PTC2324: Processamento Digital de Sinais I

Respostas: Lista de exercícios 4

MDM,FRMP-2014

1. (a)
$$\pi e^{-j\frac{\pi}{4}}\delta\left(\omega - \frac{\pi}{3}\right) + \pi e^{j\frac{\pi}{4}}\delta\left(\omega + \frac{\pi}{3}\right)$$

(b) $4\pi\delta(\omega) + \pi e^{j\frac{\pi}{8}}\delta\left(\omega - \frac{\pi}{6}\right) + \pi e^{-j\frac{\pi}{8}}\delta\left(\omega + \frac{\pi}{6}\right)$

2.
$$x(n) = 1 + \cos(\frac{\pi}{2}n)$$

3.
$$\omega_c = \frac{3\pi}{4} \text{ rad}$$

4. (b) i.
$$y(n) = 0$$

ii. $y(n) = \frac{\sin\left(\frac{\pi}{2}n\right)}{2\pi n} + \frac{\cos\left(\frac{\pi}{2}n\right) - 1}{\pi^2 n^2}$
iii. $y(n) = \frac{\sin\left(\frac{\pi}{2}n\right)}{\pi^2 n^2} - \frac{\cos\left(\frac{\pi}{2}n\right)}{2\pi n}$
iv. $y(n) = 2\left(\frac{\sin\left(\frac{\pi}{4}n\right)}{\pi n}\right)^2$
v. $y(n) = \frac{1}{4}\frac{\sin\left(\frac{\pi}{2}n\right)}{\pi n}$

5.
$$h(n) = \frac{1 - \cos(\pi n)}{\pi n} = \frac{2\sin^2\left(\frac{\pi}{2}n\right)}{\pi n} = \begin{cases} 0, & \text{se } n \text{ \'e par} \\ \frac{2}{\pi n}, & \text{se } n \text{ \'e impar} \end{cases}$$

6. (a) g(n) corresponde a um filtro passa-altas com faixa de passagem $\pi - \omega_c < |\omega| < \pi$.

(b)
$$y(n) = \sum_{k=1}^{p} (-1)^k a(k) y(n-k) + \sum_{k=0}^{q} (-1)^k b(k) x(n-k)$$

7. (a)
$$h(n) = (-1)^n \frac{\sin(\frac{\pi}{4}n)}{\pi n}$$

(b) Para
$$-\pi \le \omega < \pi$$
, temos $H_1(e^{j\omega}) = \begin{cases} \frac{1}{2}, & \text{se } |\omega| < \frac{\pi}{2} \\ 0, & \text{caso contrário} \end{cases}$

8. (a)
$$X_1(e^{j\omega}) = \frac{8e^{j\omega 3}}{1 - \frac{1}{2}e^{-j\omega}}$$

(b)
$$X_2(e^{j\omega}) = \frac{\alpha \sin(\omega_o)e^{-j\omega}}{1 - 2\alpha \cos(\omega_o)e^{-j\omega} + \alpha^2 e^{-j\omega^2}}$$

(c)
$$X_3(e^{j\omega}) = \frac{1}{1 - \frac{1}{4}e^{-j\omega^2}}$$

9.
$$x(n) = \frac{\sin\left(\frac{3\pi}{4}n\right) - \sin\left(\frac{\pi}{4}n\right)}{\pi n}$$

10. h(n) corresponde a um filtro passa-altas com faixa de passagem $\frac{\pi}{3} < |\omega| < \pi$.

11.
$$H(e^{j\omega}) = \frac{e^{-j\omega} + 3e^{-j\omega^2} + 2e^{-j\omega^3}}{2 - e^{-j\omega^2}}$$