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CHANGING ATTITUDES TO UNIVERSITY MATHEMATICS THROUGH PROBLEM SOLVING

ABSTRACT. University mathematics is often presented in a formal way that causes many students to cope by memorising what they perceive as a fixed body of knowledge rather than learning to think for themselves. This research studies the effects on students' attitudes of a course encouraging co-operative problem-solving and reflection on the thinking activities involved. The attitudinal questionnaire was shown to the students' teachers who were asked to specify the attitudes they *expect* from their students and the attitudes they *prefer*. This was used to give a 'desired direction of change' from expected to preferred. Before the course, half the students responded that university mathematics did not make sense. A majority declared negative attitudes such as anxiety, fear of new problems and lack of confidence. During the problem-solving course the changes were almost all in the *desired* direction. During the following six months of standard mathematics lecturing, almost all changes were in the *opposite* direction.

1. INTRODUCTION

> Maths education at university level, as it stands, is based like many subjects on the system of lectures. The huge quantities of work covered by each course, in such a short space of time, make it extremely difficult to take it in and understand. The pressure of time seems to take away the essence of mathematics and does not create any true understanding of the subject. From personal experience I know that most courses do not have any lasting impression and are usually forgotten directly after the examination. This is surely not an ideal situation, where a maths student can learn and pass and do well, but not have an understanding of his or her subject (3rd Year U.K. Mathematics Student, 1991).

The traditional methods of teaching mathematics at university, which are intended to inculcate rigorous standards of mathematical proof, often seem to lead students into a 'deficit model' of rote-learning material to pass examinations. The resulting procedural forms of thinking and working often prove resistant to change (Sierpiǹska, 1987; Schoenfeld, 1989; Williams, 1991). The knowledge gained may be appropriate for solving routine problems but it can fail in contexts requiring more conceptual insight (Selden, Mason and Selden, 1994). In the memorable phrases of Skemp (1971), students learn the 'product of mathematical thought' rather than the 'process of mathematical thinking'.

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There is evidence that a supportive problem-solving environment can be of help in changing students' attitudes (Schoenfeld, 1985, 1987; Davis and Mason, 1987, 1988; Rogers, 1988). A course based on Mason, Burton and Stacey (1982) has been given for over a decade by the second-named author which additionally focuses on the emotional effects of cognitive success and failure. The students are introduced to the basic ideas of the theory of Skemp (1979) which distinguishes between a goal to be achieved and an anti-goal to be avoided. Skemp theorised that a goal is associated with positive attitudes, both in pleasure of success and positive response to difficulties by seeking alternative methods of attack. But an anti-goal is associated with negative feelings of fear and anxiety. Long-term lack of success is likely to cause a change in attitude from positive goals to anti-goals avoiding failure. The subjective experience of observing the Warwick course over the years suggest that students may be helped to gain control over negative feelings by identifying cognitive difficulties and focusing on positive activities to solve the problem.

Similar difficulties occur in Malaysia (Mohd Yusof and Abd Hamid, 1990). It is the tradition that learning is based on discipline and obedience to work hard; children learn from an early age that it is best to follow the rules. At university, students are keen to succeed by learning procedures to pass examinations. As the problems involve more complex procedures, a Catch-22 situation occurs. Students who are failing seek the security of learned procedures but, because these procedures are becoming more burdensome, the students continue to fail.

It was considered that

> ...a plausible way in which students may become more successful is to become consciously aware of more successful thinking strategies and this must be done in a context designed to impose less cognitive strain (Razali and Tall, 1993, p. 219).

To test this hypothesis, the first-named author translated the course at Warwick University into Bahasa Malaysia and used it to attempt to develop positive attitudes in mathematical thinking. Universities in Malaysia include a wide range of ability (from the 50th to 90th percentile, with the top 10% going abroad for their education). However, it was considered that the same approach, using the same problems and teaching techniques, would prove effective in improving students' attitudes.

