# Parameters 2015-2018 model 5

## Rates for the home and away goals

$$\ln \lambda_k(t) = \ln \alpha_i + \ln \beta_j + \ln \gamma_h + \mathbb{I}\{\text{half} = 2\} \ln \tau + \ln \lambda_{xy} + \omega_{\text{player}}(y^*(t) - x^*(t))$$

$$\ln \mu_k(t) = \ln \alpha_j + \ln \beta_i + \mathbb{I}\{\text{half} = 2\} \ln \tau + \ln \mu_{xy} + \omega_{\text{player}}(x^*(t) - y^*(t))$$

- $i$: home team index;

- $j$: away team index;

- $\alpha$: attack strength parameter;

- $1/\beta$: defense strength parameter;

- $\gamma_h$: home advantage parameter;

- $\tau$: second half parameter;

- $x(t)$: the number of goals of the home team until minute t;

- $y(t)$: the number of goals of the away team until minute t;

- $x^*(t)$: the number of red cards of the home team until minute t;

- $y^*(t)$: the number of red cards of the away team until minute t;

- $\lambda_{xy} = \begin{cases} 1, & \text{for } x = 0, \ y = 0; \\ \lambda_{10}, & \text{for } x = 1, \ y = 0; \\ \lambda_{01}, & \text{for } x = 0, \ y = 1; \\ \lambda_{11}, & \text{for } x = 1, \ y = 1; \\ \lambda_{22}, & \text{for } x - y = 0, \ x, \ y, \geq 2; \\ \lambda_{21}, & \text{for } x - y \geq 1, \ x \geq 2; \\ \lambda_{12}, & \text{for } x - y \leq -1, \ y \geq 2. \end{cases}$

- $\omega_{\text{player}}$: parameter that measure the impact of having extra players on the field.

## Rates for the home and away red cards

$$\lambda_k^*(t) = A_\lambda \left( t + 45^{\mathbb{I}\{\text{half} = 2\}} \right)$$

$$\mu_k^*(t) = A_\mu \left( t + 45^{\mathbb{I}\{\text{half} = 2\}} \right)$$

## Stoppage time

The stoppage time for the first half, $U^1$, and the second half, $U^2$, are modeled as:

$$U^1 \sim \text{Poisson}(\eta_1 + \rho_1 r^1)$$

$$U^2 \sim \text{Poisson}(\eta_2 + \rho_2 r^2 + \kappa c)$$

- $r^t$ is the amount of red cards received in half $t$ until minute 45;

- $c = \begin{cases} 1, & \text{if } |x - y| \leq 1 \text{ at minute 45 of the second half;} \\ 0, & \text{otherwise.} \end{cases}$

## Constraint

The constraint for identificability is

$$\sum_i^n \log(\alpha_i) = \sum_i^n \log(\beta_i).$$

```
options(knitr.kable.NA = "-")
options(scipen = 999)

library(dplyr)
library(knitr)

load("data/input.RData")
load("data/mod_5.RData")
```

```
alphas_betas = tibble(Team = times$Time,
                alpha = exp(mod_5$alpha),
                beta = exp(mod_5$beta))
kable(alphas_betas, digits = 4, caption = "Alphas and betas",
      col.names = c("Team", "$\\alpha$", "$\\beta$"))
```

Table 1: Alphas and betas

| Team | $\alpha$ | $\beta$ |
|------|------|------|
| América-MG | 0.0574 | 0.1045 |
| Athletico-PR | 0.0993 | 0.0777 |
| Atlético-GO | 0.0815 | 0.1077 |
| Atlético-MG | 0.1312 | 0.0924 |
| Avaí | 0.0730 | 0.1064 |
| Bahia | 0.0974 | 0.0871 |
| Botafogo | 0.0920 | 0.0805 |
| Ceará | 0.0689 | 0.0731 |
| Chapecoense | 0.0890 | 0.0968 |
| Corinthians | 0.1129 | 0.0665 |
| Coritiba | 0.0820 | 0.0867 |
| Cruzeiro | 0.0952 | 0.0756 |
| Figueirense | 0.0728 | 0.0970 |
| Flamengo | 0.1136 | 0.0741 |

