#4 Assignment - CMPT 405

Luiz Fernando Peres de Oliveira - 301288301 - lperesde@sfu.ca

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#1

Let w_{ij} be the weight of every $(i,j) \in E$ and x_{ij} be variables such that $x_{ij} = 1$ if the shortest path contains $i \to j$ and $x_{ij} = 0$, otherwise. The shortest path from a source $s \in V$ to a target $t \in V$ in a weighted graph $G = (V, E, w)$ can be found by minimizing the summation of $w_{ij}x_{ij}$ for every (i,j). See below:

$$x_{ij} = \begin{cases} 1 & \text{if the shortest path contains } i \to j \\ 0 & \text{otherwise} \end{cases}$$

By the principle of amount of flow network, we have that for each single node i, the amount of a flow f_i is equal the difference between the amount of outgoing flow from i and the amount of incoming flow to i:

$$f_i = \sum_j x_{ij} - \sum_k x_{ki}$$

As we are looking for the shortest path from s to t, we know that our network will "travel" from the source to the target, cancelling any flow f_u for single vertices u between s and t in our network, where $u \neq s$ and $u \neq t$. Because there is no incoming flow in s, $f_s = 1$. Likewise, because there are no outgoing flow in t, $f_t = -1$. Thus:

$$f_i = \begin{cases} 1 & \text{if } i = s \\ -1 & \text{if } i = t \\ 0 & \text{otherwise} \end{cases}$$

Assuming that $x_{ij} \geq 0$, the linear program for the shortest problem is:

$$min \sum_{(i,j) \in E} w_{ij}x_{ij}$$

The resulting dual will have one variable y_u for each vertex u in the graph. The values of y have the constraint that $y_j - y_i \leq w_{ij}$ and the objective function is the maximization of $y_s - y_t$:

$$max\ y_s - y_t$$

$$y_j - y_i \leq w_{ij}\ ,\ \forall (i,j) \in E$$

Dual Encoding: The dual can be interpreted as the encoding of Bellman-Ford, because when BF terminates, it has computed for each vertex j a value y_j, such that for each edge $(i,j) \in E$, we have the same constraints as the dual: $y_j \leq y_i + w_{ij}$. The objective function is also the maximization of $y_s - y_t$.

#2

In a similar way of question #1, let w_e be the weight of every $e \in E$ and x_e be $0-1$ variables such that $x_e = 1$ if the edge e is in the matching and $x_e = 0$, otherwise.

$$x_e = \begin{cases} 1 & \text{inclusion of edge } e \text{ in the matching} \\ 0 & \text{otherwise} \end{cases}$$

We need to choose at each step an augmenting path the produces the largest possible increase in total weight in G. Thus, the objective function maximizes the the weight of all edges e in the matching and, because we have a path, we use constraints to limit one edge per vertex so that the path is created in the form $x_e \leq 1$, for all vertices u, such that $e = (u,v)$. The linear program is then:

$$max\ \sum_e w_e x_e$$

$$\sum_{e=(u,v)} x_e \leq 1\ ,\ \forall u \in V$$

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