

# Influence of Rice Factor on the Performance of Energy Detectors for Cooperative Spectrum Sensing Under Laplacian Noise

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**Abstract**—Electronic systems in general can be impaired by impulsive noise generated by a variety of sources. Spectrum sensors are of particular interest herein, since their probabilities of detection and false alarm can be severely degraded under this impairment. Several models for impulsive noise have been studied in the literature, all of them having the common characteristic of being well represented by heavy-tailed probability density functions, like Laplace and some Stable distributions. This article addresses the performances of state-of-the-art detectors for cooperative spectrum sensing when the received signal is impaired by Laplacian noise. This is made by means of estimating the probability of detection for a fixed false alarm rate, when important system parameters are varied. It is demonstrated that the robustness against impulsive noise varies significantly depending on the adopted detection strategy.

**Index Terms**—Cognitive radio, cooperative spectrum sensing, dynamic spectrum access, impulsive noise, Laplacian noise.

## I. BRIEF EXPLANATION OF THE TOPIC

**B**OTH spectrum scarcity and underutilization are experienced due to the meaningful expansion of wireless communication services operating under a fixed bandwidth allocation policy. The former consists of the lack of new free bands, whereas the latter refers to the momentary unoccupation of some band by the primary user (PU), who owns the right to use of it.

Spectrum sensing [1] arises in this scenario to allow a flexible use of the spectrum bands among the PUs and the secondary users (SUs), which do not hold the priority right of using these bands [2]. These SUs should be able to seek for vacant bands, through spectrum sensing, for shared usage with the PUs.

### A. Summary of the main concepts of spectrum sensing

Spectrum sensing is a binary hypothesis test in which the null hypothesis,  $\mathcal{H}_0$ , refers to the absence of the primary signal in the sensed band and the alternative hypothesis,  $\mathcal{H}_1$ , refers to the presence of the primary signal. By comparing

a test statistic,  $T$ , with a decision threshold,  $\gamma$ , the test is performed. It is decided in favor of  $\mathcal{H}_1$  if  $T > \gamma$ , or in favor of  $\mathcal{H}_0$  otherwise. The purpose of this test is, therefore, to decide whether the received signal was generated under the hypothesis  $\mathcal{H}_0$  or  $\mathcal{H}_1$ . There are two key performance metrics associated with this test: the probability of detection,  $P_d$ , and the probability of false alarm,  $P_{fa}$  [1]. The former is the probability of deciding that the PU signal is present in the sensed band, when it is indeed present, that is,  $P_d = P[T > \gamma | \mathcal{H}_1]$ , while the latter is the probability of deciding that such a signal is present in the sensed band when, in fact, it is absent, that is,  $P_{fa} = P[T > \gamma | \mathcal{H}_0]$ . Performance targets are, for example,  $P_d > 0.9$  and  $P_{fa} < 0.1$ , as determined by the IEEE 802.22 standard [3].

Spectrum sensing can be made by a single SU, independently from the other SUs, which is referred to as non-cooperative spectrum sensing (NCSS), or can apply several SUs working together, which is referred to as cooperative spectrum sensing (CSS) [1]. Although multipath fading and signal shadowing degrade the performance of both NCSS and CSS, the degradation is less pronounced in the second case. This is because CSS takes advantage of the spatial diversity achieved with the use of multiple SUs located in different positions.

In the centralized CSS with data fusion, which is the strategy adopted herein, the  $n$  samples collected by each of the  $m$  SUs in cooperation are transmitted to the fusion center (FC) belonging to the secondary network. Then, these samples are combined in order to allow for deciding upon the occupation state of the sensed band. In centralized CSS with decision fusion, local decisions made by each SU are sent to the FC, where these decisions are combined to form a global final decision about the occupancy state of the sensed channel [1].

In addition to the factors related to the signal propagation, it is well known that noise and different kinds of interference can degrade spectrum sensing performance. Regarding to the noise, its omnipresent form is the thermal noise or additive white Gaussian noise (AWGN), which is generated at the receiver side of every communication system. However, in specific environments, impulsive noise may be present either, and can be way more disastrous to the spectrum sensing performance than AWGN.

