

# Lecture 18

## Midterm Review

ECE 422: Reliable and Secure Systems Design



Instructor: An Ran Chen  
Term: 2024 Winter

# How was the midterm?

A quick survey: [Click here for the survey link](#)

- How hard was the midterm?
- Did you know what to study for your midterm?
- Did you have enough support (e.g., materials, sample questions) to prepare for the midterm?

## Question on (7, 4) Hamming code

**Multiple Choice Question:** Given that 0001011 is a codeword in (7, 4) Hamming code, which of the following cannot be the valid codeword in the codespace? (Hint: a (7, 4) Hamming code can correct and detect any single-bit error.)

- a. 0011101
- b. 0101100
- c. 0011010
- d. 1110100

# Code distance in error correction

To correct  $d$  bit errors, the code distance for the codewords must be larger or equal to  $2d+1$ .

## Example

Code: {000, 101}

$$C_d = 2$$

Transmitting codeword: 000

A single-bit error happens, 000 becomes 001

We cannot tell how to correct 001 (000 or 101).

## Analogy

Dictionary: {accept, except}

Distance = 2

Typed word: accept

A typo happens, accept becomes except

We cannot tell which word is mistyped.

## Question on (7, 4) Hamming code

**Multiple Choice Question:** Given that 0001011 is a codeword in (7, 4) Hamming code, which of the following **cannot be the valid codeword** in the codespace? (Hint: a (7, 4) Hamming code can correct and detect any single-bit error.)

a. 0011101

b. 0101100

c. 0011010

d. 1110100

To correct  $d$  bit errors, the code distance for the codewords **must be larger or equal to  $2d+1$** .

0   0   0   1   0   1   1

0   0   1   1   1   0   1   → distance = 3

0   1   0   1   1   0   0   → distance = 5

0   0   1   1   0   1   0   → distance = 2

1   1   1   0   1   0   0   → distance = 7

# Question on Spectrum-based Fault Localization

**Multiple Choice Question:** Which of the following statements is the most likely to be suspicious based on Spectrum-based Fault Localization?

- a.  $S_1$
- b.  $S_2$
- c.  $S_3$
- d.  $S_4$

	$T_1$	$T_2$	$T_3$
$S_1$	✓		
$S_2$		✓	✓
$S_3$	✓		
$S_4$	✓	✓	✓
Result	P	F	F

$$Ochiai(element) = \frac{e_f}{\sqrt{(e_f + n_f) \cdot (e_f + e_p)}}$$

# Lecture 6: Spectrum-based fault localization

Failed test

Source  
file A

Run tests

	T <sub>1</sub>	T <sub>2</sub>	T <sub>3</sub>
S <sub>1</sub>	✓		
S <sub>2</sub>		✓	
S <sub>3</sub>		✓	✓
S <sub>4</sub>	✓	✓	

Execution profiles

SBFL

Statement	Suspiciousness score
S <sub>1</sub>	0.00
S <sub>2</sub>	0.71
S <sub>3</sub>	1.00
S <sub>4</sub>	0.50

Hint: program elements that are covered by more failing tests but less passing tests are more suspicious.

# Question on Spectrum-based Fault Localization

**Multiple Choice Question:** Which of the following statements is the most likely to be suspicious based on Spectrum-based Fault Localization?

- a.  $S_1$
- ☒ b.  $S_2$
- c.  $S_3$
- d.  $S_4$

	$T_1$	$T_2$	$T_3$
$S_1$	✓		
$S_2$		✓	✓
$S_3$	✓		
$S_4$	✓	✓	✓
Result	P	F	F

$S_2$ : 2 failing tests  
 $S_4$ : 2 failing tests, 1 passing test

$S_2$  is the most suspicious statement

$$Ochiai(element) = \frac{e_f}{\sqrt{(e_f + n_f) \cdot (e_f + e_p)}}$$



# Question on RSA algorithm



Alice wants to send a message ( $m = 5$ ) to Bob. Assume that the two prime numbers used to generate the keys are  $p = 5$ ,  $q = 11$ , and Alice must choose a value  $e < 6$ .

**Question:** What is the ciphertext? (show your calculations and assumptions)

[2 points]

*Encryption key ( $e, n$ )*

- $m^e \bmod(n) = c$

# Question on RSA algorithm

Alice wants to send a message ( $m = 5$ ) to Bob. Assume that the two prime numbers used to generate the keys are  $p = 5$ ,  $q = 11$ , and Alice must choose a value  $e < 6$ .

