Parameters 2015-2020 model 3

Rates for the home and away goals

$$\ln \lambda_k(t) = \ln \alpha_i + \ln \beta_j + \ln \gamma_h + \mathbb{I}\{\text{half} = 2\} \ln \tau + \omega_{\text{goal}}(x(t) - y(t)) + \omega_{\text{player}}(y^*(t) - x^*(t))$$
$$\ln \mu_k(t) = \ln \alpha_j + \ln \beta_i + \mathbb{I}\{\text{half} = 2\} \ln \tau + \omega_{\text{goal}}(y(t) - x(t)) + \omega_{\text{player}}(x^*(t) - y^*(t))$$

- i: home team index;
- j: away team index;
- α: attack strength parameter;
- $1/\beta$: defense strength parameter;
- γ_h: home advantage parameter;
- τ: second half parameter;
- $x(t)$: the number of goals of the home team until minute t;
- $y(t)$: the number of goals of the away team until minute t;
- $x^*(t)$: the number of red cards of the home team until minute t;
- $y^*(t)$: the number of red cards of the away team until minute t;
- ω_{goal}: parameter that measure the impact of leading in the score in the rates;
- ω_{player}: parameter that measure the impact of having extra players on the field.

Rates for the home and away red cards

$$\lambda_k^*(t) = A_\lambda \left(t + 45^{\mathbb{I}\{\text{half} = 2\}} \right)$$
$$\mu_k^*(t) = A_\mu \left(t + 45^{\mathbb{I}\{\text{half} = 2\}} \right)$$

Stoppage time

The stoppage time for the first half, U^1, and the second half, U^2, are modeled as:

$$U^1 \sim \text{Poisson}(\eta_1 + \rho_1 r^1)$$
$$U^2 \sim \text{Poisson}(\eta_2 + \rho_2 r^2 + \kappa c)$$

- r^t is the amount of red cards received in half t until minute 45;
- $c = \begin{cases} 1, & \text{if } |x - y| \leq 1 \text{ at minute 45 of the second half;} \\ 0, & \text{otherwise.} \end{cases}$

Constraint

The constraint for identificability is

$$\sum_{i}^{n} \log(\alpha_i) = \sum_{i}^{n} \log(\beta_i).$$

```
options(knitr.kable.NA = "-")
options(scipen = 999)

library(dplyr)
library(knitr)

load("data/input.RData")
load("data/mod_3.RData")
```

```
alphas_betas = tibble(Team = times$Time,
                alpha = exp(mod_3$alpha),
                beta = exp(mod_3$beta))
kable(alphas_betas, digits = 4, caption = "Alphas and betas",
      col.names = c("Team", "$\\alpha$", "$\\beta$"))
```

Table 1: Alphas and betas

Team	α	β
América-MG	0.0576	0.1033
Athletico-PR	0.0993	0.0724
Atlético-GO	0.0865	0.0948
Atlético-MG	0.1294	0.0905
Avaí	0.0619	0.1111
Bahia	0.0998	0.0919
Botafogo	0.0833	0.0885
Ceará	0.0893	0.0816
Chapecoense	0.0854	0.0973
Corinthians	0.1082	0.0684
Coritiba	0.0797	0.0901
Cruzeiro	0.0886	0.0779
Csa	0.0513	0.1145
Figueirense	0.0728	0.0962
Flamengo	0.1345	0.0758
Fluminense	0.0962	0.0891
Fortaleza	0.0944	0.0869
Goiás	0.0918	0.1128
Grêmio	0.1162	0.0692
Internacional	0.1024	0.0682
Joinville	0.0578	0.0935
Palmeiras	0.1350	0.0705
Paraná	0.0393	0.1117
Ponte Preta	0.0933	0.0928
Red Bull Bragantino	0.1148	0.0713
Santa Cruz	0.0983	0.1399
Santos	0.1196	0.0730
São Paulo	0.1079	0.0749

Team	α	β
Sport	0.0950	0.0993
Vasco da Gama	0.0815	0.0953
Vitória	0.0992	0.1140

```
Parameter = c("$\\gamma_h$", "$\\tau$", "$\\omega_{{\\text{goal}}}$",
              "$\\omega_{{\\text{player}}}$")
goals = tibble(Parameter,
               Estimative = c(exp(mod_3$gamma), exp(mod_3$tau), mod_3$omega))
kable(goals, digits = 4, caption = "Goal rate parameters")
```

Table 2: Goal rate parameters

Parameter	Estimative
γ_h	1.5554
τ	1.2243
ω_{goal}	-0.0935
ω_{player}	0.3238

```
Parameter = c("$A_\\lambda$", "$A_\\mu$")
reds = tibble(Parameter, Estimative = exp(mod_3$a))
kable(reds, digits = 8, caption = "Red card rate parameters")
```

Table 3: Red card rate parameters

Parameter	Estimative
A_λ	0.00001973
A_μ	0.00003190

```
Parameter = c("$\\eta_1$", "$\\eta_2$", "$\\rho_1$", "$\\rho_2$", "$\\kappa$")
st = tibble(Parameter,
            Estimative = c(mod_3$eta, mod_3$rho, mod_3$kappa))
kable(st, digits = 4, caption = "Stoppage time parameters")
```

Table 4: Stoppage time parameters

Parameter	Estimative
η_1	2.4360
η_2	3.7394
ρ_1	1.1400
ρ_2	0.2267
κ	1.0101

```
mod_3$loglik
```

```
## [1] -25862.92
```