```
> # Exercicio 1
>
> #a)
>
> P = matrix(c(0,1,0,0,0,0,
+ + 0,1/5,4/5,0,0,0,
+ + 0,0,2/5,3/5,0,0,
+ + 0,0,0,3/5,2/5,0,
+ + 0,0,0,0,4/5,1/5,
+ + 0,0,0,0,0,1), nrow = 6, byrow= TRUE)
>
> colnames(P) <- c(0:5)
> rownames(P) <- c(0:5)
>
> print(P)
  0   1   2   3   4   5
0 0 1.0 0.0 0.0 0.0 0.0
1 0 0.2 0.8 0.0 0.0 0.0
2 0 0.0 0.4 0.6 0.0 0.0
3 0 0.0 0.0 0.6 0.4 0.0
4 0 0.0 0.0 0.0 0.8 0.2
5 0 0.0 0.0 0.0 0.0 1.0
>
> #b)
>
> P2 = P%*%P
> P4 = P2%*%P2
> P8 = P4%*%P4
> P5 = P4%*%P
> P10 = P5%*%P5
>
> print(P2)
  0    1    2    3    4    5
0 0 0.20 0.80 0.00 0.00 0.00
1 0 0.04 0.48 0.48 0.00 0.00
2 0 0.00 0.16 0.60 0.24 0.00
3 0 0.00 0.00 0.36 0.56 0.08
4 0 0.00 0.00 0.00 0.64 0.36
5 0 0.00 0.00 0.00 0.00 1.00
> print(P4)
  0      1      2      3      4      5
0 0 0.0080 0.2240 0.5760 0.1920 0.0000
1 0 0.0016 0.0960 0.4800 0.3840 0.0384
2 0 0.0000 0.0256 0.3120 0.5280 0.1344
3 0 0.0000 0.0000 0.1296 0.5600 0.3104
4 0 0.0000 0.0000 0.0000 0.4096 0.5904
5 0 0.0000 0.0000 0.0000 0.0000 1.0000
> print(P5)
  0       1       2       3       4       5
0 0 0.00160 0.09600 0.48000 0.38400 0.03840
1 0 0.00032 0.03968 0.34560 0.49920 0.11520
2 0 0.00000 0.01024 0.20256 0.54720 0.24000
3 0 0.00000 0.00000 0.07776 0.49984 0.42240
4 0 0.00000 0.00000 0.00000 0.32768 0.67232
```

```
5 0 0.00000 0.00000 0.00000 0.00000 1.00000
> print(P8)
  0        1          2          3         4         5
0 0 1.28e-05 0.00650240 0.14837760 0.5225472 0.3225600
1 0 2.56e-06 0.00261120 0.09292800 0.4773888 0.4270694
2 0 0.00e+00 0.00065536 0.04842240 0.4045056 0.5464166
3 0 0.00e+00 0.00000000 0.01679616 0.3019520 0.6812518
4 0 0.00e+00 0.00000000 0.00000000 0.1677722 0.8322278
5 0 0.00e+00 0.00000000 0.00000000 0.0000000 1.0000000
> print(P10)
  0         1            2           3         4         5
0 0 5.120e-07 0.0010465280 0.057323520 0.4190822 0.5225472
1 0 1.024e-07 0.0004190208 0.035022029 0.3581952 0.6063636
2 0 0.000e+00 0.0001048576 0.017825280 0.2861574 0.6959124
3 0 0.000e+00 0.0000000000 0.006046618 0.2026551 0.7912983
4 0 0.000e+00 0.0000000000 0.000000000 0.1073742 0.8926258
5 0 0.000e+00 0.0000000000 0.000000000 0.0000000 1.0000000
>
> P20 = P10%*%P10
> print(P20)
  0            1            2            3          4         5
0 0 5.242880e-14 1.099510e-07 3.652860e-04 0.05691517 0.9427194
1 0 1.048576e-14 4.398042e-08 2.192376e-04 0.04567825 0.9541025
2 0 0.000000e+00 1.099512e-08 1.096518e-04 0.03436831 0.9655220
3 0 0.000000e+00 0.000000e+00 3.656158e-05 0.02298531 0.9769781
4 0 0.000000e+00 0.000000e+00 0.000000e+00 0.01152922 0.9884708
5 0 0.000000e+00 0.000000e+00 0.000000e+00 0.00000000 1.0000000

> # As potêmcias indicam que com o aumento do número de transições, a tendência
é que todos vão para o estado 5, ou seja, 5 brinquedos.