Impulsive noise [4] is an undesired random signal that contains occasional peaks of relatively high amplitude and short duration. Electromagnetic impulsive noise, which is considered in the present context, can originate from various

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This work was partially supported by RNP, with resources from MCTIC, Grant 01245.020548/2021-07, under the Brazil 6G project of the Radiocommunication Reference Center (*Centro de Referência em Radiocomunicações* - CRR) of the National Institute of Telecommunications (*Instituto Nacional de Telecomunicações* - Inatel), Brazil, by Huawei, Grant PPA6001BRA23032110257684, under the project Advanced Academic Education in Telecommunications Networks and Systems, and by CNPq, Grant 302589/2021-0, Brazil. doi: 10.14209/jcis.2024.1.

sources.

When studying the influence of impulsive noise in electronic systems, the impulsive noise model can be modeled by the Laplacian one, which is considered in this work, aiming at evaluating its influence on some detectors for spectrum sensing. The Laplace distribution [18] is a member of the family of symmetric stable distributions, and is also known as the double-exponential distribution. It is characterized by heavy tails when compared to the Gaussian distribution, being often used to model data with outliers, which is the case of a signal corrupted by thermal plus impulsive noise [19]–[24].

### B. Contributions and organization of the work

The fundamental contribution of this work is the performance analysis of energy detector (ED), absolute value cumulating (AVC) and combining with order statistics enhanced energy detector (COS EED) for centralized CSS with data fusion when impaired by Laplacian noise. The performance comparisons are made by means of the probability of detection, for a constant false alarm rate (CFAR) of 0.1, as a function of the  $\mu_\kappa$  CSS system parameter.

The remaining of this work is structured as follows: Section II describes the system model adopted in the centralized CSS system, while Section III addresses the test statistics used in the performance analysis. Numerical results and discussions are given in Section IV. Section V concludes the work.

## II. SYSTEM MODEL

The signal model adopted herein refers to the centralized CSS with data fusion, wherein  $n$  samples proceeding from the primary signal, which has been transmitted by the PU, are collected by each of the  $m$  cooperating SUs and transmitted to the FC through an error-free control channel. At the FC,  $mn$  received samples compose the matrix  $\mathbf{Y} \in \mathbb{R}^{m \times n}$ , given by

$$\mathbf{Y} = \mathbf{h}\mathbf{x}^T + (1 - I)\mathbf{V} + I\mathbf{L}, \quad (1)$$

where  $I$  is an indicator variable such that  $I = 1$  particularizes (1) for a sensing channel with Laplacian noise and  $I = 0$  particularizes (1) for a pure AWGN channel. The vector  $\mathbf{x} \in \mathbb{R}^{n \times 1}$ , which models the PU signal, is composed of  $n$  real Gaussian samples with zero mean and variance defined according to the average SNR across the SUs. The use of samples following the Gaussian distribution is suitable for modeling the nature of envelope fluctuations of numerous modulated and filtered signals.

The channel vector  $\mathbf{h} \in \mathbb{R}^{m \times 1}$  having elements  $h_i$ ,  $i = 1, 2, \dots, m$ , portraying the gains of the sensing channel between the PU and each SU, is given by

$$\mathbf{h} = \mathbf{G}\mathbf{a}, \quad (2)$$

where  $\mathbf{a} \in \mathbb{R}^{m \times 1}$  is a vector assembled by real Gaussian random variables  $a_i \sim \mathcal{RN}[\sqrt{\kappa}/(\kappa + 1), 1/(\kappa + 1)]$ , where  $\kappa$  models the Rice factor [36] of the channel between the PU and the  $i$ -th SU. In dB, its value becomes  $\kappa_{\text{dB}} = 10\log_{10}(\kappa)$ . The elements  $h_i$  are considered constants during each sensing interval and independent and identically distributed (iid) between

successive sensing rounds. Furthermore, the bandwidth of the primary signal is considered to be smaller than the coherence bandwidth of the sensing channel, which affects all frequency components of the signal in the same manner. In other words, this scenario corresponds to a flat fading channel.

