1. **Relações importantes**

$$\cos(\theta n) = \frac{e^{j\theta n} + e^{-j\theta n}}{2} \qquad \text{sen}(\theta n) = \frac{e^{j\theta n} - e^{-j\theta n}}{2j} \qquad \sum_{k=0}^{N-1} e^{-\frac{2\pi}{N}nk} = \begin{cases} N, & n = mN \\ 0, & \text{d. v. n} \end{cases} \text{ sendo } m \text{ inteiro.}$$

2. **Transformadas de Fourier**

$$X(e^{j\omega}) = \sum_{n=-\infty}^{+\infty} x(n)e^{-j\omega n} \xleftrightarrow{\quad\text{TFTD}\quad} x(n) = \frac{1}{2\pi} \int_{-\pi}^{\pi} X(e^{j\omega})e^{j\omega n}\,d\omega$$

$$X(k) = \sum_{n=0}^{N-1} x(n)e^{-j\frac{2\pi}{N}kn} \xleftrightarrow{\quad\text{TFD}\quad} x(n) = \frac{1}{N} \sum_{n=0}^{N-1} X(k)e^{j\frac{2\pi}{N}kn}$$

**Sinal periódico**

$$\tilde{x}(n) = \tilde{x}(n+N) = \sum_{\ell=0}^{N-1} a(\ell)e^{j\frac{2\pi}{N}\ell n} \xleftrightarrow{\quad\text{TFTD}\quad} \tilde{X}(e^{j\omega}) = 2\pi \sum_{\ell=0}^{N-1} a(\ell)\delta\left(\omega - \frac{2\pi}{N}\ell\right), \quad -\pi \le \omega < \pi$$

$$\tilde{x}(n) = \tilde{x}(n+N) = \sum_{\ell=0}^{N-1} a(\ell)e^{j\frac{2\pi}{N}\ell n} \xleftrightarrow{\quad\text{SFD}\quad} \tilde{X}(k) = \tilde{X}(k+N) = N \sum_{\ell=0}^{N-1} a(\ell)\delta\left(\lfloor k - \ell \rfloor_N\right)$$

