

# Lecture 12

## Byzantine Generals Problem

ECE 422: Reliable and Secure Systems Design



Instructor: An Ran Chen  
Term: 2024 Winter

# Schedule for today

- Key concepts from last class
- Byzantine fault tolerance
- The Byzantine Generals Problem
  - Definition
  - Analogy to system design
  - The oral message solution
  - What happens when  $m = 2$ ?

# Failure classification

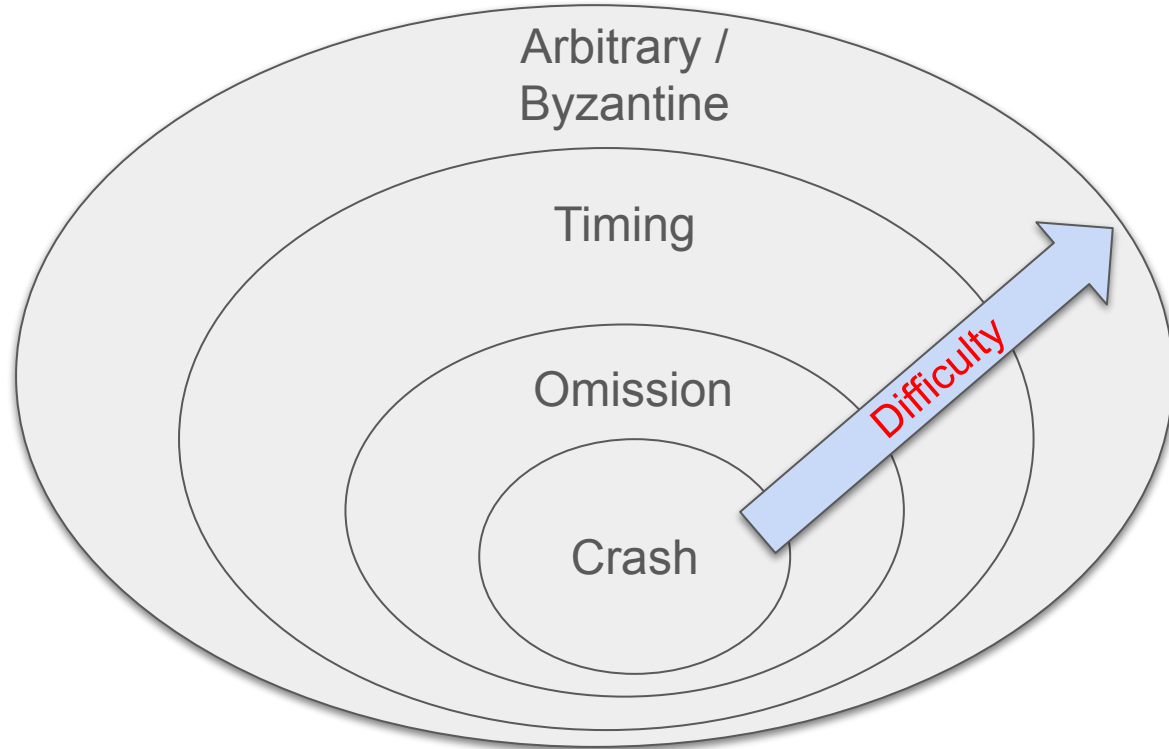
Failure classification describes the way how a system can fail that is perceived by the rest of the system.

- It aims to answer one question: “What kind of failures are we dealing with when taking the whole system into account.”
- It helps understand how to build and design for fault-tolerant systems.

There are four types of failure:

- Timing failure, also known as performance failure
- Omission failure
- Crash failure
- Arbitrary failure, also known as Byzantine failure

# Difficulties of failures



# Solution: TCP 3-way handshake

**A1**



**A2**

A1 wants to attack

*If you respond, I'll attack!*

A1 will attack;  
A2 wants to attack

A1 will attack;  
A2 will attack

*If you respond, I'll attack!*

*We'll attack!*



A1 will attack;  
A2 will attack

There is no solution to the two general problem.

# Byzantine failure

Byzantine failure: a node/component may fail arbitrary due to:

- Exhausted resources
- Conflicting information from different parts of the system

Why would nodes/components fail arbitrarily?

- Software bugs in the code
- Hardware failures
- Malicious attack on the system

There is an **inconsistency** in the functionality of the component. It presents different symptoms to different observers.

# Byzantine fault tolerance

Byzantine fault tolerance: the ability of a system to continue functioning even if some of its nodes/components fail randomly or act maliciously.

