PTC3424 - 2017

Tarefa 3

1) Sejam as sequências de tempo discreto

$$x_{\mathrm{a}}(n) = 3\cos\left(\tfrac{\pi}{11}n\right) + \mathrm{sen}^2\left(\tfrac{\pi}{11}n\right), \quad 0 \le n \le 10$$
$$x_{\mathrm{b}}(n) = e^{-2n}, \qquad\qquad 0 \le n \le 10$$

Pede-se:

(a) Determine analiticamente a expressão da TFD para cada um desses sinais supondo $N = 11$. Em outras palavras, explicite a função resultante e determine os valores de $X_{\mathrm{a}}(k)$ =TFD$\{x_{\mathrm{a}}(n)\}$ e $X_{\mathrm{b}}(k)$ =TFD$\{x_{\mathrm{b}}(n)\}$ para $0 \le k \le 10$ inteiro.

i. Use a função `fft` do MATLAB para obter $X_{\mathrm{a}}(k)$ e $X_{\mathrm{b}}(k)$ para $0 \le k \le N-1$ inteiro.

DICA: para cada um dos sinais, sugerem-se os comandos abaixo.

```
X=fft(x,N);
k=0:N-1
figure(1)
subplot(311); stem(n,x); grid
title('x(n)')
subplot(312); stem(k,abs(X)); grid
title('Módulo da TFD')
subplot(313); stem(k,angle(X)); grid
title('Fase da TFD')
```

ii. Ainda usando os comandos do MATLAB, calcule a TFD inversa com N pontos de $Y(k) = X_{\mathrm{a}}(k)\,X_{\mathrm{b}}(k)$. Denote esse resultado da convolução circular como $y_{11}(n)$.

(b) Refaça os itens (a.i) e (a.ii) aplicando uma TFD com $N = 30$ pontos. Denote o resultado da convolução circular como $y_{30}(n)$.

(c) No MATLAB, faça a convolução linear das sequências $x_{\mathrm{a}}(n)$ e $x_{\mathrm{b}}(n)$. Denote esse resultado como $y_L(n)$.

(d) Em uma mesma tela gráfica dividida em três, esboce $y_L(n)$, $y_{11}(n)$ e $y_{30}(n)$. Justifique adequadamente os resultados obtidos.

2. Figure 1 illustrates a six-point discrete-time sequence $x[n]$. Assume that $x[n]$ is zero outside the interval shown.

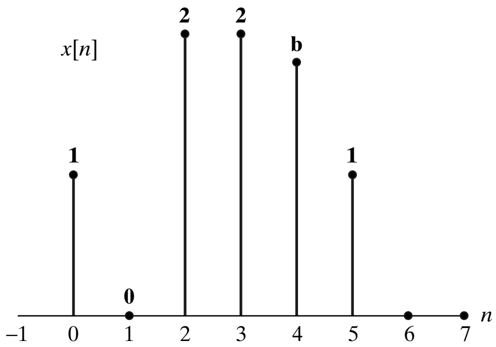

Figure 1

The value of $x[4]$ is not known and is represented as b. The sample in the figure is not shown to scale. Let $X(e^{j\omega})$ be the DTFT of $x[n]$ and $X_1[k]$ be samples of $X(e^{j\omega})$ at $\omega_k = 2\pi k/4$, i.e.,

$$X_1[k] = X(e^{j\omega})\big|_{\omega=\frac{\pi k}{2}}, \qquad 0 \le k \le 3.$$

The four-point sequence $x_1[n]$ that results from taking the four-point inverse DFT of $X_1[k]$ is shown in Figure 2. Based on the figure can you determine b uniquely? If so, give the value of b.

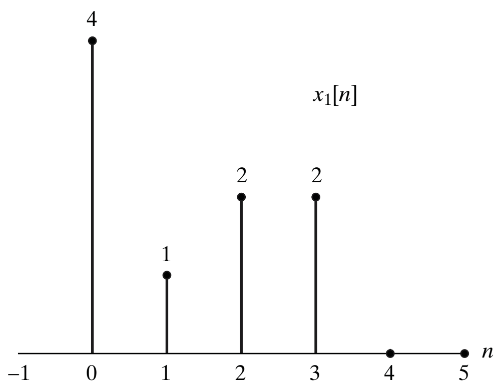

Figure 2

3. A problem that often arises in practice is one in which a distorted signal $y[n]$ is the output that results when a desired signal $x[n]$ has been filtered by an LTI system. We wish to recover the original signal $x[n]$ by processing $y[n]$. In theory, $x[n]$ can be recovered from $y[n]$ by passing $y[n]$ through an inverse filter having a system function equal to the reciprocal of the system function of the distorting filter.

Suppose that the distortion is caused by an FIR filter with impulse response

$$h[n] = \delta[n] - 0.5\delta[n - n_0],$$

where n_0 is a positive integer, i.e., the distortion of $x[n]$ takes the form of an echo at delay n_0.

(a) Determine the z-transform $H(z)$ and the N-point DFT $H[k]$ of the impulse response $h[n]$. Assume that $N = 4n_0$.

(b) Let $H_i(z)$ denote the system function of the inverse filter, and let $h_i[n]$ be the corresponding impulse response. Determine $h_i[n]$. Is this an FIR or an IIR filter? What is the duration of $h_i[n]$?

(c) Suppose that we use an FIR filter of length N in an attempt to implement the inverse filter, and let the N-point DFT of the FIR filter be

$$G[k] = 1/H[k], \qquad k = 0, 1, \ldots, N - 1.$$

What is the impulse response $g[n]$ of the FIR filter?

(d) It might appear that the FIR filter with DFT $G[k] = 1/H[k]$ implements the inverse filter perfectly. After all, one might argue that the FIR distorting filter has an N-point DFT $H[k]$ and the FIR filter in cascade has an N-point DFT $G[k] = 1/H[k]$, and since $G[k]H[k] = 1$ for all k, we have implemented an all-pass, nondistorting filter. Briefly explain the fallacy in this argument.

(e) Perform the convolution of $g[n]$ with $h[n]$, and thus determine how well the FIR filter with N-point DFT $G[k] = 1/H[k]$ implements the inverse filter.