>
> #c)
>
> #P(x5 =2,x10 = 3,x15 = 4,x20 = 5 | x0 = 0)
>
> Prob = P5["0","2"]*P5["2","3"]*P5["3","4"]*P5["4","5"]
> Prob
[1] 0.006534795
>
> #Exercicio 2
>
# a)

> PT = matrix(c(0,0,0,1/4,1/4,1/4,1/4,
+ + 0,0,0,1/4,1/4,1/4,1/4,
+ + 0,0,0,1/4,1/4,1/4,1/4,
+ + 1/3,1/3,1/3,0,0,0,0,
+ + 1/3,1/3,1/3,0,0,0,0,
+ + 1/3,1/3,1/3,0,0,0,0,
+ + 1/3,1/3,1/3,0,0,0,0), nrow=7, byrow = TRUE)
>
> colnames(PT) <- c(1:7)
> rownames(PT) <- c(1:7)
>
```

```
> print(PT)
          1         2         3    4    5    6    7
1 0.0000000 0.0000000 0.0000000 0.25 0.25 0.25 0.25
2 0.0000000 0.0000000 0.0000000 0.25 0.25 0.25 0.25
3 0.0000000 0.0000000 0.0000000 0.25 0.25 0.25 0.25
4 0.3333333 0.3333333 0.3333333 0.00 0.00 0.00 0.00
5 0.3333333 0.3333333 0.3333333 0.00 0.00 0.00 0.00
6 0.3333333 0.3333333 0.3333333 0.00 0.00 0.00 0.00
7 0.3333333 0.3333333 0.3333333 0.00 0.00 0.00 0.00
>
> # b)

> PT2 = PT%*%PT
> print(PT2)
          1         2         3    4    5    6    7
1 0.3333333 0.3333333 0.3333333 0.00 0.00 0.00 0.00
2 0.3333333 0.3333333 0.3333333 0.00 0.00 0.00 0.00
3 0.3333333 0.3333333 0.3333333 0.00 0.00 0.00 0.00
4 0.0000000 0.0000000 0.0000000 0.25 0.25 0.25 0.25
5 0.0000000 0.0000000 0.0000000 0.25 0.25 0.25 0.25
6 0.0000000 0.0000000 0.0000000 0.25 0.25 0.25 0.25
7 0.0000000 0.0000000 0.0000000 0.25 0.25 0.25 0.25
>
> PT3 = PT%*%PT2
> print(PT3)
          1         2         3    4    5    6    7
1 0.0000000 0.0000000 0.0000000 0.25 0.25 0.25 0.25
2 0.0000000 0.0000000 0.0000000 0.25 0.25 0.25 0.25
3 0.0000000 0.0000000 0.0000000 0.25 0.25 0.25 0.25
4 0.3333333 0.3333333 0.3333333 0.00 0.00 0.00 0.00
5 0.3333333 0.3333333 0.3333333 0.00 0.00 0.00 0.00
6 0.3333333 0.3333333 0.3333333 0.00 0.00 0.00 0.00
7 0.3333333 0.3333333 0.3333333 0.00 0.00 0.00 0.00
>
> PT4 = PT2%*%PT2
> print(PT4)
          1         2         3    4    5    6    7
1 0.3333333 0.3333333 0.3333333 0.00 0.00 0.00 0.00
2 0.3333333 0.3333333 0.3333333 0.00 0.00 0.00 0.00
3 0.3333333 0.3333333 0.3333333 0.00 0.00 0.00 0.00
4 0.0000000 0.0000000 0.0000000 0.25 0.25 0.25 0.25
5 0.0000000 0.0000000 0.0000000 0.25 0.25 0.25 0.25
6 0.0000000 0.0000000 0.0000000 0.25 0.25 0.25 0.25
7 0.0000000 0.0000000 0.0000000 0.25 0.25 0.25 0.25
>
> PT5 = PT%*%PT4
> print(PT5)
          1         2         3    4    5    6    7
1 0.0000000 0.0000000 0.0000000 0.25 0.25 0.25 0.25
2 0.0000000 0.0000000 0.0000000 0.25 0.25 0.25 0.25
3 0.0000000 0.0000000 0.0000000 0.25 0.25 0.25 0.25
4 0.3333333 0.3333333 0.3333333 0.00 0.00 0.00 0.00
5 0.3333333 0.3333333 0.3333333 0.00 0.00 0.00 0.00
6 0.3333333 0.3333333 0.3333333 0.00 0.00 0.00 0.00
```

```
7 0.3333333 0.3333333 0.3333333 0.00 0.00 0.00 0.00
>
> #c)
> Prob1 = PT["1","4"]
> Prob2 = PT2["1","4"]
> Prob3 = PT3["1","4"]
> Prob4 = PT4["1","4"]
>
> print(Prob1)
[1] 0.25
> print(Prob2)
[1] 0
> print(Prob3)
[1] 0.25
> print(Prob4)
[1] 0
>
> #d)

> Prob5 = PT["4","2"]
> Prob6 = PT2["4","2"]
> Prob7 = PT3["4","2"]
> Prob8 = PT4["4","2"]
>
> print(Prob5)
[1] 0.3333333
> print(Prob6)
[1] 0
> print(Prob7)
[1] 0.3333333
> print(Prob8)
[1] 0
>
>
> e) As gtransições pares são nulas e as ímpares são iguais.