- The system can keep functioning even if certain components stop working
- For example, safety-critical systems need to be able to work when if some of its components fail
  - E.g., train, airplane, spacecraft
  - E.g., heart-lung machine, robotic surgery

# Schedule for today

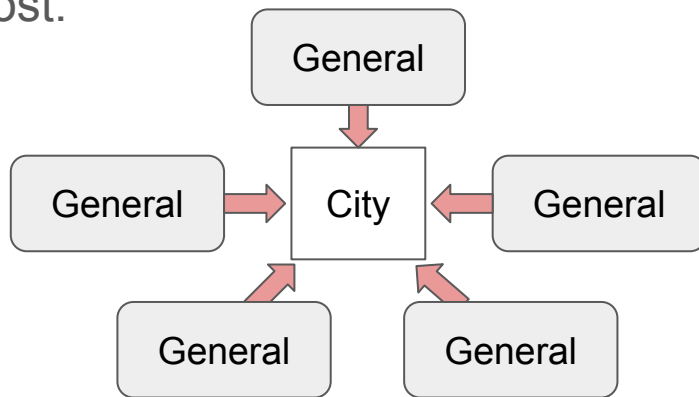
- Key concepts from last class
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# The Byzantine Generals Problem

Byzantine Generals Problem: a group of Byzantine generals who must coordinate an attack by the means of messengers to reach consensus.

- The generals must decide as a group to attack or retreat.
- If they all make the same decision, they can eventually win the battle.
- But after the decision making phase, if some of the generals attack while the other retreat, then the battle is lost.



# The Byzantine Generals Problem

New assumptions:

- If a message is sent, it is always delivered correctly.
  - Sending lots of messengers increases the probability that one getting through
- One or more generals may be “traitors”
  - Analogous to a malicious node/component in the system
  - Traitors’ mission is to break the consensus among the “royal” generals (non-traitors) so that they lose the battle.

# Consensus in the presence of a traitor

Suppose we have five generals, one of them is a traitor.

Each royal general shares their decision to others:

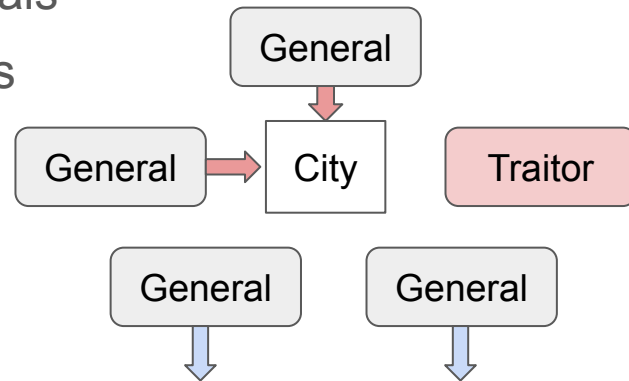
- Two generals send an “attack” message.
- Two other generals send a “retreat” message.

The traitor tries to break the consensus:

- Sends an “attack” message to the first two generals
- Send a “retreat” message to the last two generals

Next day: consensus is broken

- Two royal generals attack
- Two royal generals retreat



# Analogy to system design

In a distributed system, some nodes can be:

- Flaky nodes
- Malicious nodes (e.g., Hackers)

The Byzantine problem attempts to propose a solution to these failures from two main questions:

- How can we ensure the **consistency**?
  - All “royal” nodes produce the same results.
- How we can we ensure the **validity**?
  - All “royal” nodes produce a valid response.
- (Despite the presence of flaky or malicious nodes)

# Byzantine Generals Problem

Lamport et al. proposed to solve the Byzantine Generals Problem with two solutions:

- **Solution 1: Oral message**
- Solution 2: Signed message

[\[PDF\] The Byzantine generals problem](#)

L Lamport, R Shostak, M Pease - Concurrency: **the** works of leslie ..., 1982 - dl.acm.org

... three **generals** that coped with one traitor, then we could construct a three-**general** solution to **the Byzantine Generals Problem** that also worked in **the** presence of one traitor. Suppose ...

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The oral message solution simplifies the problem:

- A commanding general sends an order to his lieutenants.
  - A commanding general is the first person to send a decision
  - The rest become the lieutenants
  - Lieutenant can decide to execute or not execute the order from the command general
- Either of them can be traitors

# Byzantine Generals Problem

When a commanding general sends an order to his lieutenants, there are two properties:

- **Consistency**: Loyal lieutenants must execute the same order.
- **Validity**: If the commanding general is loyal, then every loyal lieutenant must obeys his order.

# Solution to Byzantine Generals Problem

Let  $m$  = # traitors (malicious nodes),  $n$  = total # generals (nodes)

**Theorem:** We need  $n = (3m + 1)$  generals to tolerate  $(m)$  traitors.

- If  $m < \frac{1}{3} n$ , then it is solvable
- We can somehow achieve both consistency and validity

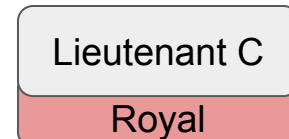
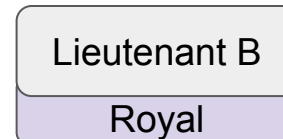
# Example: $m = 1$ , $n = 3$

Scenario 1: Why is it unsolvable?

