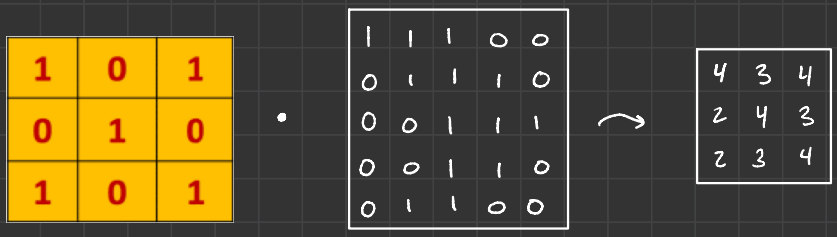
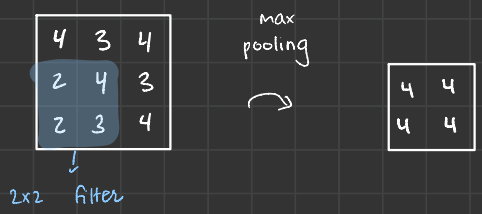
convolutional neural networks



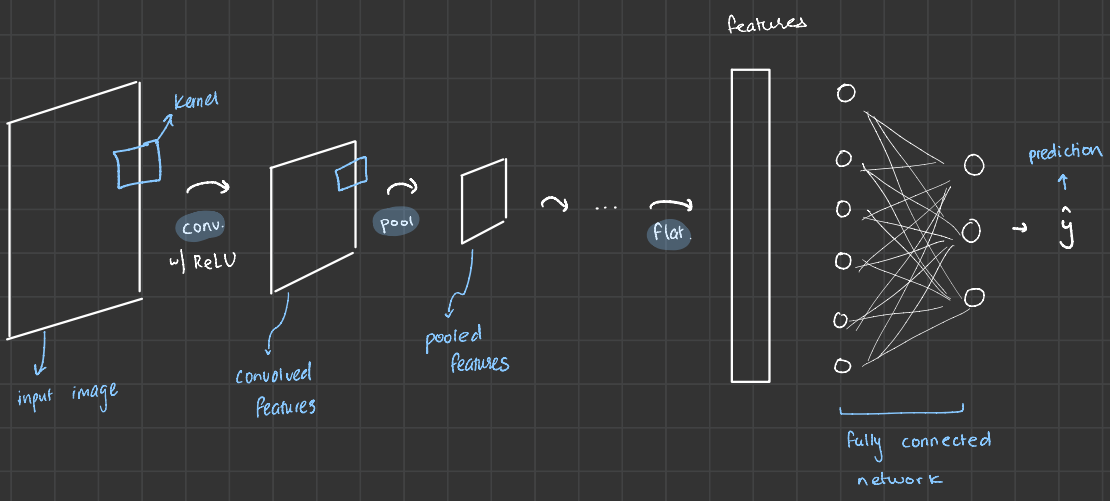
filter | kernel image convolved feature



$N = 3 \quad P = 0 \quad F = 2 \quad \text{stride} = 1$

$\Rightarrow \text{output} = \frac{3 + 2(0) - 2}{1} + 1 = 2$ ✓

for any convolution | pooling, $\text{output size} = \frac{N + 2P - F}{\text{stride}} + 1$



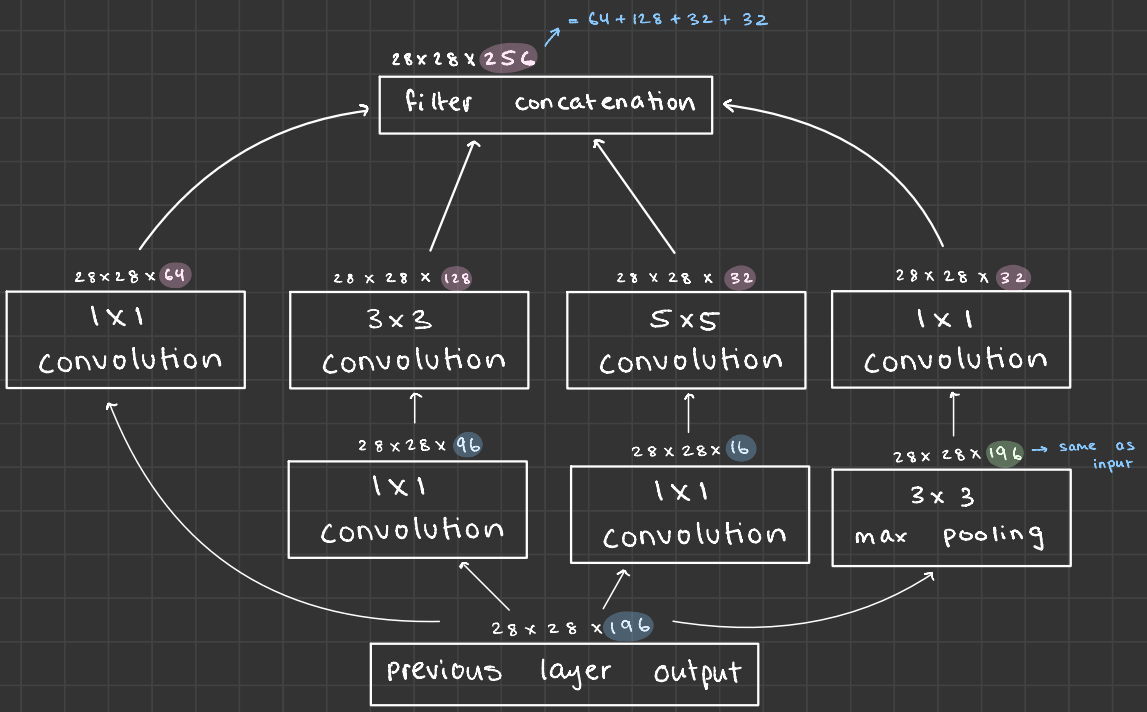
example:

- 3 x 3 input
- 1 row / column padding] 5 x 5 padded image
- 2 x 2 kernel
- stride = 1

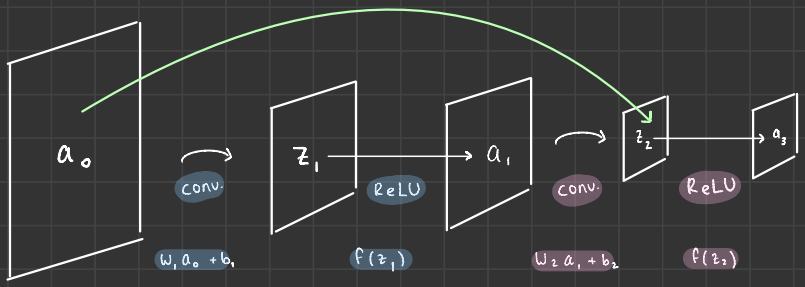
$$\Rightarrow \text{output} = \frac{3 + 2(1) - 2}{1} + 1 = 4$$

$\Rightarrow$ 4 x 4 output

inception module



Skip connection in ResNet



w/out skip

$$z_1 = w_1 a_0 + b_1$$
$$a_1 = f(z_1)$$

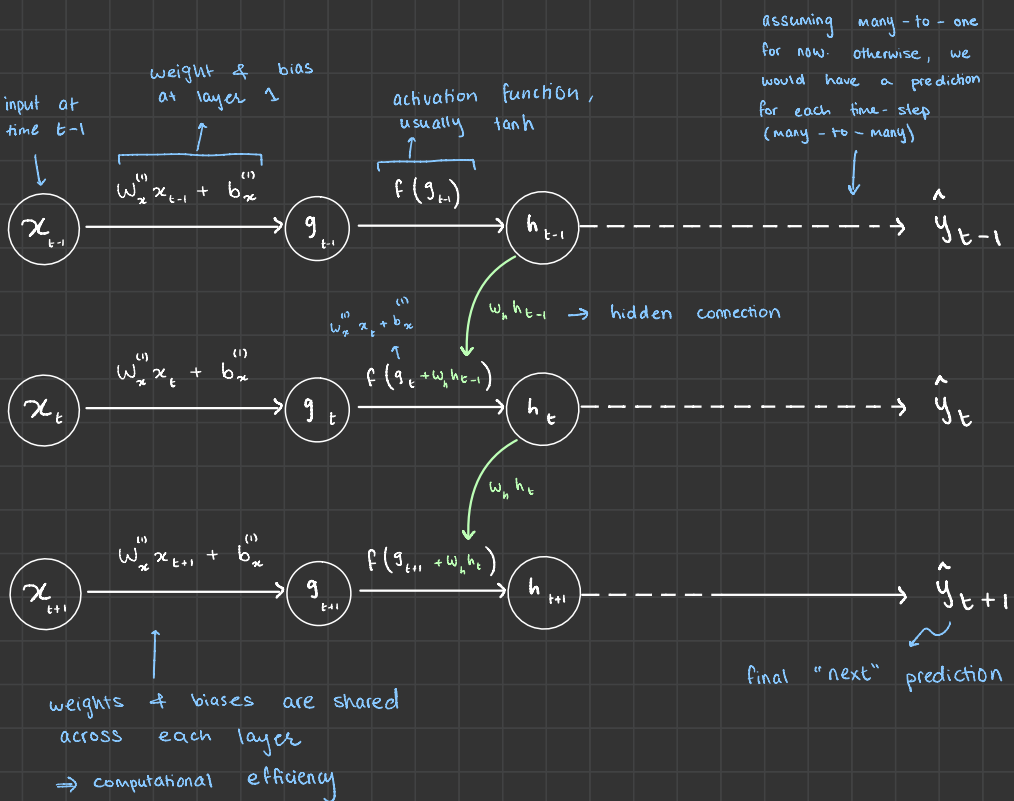
$$z_2 = w_2 a_1 + b_2$$
$$a_3 = f(z_2)$$
$$\Rightarrow a_3 = f(w_2 a_1 + b_2)$$