2. METHOD

The first-named author taught a ten week, thirty hour course, following the plan of the Warwick course, with a two hour group problem-solving session and a one hour meeting in smaller groups every week. The two hour session began with the instructor focusing on a specific aspect of problem solving, followed by students working on problems illustrating this aspect in self-selected groups of three or four. After half an hour or so the instructor reviewed the situation to see how things were progressing, ensuring that everyone was focused on the same problem and considering ideas generated by the students. She gave no clues to lead students towards a possible solution. Students were encouraged to experience all aspects of mathematical thinking – formulating, modifying, refining, reviewing problems and solutions, specialising, generalising through systematic specialisation, seeking patterns, conjecturing, testing and justifying.

During the one hour meeting, the students were encouraged to reflect on their mathematical experience and talk about their attempts to solve problems. The instructor encouraged them to consider the effectiveness of their solutions – where things may have gone wrong, where they may have failed to take advantage of certain things – ending by summarising their progress.

The students' performance and attitudes were monitored by a variety of methods:

- classroom observation,
- an attitudinal questionnaire administered to the students before, just after, and six months following the course,
- staff responses to the same questionnaire saying what attitudes they preferred and what they expected,
- invited comments from students and staff.

Classroom observation was used to chart broad changes in the students attitudes and behaviour as the course progressed. The main data was collected using an attitudinal questionnaire consisting of a list of statements to which the students were asked whether they agreed on a five point scale (definitely yes, yes, no opinion, no, definitely no). Some of the statements expressed what might be considered 'positive' attitudes and some 'negative'. Whilst we had our own ideas as to which was which, we established a 'desirable direction of change' by asking the staff to say first what attitudes they *expected* students to have and then what they *preferred*. Not only did this allow the students' attitudes to be compared directly with those expected/preferred by the staff, it also allowed us to define 'the desired direction of attitudinal change' for each statement in terms of the direction from what staff expected to what they prefer. It was hypothesised that problem solving would cause a change in students attitudes in the desired direction, but we suspected that some of these changes might be reversed when the students returned to standard mathematics.

TABLE I

Degree classification of students in the experiment

Students	Degree classification					
	I	II-1	II-2	III	P	F
SPK year 5	2	8	5	1	0	0
SPK year 4	3	11	7	1	0	0
SSI year 3	1	5	0	0	0	0
Total	6	24	12	2	0	0

The students

The 24 males and 20 females taking part in the research were a mixture of third, fourth and fifth year undergraduates aged 18 to 21 in Industrial Science, majoring in Mathematics (SSI) and Computer Education (SPK). These covered the full honours degree range (Table I). In the honours degree classification, most students are awarded a ‘good honours degree’ which is either a II-1 (‘upper second class’) or a II-2 (‘lower second’); a small minority achieve a first class (I), some obtain a third class (III), and a few are awarded a pass degree (P) without an honours classification, or a fail (F).

Classroom observation

The students were initially very confused. After a frantic attack on the given problem they would often become stuck and ask ‘What shall I do now?’, ‘Is this the right way of doing it?’. They showed enormous resistance to solving a problem for which they had no given solution procedure. It took about four weeks in which, little by little, they began to make decisions and think for themselves. By this time they were beginning to see the value of writing a ‘rubric’ commentary which kept track of their problem-solving activity.

At first they were set simple problems designed to promote a sense of success to help build self-confidence. As a policy the instructor did not work out all the problems beforehand and was willing to tackle a problem in front of the class to show that even mathematicians do not produce neat solutions at first. This was intended to encourage students to be more willing to make explicit conjectures which might prove wrong as a step along a path to finding a possible route to success. Their discussion became livelier as they found that they could explain things to their friends, rather than simply satisfying the course requirements or pleasing the instructor. Their problem solving became ‘a more creative activity, which includes the formulation of a likely conjecture, a sequence of activities testing, modifying and refining,’ (Tall, 1991).

The questionnaire

A questionnaire was designed to focus on student attitudes based on items trialled in an exploratory study at Warwick University. The Malaysian students were requested to respond to each of the following statements on a five point scale:

Y, y, –, n, N (definitely yes, yes, no opinion, no, definitely no).

Section A: Attitudes to Mathematics

1. Mathematics is a collection of facts and procedures to be remembered.
2. Mathematics is about solving problems.
3. Mathematics is about inventing new ideas.
4. Mathematics at the University is very abstract.
5. I usually understand a new idea in mathematics quickly.
6. The mathematical topics we study at University make sense to me.
7. I have to work very hard to understand mathematics.
8. I learn my mathematics through memory.
9. I am able to relate mathematical ideas learned.