According to [37], it has been verified that  $\kappa_{\text{dB}}$  is a random variable which depends on the environment where the associated system is inserted. It is modeled as a Gaussian random variable  $\kappa_{\text{dB}} \sim \mathcal{N}[\mu_\kappa, \sigma_\kappa]$ , considering both  $\mu_\kappa$  and  $\sigma_\kappa$  in dB.

The matrix  $\mathbf{G} \in \mathbb{R}^{m \times m}$  in (2) is given by

$$\mathbf{G} = \text{diag} \left( \sqrt{\frac{\mathbf{p}}{p_{\text{tx}}}} \right), \quad (3)$$

where  $\mathbf{p} = [p_1 \ p_2 \ \dots \ p_m]^T$  is the vector containing the received signal powers at the SUs,  $p_{\text{tx}}$  is the primary signal transmitted power, in watts, and  $\text{diag}(\cdot)$  returns a diagonal matrix whose main diagonal is composed by the elements of the vector present in the argument.

The area-mean received power,  $p_i$ , by the  $i$ -th SU receiver at a given distance  $d_i$  from the PU transmitter can be calculated using the log-distance propagation prediction method [36] as

$$p_i = p_{\text{tx}} \left( \frac{d_0}{d_i} \right)^\eta, \quad (4)$$

where  $d_0$  corresponds to a reference distance in the far-field region of the PU transmit antenna and  $\eta$  is the environment-dependent path-loss exponent [36]. The received power varies inversely proportional to the value of  $\eta$  at a given distance.

The elements in the  $i$ -th row of the matrix  $\mathbf{V} \in \mathbb{R}^{m \times n}$ , which models the AWGN noise, refer to the  $i$ -th SU and are iid Gaussian random variables, with zero mean and time-varying variance  $\sigma_i^2$  given by

$$\sigma_i^2 = (1 + \rho u_i) \sigma_{\text{avg}}^2, \quad (5)$$

where  $0 \leq \rho < 1$  is the fractional variation of the noise power around its mean,  $\sigma_{\text{avg}}^2 = \frac{1}{m} \sum_{i=1}^m \sigma_i^2$ , and  $u_i$  is the realization of a uniform random variable  $\mathcal{U}_i \sim [-1, 1]$ .

The instantaneous SNR,  $\gamma$ , across the SUs is given by

$$\gamma = \frac{1}{m} \sum_{i=1}^m \frac{p_i}{\sigma_i^2}. \quad (6)$$

Therefore, the average SNR across the SUs is given by

$$\text{SNR} = \mathbb{E}[\gamma], \quad (7)$$

where  $\mathbb{E}[\cdot]$  returns the expected value of the random variable located into its argument. The final formula for the average SNR across the SUs, established in [38], whose details were omitted here for concision, is given by

$$\text{SNR} = \frac{\ln \left( \frac{1+\rho}{1-\rho} \right)}{2\rho m \sigma_{\text{avg}}^2} \sum_{i=1}^m p_i. \quad (8)$$

The matrix  $\mathbf{L} \in \mathbb{R}^{m \times n}$  contains iid elements following the Laplace distribution, thus modeling the Laplacian noise samples. The Laplace (or double exponential) random variable [18,

p. 16],  $l$ , with zero mean and variance equals to  $2b^2$ , presents its PDF as

$$f(l|b) = \frac{1}{2b} \exp\left(-\frac{l}{b}\right), \quad (9)$$

where  $b > 0$  is the scale factor or diversity. Because the mean is fixed, the higher the value  $b$  is, the heavier the tail is.

### III. TEST STATISTICS

Considering centralized CSS with data fusion, this section presents the test statistics of the detectors whose numerical results are compared in Section IV, namely: the ED, the AVC and the COS EED.