w/ skip

$$z_1 = w_1 a_0 + b_1$$
$$a_1 = f(z_1)$$

$$z_2 = w_2 a_1 + b_2$$
$$a_3 = f(z_2 + a_0)$$
$$\Rightarrow a_3 = f(w_2 a_1 + b_0 + a_0)$$

recurrent neural networks



many-to-one:

forward pass

$$h_t = f(W_x x_t + W_h h_{t-1} + b)$$

backpropagation through time → to update W_h

$$W_h := W_h - \alpha \frac{\partial L}{\partial W_h}$$

$$\frac{dL}{dW_h} = \sum_{t=1}^{T} \boxed{\frac{dL_t}{W_h}}$$

$$\frac{dL_T}{W_h} = \frac{dL_T}{d\hat{y}_T} \cdot \frac{d\hat{y}_T}{dh_T} \cdot \boxed{\frac{dh_T}{dh_t}} \cdot \frac{dh_t}{dW_h}$$

$$\frac{dh_T}{dh_t} = \frac{dhT}{dh_{T-1}} \cdot \frac{dh_{T-1}}{dh_{T-2}} \cdots \cdot \boxed{\frac{dh_{t+1}}{dh_t}} = \prod_{a=t}^{T-1} \frac{dh_{a+1}}{dh_a}$$

$$\frac{dh_{t+1}}{dh_t} = \frac{d}{dh_t} \tanh\left(W_x x_{t+1} + W_h h_t + b\right)$$

$$= \text{diag}\left(\tan^{-1}\left(W_x x_{t+1} + W_h h_t + b\right) \cdot W_h\right)$$

since each $\frac{dh_{t+1}}{dh_t}$ in $\prod_{a=t}^{T-1} \frac{dh_{a+1}}{dh_a}$ has a W_h:

$$W_h^{(1)} \cdot W_h^{(2)} \cdot W_h^{(3)} \cdots = W_h^{\alpha}$$

taking eigenvalue decomposition of W_h:

$$W_h = T \Sigma T^{-1}$$

$$W_h^{\alpha} = T \boxed{\Sigma^{\alpha}} T^{-1}$$

if $\Sigma > 1 \Rightarrow$ exploding gradient

if $\Sigma < 1 \Rightarrow$ vanishing gradient

this directly affects $\frac{dL}{W_h}$ in the update rule

gated recurrent unit

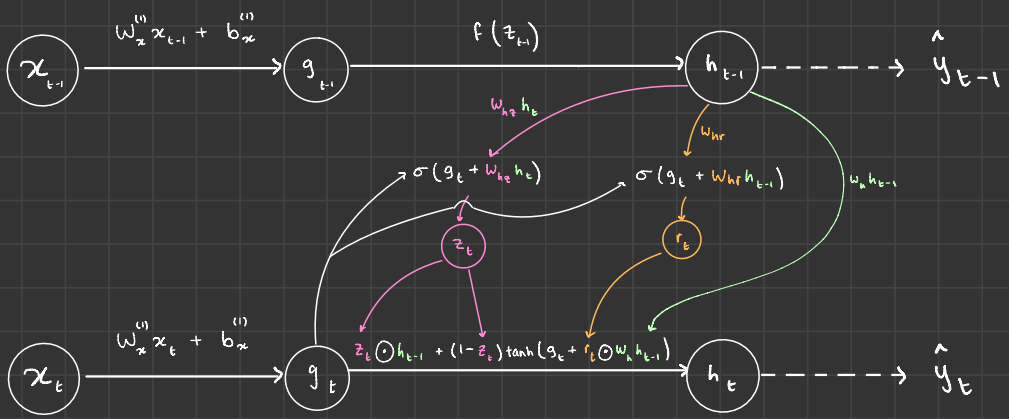
previously: $h_t = \tanh\left(w_x x_t + w_h h_{t-1} + b\right)$

now: element-wise multiplication (hadamard product)

$$h_t = z_t \odot h_{t-1} + (1 - z_t) \cdot \tanh\left(w_x x_t + r_t \odot w_h h_{t-1} + b\right)$$

$$z_t = \sigma\left(w_x x + w_{hz} h_{t-1} + b\right)$$

$$r_t = \sigma\left(w_x x + w_{hr} h_{t-1} + b\right)$$



three cases:

1. if $z_t = 0$ $\Rightarrow$ only r_t $\Rightarrow$ only short-term memory
2. if $r_t = 0$ $\Rightarrow$ only z_t $\rightarrow$ only long-term memory
3. if $z_t = 1$ $\rightarrow$ $h_t = h_{t-1}$ $\Rightarrow$ no memory