General A = commanding general, **royal**

General B = lieutenant, **royal**

General C = lieutenant, **traitor**

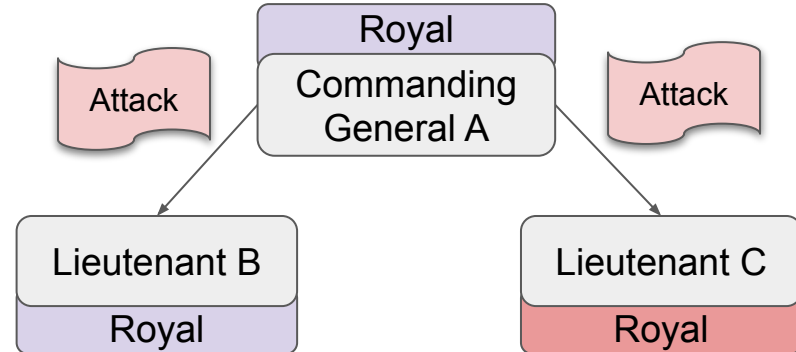




# Example: $m = 1$ , $n = 3$

Scenario 1: Why is it unsolvable?

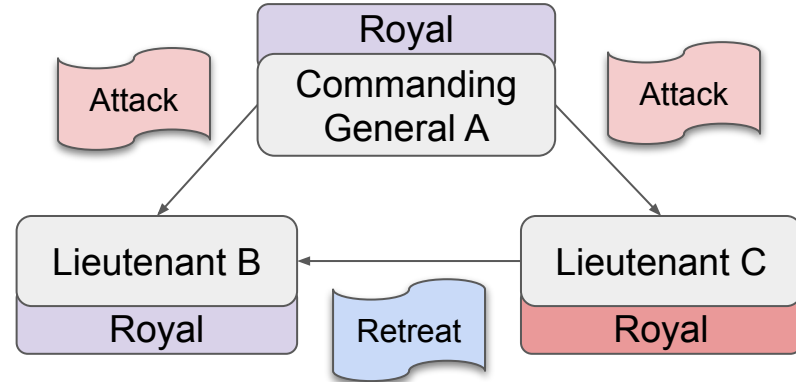
- A sends an “attack” message to the two others.
- B receives the message, and asks C to share the message that he/she received from A.



# Example: $m = 1$ , $n = 3$

## Scenario 1: Why is it unsolvable?

- A sends an “attack” message to the two others.
- B receives the message, and asks C to share the message that he/she received from A.
- C is a traitor, thus changes the message to “retreat”.
- B cannot tell who is the traitor.
- **Unsolvable**



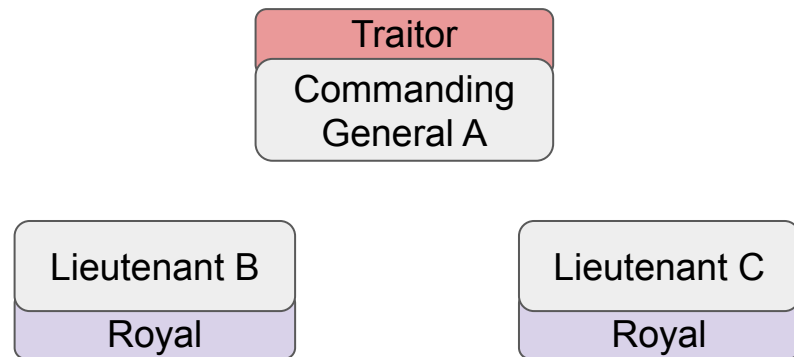
# Example: $m = 1$ , $n = 3$

Scenario 2: Why is it unsolvable?

General A = commanding general, **traitor**

General B = lieutenant, **royal**

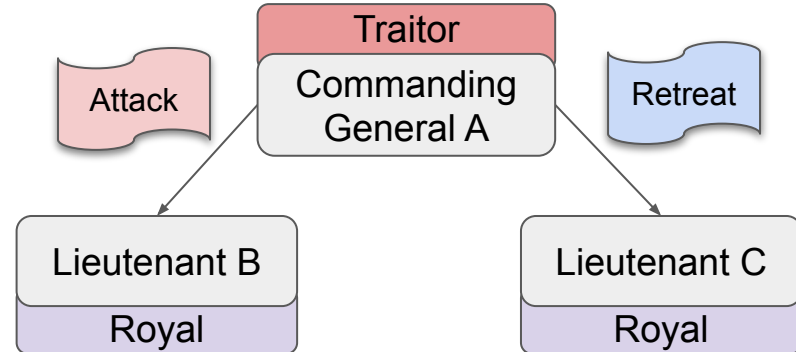
General C = lieutenant, **royal**



# Example: $m = 1$ , $n = 3$

Scenario 2: Why is it unsolvable?

- A sends different messages to others; “attack” to B, and “retreat” to C.
- B receives the message, and asks C to share the message that he/she received from A.

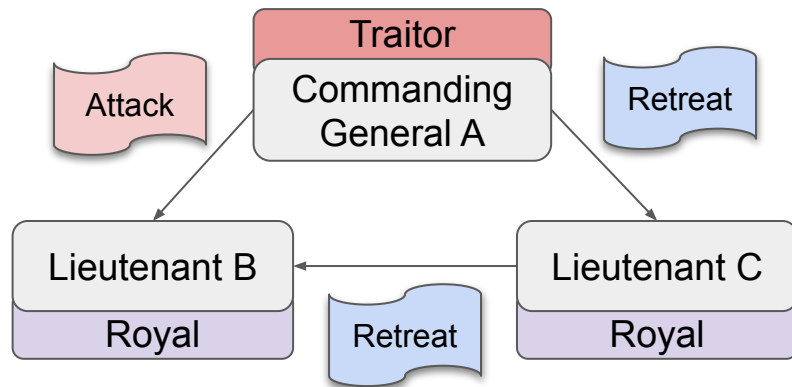


## Example: $m = 1, n = 3$

Scenario 2: Why is it unsolvable?

