#4 Assignment - CMPT 405

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#1

Let w_{ij} be the weight of every $(i, j) \in E$ and x_{ij} be variables such that $x_{ij} = 1$ if the shortest path contains $i \to j$ and $x_{ij} = 0$, otherwise. The shortest path from a source $s \in V$ to a target $t \in V$ in a weighted graph $G = (V, E, w)$ can be found by minimizing the summation of $w_{ij}x_{ij}$ for every (i, j). See below:

$$x_{ij} = \begin{cases} 1 & \text{if the shortest path contains } i \to j \\ 0 & \text{otherwise} \end{cases}$$

By the principle of amount of flow network, we have that for each single node i, the amount of a flow f_i is equal the difference between the amount of outgoing flow from i and the amount of incoming flow to i:

$$f_i = \sum_j x_{ij} - \sum_k x_{ki}$$

As we are looking for the shortest path from s to t, we know that our network will "travel" from the source to the target, cancelling any flow f_u for single vertices u between s and t in our network, where $u \neq s$ and $u \neq t$. Because there is no incoming flow in s, $f_s = 1$. Likewise, because there are no outgoing flow in t, $f_t = -1$. Thus:

$$f_i = \begin{cases} 1 & \text{if } i = s \\ -1 & \text{if } i = t \\ 0 & \text{otherwise} \end{cases}$$

Assuming that $x_{ij} \geq 0$, the linear program for the shortest problem is:

$$min \sum_{(i,j) \in E} w_{ij}x_{ij}$$

The resulting dual will have one variable y_u for each vertex u in the graph. The values of y have the constraint that $y_j - y_i \leq w_{ij}$ and the objective function is the maximization of $y_s - y_t$:

$$max\ y_s - y_t$$

$$y_j - y_i \leq w_{ij}\ ,\ \forall(i,j) \in E$$

Dual Encoding: The dual can be interpreted as the encoding of Bellman-Ford, because when BF terminates, it has computed for each vertex j a value y_j, such that for each edge $(i,j) \in E$, we have the same constraints as the dual: $y_j \leq y_i + w_{ij}$. The objective function is also the maximization of $y_s - y_t$.

#2

In a similar way of question #1, let w_e be the weight of every $e \in E$ and x_e be $0-1$ variables such that $x_e = 1$ if the edge e is in the matching and $x_e = 0$, otherwise.

$$x_e = \begin{cases} 1 & \text{inclusion of edge } e \text{ in the matching} \\ 0 & \text{otherwise} \end{cases}$$

We need to choose at each step an augmenting path p that produces the largest possible increase in weight so that the matching obtained by flipping the edges of a p has maximum weight.

The objective function maximizes the weight of all edges e in the matching and, because we have a path, we use constraints to limit one edge per vertex so that the path is created in the form $x_e \leq 1$, for all vertices u, such that $e = (u,v)$. The linear program is then:

$$max\ \sum_e w_e x_e$$

$$\sum_{e=(u,v)} x_e \leq 1\ ,\ \forall u \in V$$

#3

#4

#5