3. **Propriedades das Transformadas de Fourier**

| **TFTD** $x(n) \leftrightarrow X(e^{j\omega})$ | **SFD** $\tilde{x}(n) \leftrightarrow \tilde{X}(k)$ | **TFD** $x(n) \leftrightarrow X(k)$ |
|---|---|---|
| $x^*(n) \leftrightarrow X^*(e^{-j\omega})$ | $\tilde{x}^*(n) \leftrightarrow \tilde{X}^*(-k)$ | $x^*(n) \leftrightarrow X^*(\lfloor -k \rfloor_N)$ |
| $x(-n) \leftrightarrow X(e^{-j\omega})$ | $\tilde{x}(-n) \leftrightarrow \tilde{X}(-k)$ | $x(\lfloor -n \rfloor_N) \leftrightarrow X(\lfloor -k \rfloor_N)$ |
| $x^*(-n) \leftrightarrow X^*(e^{j\omega})$ | $\tilde{x}^*(-n) \leftrightarrow \tilde{X}^*(k)$ | $x^*(\lfloor -n \rfloor_N) \leftrightarrow X^*(k)$ |
| $x_e(n) \leftrightarrow \text{Re}(X(e^{j\omega}))$ | $\tilde{x}_e(n) \leftrightarrow \text{Re}(\tilde{X}(k))$ | $x_e(n) \leftrightarrow \text{Re}(X(k))$ |
| $x_o(n) \leftrightarrow j\text{Im}(X(e^{j\omega}))$ | $\tilde{x}_o(n) \leftrightarrow j\text{Im}(\tilde{X}(k))$ | $x_o(n) \leftrightarrow j\text{Im}(X(k))$ |
| Deslocamento no tempo $x(n-m) \leftrightarrow e^{-j\omega m}X(e^{j\omega})$ | Deslocamento no tempo $\tilde{x}(n-m) \leftrightarrow e^{-j\frac{2\pi mk}{N}}\tilde{X}(k)$ | Deslocamento no tempo $x(\lfloor n-m \rfloor_N) \leftrightarrow e^{-j\frac{2\pi mk}{N}}X(k)$ |
| Modulação $e^{j\omega_\ell n}x(n) \leftrightarrow X(e^{j(\omega-\omega_\ell)})$ | Modulação $e^{j\frac{2\pi\ell n}{N}}\tilde{x}(n) \leftrightarrow \tilde{X}(k-\ell)$ | Modulação $e^{j\frac{2\pi\ell n}{N}}x(n) \leftrightarrow X(\lfloor k-\ell \rfloor_N)$ |
| Derivada na frequência $n\,x(n) \leftrightarrow j\dfrac{dX(e^{j\omega})}{d\omega}$ | Dualidade $\tilde{X}(n) \leftrightarrow N\tilde{x}(-k)$ $\tilde{X}(-n) \leftrightarrow N\tilde{x}(k)$ | Dualidade $X(n) \leftrightarrow Nx(\lfloor -k \rfloor_N)$ $X(\lfloor -n \rfloor_N) \leftrightarrow Nx(k)$ |
| Convolução no tempo $x(n)*y(n) \leftrightarrow X(e^{j\omega})Y(e^{j\omega})$ $\sum_{\ell=-\infty}^{+\infty} x(\ell)y(n-\ell) \leftrightarrow X(e^{j\omega})Y(e^{j\omega})$ | Conv. periódica no tempo $\tilde{x}(n)*\tilde{y}(n) \leftrightarrow \tilde{X}(k)\tilde{Y}(k)$ $\sum_{\ell=0}^{N-1} \tilde{x}(\ell)\tilde{y}(n-\ell) \leftrightarrow \tilde{X}(k)\tilde{Y}(k)$ | Conv. circular no tempo $x(n) \circledast_N y(n) \leftrightarrow X(k)Y(k)$ $\sum_{\ell=0}^{N-1} x(\ell)y(\lfloor n-\ell \rfloor_N) \leftrightarrow X(k)Y(k)$ |
| Multiplicação no tempo $x(n)y(n) \leftrightarrow \dfrac{1}{2\pi}X(e^{j\omega})*Y(e^{j\omega})$ $x(n)y(n) \leftrightarrow \dfrac{1}{2\pi}\int_{-\pi}^{\pi} X(e^{j\theta})Y(e^{j(\omega-\theta)})d\theta$ | Multiplicação no tempo $\tilde{x}(n)\tilde{y}(n) \leftrightarrow \dfrac{1}{N}\tilde{X}(k)*\tilde{Y}(k)$ $\tilde{x}(n)\tilde{y}(n) \leftrightarrow \dfrac{1}{N}\sum_{\ell=0}^{N-1}\tilde{X}(\ell)\tilde{Y}(k-\ell)$ | Multiplicação no tempo $x(n)y(n) \leftrightarrow \dfrac{1}{N}X(k)\circledast_N Y(k)$ $x(n)y(n) \leftrightarrow \dfrac{1}{N}\sum_{\ell=0}^{N-1}X(\ell)Y(\lfloor k-\ell \rfloor_N)$ |
| Igualdade de Parseval $\sum_{n=-\infty}^{+\infty}|x(n)|^2 = \dfrac{1}{2\pi}\int_{-\pi}^{\pi}|X(e^{j\omega})|^2 d\omega$ | Igualdade de Parseval $\sum_{n=0}^{N-1}|\tilde{x}(n)|^2 = \dfrac{1}{N}\sum_{k=0}^{N-1}|\tilde{X}(k)|^2$ | Igualdade de Parseval $\sum_{n=0}^{N-1}|x(n)|^2 = \dfrac{1}{N}\sum_{k=0}^{N-1}|X(k)|^2$ |