> #Exercicio 3
>
> PL3=matrix(c(1/2,1/2,0,0,0,0,0,
+ 3/4,1/4,0,0,0,0,0,
+ 1/8,0,1/8,0,1/2,1/8,1/8,
+ 0,1/8,1/8,1/2,1/8,1/8,0,
+ 1/8,0,1/2,0,1/4,0,1/8,
+ 0,0,0,0,0,1/3,2/3,
+ 0,0,0,0,0,2/3,1/3), nrow=7,byrow=TRUE)
>
> #Exercicio 3

> #a) Estados transitórios: 3, 4, 5
      Estados recorrentes: 1, 2, 6, 7

> #b) Classes recorrentes R1 = {1, 2} e R2 = {6, 7}
>
```

```
> PL3=matrix(c(1/2,1/2,0,0,0,0,0,
+ 3/4,1/4,0,0,0,0,0,
+ 1/8,0,1/8,0,1/2,1/8,1/8,
+ 0,1/8,1/8,1/2,1/8,1/8,0,
+ 1/8,0,1/2,0,1/4,0,1/8,
+ 0,0,0,0,0,1/3,2/3,
+ 0,0,0,0,0,2/3,1/3), nrow=7,byrow=TRUE)
>
> PL3
      [,1]  [,2]  [,3] [,4]  [,5]      [,6]      [,7]
[1,] 0.500 0.500 0.000  0.0 0.000 0.0000000 0.0000000
[2,] 0.750 0.250 0.000  0.0 0.000 0.0000000 0.0000000
[3,] 0.125 0.000 0.125  0.0 0.500 0.1250000 0.1250000
[4,] 0.000 0.125 0.125  0.5 0.125 0.1250000 0.0000000
[5,] 0.125 0.000 0.500  0.0 0.250 0.0000000 0.1250000
[6,] 0.000 0.000 0.000  0.0 0.000 0.3333333 0.6666667
[7,] 0.000 0.000 0.000  0.0 0.000 0.6666667 0.3333333
>
>
> Q=matrix(c(1/8,0,1/2,1/8,1/2,1/8,1/2,0,1/4),nrow=3,byrow=TRUE)
>
> I = matrix(c(1,0,0,0,1,0,0,0,1),nrow=3)
> I
     [,1] [,2] [,3]
[1,]    1    0    0
[2,]    0    1    0
[3,]    0    0    1
> IQ=I-Q
> IQ
       [,1] [,2]   [,3]
[1,]  0.875  0.0 -0.500
[2,] -0.125  0.5 -0.125
[3,] -0.500  0.0  0.750
>
> inv=solve(IQ)
> inv
          [,1] [,2]      [,3]
[1,] 1.8461538    0 1.2307692
[2,] 0.7692308    2 0.8461538
[3,] 1.2307692    0 2.1538462
>
> B=matrix(c(1/8,1/4,1/8,1/8,1/8,1/8),nrow=3,byrow=TRUE)
>
> B
      [,1]  [,2]
[1,] 0.125 0.250
[2,] 0.125 0.125
[3,] 0.125 0.125
>
> R=inv%*%B
>
> R
          [,1]      [,2]
[1,] 0.3846154 0.6153846
```

```
[2,] 0.4519231 0.5480769
[3,] 0.4230769 0.5769231
>
>
> colnames(R) = c("R1", "R2")
> rownames(R) = c(3,4,5)
>
> R
         R1        R2
> 3 0.3846154 0.6153846
> 4 0.4519231 0.5480769
> 5 0.4230769 0.5769231

>
> # Exercicio 4
>
> PR = matrix(c(0,1/2,1/2,0,0,
+ 0,0,0,0,0,
+ 1/3,1/3,0,1/3,0,
+ 0,0,0,0,1,
+ 0,0,1/2,1/2,0), nrow = 5, byrow = TRUE)
>
> PR
          [,1]      [,2] [,3]      [,4] [,5]
[1,] 0.0000000 0.5000000  0.5 0.0000000    0
[2,] 0.0000000 0.0000000  0.0 0.0000000    0
[3,] 0.3333333 0.3333333  0.0 0.3333333    0
[4,] 0.0000000 0.0000000  0.0 0.0000000    1
[5,] 0.0000000 0.0000000  0.5 0.5000000    0
```