Parameters Série A 2019

Rates for the home and away teams

Model 0

$$\lambda_k = \alpha_i \beta_j \gamma_h$$
$$\mu_k = \alpha_j \beta_i$$

- i: home team index;
- j: away team index;
- α: attack strength parameter;
- $1/\beta$: defense strength parameter;
- γ_h: home advantage parameter.

Model 1

$$\lambda_k(t) = \alpha_i \beta_j \gamma_h$$
$$\mu_k(t) = \alpha_j \beta_i$$

Model 2

$$\lambda_k(t) = \alpha_i \beta_j \gamma_h \tau^{\mathbb{I}\{\text{half} = 2\}}$$
$$\mu_k(t) = \alpha_j \beta_i \tau^{\mathbb{I}\{\text{half} = 2\}}$$

- τ: second half parameter.

Model 3

$$\lambda_k(t) = \alpha_i \beta_j \gamma_h \tau^{\mathbb{I}\{\text{half} = 2\}} \lambda_{xy}$$
$$\mu_k(t) = \alpha_j \beta_i \tau^{\mathbb{I}\{\text{half} = 2\}} \mu_{xy}$$

- $\lambda_{xy} = \begin{cases} 1, & \text{if } x = y; \\ \lambda_{10}, & \text{if } x > y; \\ \lambda_{01}, & \text{if } x < y; \end{cases}$

- $\mu_{xy} = \begin{cases} 1, & \text{if } x = y; \\ \mu_{10}, & \text{if } x > y; \\ \mu_{01}, & \text{if } x < y. \end{cases}$

Stoppage time

For all models except model 0, the stoppage time for the first half, U^1, and the second half, U^2, are modeled as:

$$U^1 \sim \text{Poisson}(\eta_1 + \phi_1 g^1)$$

$$U^2 \sim \text{Poisson}(\eta_2 + \phi_2 g^2 + \kappa c)$$

- g^t is the amount of goals scored in half t until minute 45;

- $c = \begin{cases} 1, & \text{if } |x - y| \leq 1 \text{ at minute 45 of the second half;} \\ 0, & \text{otherwise.} \end{cases}$

Constraint

The constraint for identificability in all models is

$$\sum_i^n \log(\alpha_i) = \sum_i^n \log(\beta_i)$$

```r
options(knitr.kable.NA = "-")

library(dplyr)
library(knitr)

load("data/input.RData")
load("data/mod_0.RData")
load("data/mod_1.RData")
load("data/mod_2.RData")
load("data/mod_3.RData")
```

```r
alphas = tibble(Time = times$Time,
                mod_0 = exp(mod_0$alpha),
                mod_1 = exp(mod_1$alpha),
                mod_2 = exp(mod_2$alpha),
                mod_3 = exp(mod_3$alpha))
kable(alphas, digits = 4, caption = "Alphas")
```

Table 1: Alphas

Time	mod_0	mod_1	mod_2	mod_3
Athletico-PR	1.1254	0.1134	0.1074	0.1055
Atlético-MG	1.0132	0.1014	0.0960	0.0917
Avaí	0.4108	0.0417	0.0395	0.0360
Bahia	0.9835	0.0991	0.0938	0.0893
Botafogo	0.6940	0.0703	0.0665	0.0635
Ceará	0.8023	0.0805	0.0761	0.0715
Chapecoense	0.6998	0.0709	0.0670	0.0642
Corinthians	0.9286	0.0939	0.0888	0.0864
Cruzeiro	0.6050	0.0615	0.0582	0.0540
CSA	0.5454	0.0554	0.0524	0.0483

Time	mod_0	mod_1	mod_2	mod_3
Flamengo	1.9129	0.1935	0.1833	0.1916
Fluminense	0.8521	0.0857	0.0811	0.0768
Fortaleza	1.1262	0.1139	0.1078	0.1070
Goiás	1.0551	0.1058	0.1002	0.0953
Grêmio	1.4253	0.1448	0.1371	0.1360
Internacional	0.9787	0.0980	0.0928	0.0903
Palmeiras	1.3467	0.1360	0.1287	0.1301
Santos	1.3261	0.1348	0.1276	0.1312
São Paulo	0.8581	0.0861	0.0814	0.0805
Vasco da Gama	0.8735	0.0876	0.0829	0.0807

```r
betas = tibble(Time = times$Time,
               mod_0 = exp(mod_0$beta),
               mod_1 = exp(mod_1$beta),
               mod_2 = exp(mod_2$beta),
               mod_3 = exp(mod_3$beta))
kable(betas, digits = 4, caption = "Betas")
```

