Case Study of *M/M/1* Queues with Batch Arrivals

Aline Aires Teixeira, Luiz Gustavo Barros Guedes and Masoud Khazaee
Prof. Dr. Samuel Baraldi Mafra

Junho / 2024

Inatel
Instituto Nacional de Telecomunicações

CRR
Inatel - Brasil
Centro de Referência em Radiocomunicação

Summary

Inatel
Instituto Nacional de Telecomunicações

Summary

www.inatel.br

Introduction

- **History:** Pioneering work by Erlang (1909) and Molina (1927);
- **Relevance:** Queue systems in telecommunications, banks, restaurants, hospitals and so on;
- **What is the problem we are dealing with?** *M/M/1* queues with batch arrivals;
- **How can it be applied in telecommunications?** When data is broadcast in bursts or bundled together in bigger units to improve efficiency;
- **What is the purpose of this work?** Assess *M/M/1* queues with batch arrivals performances under different levels of utilization and buffer sizes.

Summary

Modeling

- **Model Components:**
  - Batch arrival rate, λ_b;
  - Packet arrival rate, λ;
  - Service rate, μ;
  - Batch size (number of packets by each batch) determined by Bernoulli process;
  - Utilization Factor, ρ;
  - Buffer size, N;
  - Server State;

- **Performance metrics:**
  - Blocking probability, mean time in system and mean packets in system.

Summary

Case study: two batch sizes with the same probability of occurrence

- **Problem:**

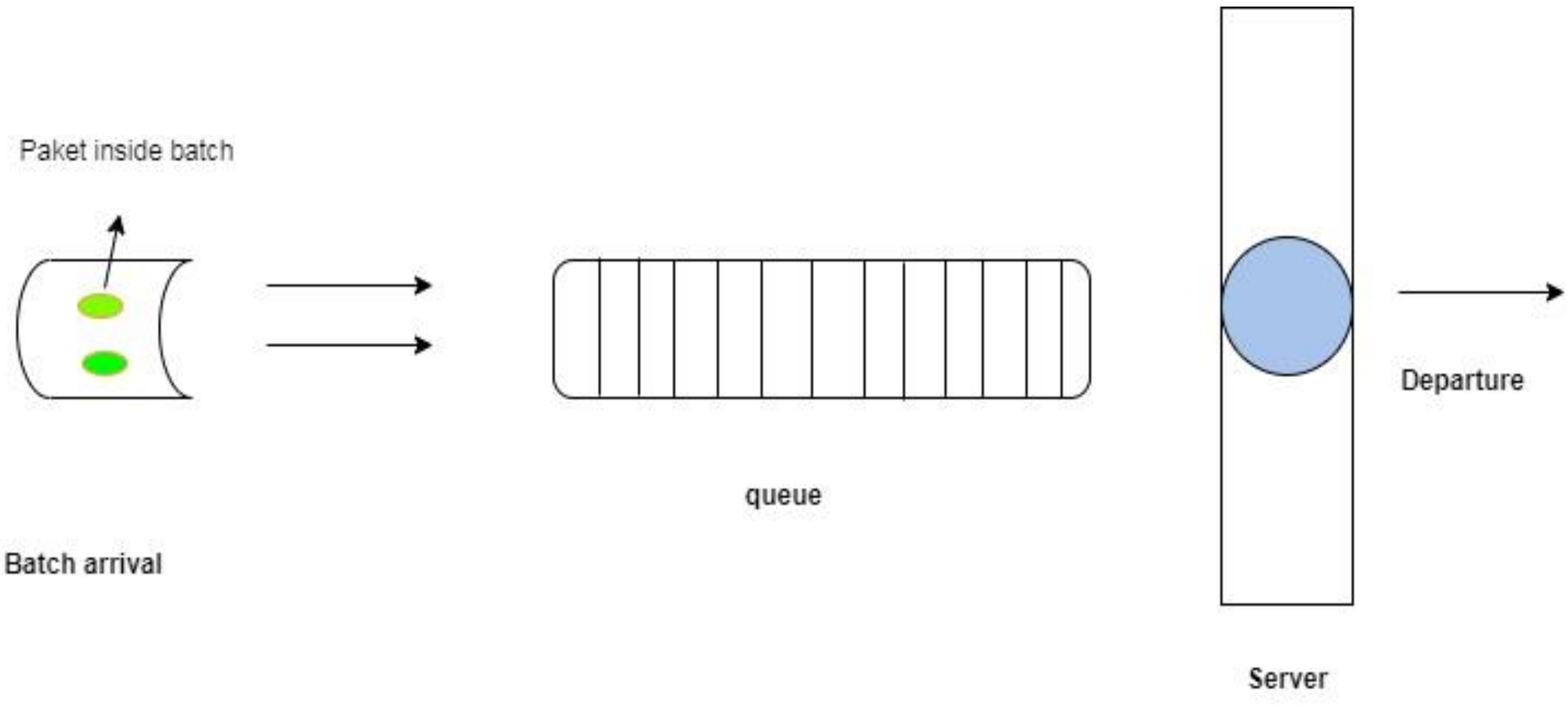
Consider an *M/M/1* queue with batch arrivals (variable number of packets arrive at the server). The batches have different probabilities of arrival. For example, we can have a network with only two types of batches: a batch of size 1 packet, as in the *M/M/1* case, and a batch of size 2 packets. In this case, the arrival probabilities of batches of sizes 1 and 2 can be equal. The arrival rate of the batches is λ_b. The service time has an average of 1/μ (the service is performed for each packet, not for each batch).

