# 1.1 Four Ways to Represent a Function

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1. A box with an open top is to be constructed from a rectangular piece of cardboard with dimensions 18 in. by 30 in. by cutting out equal squares of side x at each corner and then folding up the sides as in the figure. Express the volume V of the box as a function of x.

The volume of a box is given by $L \times W \times H$, thus:

$$\begin{aligned} V(x) &= (30-2x)(18-2x)(x) \\ &= 2(15-x)2(9-x)(x) \\ &= 4(135-15x-9x+x^2)(x) \\ &= 4x(135-15x-9x+x^2) \\ &= 4x^3-96x^2+540x \end{aligned} \tag{1}$$

2. If $f(x) = 5x^2 - x + 4$,find the following.

$f(2) =?$

$$\begin{aligned} f(2) &= 5(2)^2 - 2 + 4 \\ &= 5 \times 4 - 2 + 4 \\ &= 20 + 2 \\ &= 22 \end{aligned} \tag{2}$$

$f(-2) =?$

$$\begin{aligned} f(-2) &= 5(-2)^2 - (-2) + 4 \\ &= 5 \times 4 + 2 + 4 \\ &= 20 + 6 \\ &= 26 \end{aligned} \tag{3}$$

$f(a) =?$

$$f(a) = 5a^2 - a + 4 \tag{4}$$

$f(-a) =?$

$$\begin{aligned} f(-a) &= 5(-a)^2 - (-a) + 4 \\ &= 5a^2 + a + 4 \end{aligned} \tag{5}$$

$f(a+1) =?$

$$\begin{aligned} f(a+1) &= 5(a+1)^2 - (a+1) + 4 \\ &= 5(a+1)(a+1) - a - 1 + 4 \\ &= 5(a^2 + 2a + 1) - a - 1 + 4 \\ &= 5a^2 + 10a + 5 - a + 3 \\ &= 5a^2 + 9a + 8 \end{aligned} \tag{6}$$

$2f(a) =?$

$$\begin{aligned} 2f(a) &= 2 \times f(a) \\ &= 2 \times (5a^2 - a + 4) \\ &= 10a^2 - 2a + 8 \end{aligned} \tag{7}$$

$f(2a) =?$

$$\begin{aligned} f(2a) &= 5(2a)^2 - 2a + 4 \\ &= 5(4a^2) - 2a + 4 \\ &= 20a^2 - 2a + 4 \end{aligned} \tag{8}$$

$f(a^2) =?$

$$\begin{aligned} f(a^2) &= 5(a^2)^2 - a^2 + 4 \\ &= 5(a^4) - a^2 + 4 \\ &= 5a^4 - a^2 + 4 \end{aligned} \tag{9}$$

$[f(a)]^2 =?$

$$
\begin{aligned}
[f(a)]^2 &= f(a)^2 \\
&= (5a^2 - a + 4)^2 \\
&= (5a^2 - a + 4)(5a^2 - a + 4) \\
&= 25a^4 - 5a^3 + 20a^2 - 5a^3 + a^2 - 4a + 20a^2 - 4a + 16 \\
&= 25a^4 - 10a^3 + 41a^2 - 8a + 16
\end{aligned} \tag{10}
$$

$f(a + h) =?$

$$
\begin{aligned}
f(a + h) &= 5(a + h)^2 - (a + h) + 4 \\
&= 5(a + h)(a + h) - a - h + 4 \\
&= 5(a^2 + 2ah + h^2) - a - h + 4 \\
&= 5a^2 + 10ah + 5h^2 - a - h + 4
\end{aligned} \tag{11}
$$

3. Find the domain of the function. (Enter your answer using interval notation.)

$$
f(x) = \frac{x + 4}{x^2 - 9} \tag{12}
$$

For that, $x^2 - 9 \neq 0$, once we cannot divide by zero. Thus:

$$
\begin{aligned}
x^2 &\neq 9 \\
x &\neq \sqrt{9} \\
x &\neq 3
\end{aligned} \tag{13}
$$

However, we will need to consider $-3$ as well as: $-3^2 = 9$. So the interval notation is:

$$
(-\infty, -3) \cup (-3, 3) \cup (3, \infty)
$$

4. Sketch the graph of the function.

$$
f(x) = 4x + |4x| \tag{14}
$$

We know that $|y| = -y$ for negative numbers and $|y| = y$ for non-negative ones. In other words, it is the positive version of the number, thus:

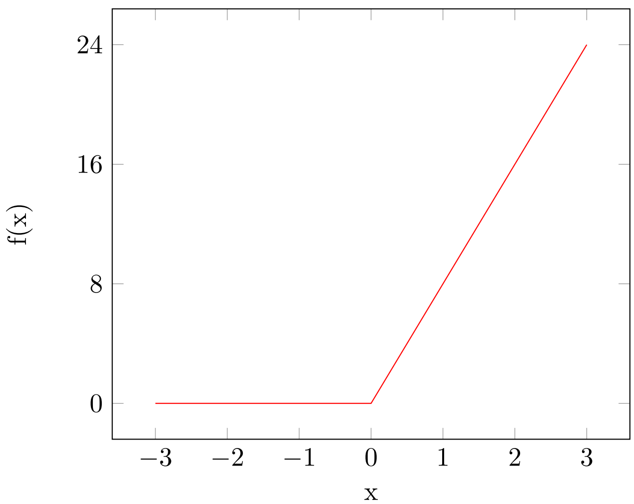
$$
\begin{aligned}
f(y) &= y + |y| = y + y, \forall x > 0 \\
\therefore f(x) &= 4x + 4x = 8x, \forall x > 0
\end{aligned} \tag{15}
$$

and

$$\begin{aligned} f(y) &= y + |y| = -y + y = 0, \forall x < 0 \\ \therefore f(x) &= -4x + 4x = 0, \forall x < 0 \end{aligned} \tag{16}$$

and we cannot forget that $a \times b = 0$ when $a = 0$ or $b = 0$, thus:

$$f(x) = 0, x = 0 \tag{17}$$




Notes: A functions is said even when $f(x) = f(-x)$, i.e, it is "mirroed" in the y-axis and said odd when $f(x) = -f(x)$ and it passes through the origin.