Section B: Attitudes to Problem Solving

1. I feel confident in my ability to solve mathematics problems.
2. Solving mathematics problem is a great pleasure for me.
3. I only solve mathematics problems to get through the course.
4. I feel anxious when I am asked to solve mathematics problems.
5. I often fear unexpected mathematics problems.
6. I feel the most important thing in mathematics is to get correct answers.
7. I am willing to try a different approach when my attempt fails.
8. I give up fairly easily when the problem is difficult.

Students were also asked to respond to the following question:

In a few sentences describe your feelings about mathematics.

TABLE II

Lecturers' perceptions of students preferred and expected attitudes

	Attitude	Desired		Expect					Prefer				
				Yes	(Y)	No	(N)	–	Yes	(Y)	No	(N)	–
Mathematics	Facts and procedures	$\downarrow^{+++}_{+}$	<1%	20	(8)	2	(0)	0	13	(4)	9	(2)	0
	Solving problems	$\uparrow^{+++}_{+++}$	n.s.	19	(9)	3	(0)	0	22	(9)	0	(0)	0
	Interventing new ideas	$\uparrow^{+}_{-}$	n.s.	8	(2)	14	(1)	0	11	(3)	11	(1)	0
	Abstract	$\downarrow^{+++}_{--}$	<1%	20	(6)	2	(0)	0	7	(0)	15	(4)	0
	Understand quickly	$\uparrow^{+}_{---}$	<1%	3	(0)	19	(6)	0	15	(1)	7	(1)	0
	Make sense	$\uparrow^{++}_{-}$	<1%	8	(0)	14	(2)	0	19	(3)	3	(0)	0
	Work hard	$\downarrow^{+++}_{++}$	<1%	21	(13)	1	(0)	0	18	(4)	4	(0)	0
	Learn by memory	$\downarrow^{++}_{---}$	<1%	15	(5)	7	(1)	0	2	(1)	20	(6)	0
	Able to relate ideas	$\uparrow^{+++}_{--}$	<1%	5	(0)	17	(5)	0	22	(5)	0	(0)	0
Problem solving	Confidence	$\uparrow^{+++}_{-}$	<1%	10	(1)	12	(0)	0	22	(3)	0	(0)	0
	Pleasure	$\uparrow^{+++}_{+}$	<5%	15	(0)	7	(2)	0	21	(4)	1	(0)	0
	Only to get through	$\downarrow^{+++}_{-}$	<1%	21	(9)	1	(0)	0	7	(2)	15	(3)	0
	Anxiety	$\downarrow^{++}_{---}$	<1%	16	(5)	6	(0)	0	2	(0)	20	(5)	0
	Fear unexpected	$\downarrow^{++}_{--}$	<1%	15	(7)	7	(0)	0	3	(0)	19	(5)	0
	Correct answers	$\downarrow^{++}_{-}$	<1%	19	(3)	3	(0)	0	6	(2)	16	(2)	0
	Try different approach	$\uparrow^{+++}_{+}$	<1%	12	(1)	10	(0)	0	22	(4)	0	(0)	0
	Give up	$\downarrow^{++}_{--}$	<1%	16	(2)	6	(0)	0	2	(0)	20	(2)	0

3. RESULTS

The 'desired direction of attitudinal change' perceived by mathematics staff

Table II shows the responses of 22 mathematics lecturers asked to respond to the same questions, stating how they *expect* the students to reply and how they *prefer* the response to be. In the columns for 'Expect' and 'Prefer' the total 'Yes' responses (Y + y) are given (added together) with the subset of 'definite yes' responses (Y) given in brackets in the same column. A similar convention is used for 'No (N)'.

The column marked 'desired change' contains an arrow for each item, marking the direction of change (ascending from 'N' to 'Y' and descending from 'Y' to 'N'). Each arrow has two strings of '+' or '–' signs to indicate the strength of agreement or disagreement before and after, given in terms of the average weighted strength (calculated with 'Y' as 2, 'y' as 1, 'n' as –1 and 'N' as –2). An average response of 1 or more is considered 'strong' and denoted '+++' or '---', from 0.5 and less than 1 is denoted by '++' or '--', and less than 0.5 is considered 'weak', denoted '+' or

'–'. For instance, in line 1, 'facts and procedures' changes down from an expected strong agreement (+++) to a preferred weak agreement (+). In line 4, 'being abstract' diminishes from an expected strong agreement (+++) to a preferred disagreement (- -).

