Parameters 2015-2020 model 8

Rates for the home and away goals

$$\ln \lambda_k(t) = \ln \alpha_i + \ln \beta_j + \ln \gamma_h + \mathbb{I}\{\text{half} = 2\} \ln \tau + \ln \lambda_{xy} + \omega_{\text{player}}(y^*(t) - x^*(t))$$

$$\ln \mu_k(t) = \ln \alpha_j + \ln \beta_i + \mathbb{I}\{\text{half} = 2\} \ln \tau + \ln \mu_{xy} + \omega_{\text{player}}(x^*(t) - y^*(t))$$

- i: home team index;

- j: away team index;

- α: attack strength parameter;

- $1/\beta$: defense strength parameter;

- γ_h: home advantage parameter;

- τ: second half parameter;

- $x(t)$: the number of goals of the home team until minute t;

- $y(t)$: the number of goals of the away team until minute t;

- $x^*(t)$: the number of red cards of the home team until minute t;

- $y^*(t)$: the number of red cards of the away team until minute t;

- $\lambda_{xy} = \begin{cases} 1, & \text{for } x = y; \\ \omega_{\text{ahead}}, & \text{for } x - y \geq 1; \\ \omega_{\text{behind}}, & \text{for } x - y \leq -1; \end{cases}$

- $\mu_{xy} = \begin{cases} 1, & \text{for } x = y; \\ \lambda_{\text{ahead}}, & \text{for } y - x \geq 1; \\ \lambda_{\text{behind}}, & \text{for } y - x \leq -1; \end{cases}$

- ω_{player}: parameter that measure the impact of having extra players on the field.

Rates for the home and away red cards

$$\lambda_k^*(t) = A_\lambda \left(t + 45^{\mathbb{I}\{\text{half} = 2\}} \right)$$

$$\mu_k^*(t) = A_\mu \left(t + 45^{\mathbb{I}\{\text{half} = 2\}} \right)$$

Stoppage time

The stoppage time for the first half, U^1, and the second half, U^2, are modeled as:

$$U^1 \sim \text{Poisson}(\eta_1 + \rho_1 r^1)$$

$$U^2 \sim \text{Poisson}(\eta_2 + \rho_2 r^2 + \kappa c)$$

- r^t is the amount of red cards received in half t until minute 45;

- $c = \begin{cases} 1, & \text{if } |x - y| \leq 1 \text{ at minute 45 of the second half;} \\ 0, & \text{otherwise.} \end{cases}$

Constraint

The constraint for identificability is

$$\sum_i^n \log(\alpha_i) = \sum_i^n \log(\beta_i).$$

```r
options(knitr.kable.NA = "-")
options(scipen = 999)

library(dplyr)
library(knitr)

load("data/input.RData")
load("data/mod_8.RData")
```

```r
alphas_betas = tibble(Team = times$Time,
                alpha = exp(mod_8$alpha),
                beta = exp(mod_8$beta))
kable(alphas_betas, digits = 4, caption = "Alphas and betas",
      col.names = c("Team", "$\\alpha$", "$\\beta$"))
```

Table 1: Alphas and betas

Team	α	β
América-MG	0.0584	0.1045
Athletico-PR	0.0998	0.0731
Atlético-GO	0.0871	0.0950
Atlético-MG	0.1300	0.0918
Avaí	0.0630	0.1120
Bahia	0.1005	0.0924
Botafogo	0.0844	0.0889
Ceará	0.0893	0.0825
Chapecoense	0.0864	0.0983
Corinthians	0.1090	0.0689
Coritiba	0.0805	0.0904
Cruzeiro	0.0891	0.0791
Csa	0.0525	0.1147
Figueirense	0.0738	0.0970

Team	α	β
Flamengo	0.1356	0.0770
Fluminense	0.0968	0.0900
Fortaleza	0.0952	0.0868
Goiás	0.0932	0.1126
Grêmio	0.1166	0.0702
Internacional	0.1027	0.0691
Joinville	0.0585	0.0946
Palmeiras	0.1359	0.0716
Paraná	0.0399	0.1133
Ponte Preta	0.0942	0.0941
Red Bull Bragantino	0.1123	0.0718
Santa Cruz	0.0999	0.1424
Santos	0.1202	0.0739
São Paulo	0.1088	0.0756
Sport	0.0963	0.0998
Vasco da Gama	0.0829	0.0952
Vitória	0.1004	0.1152

```
Parameter = c("$\\gamma_h$", "$\\tau$",
              "$\\omega_{{\\text{ahead}}}$",
              "$\\omega_{{\\text{behind}}}$",
              "$\\omega_{{\\text{player}}}$")
goals = tibble(Parameter,
               Estimative = c(exp(mod_8$gamma),
                              exp(mod_8$tau),
                              exp(mod_8$omega_xy),
                              mod_8$omega))
kable(goals, digits = 4, caption = "Goal rate parameters")
```

Table 2: Goal rate parameters

Parameter	Estimative
γ_h	1.5447
τ	1.2331
ω_ahead	0.8355
ω_behind	1.1036
ω_player	0.3159

```
Parameter = c("$A_\\lambda$", "$A_\\mu$")
reds = tibble(Parameter, Estimative = exp(mod_8$a))
kable(reds, digits = 8, caption = "Red card rate parameters")
```

Table 3: Red card rate parameters

Parameter	Estimative
A_λ	0.00001973
A_μ	0.00003190

```r
Parameter = c("$\\eta_1$", "$\\eta_2$", "$\\rho_1$", "$\\rho_2$", "$\\kappa$")
st = tibble(Parameter,
            Estimative = c(mod_8$eta, mod_8$rho, mod_8$kappa))
kable(st, digits = 4, caption = "Stoppage time parameters")
```

Table 4: Stoppage time parameters

Parameter	Estimative
η_1	2.4360
η_2	3.7394
ρ_1	1.1400
ρ_2	0.2267
κ	1.0101

```r
mod_8$loglik
```

```
## [1] -12912.89
```