Generating red cards

$$\lambda_k^*(t) = A_\lambda t$$

where

$$A_\lambda = \begin{cases} A_\lambda^1, & \text{if half} = 1; \\ A_\lambda^2, & \text{if half} = 2. \end{cases}$$

$$\mu_k^*(t) = A_\mu t$$

where

$$A_\mu = \begin{cases} A_\mu^1, & \text{if half} = 1; \\ A_\mu^2, & \text{if half} = 2. \end{cases}$$

$$\hat{A}_\lambda^1 = 0.00001555931$$

$$\hat{A}_\lambda^2 = 0.00005941237$$

$$\hat{A}_\mu^1 = 0.00002295013$$

$$\hat{A}_\mu^2 = 0.00009806610$$

$$\Lambda^*(t) = \int_0^t \lambda^*(y)dy = A_\lambda \frac{t^2}{2}$$

$$\Upsilon^*(t) = \int_0^t \mu^*(y)dy = A_\mu \frac{t^2}{2}$$

$$\Lambda^{*-1}(t) = \left(\frac{2t}{A_\lambda}\right)^{1/2}$$

$$\Upsilon^{*-1}(t) = \left(\frac{2t}{A_\mu}\right)^{1/2}$$

```
library(dplyr)
library(knitr)

A_lambda_1 = 0.00001555931
A_lambda_2 = 0.00005941237
A_mu_1 = 0.00002295013
A_mu_2 = 0.00009806610

inv_lambda_1 <- function(x) {
  (2*x/A_lambda_1)^(1/2)
}
```

```r
inv_lambda_2 <- function(x) {
  (2*x/A_lambda_2)^(1/2)
}

inv_mu_1 <- function(x) {
  (2*x/A_mu_1)^(1/2)
}

inv_mu_2 <- function(x) {
  (2*x/A_mu_2)^(1/2)
}
```

Generating for the home team in the first half

```r
nit = 1000000

home_1st = list()
for(it in 1:nit) {
  t = 0
  s = 0
  tmp = NULL
  while(t < 47) {
    u = runif(1)
    s = s - log(u)
    t = inv_lambda_1(s)
    if(t < 47) {
      tmp = c(tmp, t)
    }
  }
  home_1st[[it]] = tmp
}
```

Generating for the away team in the first half

```r
away_1st = list()
for(it in 1:nit) {
  t = 0
  s = 0
  tmp = NULL
  while(t < 47) {
    u = runif(1)
    s = s - log(u)
    t = inv_mu_1(s)
    if(t < 47) {
      tmp = c(tmp, t)
    }
  }
  away_1st[[it]] = tmp
}
```

Generating for the home team in the second half

```
home_2nd = list()
for(it in 1:nit) {
  t = 0
  s = 0
  tmp = NULL
  while(t < 49) {
    u = runif(1)
    s = s - log(u)
    t = inv_lambda_2(s)
    if(t < 49) {
      tmp = c(tmp, t)
    }
  }
  home_2nd[[it]] = tmp
}
```

Generating for the away team in the second half

```
away_2nd = list()
for(it in 1:nit) {
  t = 0
  s = 0
  tmp = NULL
  while(t < 49) {
    u = runif(1)
    s = s - log(u)
    t = inv_mu_2(s)
    if(t < 49) {
      tmp = c(tmp, t)
    }
  }
  away_2nd[[it]] = tmp
}
```

Comparing the percentage of generated red cards with the real data

```
load("scrape/data/reds.RData")
load("2015-2020/data/input.RData")
Generated = c(length(unlist(home_1st))/nit, length(unlist(away_1st))/nit,
              length(unlist(home_2nd))/nit, length(unlist(away_2nd))/nit,
              length(unlist(home_1st))/nit + length(unlist(away_1st))/nit +
                length(unlist(home_2nd))/nit + length(unlist(away_2nd))/nit)
Real = c(reds %>% filter(Team == 1, Half == 1) %>% nrow() / N,
         reds %>% filter(Team == 2, Half == 1) %>% nrow() / N,
         reds %>% filter(Team == 1, Half == 2) %>% nrow() / N,
         reds %>% filter(Team == 2, Half == 2) %>% nrow() / N,
         nrow(reds)/N)
```

```r
Team = c("Home", "Away", "Home", "Away", "Both")
Half = c("1st", "1st", "2nd", "2nd", "Both")
tib = tibble(Team, Half, Generated, Real)
kable(tib, digits = 4)
```

Team	Half	Generated	Real
Home	1st	0.0172	0.0176
Away	1st	0.0252	0.0259
Home	2nd	0.0714	0.0733
Away	2nd	0.1178	0.1202
Both	Both	0.2316	0.2369