This representation is, of course, only a rough guide; a moderately large change within an interval (say from 0.5 to 0.99) will not show a change in symbol (remaining at '++') whilst a tiny change across a boundary (say from 0.49 to 0.5) results in a recorded change (from '+' to '++'). More discerning is the statistical significance of the change, computed using the Wilcoxon Matched-pairs Signed-rank Test (as in Siegel, 1956). For each item the statistic is denoted by one of the three symbols '$<1\%$' (highly significant), '$<5\%$' (significant), or 'n.s.' (not significant).

The desired change of direction was in general in the way we had expected (Mohd Yusof and Tall, 1994). However, we had not expected to find so many items (twelve out of seventeen) in which the teachers expected the students to have attitudes which were the opposite of what they desired.

Only one of these twelve differences is statistically insignificant. The staff have a weak positive preference that students will consider that mathematics involves *inventing new ideas* but a weak negative expectation. The remaining eleven items with opposite preference and expectation are statistically significant at the 1% level. The lecturers *expect* typical students to think mathematics is very *abstract*, that they will *not understand quickly*, they will consider that mathematics does *not make sense*, and they will *not relate mathematical ideas*. Typical students are expected only to *learn through memory*, solving problems only *to get through the course* and consider getting *correct answers* to be the most important thing in mathematics. They are expected *not to have confidence*, to feel *anxious* when faced with problems, to *fear* the unexpected, and to *give up* in the face of difficulty. In every case the lecturers *prefer* the student to have the opposite attitude.

In the remaining five items the staff preference for student attitudes was in the same direction as their expectation (but with different strengths). The attitude to mathematics as *solving problems* was very high on both counts and the students were preferred to obtain great *pleasure*, though expected to get less. The three other items were positive on both counts but were significantly different at the 1% level. The staff prefer students to think mathematics consists of *procedures to be remembered* and to *work hard*, but they expect students to put even greater emphasis on memory and hard work. They prefer students to *try a new approach* but have a lower expectation.

TABLE III

The changing attitudes of students before and after problem solving and ‘after math’

		Before P S					After P S					After Math				
		Yes	(Y)	No	(N)	–	Yes	(Y)	No	(N)	–	Yes	(Y)	No	(N)	–
Mathematics	Facts and procedures	23	(18)	8	(2)	2	11	(3)	32	(8)	1	30	(9)	14	(1)	0
	Solving problems	27	(10)	16	(4)	1	42	(21)	0	(0)	2	32	(22)	12	(0)	0
	Inventing new ideas	21	(4)	21	(6)	2	37	(15)	5	(0)	2	24	(4)	18	(1)	2
	Abstract	25	(13)	17	(0)	2	15	(8)	27	(3)	2	22	(7)	21	(0)	1
	Understand quickly	9	(0)	30	(5)	5	20	(3)	21	(2)	3	16	(2)	26	(1)	2
	Make sense	22	(4)	22	(5)	0	35	(5)	7	(0)	2	29	(4)	14	(0)	1
	Work hard	37	(15)	5	(1)	2	28	(8)	13	(0)	3	32	(8)	12	(1)	0
	Learn by memory	30	(1)	12	(2)	2	11	(0)	31	(7)	2	20	(2)	21	(1)	2
	Able to relate ideas	24	(8)	18	(2)	2	35	(11)	8	(0)	1	31	(5)	10	(0)	3
Problem solving	Confidence	26	(7)	17	(2)	1	36	(12)	6	(0)	2	34	(7)	10	(0)	0
	Pleasure	43	(25)	1	(1)	0	42	(21)	0	(0)	2	42	(21)	1	(0)	1
	Only to get through	16	(4)	27	(8)	1	4	(0)	37	(10)	3	14	(1)	30	(5)	0
	Anxiety	17	(1)	24	(4)	3	6	(0)	36	(9)	2	9	(0)	33	(4)	2
	Fear unexpected	30	(10)	12	(3)	2	10	(3)	31	(9)	3	16	(3)	28	(2)	0
	Correct answers	21	(4)	21	(3)	2	5	(1)	36	(11)	3	17	(0)	25	(7)	2
	Try different approach	42	(17)	0	(0)	2	43	(20)	0	(0)	1	43	(16)	1	(0)	0
	Give up	19	(3)	24	(9)	1	5	(0)	37	(20)	2	9	(0)	33	(12)	2