Table 2: Betas

Time	mod_0	mod_1	mod_2	mod_3
Athletico-PR	0.6954	0.0698	0.0660	0.0620
Atlético-MG	1.0584	0.1061	0.1005	0.0973
Avaí	1.2971	0.1322	0.1252	0.1297
Bahia	0.9273	0.0933	0.0883	0.0852
Botafogo	0.9556	0.0968	0.0916	0.0909
Ceará	0.8757	0.0882	0.0834	0.0821
Chapecoense	1.1046	0.1115	0.1055	0.1048
Corinthians	0.7311	0.0738	0.0698	0.0654
Cruzeiro	0.9722	0.0990	0.0937	0.0928
CSA	1.2220	0.1243	0.1177	0.1210
Flamengo	0.8400	0.0847	0.0802	0.0730
Fluminense	0.9851	0.0994	0.0940	0.0917
Fortaleza	1.0649	0.1075	0.1017	0.0974
Goiás	1.3856	0.1391	0.1317	0.1307
Grêmio	0.8616	0.0869	0.0823	0.0793
Internacional	0.8409	0.0847	0.0802	0.0768
Palmeiras	0.7039	0.0712	0.0674	0.0639
Santos	0.7251	0.0736	0.0697	0.0646
São Paulo	0.6426	0.0645	0.0610	0.0576
Vasco da Gama	0.9648	0.0966	0.0914	0.0884

```r
param = tibble(Model = 0:3,
               gamma = c(exp(mod_0$gamma), exp(mod_1$gamma), exp(mod_2$gamma),
                     exp(mod_3$gamma)),
               tau = c(NA, NA, exp(mod_2$tau), exp(mod_3$tau)),
               lambda_10 = c(NA, NA, NA, exp(mod_3$lambda_xy["10"])),
               lambda_01 = c(NA, NA, NA, exp(mod_3$lambda_xy["01"])),
               mu_10 = c(NA, NA, NA, exp(mod_3$mu_xy["10"])),
```

```
                mu_01 = c(NA, NA, NA, exp(mod_3$mu_xy["01"])),
                eta_1 = c(NA, mod_1$eta[1], mod_2$eta[1], mod_3$eta[1]),
                eta_2 = c(NA, mod_1$eta[2], mod_2$eta[2], mod_3$eta[2]),
                phi_1 = c(NA, mod_1$phi[1], mod_2$phi[1], mod_3$phi[1]),
                phi_2 = c(NA, mod_1$phi[2], mod_2$phi[2], mod_3$phi[2]),
                kappa = c(NA, mod_1$kappa, mod_2$kappa, mod_3$kappa))
kable(param, digits = 4, caption = "Other parameters",
      col.names = c("Model", "$\\gamma_h$", "$\\tau$", "$\\lambda_{10}$",
                    "$\\lambda_{01}$", "$\\mu_{10}$", "$\\mu_{01}$", "$\\eta_1$",
                    "$\\eta_2$", "$\\phi_1$", "$\\phi_2$", "$\\kappa$"))
```

Table 3: Other parameters

Model	γ_h	τ	λ_{10}	λ_{01}	μ_{10}	μ_{01}	η_1	η_2	ϕ_1	ϕ_2	κ
0	1.4957	–	–	–	–	–	–	–	–	–	–
1	1.4990	–	–	–	–	–	2.8341	3.6016	0.2261	0.2075	1.5566
2	1.4994	1.2284	–	–	–	–	2.8341	3.6017	0.2261	0.2075	1.5566
3	1.6930	1.2709	0.7617	1.0247	1.2748	0.7623	2.8341	3.6016	0.2261	0.2075	1.5566