- A sends different messages to others; “attack” to B, and “retreat” to C.
- B receives the message, and asks C to share the message that he/she received from A.
- C shares the message “retreat” with B.
- B still cannot tell who is the traitor.
- **Unsolvable**

If  $m < \frac{1}{3} n$ , then it is solvable,  
Otherwise it is unsolvable.



# Example: $m = 1$ , $n = 4$

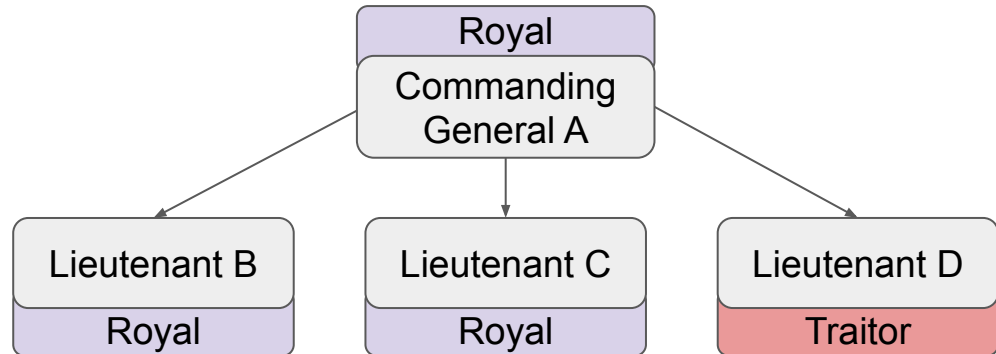
Scenario 1: Why is it solvable?

General A = commanding general, **royal**

General B = lieutenant, **royal**

General C = lieutenant, **royal**

General D = lieutenant, **traitor**



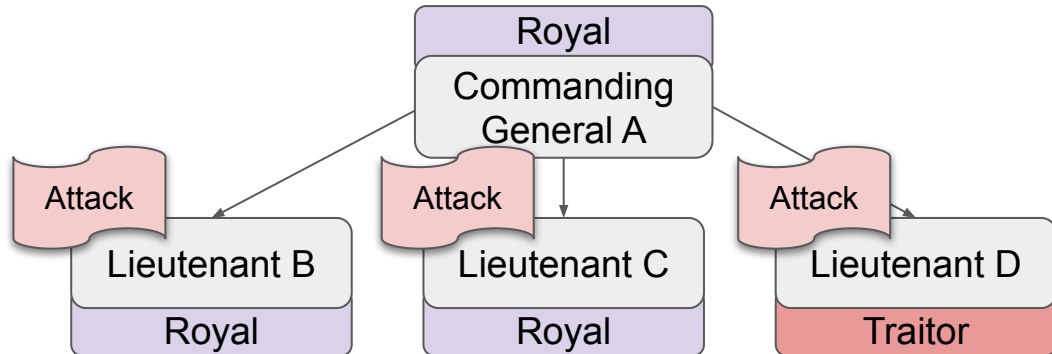
# Example: $m = 1, n = 4$

## Scenario 1: Why is it solvable?

- A sends an “attack” message to others.

- B decides to attack,  
 $V(B) = (A, A, R)$

- B asks C and D for the message they received.
- B receives: “attack” from A, “attack” from C, and “retreat” from D
  - $V(B) = (A, A, R)$
- B decides to attack

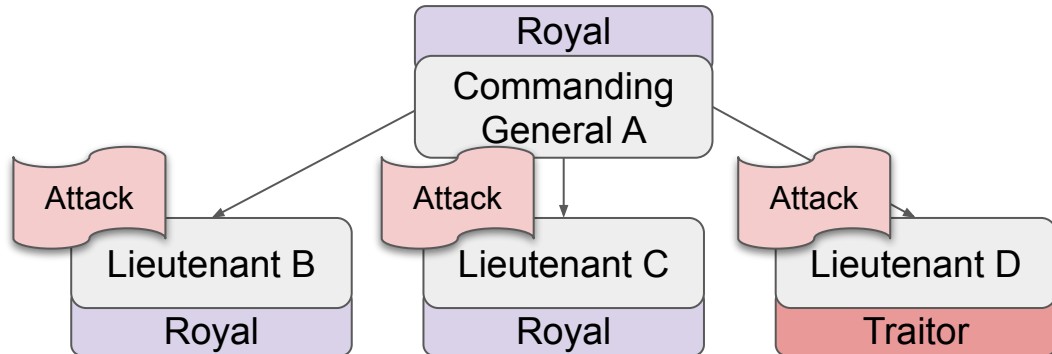


# Example: $m = 1, n = 4$

Scenario 1: Why is it solvable?