These responses show the staff expecting the students to work hard to commit facts and procedures to memory. The fact that students are expected to get pleasure from this kind of learning is consistent with their viewing procedural learning as a goal to be attained. But the expectation of emotions such as anxiety and fear of the unexpected intimates an alternative anti-goal of avoiding failure rather than a goal of achieving success. This, coupled with the expectation that they will give up when faced with difficulty, is consistent with the idea that the students are expected to settle for the lesser goal of being able to carry out procedures to solve routine problems rather than attempting to build a broader conceptual understanding.

Changes in student attitudes in problem solving and mathematics lectures

To chart the changes in attitude expressed by the students, the attitudinal questionnaire was given before and after the problem-solving course, then six months later after a semester of standard mathematics lectures (Table III).

TABLE IV

Desired changes compared with changes after problem solving and after mathematics lectures

		Desired change		After P S		After Math		Total change	
Mathematics	Facts and procedures	$\downarrow^{+++}_{+}$	<1%	$\downarrow^{++}_{--}$	<1%	$\downarrow^{++}_{--}$	<1%	$\downarrow^{++}_{++}$	n.s.
	Solving problems	$\uparrow^{+++}_{+++}$	n.s.	$\uparrow^{+++}_{+}$	<1%	$\downarrow^{+++}_{++}$	<1%	$\uparrow^{+++}_{+++}$	n.s.
	Inventing new ideas	$\uparrow^{+}_{-}$	n.s.	$\uparrow^{+++}_{-}$	<1%	$\downarrow^{+++}_{+}$	<1%	$\uparrow^{+}_{-}$	n.s.
	Very abstract	$\downarrow^{+++}_{--}$	<1%	$\downarrow^{+}_{-}$	n.s.*	$\uparrow^{+}_{-}$	n.s.	$\downarrow^{+}_{+}$	n.s.
	Understand quickly	$\uparrow^{+}_{---}$	<1%	$\uparrow^{o}_{-}$	<1%	$\downarrow^{o}_{-}$	<1%	$\uparrow^{-}_{--}$	n.s.
	Make sense	$\uparrow^{++}_{-}$	<1%	$\uparrow^{++}_{-}$	<1%	$\downarrow^{++}_{+}$	n.s.%	$\downarrow^{+}_{-}$	n.s.*
	Work very hard	$\downarrow^{+++}_{++}$	n.s.	$\downarrow^{+++}_{+++}$	n.s.*	$\uparrow^{++}_{++}$	n.s.	$\downarrow^{+++}_{++}$	n.s.
	Learn by memory	$\downarrow^{++}_{---}$	<1%	$\downarrow^{+}_{--}$	<1%	$\uparrow^{-}_{--}$	<5%	$\downarrow^{+}_{-}$	n.s.*
	Able to relate ideas	$\uparrow^{+++}_{--}$	<1%	$\uparrow^{++}_{+}$	<5%	$\downarrow^{++}_{++}$	n.s.	$\uparrow^{++}_{+}$	n.s.
Problem solving	Confidence	$\uparrow^{+++}_{-}$	<1%	$\uparrow^{++}_{+}$	<5%	$\downarrow^{++}_{++}$	n.s.	$\uparrow^{++}_{+}$	<5%
	Pleasure	$\uparrow^{+++}_{+}$	n.s.	$\boxed{\downarrow^{+++}_{+++}}$	n.s.	$\downarrow^{+++}_{+++}$	n.s.	$\downarrow^{+++}_{+++}$	n.s.
	Get through	$\downarrow^{+++}_{-}$	<1%	$\downarrow^{-}_{---}$	<1%	$\uparrow^{--}_{---}$	<1%	$\downarrow^{-}_{-}$	n.s.
	Anxiety	$\downarrow^{++}_{---}$	<1%	$\downarrow^{-}_{--}$	<5%	$\boxed{\downarrow^{--}_{--}}$	n.s.	$\downarrow^{-}_{--}$	n.s.
	Fear unexpected	$\downarrow^{++}_{--}$	<1%	$\downarrow^{++}_{--}$	<1%	$\uparrow^{-}_{--}$	n.s.	$\downarrow^{++}_{-}$	<1%
	Correct answers	$\downarrow^{++}_{-}$	<1%	$\downarrow^{+}_{--}$	<1%	$\uparrow^{-}_{--}$	<1%	$\downarrow^{+}_{-}$	n.s.
	Try new approach	$\uparrow^{+++}_{+}$	<1%	$\uparrow^{+++}_{+++}$	n.s.	$\downarrow^{+++}_{+++}$	n.s.	$\downarrow^{+++}_{+++}$	n.s.
	Give up	$\downarrow^{++}_{--}$	<5%	$\downarrow^{-}_{---}$	<1%	$\downarrow^{--}_{---}$	n.s.	$\downarrow^{-}_{--}$	<5%