| Team | $\alpha$ | $\beta$ |
|------|------|------|
| Fluminense | 0.0916 | 0.0930 |
| Goiás | 0.0844 | 0.0969 |
| Grêmio | 0.1073 | 0.0668 |
| Internacional | 0.0914 | 0.0697 |
| Joinville | 0.0575 | 0.0945 |
| Palmeiras | 0.1382 | 0.0741 |
| Paraná | 0.0394 | 0.1148 |
| Ponte Preta | 0.0924 | 0.0931 |
| Santa Cruz | 0.0980 | 0.1401 |
| Santos | 0.1149 | 0.0713 |
| São Paulo | 0.1071 | 0.0806 |
| Sport | 0.1014 | 0.1017 |
| Vasco da Gama | 0.0800 | 0.0959 |
| Vitória | 0.0980 | 0.1134 |

```
Parameter = c("$\\gamma_h$", "$\\tau$",
              "$\\lambda_{10}$", "$\\lambda_{01}$",
              "$\\lambda_{11}$", "$\\lambda_{22}$",
              "$\\lambda_{21}$", "$\\lambda_{12}$",
              "$\\mu_{10}$", "$\\mu_{01}$",
              "$\\mu_{11}$", "$\\mu_{22}$",
              "$\\mu_{21}$", "$\\mu_{12}$",
              "$\\omega_{{\\text{player}}}$")
goals = tibble(Parameter,
               Estimative = c(exp(mod_5$gamma), exp(mod_5$tau),
                              exp(mod_5$lambda_xy), exp(mod_5$mu_xy),
                              mod_5$omega))
kable(goals, digits = 4, caption = "Goal rate parameters")
```

Table 2: Goal rate parameters

| Parameter | Estimative |
|------|------|
| $\gamma_h$ | 1.5863 |
| $\tau$ | 1.2537 |
| $\lambda_{10}$ | 0.8040 |
| $\lambda_{01}$ | 1.1092 |
| $\lambda_{11}$ | 1.1084 |
| $\lambda_{22}$ | 1.0277 |
| $\lambda_{21}$ | 0.8376 |
| $\lambda_{12}$ | 1.1979 |
| $\mu_{10}$ | 1.0136 |
| $\mu_{01}$ | 0.8760 |
| $\mu_{11}$ | 1.0177 |
| $\mu_{22}$ | 1.0566 |
| $\mu_{21}$ | 1.1188 |
| $\mu_{12}$ | 0.8409 |
| $\omega_{\text{player}}$ | 0.2571 |

```r
Parameter = c("$A_\\lambda$", "$A_\\mu$")
reds = tibble(Parameter, Estimative = exp(mod_5$a))
kable(reds, digits = 8, caption = "Red card rate parameters")
```

Table 3: Red card rate parameters

| Parameter | Estimative |
|-----------|------------|
| $A_\lambda$ | 0.00001788 |
| $A_\mu$ | 0.00003212 |

```r
Parameter = c("$\\eta_1$", "$\\eta_2$", "$\\rho_1$", "$\\rho_2$", "$\\kappa$")
st = tibble(Parameter,
            Estimative = c(mod_5$eta, mod_5$rho, mod_5$kappa))
kable(st, digits = 4, caption = "Stoppage time parameters")
```

Table 4: Stoppage time parameters

| Parameter | Estimative |
|-----------|------------|
| $\eta_1$ | 2.1709 |
| $\eta_2$ | 3.4403 |
| $\rho_1$ | 0.7630 |
| $\rho_2$ | 0.2504 |
| $\kappa$ | 0.8641 |

```r
mod_5$loglik
```

```
## [1] -9744.972
```