

1.2 Mathematical Models

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1. Find expressions for the quadratic functions whose graphs are shown (the images for this assignment are:

<http://www.webassign.net/waplots/1/0/87e17e436a7105b360de437ef9dd6e.gif>
and

<http://www.webassign.net/waplots/e/f/059c1ccdb452a64e9c2e5842cc3114.gif>

$f(x) = ?$

We know that quadratic functions can be described as $y = a(x+h)^2 + k$ because 6 is the root, thus: $y = a(x-6)^2 + k = 0$ when $x = 6 \therefore k = 0$. Now, we just need to substitute y and x with the point given (7, 2):

$f(x) = y$

$$y = a(x-6)^2 + 0$$

$$2 = a(7-6)^2$$

$$a = 2 \tag{1}$$

\therefore

$$f(x) = 2(x-6)^2$$

$g(x) = ?$

The formula for quadratic functions is $y = ax^2 + bx + c$, thus:

First we will find c by switching it with the given point (0, 1):

$$1 = a \times 0 + b \times 0 + c \tag{2}$$

$$c = 1$$

Now we must substitute them with the other two given points (-4, 11) and (1, -6.5):

$$11 = a(-4)^2 + -4b + 1, \text{ } c=1 \text{ as per (2)}$$

$$\therefore \tag{3}$$

$$10 = 16a - 4b$$

$$\begin{aligned}
-6.5 &= a(1)^2 + 1b + 1, \quad c=1 \text{ as per (2)} \\
&\therefore \\
-7.5 &= a + b
\end{aligned} \tag{4}$$

To find a and b now, we will need to eliminate one of them:

$$\begin{aligned}
10 &= 16a - 4b \\
&+ \\
-7.5 &= a + b \text{ we will eliminate } b \therefore \\
-7.5 \times (4) &= (a + b) \times (4) \\
&\therefore \\
10 &= 16a - 4b \\
(+) \\
-30 &= 4a + 4b
\end{aligned} \tag{5}$$

$$\begin{aligned}
10 + (-30) &= 16a + 4a - 4b + 4b \\
-20 &= 20a \\
a &= -1
\end{aligned}$$

And now, we find b with $a = 1$ and $c = 1$ by replacing with any of these points. I will take $(1, -6.5)$:

$$\begin{aligned}
-6.5 &= -1(1)^2 + b(1) + 1 \\
-6.5 &= b
\end{aligned} \tag{6}$$

With that $g(x) = -(x)^2 + (-6.5)x + 1$.

2. Many physical quantities are connected by inverse square laws, that is, by power functions of the form $f(x) = kx^{-2}$. In particular, the illumination of an object by a light source is inversely proportional to the square of the distance from the source. Suppose that after dark you are in a room with just one lamp and you are trying to read a book. The light is too dim and so you move halfway to the lamp. How much brighter is the light?

The equation is something like $\frac{1}{d^2}$, which satisfies $f(x) = kx^{-2}$, once $\frac{1}{d^2} = 1 \times \text{distance}^{-2}$, thus:

If you move halfway, we have something like:

$$\begin{aligned}
 f(x) &= \frac{1}{d^{-2}}, \text{ distance} = \frac{1}{2} \\
 &= \frac{1}{\left(\frac{1}{2}\right)^{-2}} \\
 &= \frac{1}{\frac{1}{4}} \\
 &= \frac{1}{1} \div \frac{1}{4} \\
 &= 4
 \end{aligned} \tag{7}$$

3. Find an expression for a cubic function f if $f(5) = 200$ and $f(5) = f(0) = f(6) = 0$.

$f(x) = ?$

$$\begin{aligned}
 y &= a(x+5)(x-6)(x) \\
 200 &= a(10)(-1)(5) \\
 200 &= -50a \\
 a &= -4 \\
 \therefore \\
 f(x) &= -4x(x+5)(x-6)
 \end{aligned} \tag{8}$$