Calculating the significance in the change of the responses and using the same techniques as in Table II, we find the changes given in Table IV. These show a remarkable story of two quite distinct changes in attitude.

During the problem-solving course, the weighted attitudinal changes are in the direction desired by the lecturers, (except only for ‘pleasure’ which is almost a maximum both times and changes only marginally from 43 to 42 (out of 44)).

During the return to mathematics lectures, the weighted attitudinal changes are in the opposite direction from that desired by the lecturers (except only for ‘anxiety’, and even here the *total* feeling anxious increases from 6 to 9 and those not anxious decrease from 36 to 39).

During the problem-solving course, only four changes are not statistically significant. *Pleasure, willingness to work hard, willingness to try a new approach* all start so high that there is little room for improvement. Meanwhile, the sense that *mathematics is abstract* hovers around the central measure, changing slightly from positive to negative.

Three items change significantly at the 5% level, all in the 'desired direction of change': *ability to relate ideas* and *confidence* both increase, whilst *anxiety* diminishes.

All other items have highly significant changes in the desired direction. Some beliefs are reversed so that after problem-solving the weight of student opinion no longer supports the idea that mathematics is just facts and procedures. The overall attitude has moved so that it now involves *inventing new ideas*, it *makes sense*, it is *not learnt just through memory*, there is *less fear of the unexpected*, it is *not just getting correct answers*. Other positive attitudes are greatly increased: *mathematics is more about solving problems*, it can be *understood more quickly*, and students are *less likely to give up* when encountering a difficulty.

However, six months later, after returning to the mathematics course many opinions have reverted back in the old direction. There is a significant increase in belief that *mathematics is to be learned through memorisation*, and a highly significant reversal in belief back to the notion that *mathematics is just facts and procedures*. It is *less about solving problems, less about inventing new ideas, less about doing the work for reasons other than to get through the course* and *less about things other than correct answers*.

Comparing the situation from before the problem-solving course with the status after six months back at regular mathematics lectures, many of the indicators revert back towards their old position. However, three problem-solving attributes broadly retain their improvement: *confidence* and *unwillingness to give up* remain significantly high, while *fear of the unexpected* remains highly significantly reversed. Thus the emotional attitudes of confidence, persistence and overcoming fear all retain their improvement six months later. In terms of Skemp's theory this is consistent with anti-goals avoiding failure diminishing whilst goals of achieving success increase.

Smaller changes are evident in the belief that mathematics *make sense* and that *it is not necessary just to learn by memory*. (These are improved by a factor that would be significant at the 10% level, marked 'n.s.*' in Table IV.)

In addition, a number of desirable attitudes rated at least '++' or '--' carry over from before the problem-solving course. These are *mathematics is about facts and procedures*, and *solving problems*, that students *work hard*, are *able to relate ideas*, take great *pleasure* in their work, have *low anxiety*, and are *willing to try a new approach*. This combination of attitudes is consistent with having the goal of achieving success through procedural learning.

