1.4 Exponential Functions

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1. Consider the graph of $y = e^x$.
(a) Find the equation of the graph that results from reflecting about the line $y = 3$.

$$\begin{aligned}
y &= e^x \\
&\textit{put on zero} \\
&= e^x - 3 \\
&\textit{now we reflect the graph} \\
&= -e^x + 3 \\
&\textit{then, we move it back to its place (3 positions)} \\
&= -e^x + 6
\end{aligned} \tag{1}$$

(b) Find the equation of the graph that results from reflecting about the line $x = 5$.

$$\begin{aligned}
y &= e^x \\
&\textit{reflect x and put it on zero} \\
&= e^{x-5} \\
&\textit{now we reflect the graph} \\
&= e^{-x+5} \\
&\textit{then, we move it back to its place (5 positions)} \\
&= e^{-x+10}
\end{aligned} \tag{2}$$

2. Find the domain of each function. (Enter your answer using interval notation.)
(a)

$$
\begin{aligned}
f(x) = &\frac{64 - e^{x^2}}{1 - e^{64-x^2}} \\
&D_f \,:\, \mathbb{R}, (1 - e^{64-x^2}) \neq 0 \\
&1 - e^{64-x^2} = 0 \\
&e^{64-x^2} = 1 | (ln) \\
&ln(e^{64-x^2}) = ln(1) \\
&64 - x^2 = 0 \\
&x^2 = -64 \\
&x = \sqrt{64} \\
&\textit{we must consider -8 and 8, because of the sqrt.} \\
&(-\infty,\ -8) \cup (-8,\ 8) \cup (8,\ \infty)
\end{aligned} \tag{3}
$$

(b)

$$
\begin{aligned}
f(x) = &\frac{3 + x}{e^{cos(x)}} \\
&\textit{exponential functions are always positive} \\
&\therefore \\
&(-\infty, \infty)
\end{aligned} \tag{4}
$$

3. Find the exponential function $f(x) = Cb^x$ whose graph is given.

$$
\begin{aligned}
15 &= Cb^x \\
C &= \frac{15}{b^1} \\
\\
135 &= Cb^3 \textit{ switch C and the fraction above} \\
135 &= \frac{15b^3}{b} \\
135 &= 15b^2 \\
\frac{135}{15} &= b^2 \\
b &= \sqrt{9} \\
b &= 3 \\
\\
15 &= C \times 3^1 \\
C &= 3
\end{aligned} \tag{5}
$$

The equation is $5(3^x)$

4. If 6^x, show that:
$\frac{f(x+h)-f(x)}{h} = 6x(\frac{6^h-1}{h})$

$$
\begin{aligned}
&= \frac{f(x+h) - 6^x}{h} \\
&= \frac{6^{x+h} - 6^x}{h} \\
&= \frac{6^x(6^h) - 6^x}{h} \\
&= \frac{6^x(6^h - 1)}{h}
\end{aligned} \tag{6}
$$

5. A bacteria culture starts with 300 bacteria and doubles in size every half hour.
(a) How many bacteria are there after 2 hours?

$$
\begin{aligned}
&300 \times 2^4 \\
&300 \times 16 \\
&4800
\end{aligned} \tag{7}
$$

(b) How many bacteria are there after t hours?

$$300 \times 2^{2t} \textit{ each hour is worth 2.} \tag{8}$$

(c) How many bacteria are there after 40 minutes? (Round your answer to the nearest whole number.)

$$\begin{aligned}
&300 times 2^{\frac{3}{4}}\\
&300\sqrt[3]{2^4}\\
&300\sqrt[3]{2^3 \times 2}\\
&600\sqrt[3]{2}\\
&\quad\approx 756
\end{aligned} \tag{9}$$

(e) Estimate the time for the population to reach 40,000. (Round your answer to one decimal place.)

$$\begin{aligned}
300 \times 2^{2t} &= 40000\\
2^{2t} &= \frac{40000}{300}\\
2^{2t} &= \frac{400}{3}\\
\log_2(2^{2t}) &= \log_2(\frac{400}{3})\\
2t &= \log_2(400) - \log(3)\\
t &= \frac{8.642356 - 1.584963}{2}\\
&\approx 3.5
\end{aligned} \tag{10}$$