The ED test statistic is given by [1]

$$T_{ED} = \sum_{i=1}^m \frac{1}{\sigma_i^2} \sum_{j=1}^n |y_{ij}|^2, \quad (10)$$

where  $\sigma_i^2$  is the Gaussian noise variance at each SU and  $y_{ij}$  composes the matrix  $\mathbf{Y}$  established in (1), referring to the  $j$ -th sample gathered by the  $i$ -th SU.

The test statistic of the AVC detector [26] is given by

$$T_{AVC} = \sum_{i=1}^m \frac{1}{\sigma_i} \sum_{j=1}^n |y_{ij}|, \quad (11)$$

where  $\sigma_i$  is the Gaussian noise standard deviation at each SU.

The COS EED test statistic is build as follows

$$v(i) = \frac{1}{\sigma_i^p} \sum_{j=1}^n |y_{ij}|^p, \quad (12)$$

where  $v(i)$  is the output of the  $i$ -th EED,  $i = 1, 2, \dots, m$ ,  $\sigma_i$  is the Gaussian noise standard deviation at the  $i$ -th SU and  $y_{ij}$  composes the matrix  $\mathbf{Y}$  established in (1), referring to the  $j$ -th sample gathered by the  $i$ -th SU. The EED arises from the fact that the exponent  $p$  generalizes the ED, potentially enhancing its performance. Having the vector  $\mathbf{v} = [v(1) \ v(2) \ \dots \ v(m)]$  available, it is sufficient to reorder it in ascending order, that is,  $v(1) < v(2) < \dots < v(m)$ . The final expression for the COS EED test statistic is

$$T_{COSEED} = \sum_{k=1}^K v(i_k), \quad (13)$$

where  $K = 1, 2, \dots, m$  and  $1 \leq i_1 \leq \dots \leq i_K \leq m$ . The notation COS EED  $(i_1, i_2, \dots, i_K)$  will be used to name COS scheme with the EED and the test statistic in (13). For example, as used in the numerical assessment, COS EED (1,4) corresponds to the selection and summation of  $v(1)$  and  $v(4)$  to form the test statistic that will be compared to the decision threshold. COS EED (3) builds the test statistic only by the presence of  $v(3)$ . It was adopted  $p = 2$  to consider conventional ED in each SU.

### IV. NUMERICAL RESULTS

This section presents numerical results of the centralized CSS with data fusion. Systems in the absence and presence of impulsive noise, respectively, under Gaussian noise only and under Laplacian noise, are compared, for the detectors ED, AVC, COS EED (1,4) and COS EED (3). The last two were added in the Python simulation, not present in the original reference article.

The results expressed herein return the values of  $P_d$  achieved in accordance with the variation of  $\mu_\kappa$  system parameter, assuming  $P_{fa} = 0.1$  [3]. Each point on a curve has been generated from 10000 Monte Carlo events using the Matlab R2019a (in the original paper) and reproduced using Python 3 (for TP547 subject).

By making the absence of Laplacian noise scenario as reference, the value of the average SNR or the number of samples,  $n$ , has been adapted, in part of the cases, so that the respective best detector yields  $P_d \approx 0.9$  at the mid-value of the system parameter being analyzed. Thus, variations in  $P_d$  can be clearly perceived. When fixed, for a better adequacy of situations more likely to occur in practice, the system configuration parameters are:  $m = 6$  SUs, which corresponds to a small number of cooperative cognitive radios, leading to an efficient utilization of control channel resources. Additionally, the following curves show that an increase in the number of SUs results in a diminishing returns fashion of performance improvement;  $n = 250$  samples to achieve the targeted performance metrics; SNR = -10 dB, considering that the system's operation may occur in scenarios characterized by significantly low SNR; fraction of noise variations,  $\rho = 0.5$ , which was arbitrarily chosen to model variations in thermal noise power due to changes in ambient temperature, some form of reception circuit descaling or interfering signals in the bandwidth of interest; path-loss exponent,  $\eta = 2.5$ , chosen to fit a typically urban scenario; normalized coverage radius,  $r = 1$  m; reference distance for path-loss calculation,  $d_0 = 0.001r$ ;  $p_{tx} = 5$  W, adapting to practical requirements of power for real PU transmitters; and random Rice factor, with mean  $\mu_\kappa = 1.88$  dB and standard deviation  $\sigma_\kappa = 4.13$  dB, considering urban area [37]. The parameters related to the Laplacian noise have been calculated and generated as described in Section II.