- **Determination of the batch size:** Using a Bernoulli distribution;
- **Probabilities (p_1, p_2):** For selecting one or two packet batches, respectively;
- **Determination of arrival rate of the batches considering different probabilities of arrival**:

$$\lambda_b = \frac{\lambda}{1 \cdot p_1 + 2 \cdot p_2} = \frac{\rho\mu}{1 \cdot p_1 + 2 \cdot p_2}$$

Case study: two batch sizes with the same probability of occurrence

- ***M/M/1*** **queue with batch arrivals:**



Summary

Theoretical reference: conventional M/M/1

- **Performance metrics**

➢ **Mean packets in system:**

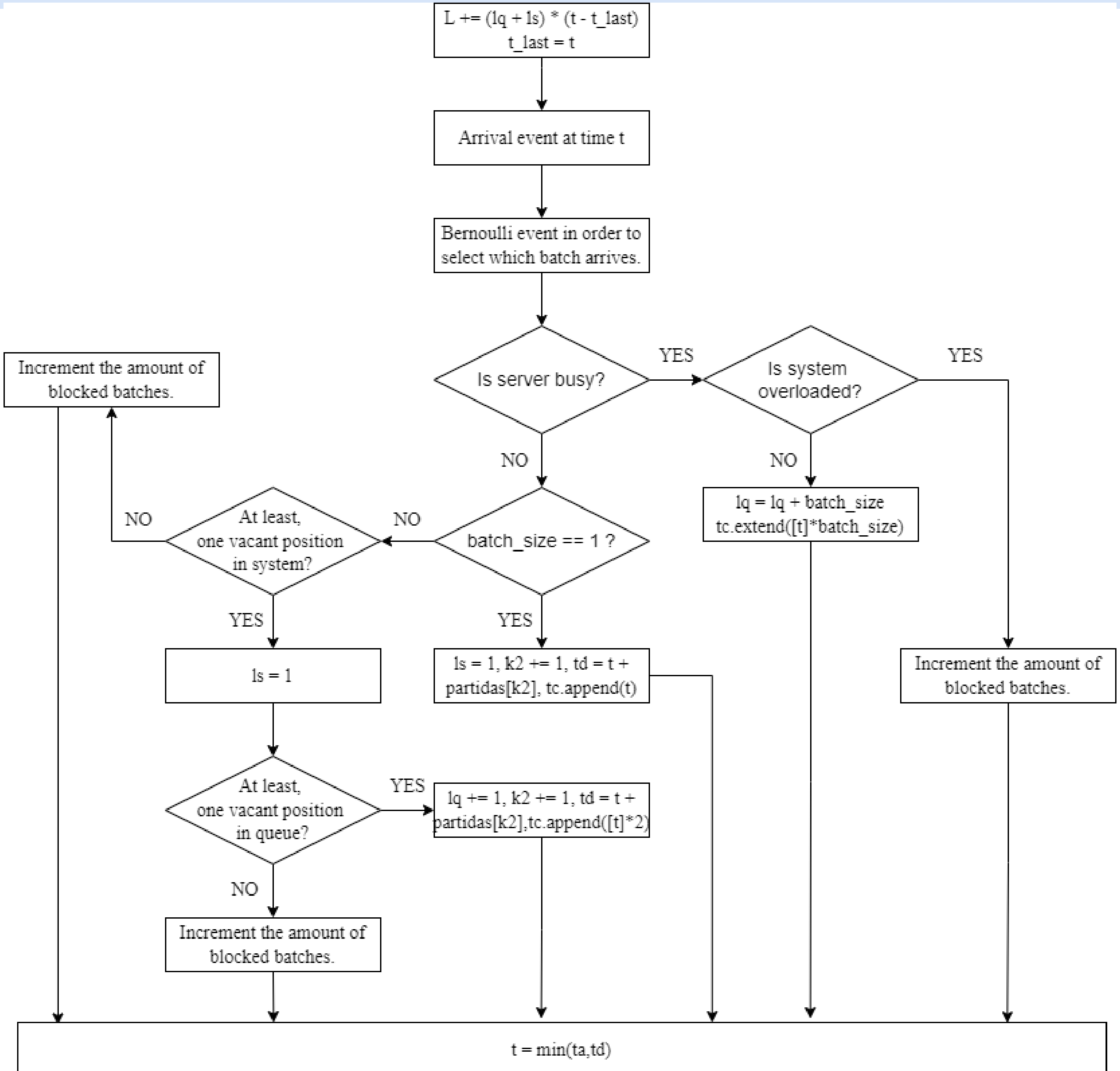
$$\mathbb{E}[q] = \frac{\rho}{1-\rho}$$

➢ **Mean time in system:**

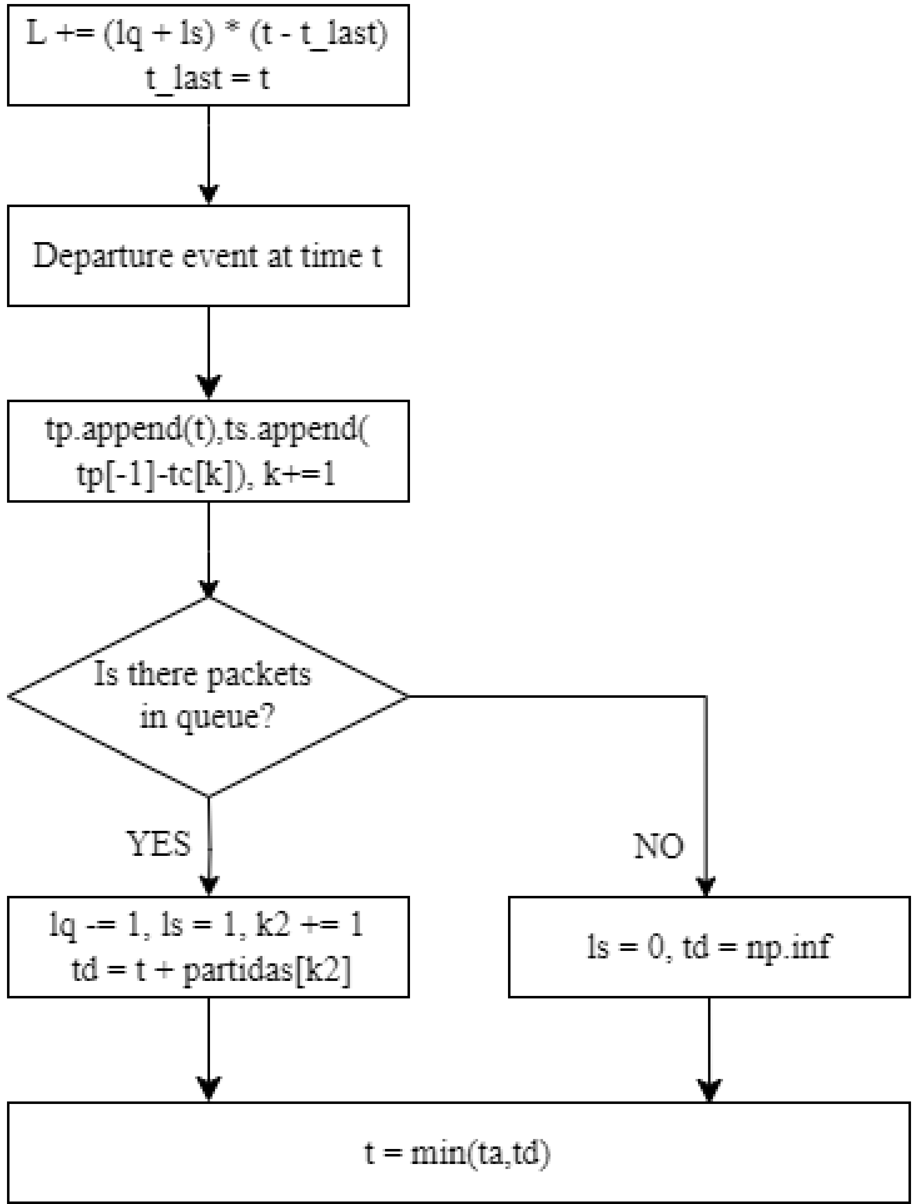
$$\mathbb{E}[T_q] = \frac{E[q]}{\lambda} = \frac{1}{\mu - \lambda}$$

Summary

Numerical Analysis: Flowchart for Arrival Event



Numerical Analysis: Flowchart for Departure Event



Summary

1. Introduction

2. Modeling
   A. Case study: two batch sizes with the same probability of occurrence
   B. Theoretical reference: conventional *M/M/1*

3. Numerical Analysis
   A. Flowchart
   B. Simulation setup
   C. Results

4. Conclusions

Numerical Analysis: Simulation Setup

- **Parameters:**
  - **Service Rate, μ:** 12 packets per second;
  - **Utilization Factors, ρ:** 0.2, 0.4, 0.6, 0.8, 1.0;
  - **Simulation Time:** 5000 seconds**;**
  - **Buffer size, *N***: infinite for *M/M/1* and finite (N = 10).
- **Performance metrics:**
  - Average Time in System, E[q];
  - Average Number of Packets in System, E[T_q];
  - Blocking Probability, P_b.

