

Parameters 2015-2018 model 3

Rates for the home and away goals

$$\begin{aligned}\ln \lambda_k(t) &= \ln \alpha_i + \ln \beta_j + \ln \gamma_h + \mathbb{I}\{\text{half} = 2\} \ln \tau + \omega_{\text{goal}}(x(t) - y(t)) + \omega_{\text{player}}(y^*(t) - x^*(t)) \\ \ln \mu_k(t) &= \ln \alpha_j + \ln \beta_i + \mathbb{I}\{\text{half} = 2\} \ln \tau + \omega_{\text{goal}}(y(t) - x(t)) + \omega_{\text{player}}(x^*(t) - y^*(t))\end{aligned}$$

- i : home team index;
- j : away team index;
- α : attack strength parameter;
- $1/\beta$: defense strength parameter;
- γ_h : home advantage parameter;
- τ : second half parameter;
- $x(t)$: the number of goals of the home team until minute t ;
- $y(t)$: the number of goals of the away team until minute t ;
- $x^*(t)$: the number of red cards of the home team until minute t ;
- $y^*(t)$: the number of red cards of the away team until minute t ;
- ω_{goal} : parameter that measure the impact of leading in the score in the rates;
- ω_{player} : parameter that measure the impact of having extra players on the field.

Rates for the home and away red cards

$$\begin{aligned}\lambda_k^*(t) &= A_\lambda \left(t + 45^{\mathbb{I}\{\text{half} = 2\}} \right) \\ \mu_k^*(t) &= A_\mu \left(t + 45^{\mathbb{I}\{\text{half} = 2\}} \right)\end{aligned}$$

Stoppage time

The stoppage time for the first half, U^1 , and the second half, U^2 , are modeled as:

$$\begin{aligned}U^1 &\sim \text{Poisson}(\eta_1 + \rho_1 r^1) \\ U^2 &\sim \text{Poisson}(\eta_2 + \rho_2 r^2 + \kappa c)\end{aligned}$$

- r^t is the amount of red cards received in half t until minute 45;
- $c = \begin{cases} 1, & \text{if } |x - y| \leq 1 \text{ at minute 45 of the second half;} \\ 0, & \text{otherwise.} \end{cases}$

Constraint

The constraint for identificability is

$$\sum_i^n \log(\alpha_i) = \sum_i^n \log(\beta_i).$$

```
options(knitr.kable.NA = "-")
options(scipen = 999)

library(dplyr)
library(knitr)

load("data/input.RData")
load("data/mod_3.RData")

alphas_betas = tibble(Team = times$Time,
                      alpha = exp(mod_3$alpha),
                      beta = exp(mod_3$beta))
kable(alphas_betas, digits = 4, caption = "Alphas and betas",
      col.names = c("Team", "$\\alpha$", "$\\beta$"))
```

Table 1: Alphas and betas

Team	α	β
América-MG	0.0564	0.1026
Athletico-PR	0.0980	0.0764
Atlético-GO	0.0808	0.1070
Atlético-MG	0.1298	0.0916
Avaí	0.0718	0.1050
Bahia	0.0964	0.0858
Botafogo	0.0907	0.0797
Ceará	0.0685	0.0727
Chapecoense	0.0881	0.0958
Corinthians	0.1114	0.0655
Coritiba	0.0816	0.0856
Cruzeiro	0.0942	0.0745
Figueirense	0.0713	0.0954
Flamengo	0.1121	0.0734
Fluminense	0.0905	0.0924
Goiás	0.0843	0.0960
Grêmio	0.1063	0.0660
Internacional	0.0902	0.0688
Joinville	0.0567	0.0929
Palmeiras	0.1360	0.0733
Paraná	0.0386	0.1121
Ponte Preta	0.0913	0.0915
Santa Cruz	0.0962	0.1377
Santos	0.1134	0.0702
São Paulo	0.1052	0.0793
Sport	0.1001	0.1002
Vasco da Gama	0.0783	0.0953
Vitória	0.0973	0.1126

```

Parameter = c("$\\gamma_h$", "$\\tau$", "$\\omega_{\\text{goal}}$",
              "$\\omega_{\\text{player}}$")
goals = tibble(Parameter,
               Estimative = c(exp(mod_3$gamma), exp(mod_3$tau), mod_3$omega))
kable(goals, digits = 4, caption = "Goal rate parameters")

```

Table 2: Goal rate parameters

Parameter	Estimative
γ_h	1.6155
τ	1.2506
ω_{goal}	-0.0910
ω_{player}	0.2564

```

Parameter = c("$A_{\\lambda}$", "$A_{\\mu}$")
reds = tibble(Parameter, Estimative = exp(mod_3$a))
kable(reds, digits = 8, caption = "Red card rate parameters")

```

Table 3: Red card rate parameters

Parameter	Estimative
A_{λ}	0.00001788
A_{μ}	0.00003212

```

Parameter = c("$\\eta_1$", "$\\eta_2$", "$\\rho_1$", "$\\rho_2$", "$\\kappa$")
st = tibble(Parameter,
            Estimative = c(mod_3$eta, mod_3$rho, mod_3$kappa))
kable(st, digits = 4, caption = "Stoppage time parameters")

```

Table 4: Stoppage time parameters

Parameter	Estimative
η_1	2.1709
η_2	3.4403
ρ_1	0.7630
ρ_2	0.2504
κ	0.8641

```
mod_3$loglik
```

```
## [1] -9750.899
```