1.3 New Functions from Old Functions

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1. Graph the function by hand, not by plotting points, but by starting with the graph of one of the standard functions given in Section 1.2, and then applying the appropriate transformations.

$y = sin(\frac{1}{6}x)$

This equation has a form of $y = f(\frac{x}{6})$ and therefore must be stretched.

2. Find the functions and their domains. (Enter the domains in interval notation.) $f(x) = x + \frac{1}{x}$, $g(x) = \frac{x+8}{x+2}$

(a) $f \circ g = f(g(x))$

$$\begin{aligned} f(g(x)) &= \frac{x+8}{x+2} + \frac{1}{\frac{x+8}{x+2}} \\ &= \frac{x+8}{x+2} + \frac{x+2}{x+8} \\ &= \frac{(x+8)(x+8)}{(x+2)(x+8)} + \frac{(x+2)(x+2)}{(x+2)(x+8)} \\ &= \frac{x^2+16x+64+x^2+4x+4}{(x+2)(x+8)} \\ &= \frac{2x^2+20x+68}{(x+2)(x+8)} \end{aligned} \tag{1}$$

The Domain is $(-\infty, -8) \cup (-8, -2) \cup (-2, \infty)$

(b) $g \circ f = g(f(x))$

$$
\begin{aligned}
g(f(x)) &= (\frac{x+\frac{1}{x}+\frac{8x}{x}}{x+\frac{1}{x}+2}) \\
&= \frac{\frac{x^2+8x+1}{x}}{\frac{x^2+2x+1}{x}} \\
&= \frac{x^2+8x+1}{x} \div \frac{x^2+2x+1}{x} \\
&= \frac{x(x^2+8x+1)}{x(x^2+2x+1)} \\
&= \frac{x^2+8x+1}{x^2+2x+1} \\
&= \frac{x^2+8x+1}{(x+1)^2}
\end{aligned} \tag{2}
$$

The Domain is $(-\infty,-1)\cup(-1,0)\cup(0,\infty)$

(c) $f \circ f = f(f(x))$

$$
\begin{aligned}
f(f(x)) &= (x+\frac{1}{x}) + \frac{1}{x+\frac{1}{x}} \\
&= \frac{x^2+1}{x} + \frac{1}{\frac{x^2+1}{x}} \\
&= \frac{x^2+1}{x} + (\frac{1}{1} \div \frac{x^2+1}{x}) \\
&= \frac{(x^2+1)(x^2+1)}{(x^2+1)x} + \frac{(x)x}{(x^2+1)x} \\
&= \frac{(x^2+1)(x^2+1)}{(x^2+1)x} + \frac{x^2}{(x^2+1)x} \\
&= \frac{x^4+x^2+x^2+1+x^2}{(x^2+1)x} \\
&= \frac{x^4+3x^2+1}{(x^2+1)x}
\end{aligned} \tag{3}
$$

The Domain is $(-\infty,0)\cup(0,\infty)$

(d) $g \circ g = g(g(x))$

$$\begin{aligned}
g(g(x)) &= \frac{\frac{x+8}{x+2}+8}{\frac{x+8}{x+2}+2} \\
&= \frac{\frac{x+8}{x+2}+\frac{8(x+2)}{x+2}}{\frac{x+8}{x+2}+\frac{2(x+2)}{x+2}} \\
&= \frac{\frac{x+8}{x+2}+\frac{8x+16}{x+2}}{\frac{x+8}{x+2}+\frac{2x+4}{x+2}} \\
&= \frac{\frac{9x+24}{x+2}}{\frac{3x+12}{x+2}} \\
&= \frac{\frac{3(3x+8)}{x+2}}{\frac{3(x+4)}{x+2}} \\
&= \frac{3(3x+8)}{x+2} \div \frac{3(x+4)}{x+2} \\
&= \frac{3(3x+8)(x+2)}{3(x+4)(x+2)} \\
&= \frac{3x+8}{x+4}
\end{aligned} \tag{4}$$

The Domain is $(-\infty,-4)\cup(-4,-2)\cup(-2,\infty)$

3. Express the function in the form $f \circ g$.(Use non-identity functions for f and g.)

$u(t) = \frac{cos(t)}{7+cos(t)}$

$$\begin{aligned}
g(t) &= cos(t) \\
f(t) &= \frac{t}{7+t} \\
f(g(t)) = f(cos(t)) &= \frac{cos(t)}{7+cos(t)}
\end{aligned} \tag{5}$$

4. A ship is moving at a speed of 40 km/h parallel to a straight shoreline. The ship is 5 km from shore and it passes a lighthouse at noon.

(a) Express the distance s between the lighthouse and the ship as a function of d, the distance the ship has travelled since noon; that is, find f so that $s = f(d)$.

$$\begin{aligned}
s &= \sqrt{d^2+5^2} \\
&= \sqrt{d^2+25}
\end{aligned} \tag{6}$$

(b) Express d as a function of t, the time elapsed since noon; that is, find g so that $d = g(t)$.

$$d = g(t) = 40t \tag{7}$$

(c) Find $f \circ g$:

$$\begin{aligned} (f \circ g)(t) &= f(g(t)) \\ f(40t) &= \sqrt{40t^2 + 25} \\ &= \sqrt{1600t^2 + 25} \end{aligned} \tag{8}$$