

## Parameters 2015-2019 model 5

### Rates for the home and away goals

$$\ln \lambda_k(t) = \ln \alpha_i + \ln \beta_j + \ln \gamma_h + \mathbb{I}\{\text{half} = 2\} \ln \tau + \ln \lambda_{xy} + \omega_{\text{player}}(y^*(t) - x^*(t))$$

$$\ln \mu_k(t) = \ln \alpha_j + \ln \beta_i + \mathbb{I}\{\text{half} = 2\} \ln \tau + \ln \mu_{xy} + \omega_{\text{player}}(x^*(t) - y^*(t))$$

- $i$ : home team index;
- $j$ : away team index;
- $\alpha$ : attack strength parameter;
- $1/\beta$ : defense strength parameter;
- $\gamma_h$ : home advantage parameter;
- $\tau$ : second half parameter;
- $x(t)$ : the number of goals of the home team until minute  $t$ ;
- $y(t)$ : the number of goals of the away team until minute  $t$ ;
- $x^*(t)$ : the number of red cards of the home team until minute  $t$ ;
- $y^*(t)$ : the number of red cards of the away team until minute  $t$ ;

$$\lambda_{xy} = \begin{cases} 1, & \text{for } x = 0, y = 0; \\ \lambda_{10}, & \text{for } x = 1, y = 0; \\ \lambda_{01}, & \text{for } x = 0, y = 1; \\ \lambda_{11}, & \text{for } x = 1, y = 1; \\ \lambda_{22}, & \text{for } x - y = 0, x, y \geq 2; \\ \lambda_{21}, & \text{for } x - y \geq 1, x \geq 2; \\ \lambda_{12}, & \text{for } x - y \leq -1, y \geq 2. \end{cases}$$

- $\omega_{\text{player}}$ : parameter that measure the impact of having extra players on the field.

### Rates for the home and away red cards

$$\lambda_k^*(t) = A_\lambda \left( t + 45^{\mathbb{I}\{\text{half} = 2\}} \right)$$

$$\mu_k^*(t) = A_\mu \left( t + 45^{\mathbb{I}\{\text{half} = 2\}} \right)$$

## Stoppage time

The stoppage time for the first half,  $U^1$ , and the second half,  $U^2$ , are modeled as:

$$U^1 \sim \text{Poisson}(\eta_1 + \rho_1 r^1)$$

$$U^2 \sim \text{Poisson}(\eta_2 + \rho_2 r^2 + \kappa c)$$

- $r^t$  is the amount of red cards received in half  $t$  until minute 45;
- $c = \begin{cases} 1, & \text{if } |x - y| \leq 1 \text{ at minute 45 of the second half;} \\ 0, & \text{otherwise.} \end{cases}$

## Constraint

The constraint for identificability is

$$\sum_i^n \log(\alpha_i) = \sum_i^n \log(\beta_i).$$

```
options(knitr.kable.NA = "-")
options(scipen = 999)
```

```
library(dplyr)
library(knitr)
```

```
load("data/input.RData")
load("data/mod_5.RData")
```

```
alphas_betas = tibble(Team = times$Time,
                      alpha = exp(mod_5$alpha),
                      beta = exp(mod_5$beta))
kable(alphas_betas, digits = 4, caption = "Alphas and betas",
      col.names = c("Team", "$\\alpha$", "$\\beta$"))
```

Table 1: Alphas and betas

Team	$\alpha$	$\beta$
América-MG	0.0582	0.1028
Athletico-PR	0.1026	0.0731
Atlético-GO	0.0828	0.1071
Atlético-MG	0.1260	0.0910
Avaí	0.0623	0.1097
Bahia	0.0977	0.0844
Botafogo	0.0872	0.0810
Ceará	0.0741	0.0752
Chapecoense	0.0857	0.0959
Corinthians	0.1100	0.0651
Coritiba	0.0829	0.0857
Cruzeiro	0.0886	0.0771
Csa	0.0521	0.1136
Figueirense	0.0734	0.0950

Team	$\alpha$	$\beta$
Flamengo	0.1310	0.0724
Fluminense	0.0910	0.0902
Fortaleza	0.1130	0.0923
Goiás	0.0936	0.1093
Grêmio	0.1155	0.0678
Internacional	0.0934	0.0696
Joinville	0.0580	0.0926
Palmeiras	0.1396	0.0704
Paraná	0.0399	0.1126
Ponte Preta	0.0933	0.0915
Santa Cruz	0.0992	0.1383
Santos	0.1208	0.0683
São Paulo	0.1040	0.0740
Sport	0.1026	0.0999
Vasco da Gama	0.0820	0.0917
Vitória	0.0992	0.1120

```

Parameter = c("$\\gamma_h$", "$\\tau$",
              "$\\lambda_{10}$", "$\\lambda_{01}$",
              "$\\lambda_{11}$", "$\\lambda_{22}$",
              "$\\lambda_{21}$", "$\\lambda_{12}$",
              "$\\mu_{10}$", "$\\mu_{01}$",
              "$\\mu_{11}$", "$\\mu_{22}$",
              "$\\mu_{21}$", "$\\mu_{12}$",
              "$\\omega_{\\{\\text{player}\\}}$")
goals = tibble(Parameter,
               Estimative = c(exp(mod_5$gamma), exp(mod_5$tau),
                             exp(mod_5$lambda_xy), exp(mod_5$mu_xy),
                             mod_5$omega))
kable(goals, digits = 4, caption = "Goal rate parameters")

```

Table 2: Goal rate parameters

Parameter	Estimative
$\gamma_h$	1.5956
$\tau$	1.2408
$\lambda_{10}$	0.8074
$\lambda_{01}$	1.1066
$\lambda_{11}$	1.1257
$\lambda_{22}$	1.0180
$\lambda_{21}$	0.8485
$\lambda_{12}$	1.1688
$\mu_{10}$	1.0767
$\mu_{01}$	0.8706
$\mu_{11}$	1.0271
$\mu_{22}$	1.1137
$\mu_{21}$	1.1567
$\mu_{12}$	0.8467
$\omega_{\text{player}}$	0.3121

```
Parameter = c("$A_\\lambda$", "$A_\\mu$")
reds = tibble(Parameter, Estimative = exp(mod_5$a))
kable(reds, digits = 8, caption = "Red card rate parameters")
```

Table 3: Red card rate parameters

Parameter	Estimative
$A_\lambda$	0.00001815
$A_\mu$	0.00003237

```
Parameter = c("$\\eta_1$", "$\\eta_2$", "$\\rho_1$", "$\\rho_2$", "$\\kappa$")
st = tibble(Parameter,
             Estimative = c(mod_5$eta, mod_5$rho, mod_5$kappa))
kable(st, digits = 4, caption = "Stoppage time parameters")
```

Table 4: Stoppage time parameters

Parameter	Estimative
$\eta_1$	2.3404
$\eta_2$	3.5377
$\rho_1$	0.8663
$\rho_2$	0.2537
$\kappa$	0.9764

```
mod_5$loglik
```

```
## [1] -11374.15
```