A 2 Jianman Lu 1577618

P_1: $\|z-x\| = \|\lambda x + (1-\lambda)y - x\|$

$= \|(\lambda-1)x + (1-\lambda)y\|$

$= \|(1-\lambda)y - (1-\lambda)x\|$

$= |(1-\lambda)|\,\|y-x\|$; $\lambda = 1 - \frac{\varepsilon}{2\|y-x\|}$

$= \frac{\varepsilon}{2\|y-x\|}\|y-x\|$

$= \frac{\varepsilon}{2} < \varepsilon$

P_2: $\forall x_1, x_2 \in S$, $f(x_2) \geq f(x_1) + \nabla f(x_1)^T (x_2 - x_1)$.

let $\exists x^* \in D(f)$ is local opt. $\forall x \in D(f)$,

$$f(x) \geq f(x^*) + \nabla f(x^*)^T (x - x^*).$$

$\because \nabla f(x^*) = 0$.

$\therefore f(x) \geq f(x^*)$

$\Rightarrow \therefore \forall x \in D(f)$, $f(x) \geq f(x^*)$.

$\therefore$ it is a global opt. of f.

P3. ① $J = \frac{1}{2M} \sum_{m=1}^{M} \left(\sum_{i=0}^{d} w_i x_i^{(m)} - t^{(m)} \right)^2 + \lambda \sum_{i=0}^{d} w_i^2$

$= \frac{1}{2M} \| Xw - t \|^2 + \lambda \|w\|^2$

$= \frac{1}{2M} (Xw - t)^T (Xw - t) + \lambda w^T w$

② $\frac{\partial}{\partial w} J(w) = \frac{1}{M} X^T (Xw - t) + 2\lambda w$

$Hf(w) = \left[\frac{\partial^2 J(w)}{\partial w^2} \right] = \left(\frac{X^T X}{M} w + 2\lambda w - \frac{X^T t}{M} \right)' = \frac{X^T X}{M} + 2\lambda.$

$\because \lambda > 0 \quad \frac{X^T X}{M} \geq 0.$

$\therefore \frac{\partial^2 J(w)}{\partial w^2} > 0.$, convex on w.

③ $\frac{\partial}{\partial w} J(w) = 0.$

$\left(\frac{1}{M} X^T X + 2\lambda \right) w = \frac{1}{M} X^T t$

$w = \left(\frac{1}{M} \|X\|^2 + 2\lambda \right)^{-1} \frac{1}{M} X^T t$