Parameters Série A 2018

Rates for the home and away teams

Model 0

$$\lambda_k = \alpha_i \beta_j \gamma_h$$
$$\mu_k = \alpha_j \beta_i$$

- i: home team index;
- j: away team index;
- α: attack strength parameter;
- $1/\beta$: defense strength parameter;
- γ_h: home advantage parameter.

Model 1

$$\lambda_k(t) = \alpha_i \beta_j \gamma_h$$
$$\mu_k(t) = \alpha_j \beta_i$$

Model 2

$$\lambda_k(t) = \alpha_i \beta_j \gamma_h \tau^{\mathbb{I}\{\text{half} = 2\}}$$
$$\mu_k(t) = \alpha_j \beta_i \tau^{\mathbb{I}\{\text{half} = 2\}}$$

- τ: second half parameter.

Model 3

$$\lambda_k(t) = \alpha_i \beta_j \gamma_h \tau^{\mathbb{I}\{\text{half} = 2\}} \lambda_{xy}$$
$$\mu_k(t) = \alpha_j \beta_i \tau^{\mathbb{I}\{\text{half} = 2\}} \mu_{xy}$$

- $\lambda_{xy} = \begin{cases} 1, \text{ if } x = y; \\ \lambda_{10}, \text{ if } x > y; \\ \lambda_{01}, \text{ if } x < y; \end{cases}$
- $\mu_{xy} = \begin{cases} 1, \text{ if } x = y; \\ \mu_{10}, \text{ if } x > y; \\ \mu_{01}, \text{ if } x < y. \end{cases}$

Stoppage time

For all models except model 0, the stoppage time for the first half, U^1, and the second half, U^2, are modeled as:

$$U^1 \sim \text{Poisson}(\eta_1 + \phi_1 g^1)$$
$$U^2 \sim \text{Poisson}(\eta_2 + \phi_2 g^2 + \kappa c)$$

- g^t is the amount of goals scored in half t until minute 45;
- $c = \begin{cases} 1, & \text{if } |x-y| \leq 1 \text{ at minute 45 of the second half;} \\ 0, & \text{otherwise.} \end{cases}$

Constraint

The constraint for identificability in all models is

$$\sum_i^n \log(\alpha_i) = \sum_i^n \log(\beta_i)$$

```r
options(knitr.kable.NA = "-")

library(dplyr)
library(knitr)

load("data/input.RData")
load("data/mod_0.RData")
load("data/mod_1.RData")
load("data/mod_2.RData")
load("data/mod_3.RData")
```

```r
alphas = tibble(Time = times$Time,
                mod_0 = exp(mod_0$alpha),
                mod_1 = exp(mod_1$alpha),
                mod_2 = exp(mod_2$alpha),
                mod_3 = exp(mod_3$alpha))
kable(alphas, digits = 4, caption = "Alphas")
```

Table 1: Alphas

Time	mod_0	mod_1	mod_2	mod_3
América-MG	0.6543	0.0670	0.0617	0.0566
Athletico-PR	1.1646	0.1191	0.1098	0.1098
Atlético-MG	1.2174	0.1235	0.1137	0.1109
Bahia	0.8446	0.0860	0.0793	0.0722
Botafogo	0.8282	0.0838	0.0772	0.0743
Ceará	0.6901	0.0696	0.0642	0.0586
Chapecoense	0.7446	0.0757	0.0697	0.0612
Corinthians	0.7306	0.0741	0.0683	0.0641
Cruzeiro	0.7296	0.0749	0.0689	0.0641
Flamengo	1.2597	0.1280	0.1179	0.1199

