P1. $P(X=x) = \begin{cases} p & x=1 \\ 1-p & x=0. \end{cases}$

$E(X) = 0 \times (1-p) + 1 \times p = p.$

P2. $\sigma(z) = 1 - \sigma(-z).$

$\Rightarrow \sigma(z) + \sigma(-z) = 1.$

$\frac{1}{1+e^{-z}} + \frac{1}{1+e^{z}}$

$= \frac{1+e^{z}+1+e^{-z}}{(1+e^{-z})(1+e^{z})}$

$= \frac{1+e^{z}+1+e^{-z}}{1+e^{z}+e^{-z}+1}$

$= 1.$

$\therefore$ proved.

P3. $D_{KL}(t\|y) = t\log\left(\frac{t}{y}\right) + (1-t)\log\left(\frac{1-t}{1-y}\right).$

$= t\log(t) - t\log(y) + (1-t)\log(1-t) - (1-t)\log(1-y).$

$\underset{y}{\operatorname{argmin}}\, D_{KL}(t\|y) = \underset{y}{\operatorname{argmin}}\left[-t\log(y) - (1-t)\log(1-y)\right].$

$= \underset{y}{\operatorname{argmin}}\, J.$

P4: $(\sigma(z))' = -\frac{1}{(1+e^{-z})^2} \times (-e^{-z}). = \frac{e^{-z}}{(1+e^{-z})^2}$

$\sigma(z)(1-\sigma(z)) = \sigma(z) - \sigma^2(z) = \frac{1}{1+e^{-z}} - \frac{1}{(1+e^{-z})^2} = \frac{\cancel{1}+e^{-z}-\cancel{1}}{(1+e^{-z})^2} = (\sigma(z))'$

$\therefore [\sigma(z)]' = \sigma(z)(1-\sigma(z)).$