TABLE V

Classification of written responses

Student comments ($N = 44$)	Positive Pre	Positive Post	Negative Pre	Negative Post
Nature of mathematics	5	12	7	2
Personal	15	28	12	5
Teaching	0	0	8	4
Total comments	20	40	27	11

The nature of the change in students' mathematical thinking

The data reveals the astonishing result that, when students are doing problem-solving, their attitudes change in the direction desired by the mathematics lecturers yet, when the students return to be taught by the mathematics lecturers, the students attitudes now move in the opposite direction from that desired by those teaching them. How can this be? There is certainly the possibility that the problems faced in the problem-solving course are less demanding than the mastery required of new ideas in the regular courses. It may be that the regular course involves difficulties for which students can no longer operate in any way other than rote-learning the material for reproduction in the examinations. Such an opinion had been expressed at Warwick University:

> Clearly it is better if students understand the mathematics they have studied. But in practice this is difficult to attain because of the nature of the subject and time constraints. Often many students find it is not always possible to understand the course so they may choose instead to memorise the syllabus and solutions to the example sheets. ... It is perfectly possible to gain a good grade by rote-learning (3rd Year U.K. Mathematics Student).

Student comments before and after problem-solving

In the questionnaire, the students were asked to write a few sentences describing their feelings about mathematics. These were grouped under three headings: the nature of mathematics, personal feelings (such as motivation, interest, pressure etc.) and teaching methods. For example, items classified under the 'nature of mathematics' on the pre-test included negative comments such as 'too abstract', 'seems pointless', and 'theory more difficult than practice'. Negative 'personal factors' included 'lack of motivation', 'put off by amount that needs to be done' and 'puzzled by what is going on'. Positive feelings were mostly about the course being 'enjoyable and challenging', 'great sense of satisfaction when able to understand new concepts and to solve problems' and 'effort put in is worthwhile'. All responses relating to teaching were all negative – such as 'difficult to follow' and 'delivered in a dull atmosphere'.

After problem solving, comments shifted in a positive direction (Table V). Comments written after the problem-solving course include:

> I am beginning to think instead of just doing the tutorial questions. . . . I think I am learning more because I understand what is going on (3rd Year Industrial Science Student, majoring in Mathematics).

> Mathematics has always given me a lot of problems because I don't have the ability for memorisation. . . . Now that I know about mathematical thinking, my interest and desire to learn maths have increased (4th Year Computer Education Student).

> The course should have been introduced earlier. . . . After following the course I am more confident to solve any maths problem that is given (5th Year Computer Education Student).

These are consistent with the classroom observations and the changes intimated by the questionnaire support the hypothesis that the course in problem-solving changes the attitudes of students from mathematics as a body of procedures to be learned to mathematics as a process of thinking.

Student comments six months after returning to mathematics

The students comments after returning to standard mathematics learning suggest a number of factors that could explain their changes in attitudes. For instance, about a third (32%) reported that the regular mathematics did not allow them to think in a problem-solving manner:

> Since following the course I know mathematics is about solving problems. But whatever mathematics I am doing now doesn't allow me to do all those things. They are just more things to be remembered (5th Year Computer Education Student).

> I believed mathematics is useful in that it helps me to think. Having said that it is hard to say how I can do this with the maths I am doing. Most of the questions given can be solved by applying directly the procedures we had just learned. There is nothing to think about (3rd Year Industrial Science Student, majoring in Mathematics).

> There is little discussion and it provides no encouragement to do maths. The content is emphasised over everything else. We are crammed full of lots of bland mathematical abstract theory (3rd Year Industrial Science Student, majoring in Mathematics).

Some emphasise the speed of presentation to complete the content:

> I did not enjoy most of the maths courses – too dependent on the lecturers. I don't find the way most of them teach particularly inspiring. We find ourselves hurrying through to keep up. There is no time to think about the mathematics we are doing (3rd Year Industrial Science Student, majoring in Mathematics).

Some appreciate their knowledge in problem solving, suggesting it helps them to learn their mathematics and solve problems more effectively:

> The problem solving techniques help me come to terms with the abstract nature of the maths I am doing. I try to connect the ideas together and talk about them with my friends. It is not that easy though. But I feel all the effort worth it when I am able to do so (3rd Year Industrial Science Student, majoring in Mathematics).

> I find the problem solving knowledge very useful in helping me understand the whys and the hows of advanced mathematics. It is much more satisfying than rote-learning. Furthermore it is actually easier to remember something that you understand (4th Year Computer Education Student).

