR	0.0793	0.0749
Atlético-GO	0.0847	0.0941
Atlético-MG	0.1403	0.0921
Bahia	0.0988	0.1305
Botafogo	0.0650	0.1323
Ceará	0.1145	0.1071
Corinthians	0.0951	0.0915
Coritiba	0.0638	0.1154
Flamengo	0.1490	0.1000
Fluminense	0.1174	0.0876
Fortaleza	0.0721	0.0918
Goiás	0.0825	0.1338
Grêmio	0.1148	0.0827
Internacional	0.1357	0.0686
Palmeiras	0.1108	0.0758
Red Bull Bragantino	0.1093	0.0815
Santos	0.1090	0.1055
São Paulo	0.1252	0.0859
Sport	0.0624	0.1076
Vasco da Gama	0.0787	0.1188

```
Parameter = c("$\\gamma_h$", "$\\tau$", "$\\omega_{\\text{goal}}$",
              "$\\omega_{\\text{player}}$", "$\\omega_{\\text{goal}}^2$")
goals = tibble(Parameter,
                Estimative = c(exp(mod_4$gamma), exp(mod_4$tau), mod_4$omega))
kable(goals, digits = 4, caption = "Goal rate parameters")
```

Table 2: Goal rate parameters

Parameter	Estimative
$\gamma_h$	1.3803
$\tau$	1.1433
$\omega_{\text{goal}}$	-0.1260
$\omega_{\text{player}}$	0.3883
$\omega_{\text{goal}^2}$	-0.0055

```
Parameter = c("$A_\\lambda$", "$A_\\mu$")
reds = tibble(Parameter, Estimative = exp(mod_4$a))
kable(reds, digits = 8, caption = "Red card rate parameters")
```

Table 3: Red card rate parameters

Parameter	Estimative
$A_\lambda$	0.00002736
$A_\mu$	0.00002960

```
Parameter = c("$\\eta_1$", "$\\eta_2$", "$\\rho_1$", "$\\rho_2$", "$\\kappa$")
st = tibble(Parameter,
             Estimative = c(mod_4$eta, mod_4$rho, mod_4$kappa))
kable(st, digits = 4, caption = "Stoppage time parameters")
```

Table 4: Stoppage time parameters

Parameter	Estimative
$\eta_1$	2.9222
$\eta_2$	4.7355
$\rho_1$	1.8709
$\rho_2$	0.1346
$\kappa$	1.1871

```
mod_4$loglik
```

```
## [1] -1410.828
```