- A sends an “attack” message to others.
- C asks B and D for the message they received.
- C receives: “attack” from A, “attack” from B, and “retreat” from D
  - $V(C) = (A A R)$
- C decides to attack

- B decides to attack,  
 $V(B) = (A A R)$
- C decides to attack  
 $V(C) = (A A R)$





# Example: $m = 1, n = 4$

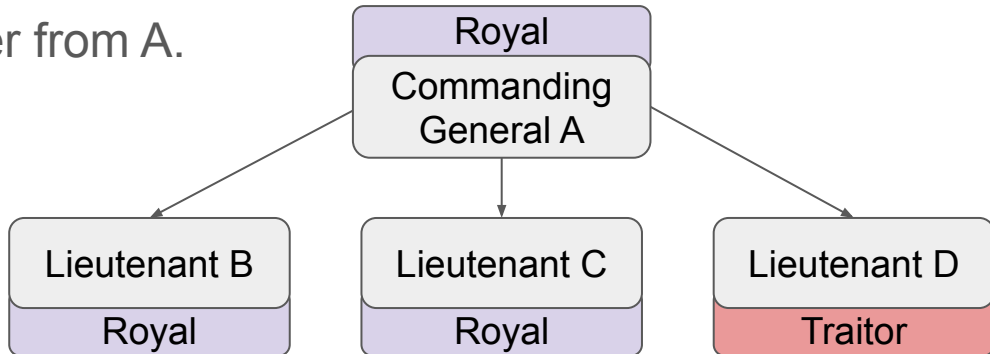
Scenario 1: Why is it solvable?

- A sends an “attack message” to other.

Problem solved!

- **Consistency**: B and C executed the same order.
- **Validity**: B and C obeyed the order from A.

- B decides to attack,  
 $V(B) = (A A R)$
- C decides to attack  
 $V(C) = (A A R)$



# Example: $m = 1$ , $n = 4$



Scenario 2: Is this solvable? Can we still achieve both properties?

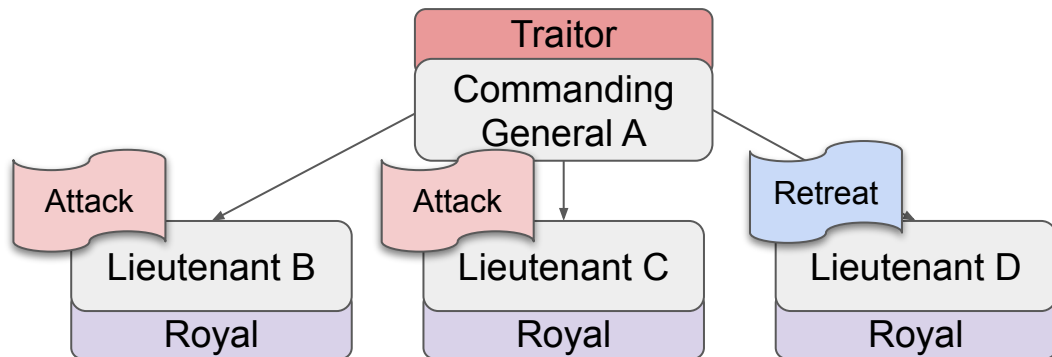
General A = commanding general, **traitor**

General B = lieutenant, **royal**

General C = lieutenant, **royal**

General D = lieutenant, **royal**

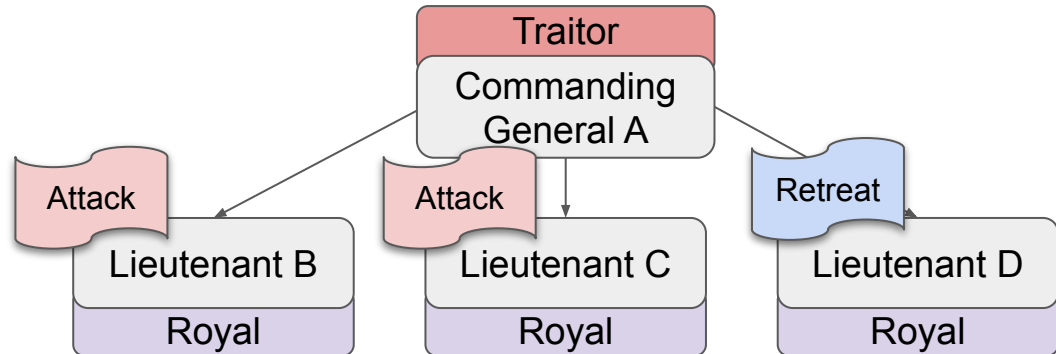
**Validity:** if and only if the commanding general is loyal, then every loyal lieutenant must obeys his order.



# Example: $m = 1, n = 4$

Scenario 2: Why is it solvable?