Fig. 1 shows  $P_d$  as a function of the mean of Rice factor in dB,  $\mu_\kappa$ . In the absence (left-hand side graph) and in the presence (right-hand side graph) of Laplacian noise, it can be seen that the mean of Rice factor has a certain influence on the performance of the various detectors just from around  $\mu_\kappa \approx -5$  dB. This value corresponds to a situation in which the dominant component of the primary signal starts to stand out in relation to the scattered component, thus increasing the performance of most detectors, in different proportions. For mean of Rice factor values greater than 5 dB, the detectors ED and AVC tend to show similar and optimal performances. The exception is found in the performance drop shown by the ED in the presence of Laplacian noise.

Fig. 1a, referring to the absence of Laplacian noise, shows ED outperforming the other detectors, as expected, and fol-

lowed by the AVC detector.

From Fig. 1b, it is noticed that there is a maintenance in the pattern of the curves, only with a performance reduction for most of the detectors due to the presence of Laplacian noise, in comparison to the absence of Laplacian noise scenario. The exception is for the AVC detector, which presents a performance improvement and excels the other detectors for all values of  $\mu_K$ . A more evident deterioration in the performance of the ED, in relation to the other detectors, when compared with the scenario of absence of Laplacian noise.

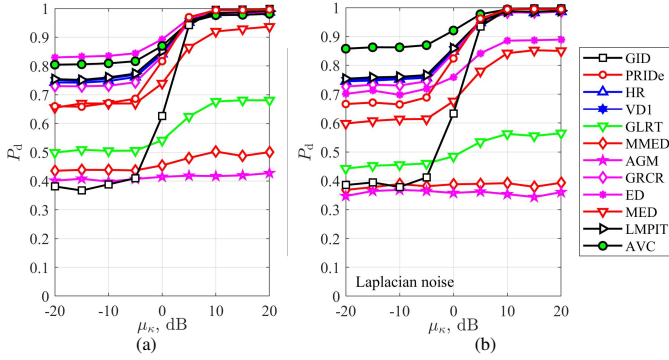


Fig. 1. Probability of detection,  $P_d$ , versus mean of Rice factor (dB),  $\mu_K$ , for SNR = -8 dB: system under Gaussian noise (left) and system under Laplacian noise (right). This figure is better viewed in color.

## V. CONCLUSIONS

This work has assessed the performance of centralized cooperative spectrum sensing with data fusion, under Laplacian impulsive noise. The performances of the detectors ED, AVC, COS EED (1,4) and COS EED (3) were compared under Gaussian noise and under Laplacian noise models.

The AVC detector has shown excellent performance under Laplacian noise, but, from what was concluded in the Appendix of the original paper, it will no longer be optimal under other types of noise. For any values of the analyzed parameter, it can be noted a great robustness for the AVC detector in the presence of Laplacian noise, since there was an outperforming pattern, in comparison with the other detectors.

The ED has demonstrated a significant sensibility against the Laplacian impulsive noise, which went from a reference behavior, in the absence of impulsive noise, to an intermediate behavior, in the presence of the Laplacian noise. For that reason, the use of ED, COS EED (1,4) and COS EED (3) detectors proved to be inappropriate for spectrum sensing under Laplacian impulsive noise, in contrast with the AVC.

As an opportunity for contribution in future researches, this performance analysis can be extended to other types of impulsive noise. It becomes interesting either to develop or improve a given detector in order to make it optimal or even more robust for any type of impulsive noise. Another opportunity that can be considered is the assessment of the detectors' performances under impulsive noise, but considering other channel fading models, or even other impulsive noise models.

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