Inatel
Instituto Nacional de Telecomunicações

Summary

www.inatel.br

Numerical Analysis: conventional *M/M/1* with infinite buffer

TABLE I

PERFORMANCE COMPARISON USING DIFFERENT ρ VALUES IN INFINITE CASE, CONSIDERING PACKET ARRIVALS.

ρ	$\mathbb{E}[T_q]_s$, s	P_b	$\mathbb{E}[q]_s$, packets	$\mathbb{E}[T_q]_t$, $\mathbb{E}[q]_t$
0.2	0.1054	0.0000	0.2565	0.1042, 0.25
0.4	0.1407	0.0000	0.6809	0.1389, 0.6667
0.6	0.2049	0.0000	1.4643	0.2083, 1.5
0.8	0.3975	0.0000	3.7570	0.4167, 4
1	17.2847	0.0000	206.6625	∞, ∞

Numerical Analysis: *M/M/1* with batch arrivals

TABLE II

PERFORMANCE COMPARISON USING DIFFERENT ρ VALUES IN INFINITE CASE, CONSIDERING BATCH ARRIVALS.

ρ	$\mathbb{E}[T_q]_s$, s	P_b	$\mathbb{E}[q]_s$, packets
0.2	0.1033	0.0000	0.2476
0.4	0.1410	0.0000	0.6818
0.6	0.2063	0.0000	1.4818
0.8	0.4272	0.0000	4.1141
1	12.9260	0.0000	155.2688

Numerical Analysis : *M/M/1* with batch arrivals

TABLE III

PERFORMANCE COMPARISON USING DIFFERENT ρ VALUES IN FINITE CASE ($N = 5$), CONSIDERING BATCH ARRIVALS.

ρ	$\mathbb{E}[T_q]_s$, s	P_b	$\mathbb{E}[q]_s$, packets
0.2	0.1343	0.0035	0.3212
0.4	0.1649	0.0273	0.7685
0.6	0.1926	0.0687	1.2727
0.8	0.2169	0.1281	1.7622
1	0.2441	0.2004	2.2482

Numerical Analysis : *M/M/1* with batch arrivals

TABLE IV

PERFORMANCE COMPARISON USING DIFFERENT ρ VALUES IN FINITE CASE ($N = 10$), CONSIDERING BATCH ARRIVALS.

ρ	$\mathbb{E}[T_q]_s$, s	P_b	$\mathbb{E}[q]_s$, packets
0.2	0.1371	0.0000	0.3199
0.4	0.1777	0.0007	0.8546
0.6	0.2533	0.0102	1.8213
0.8	0.3426	0.0415	3.1384
1	0.4531	0.1073	4.7663

Inatel
Instituto Nacional de Telecomunicações

Summary

1. Introduction

2. Modeling
   A. Case study: two batch sizes with the same probability of occurrence
   B. Theoretical reference: conventional *M/M/1*

3. Numerical Analysis
   A. Flowchart
   B. Simulation setup
   C. Results

4. <u>Conclusions</u>

www.inatel.br

Conclusions

- O desempenho alcançado pela fila *M/M/1*, considerando chegadas em lotes, é o mesmo que o do sistema convencional, que analisa chegadas de pacotes. Tanto o tempo médio quanto o número de elementos no sistema consideram pacotes, não lotes, e essa é a razão. Além disso, as taxas efetivas de chegada de pacotes são as mesmas em ambas as simulações;

- Baixa utilização: ocorre quando a taxa de serviço é consideravelmente maior do que a taxa de chegada. Assim, o sistema pode operar adequadamente sem rejeitar lotes ou pacotes. Se reduzirmos o tamanho do buffer, é provável que essa condição se torne limitada, aumentando a chance de bloqueio mesmo em cenários com baixa utilização;

- À medida que a utilização aumenta, considera-se que a taxa de chegada aumentou para uma taxa de serviço fixa. Assim, um buffer limitado apresentará uma maior chance de rejeitar lotes, aumentando a probabilidade de bloqueio;

- Trabalho futuro: Verificar expressões teóricas para análise de desempenho considerando chegadas em lotes.

Thanks!

luizgustavo.barros@inatel.br
masoud.khazaee@dtel.Inatel.br
aline.teixeira@dtel.Inatel.br