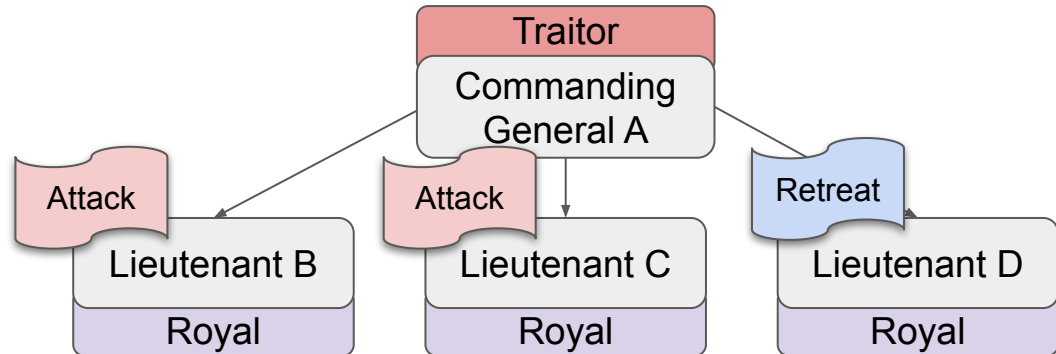
- A sends “attack” to the first two lieutenants, and “retreat” to the last.
- B receives: “attack” from A, “attack” from C, and “retreat” from D
  - $V(B) = (A A R)$
- B decides to attack



# Example: $m = 1, n = 4$

Scenario 2: Why is it solvable?

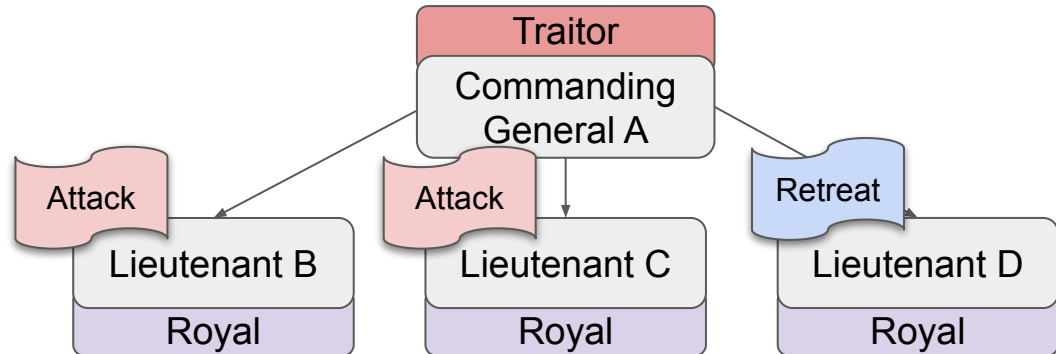
- A sends “attack” to the first two lieutenants, and “retreat” to the last.
- C receives: “attack” from A, “attack” from B, and “retreat” from D
  - $V(C) = (A A R)$
- C decides to attack



# Example: $m = 1, n = 4$

Scenario 2: Why is it solvable?

- A sends “attack” to the first two lieutenants, and “retreat” to the last.
- D receives: “retreat” from A, “attack” from B, and “attack” from C
  - $V(D) = (R A A)$
- D decides to attack



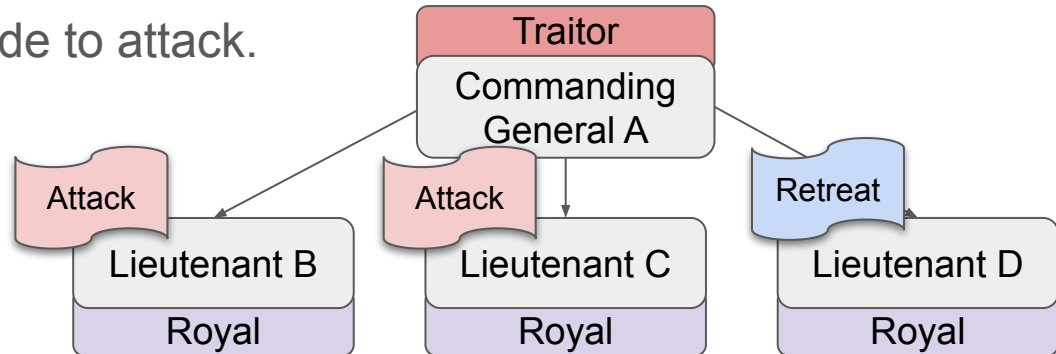
# Example: $m = 1, n = 4$

Scenario 2: Why is it solvable?

- A sends “attack” to the first two lieutenants, and “retreat” to the last.
- $V(B) = (A A R)$
- $V(C) = (R A A)$
- $V(D) = (R A A)$
- All three royal lieutenants decide to attack.

Problem solved!

- **Consistency** fulfilled.



# Schedule for today

- Key concepts from last class
- Byzantine fault tolerance
- The Byzantine Generals Problem
  - Definition
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  - The oral message solution
  - What happens when  $m = 2$ ?

