

1.1 Four Ways to Represent a Function

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1. A box with an open top is to be constructed from a rectangular piece of cardboard with dimensions 18 in. by 30 in. by cutting out equal squares of side x at each corner and then folding up the sides as in the figure. Express the volume V of the box as a function of x .

The volume of a box is given by $L \times W \times H$, thus:

$$\begin{aligned}
 V(x) &= (30 - 2x)(18 - 2x)(x) \\
 &= 2(15 - x)2(9 - x)(x) \\
 &= 4(135 - 15x - 9x + x^2)(x) \\
 &= 4x(135 - 15x - 9x + x^2) \\
 &= 4x^3 - 96x^2 + 540x
 \end{aligned} \tag{1}$$

2. If $f(x) = 5x^2 - x + 4$, find the following.

$$f(2) = ?$$

$$\begin{aligned}
 f(2) &= 5(2)^2 - 2 + 4 \\
 &= 5 \times 4 - 2 + 4 \\
 &= 20 + 2 \\
 &= 22
 \end{aligned} \tag{2}$$

$$f(-2) = ?$$

$$\begin{aligned}
 f(-2) &= 5(-2)^2 - (-2) + 4 \\
 &= 5 \times 4 + 2 + 4 \\
 &= 20 + 6 \\
 &= 26
 \end{aligned} \tag{3}$$

$$f(a) = ?$$

$$f(a) = 5a^2 - a + 4 \tag{4}$$

$$f(-a) = ?$$

$$\begin{aligned}
 f(-a) &= 5(-a)^2 - (-a) + 4 \\
 &= 5a^2 + a + 4
 \end{aligned} \tag{5}$$

$$f(a+1)=?$$

$$\begin{aligned}
 f(a+1) &= 5(a+1)^2 - (a+1) + 4 \\
 &= 5(a+1)(a+1) - a - 1 + 4 \\
 &= 5(a^2 + 2a + 1) - a - 1 + 4 \\
 &= 5a^2 + 10a + 5 - a + 3 \\
 &= 5a^2 + 9a + 8
 \end{aligned} \tag{6}$$

$$2f(a)=?$$

$$\begin{aligned}
 2f(a) &= 2 \times f(a) \\
 &= 2 \times (5a^2 - a + 4) \\
 &= 10a^2 - 2a + 8
 \end{aligned} \tag{7}$$

$$f(2a)=?$$

$$\begin{aligned}
 f(2a) &= 5(2a)^2 - 2a + 4 \\
 &= 5(4a^2) - 2a + 4 \\
 &= 20a^2 - 2a + 4
 \end{aligned} \tag{8}$$

$$f(a^2)=?$$

$$\begin{aligned}
 f(a^2) &= 5(a^2)^2 - a^2 + 4 \\
 &= 5(a^4) - a^2 + 4 \\
 &= 5a^4 - a^2 + 4
 \end{aligned} \tag{9}$$

$$[f(a)]^2=?$$

$$\begin{aligned}
 [f(a)]^2 &= f(a)^2 \\
 &= (5a^2 - a + 4)^2 \\
 &= (5a^2 - a + 4)(5a^2 - a + 4) \\
 &= 25a^4 - 5a^3 + 20a^2 - 5a^3 + a^2 - 4a + 20a^2 - 4a + 16 \\
 &= 25a^4 - 10a^3 + 41a^2 - 8a + 16
 \end{aligned} \tag{10}$$

$$f(a+h)=?$$

$$\begin{aligned} f(a+h) &= 5(a+h)^2 - (a+h) + 4 \\ &= 5(a+h)(a+h) - a - h + 4 \\ &= 5(a^2 + 2ah + h^2) - a - h + 4 \\ &= 5a^2 + 10ah + 5h^2 - a - h + 4 \end{aligned} \tag{11}$$

3. Find the domain of the function. (Enter your answer using interval notation.)

$$f(x) = \frac{x+4}{x^2-9} \tag{12}$$

For that, $x^2 - 9 \neq 0$, once we cannot divide by zero. Thus:

$$\begin{aligned} x^2 &\neq 9 \\ x &\neq \sqrt{9} \\ x &\neq 3 \end{aligned} \tag{13}$$

However, we will need to consider -3 as well as: $-3^2 = 9$. So the interval notation is:

$$(-\infty, -3) \cup (-3, 3) \cup (3, \infty)$$

4. Sketch the graph of the function.

$$f(x) = 4x + |4x| \tag{14}$$

We know that $|y| = -y$ for negative numbers and $|y| = y$ for non-negative ones. In other words, it is the positive version of the number, thus:

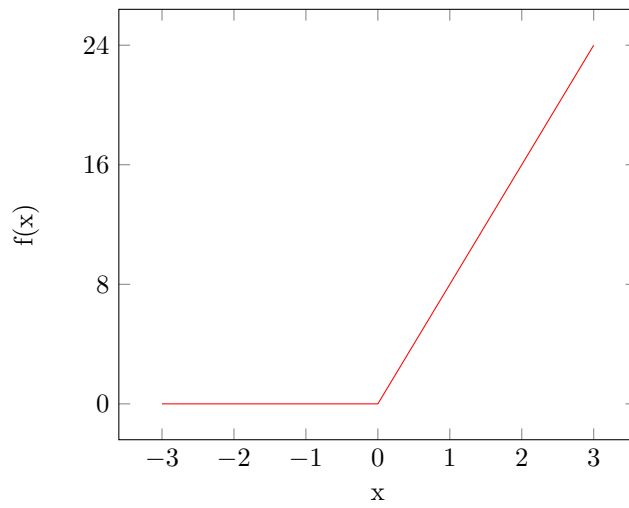
$$\begin{aligned} f(y) &= y + |y| = y + y, \forall x > 0 \\ \therefore f(x) &= 4x + 4x = 8x, \forall x > 0 \end{aligned} \tag{15}$$

and

$$\begin{aligned} f(y) &= y + |y| = -y + y = 0, \forall x < 0 \\ \therefore f(x) &= -4x + 4x = 0, \forall x < 0 \end{aligned} \tag{16}$$

and we cannot forget that $a \times b = 0$ when $a = 0$ or $b = 0$, thus:

$$f(x) = 0, x = 0 \tag{17}$$



Notes: A functions is said even when $f(x) = f(-x)$, i.e, it is "mirrored" in the y-axis and said odd when $f(x) = -f(-x)$ and it passes through the origin.