Time	mod_0	mod_1	mod_2	mod_3
Fluminense	0.6971	0.0709	0.0653	0.0618
Grêmio	1.0217	0.1040	0.0957	0.0915
Internacional	1.0885	0.1103	0.1015	0.0965
Palmeiras	1.3615	0.1384	0.1276	0.1304
Paraná	0.3972	0.0403	0.0371	0.0315
Santos	0.9954	0.1015	0.0934	0.0880
São Paulo	0.9878	0.1002	0.0923	0.0913
Sport	0.7735	0.0786	0.0724	0.0648
Vasco da Gama	0.8961	0.0900	0.0828	0.0747
Vitória	0.8019	0.0815	0.0751	0.0675

```r
betas = tibble(Time = times$Time,
               mod_0 = exp(mod_0$beta),
               mod_1 = exp(mod_1$beta),
               mod_2 = exp(mod_2$beta),
               mod_3 = exp(mod_3$beta))
kable(betas, digits = 4, caption = "Betas")
```

Table 2: Betas

Time	mod_0	mod_1	mod_2	mod_3
América-MG	0.9961	0.1020	0.0940	0.0914
Athletico-PR	0.8081	0.0824	0.0759	0.0687
Atlético-MG	0.9421	0.0955	0.0879	0.0795
Bahia	0.8787	0.0894	0.0824	0.0793
Botafogo	0.9849	0.0997	0.0919	0.0869
Ceará	0.8070	0.0812	0.0749	0.0698
Chapecoense	1.0653	0.1087	0.1001	0.0974
Corinthians	0.7451	0.0758	0.0698	0.0667
Cruzeiro	0.7238	0.0741	0.0682	0.0653
Flamengo	0.6370	0.0646	0.0595	0.0539
Fluminense	0.9773	0.0998	0.0919	0.0880
Grêmio	0.5847	0.0595	0.0548	0.0490
Internacional	0.6305	0.0639	0.0588	0.0520
Palmeiras	0.5746	0.0584	0.0538	0.0448
Paraná	1.1903	0.1212	0.1116	0.1179
Santos	0.8649	0.0880	0.0810	0.0761
São Paulo	0.7348	0.0743	0.0685	0.0606
Sport	1.2165	0.1238	0.1141	0.1113
Vasco da Gama	1.0318	0.1037	0.0954	0.0915
Vitória	1.3467	0.1373	0.1265	0.1258

```r
param = tibble(Model = 0:3,
              gamma = c(exp(mod_0$gamma), exp(mod_1$gamma), exp(mod_2$gamma),
                        exp(mod_3$gamma)),
              tau = c(NA, NA, exp(mod_2$tau), exp(mod_3$tau)),
              lambda_10 = c(NA, NA, NA, exp(mod_3$lambda_xy["10"])),
              lambda_01 = c(NA, NA, NA, exp(mod_3$lambda_xy["01"])),
              mu_10 = c(NA, NA, NA, exp(mod_3$mu_xy["10"])),
```

```r
                mu_01 = c(NA, NA, NA, exp(mod_3$mu_xy["01"])),
                eta_1 = c(NA, mod_1$eta[1], mod_2$eta[1], mod_3$eta[1]),
                eta_2 = c(NA, mod_1$eta[2], mod_2$eta[2], mod_3$eta[2]),
                phi_1 = c(NA, mod_1$phi[1], mod_2$phi[1], mod_3$phi[1]),
                phi_2 = c(NA, mod_1$phi[2], mod_2$phi[2], mod_3$phi[2]),
                kappa = c(NA, mod_1$kappa, mod_2$kappa, mod_3$kappa))
kable(param, digits = 2, caption = "Other parameters",
      col.names = c("Model", "$\\gamma_h$", "$\\tau$", "$\\lambda_{10}$",
                    "$\\lambda_{01}$", "$\\mu_{10}$", "$\\mu_{01}$", "$\\eta_1$",
                    "$\\eta_2$", "$\\phi_1$", "$\\phi_2$", "$\\kappa$"))
```

Table 3: Other parameters

Model	γ_h	τ	λ_{10}	λ_{01}	μ_{10}	μ_{01}	η_1	η_2	ϕ_1	ϕ_2	κ
0	1.74	–	–	–	–	–	–	–	–	–	–
1	1.74	–	–	–	–	–	2.22	3.63	0.22	-0.01	1.03
2	1.74	1.35	–	–	–	–	2.22	3.63	0.22	-0.01	1.03
3	2.05	1.39	0.68	1.41	1.34	0.98	2.22	3.63	0.22	-0.01	1.03