# What happens when $m = 2$ ?

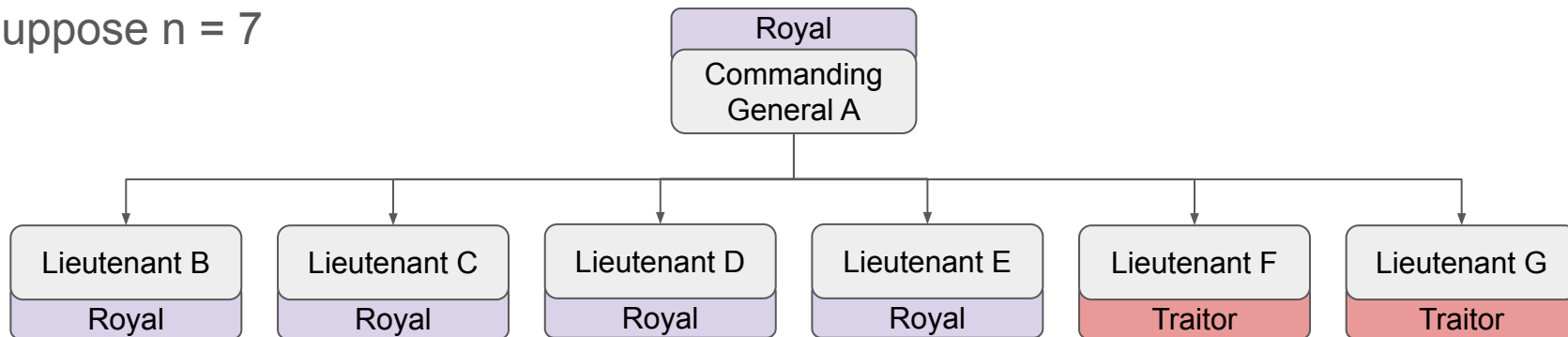
A recursive solution must be used to solve  $m > 1$ :

- The commanding general sends the message to the other lieutenants.
- Each lieutenant becomes a commanding general and sends the message they received from the original commanding general to the other lieutenants.
- ...



# What happens when $m = 2$ ?

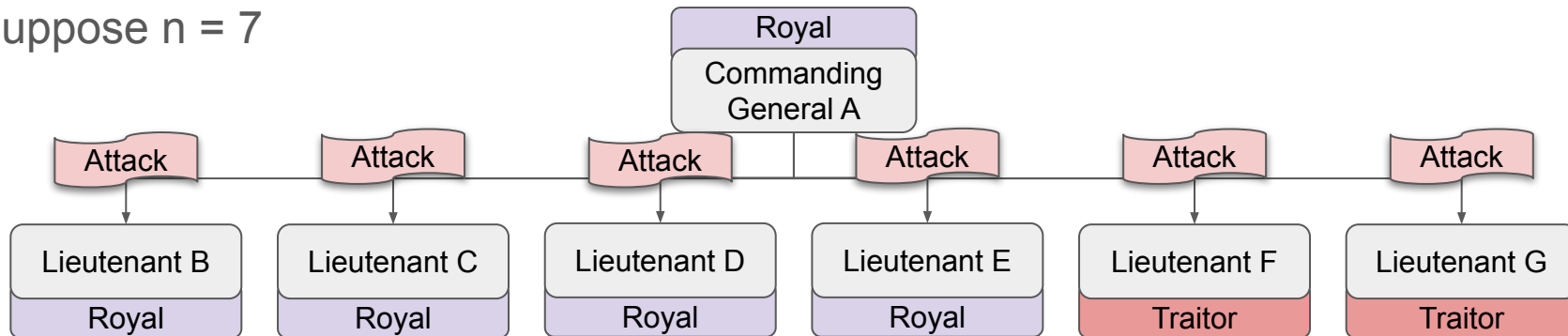
Suppose  $n = 7$



- General A = commanding general, **royal**
- General B = lieutenant, **royal**
- General C = lieutenant, **royal**
- General D = lieutenant, **royal**
- General E = lieutenant, **royal**
- General F = lieutenant, **traitor**
- General G = lieutenant, **traitor**