There are some who have reservations about using problem solving in regular mathematics, but see the possibility of improvement:

> The main disadvantage is time. It would take several hours maybe days to understand each new concept. Under the current circumstances we are finding ourselves rapidly hurrying to keep up. Sometime we were too bogged down in the technical details and we end up purely taking down the notes without even concentrating. This really defeats the problem-solving techniques. ... But I think with further support from good teaching as well as tailoring the courses to suit the needs of the students the situation can be improved (5th Year Computer Education Student).

Individual interviews with lecturers

Interviews revealed substantial differences in meaning of ideas expressed in the questionnaire from the ideas of 'mathematical thinking' in the problem-solving course. For instance, Kilpatrick and Stanic (1989) suggest three different perceptions of problem solving – as a means to a focused end, as a skill, and as an art. It soon became apparent that the lecturers see it more as a means to achieve a specific end or a skill to be learned rather than the art of thinking mathematically. 'Inventing new ideas' was perceived as original research rather than just ideas that are new to the individual:

> To me mathematics is a tool for solving problems. One way of motivating the students is by showing them applications in the real world. In this way they get the knowledge and the skills for solving problems. ... I do not think the students are capable of creating new ideas on their own.

Lecturers are not certain of the strategies used in the problem-solving course:

> ... I am not sure of these [processes]. I have not thought about them and I don't know how to go about [teaching] them. I think I need to learn more about them before I can implement them. We developed certain abilities to look at problems but we are not sure how those abilities came to be with you.

Instead they show students how to do examples in the hope that they will develop their own techniques:

The experience of making conjectures, generalising and the like, I think the students can get themselves on their own, from doing their project work. We do not have the time to teach them everything. We tell them how to do it – for example, what are the criteria that should be fulfilled in the formula before they can use it. Normally I explain only part of it then I think the students can complete it themselves. . . . I think that is sufficient for the students.

Some lecturers genuinely want to change the system but are not sure how:

I would like students not only to see mathematics as a subject that they need to learn and pass in an exam but also as a discipline which enables them to think for themselves. My main aim is not in trying to finish the syllabus but rather in making the students learn the mathematics in a more meaningful way. . . . I am not really sure how but I am trying to do it.

To me mathematics is a mental activity but I should say that at this level I presented it more as a formal system. Because we are confined by the syllabus and also depending on the students' background. . . . I would like it to change. How do I do that? I don't know.

The system has been proven a failure. It has not been successful in producing good mathematicians, or engineers that can use mathematics effectively. They only know how to use procedures or computer packages without really understanding why they use them. . . . It's all down to the system. We are not training students to discover patterns, or how to prove a statement is true, for example. What we teach them is mainly how to use the procedures.

4. SUMMARY

Although lecturers prefer students to have a range of positive attitudes to mathematics, they expect the reality to be different. They *prefer* students to desire to work hard to see mathematics as a challenge of solving problems, making sense through relating new ideas without needing to learn solely through memory, having confidence, deriving pleasure, with low anxiety and fear, ready to try a new approach and being undeterred by difficulty. On the other hand, they *expect* students to perceive mathematics as an abstract subject which they (initially) fail to understand, responding to their fear, anxiety and lack of confidence in tackling novel problems by working hard to finish the course by memorising facts and procedures, seeking only the security of getting correct answers, and being all too ready to give up when things get difficult.

The findings suggest that the lecturers have little confidence in the students' ability to cope with a formal mathematics course and teach them accordingly. The students acquiesce in this approach, setting their sights on the lower target of being successful in routine tasks by learning procedurally. This succeeds in giving them a sense of pleasure in achieving their aim, yet many comment that the quantity and difficulty of mathematics covered gives them little room for creative thinking.

By assigning a 'desired direction of change' from what lecturers expect to what they prefer, it transpires that when doing a problem-solving course almost all the changes are in the desired direction and when returning to mathematics lectures, almost all the changes are in the reverse direction.

A major problem not addressed at all in this research is how to complement problem-solving strategies with curriculum content in a way which results in appropriate attainments in both. What has been shown in this research is that standard formal teaching methods in higher education can cause attitudinal changes which are the reverse of what is considered desirable by mathematicians. This dilemma causes us to pose the following central question:

Given this evidence, do professors wish to continue to get what they expect, or is it possible to effect changes towards what they prefer?

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