# What happens when $m = 2$ ?

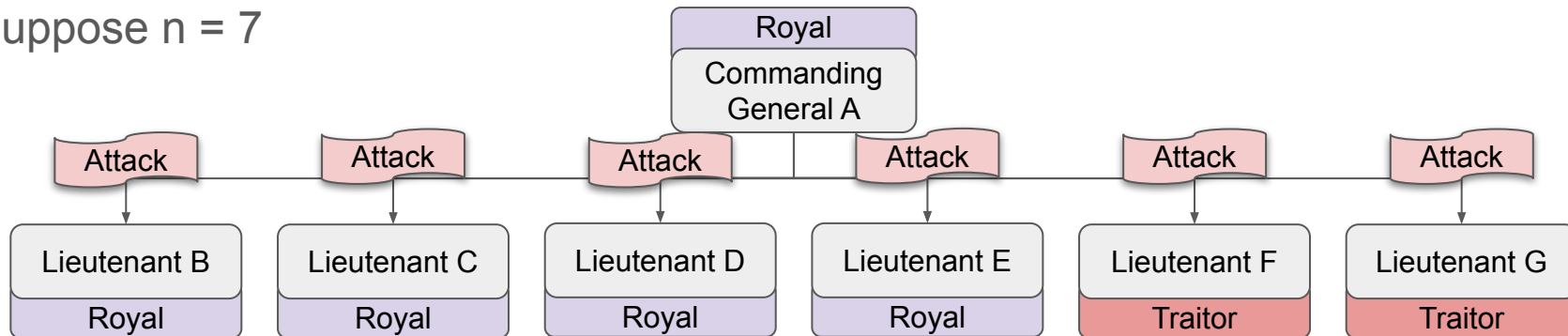
Suppose  $n = 7$



- A sends “attack”
- B asks C the message he/she received
  - C replies “attack”
- B asks D, E, F, G: “what C received from A?”
  - D replies “attack”, E replies “attack”, F replies “retreat”, G replies “retreat”
  - $V_C = (A, A, A, R, R)$ , B confirms that C received “attack”

# What happens when $m = 2$ ?

Suppose  $n = 7$



- A sends "attack"
- B asks C, D, E, F, G: "what D received from A?"
  - C replies "attack", D replies "attack", E replies "attack", F replies "retreat", G replies "retreat"
  - $V_D = (A, A, A, R, R)$ , B confirms that D received "attack"

# What happens when $m = 2$ ?

Suppose  $n = 7$

## For Lieutenant B

- Lieutenant B pick the most popular action from the other generals:
  - $V_B = A$ , B receives “attack” from A
  - $V_C = (A, A, A, R, R)$ , B confirms that C received “attack”
  - $V_D = (A, A, A, R, R)$ , B confirms that D received “attack”
  - $V_E = (A, A, A, R, R)$ , B confirms that E received “attack”
  - $V_F = (R, R, R, R, R)$ , B confirms that F received “retreat”
  - $V_G = (R, R, R, R, R)$ , B confirms that G received “retreat”
- Lieutenant B decides to attack, because there are 4 attacks and 2 retreats

## For Lieutenant C ...

# What happens when $m = 2$ ?

Suppose  $n = 7$

- Commanding general A will attack.
- Lieutenant B decides to attack, because there are 4 attacks and 2 retreats
- Lieutenant C decides to attack, because there are 4 attacks and 2 retreats
- Lieutenant D decides to attack, because there are 4 attacks and 2 retreats
- Lieutenant E decides to attack, because there are 4 attacks and 2 retreats

All four royal lieutenants decide to attack, obeying A.

Problem solved!

- **Consistency** fulfilled
- **Validity** fulfilled

# Cost of solving Byzantine Generals Problem

Solving the Byzantine Generals Problem is expensive

- $m = 0$ , sending  $n$  messages to every lieutenant
- $m = 1$ , sending  $n^2$  messages to every lieutenant
- $m = 2$ , sending  $n^3$  messages to every lieutenant
- ...

m	Messages Sent
0	$O(n)$
1	$O(n^2)$
2	$O(n^3)$
3	$O(n^4)$

# Timeline

- In 1978, Robert Shostak worked on a NASA-sponsored project about fault-tolerant for aircraft control.

## **SIFT: Design and analysis of a fault-tolerant computer for aircraft control**

JH Wensley, L **Lamport**, J Goldberg... - Proceedings of the ..., 1978 - [ieeexplore.ieee.org](#)

SIFT (Software Implemented Fault Tolerance) is an ultrareliable computer for critical aircraft control applications that achieves fault tolerance by the replication of tasks among

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- In 1980, Leslie Lamport, Robert Shostak, and Marshall Pease together worked on an algorithm for processors to reach consensus in the presence of faults.

## **Reaching agreement in the presence of faults**

M Pease, R Shostak, L Lamport - Journal of the ACM (JACM), 1980 - [dl.acm.org](#)

... In the absence of **faults reaching** a satisfactory mutual **agreement** is usually an easy matter.

In ... include those of **reaching** approximate **agreement** and **reaching agreement** under various ...

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- In 1982, they later come up with the analogy of the Byzantine generals problem.

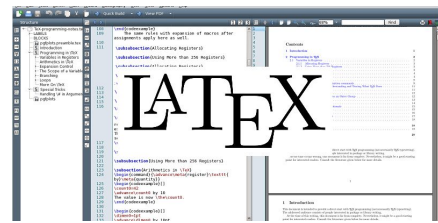
## **[PDF] The Byzantine generals problem**

L Lamport, R Shostak, M Pease - Concurrency: the works of leslie ..., 1982 - [dl.acm.org](#)

... three **generals** that coped with one traitor, then we could construct a three-**general** solution to the **Byzantine Generals Problem** that also worked in **the** presence of one traitor. Suppose ...

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Leslie Lamport is also the initial developer of LaTeX.



# Timeline

- In 1998, “Practical Byzantine Fault Tolerance” (PBFT) was published.
  - PBFT was proposed as a practical solution for Byzantine fault tolerance that takes into consideration there can be delays in the network.
- In 2008, “Bitcoin: A peer-to-peer electronic cash system” was published.
  - Key idea: A ledger via some complex math records the transactions.
  - Proof of work (PoW) to validate transactions on a blockchain.
  - Individuals around the world can come to a consensus without relying on any third-party as an intermediate for trust.