**Question:** What is the ciphertext? (show your calculations and assumptions)

**Thought process:** We have the values  $m$ ,  $p$ ,  $q$ , asked to calculate  $c$

- We need to calculate the public key  $(e, n)$  to find  $c$ 
  - Step 1: calculate  $n$
  - Step 2: calculate  $\phi(n)$
  - Step 3: find  $e$  that satisfies the conditions
  - Step 4: calculate  $c$  with  $(e, n)$

# Lecture 17: RSA algorithm

## Part I: Bob's **public and private key setup**

- Chooses two prime numbers,  $p$  and  $q$
- Calculate the product  $n = pq$
- Solve  $\varphi(n) = (p-1)(q-1)$
- Choose numbers  $e$  and  $d$  so that  $ed$  has a remainder of 1 when divided by  $\varphi(n)$ 
  - $1 < e < \varphi(n)$ , where  $e$  must be an integer
  - $e$  and  $\varphi(n)$  must be coprime
- Publish the public key  $(e, n)$

## Example

- $p = 11, q = 3$
- $n = pq = 33$
- $\varphi(n) = 10 \times 2 = 20$
- Pick  $e$  and  $d$  so that  $ed = 20 + 1$

e.g.,:  $e = 3, d = 7$

- $1 < 3 < 20$
- 3 and 20 are coprime
- Publish  $(e, n) = (3, 33)$

# Question on RSA algorithm

**Solution:** We need to calculate the public key  $(e, n)$  to find  $c$

Step 1: calculate  $n$

- If  $p = 5$ ,  $q = 11$ , then  $n = pq = 55$  [0.5 pts]

Step 2: calculate  $\phi(n)$

- $\phi(n) = (p-1)(q-1) = 4 \times 10 = 40$  [0.5 pts]

# Question on RSA algorithm

## Solution (cont.):

Step 3: find  $e$  that satisfies the conditions

- Condition:  $1 < e < \phi(n)$ , and given that  $e < 6$ 
  - $1 < e < 6$
- Condition:  $e$  and  $\phi(n)$  must be coprime
  - $e$  and 40 must be coprime
  - $e$  cannot be a divisor of 40 including 2, 4, 5
- Since  $e$  must be less than 6, it must be 3
- Public key: (3, 55)

# Question on RSA algorithm

## Solution (cont.):

Step 4: calculate  $c$  with  $(e, n)$

Version A: If  $e = 3$ ,  $m = 5$

- $m^e \bmod(n) = c$
- $5^3 \bmod(55) = 15$

[1 pt]

Version B: If  $e = 3$ ,  $m = 7$

- $m^e \bmod(n) = c$
- $7^3 \bmod(55) = 13$

[1 pt]

# Question on RSA algorithm



In previous question, Alice used the RSA algorithm to send the message ( $m = 5$ ) to Bob so that others could not read the message. Suppose that you have been listening to their communication channel:

**Question:** Do you know Alice's public key? If yes, what is it? (1~2 sentences)

[1 point]

# Question on RSA algorithm

In previous question, Alice used the RSA algorithm to send the message ( $m = 5$ ) to Bob so that others could not read the message. Suppose that you have been listening to their communication channel:

**Question:** Do you know Alice's public key? If yes, what is it? (1~2 sentences)

**Thought process:** Alice wants to send a message to Bob

- Alice should use Bob's public key to encrypt the message
  - Analogy: only Bob can open the envelope with his private key



# Lecture 17: Asymmetric encryption

Asymmetric encryption uses a public key to encrypt and a private key to decrypt.

- Public key: anyone can see and use this key
- Private key: kept private
- Private and public keys come in pairs
- Data encrypted with the public key can only be decrypted with the private key

Suppose Alice needs to send a message to Bob

- Alice will use **Bob's public key** to encrypt the message
- Bob will use **his own private key** to decrypt the message

# Question on RSA algorithm

In previous question, Alice used the RSA algorithm to send the message ( $m = 5$ ) to Bob so that others could not read the message. Suppose that you have been listening to their communication channel:

**Question:** Do you know Alice's public key? If yes, what is it? (1~2 sentences)

**Thought process:** Alice wants to send a message to Bob

- Alice should use Bob's public key to encrypt the message
  - Analogy: only Bob can open the envelope with his private key
- Do we need Alice's public key? No

**Solution:** No, the public and private key needed for encryption belong to Bob. Nothing is known about Alice's public key.

# Question on RSA algorithm



In previous question, Alice used the RSA algorithm to send the message ( $m = 5$ ) to Bob so that others could not read the message. Suppose that you have been listening to their communication channel:

**Question:** Do you know Bob's public key? If yes, what is it? (1~2 sentences)

[1 point]

# Question on RSA algorithm

In previous question, Alice used the RSA algorithm to send the message ( $m = 5$ ) to Bob so that others could not read the message. Suppose that you have been listening to their communication channel:

**Question:** Do you know Bob's public key? If yes, what is it? (1~2 sentences)

**Solution:** Yes, Bob's public key:  $(e, n) = (3, 55)$

# Question on RSA algorithm



In previous question, Alice used the RSA algorithm to send the message ( $m = 5$ ) to Bob so that others could not read the message. Suppose that you have been listening to their communication channel:

**Question:** If Bob uses a digital signature, then what is its purpose? (i.e., what does the digital signature do?)

[1 point]

# Question on RSA algorithm

In previous question, Alice used the RSA algorithm to send the message ( $m = 5$ ) to Bob so that others could not read the message. Suppose that you have been listening to their communication channel:

**Question:** If Bob uses a digital signature, then what is its purpose? (i.e., what does the digital signature do?)

**Solution:** The digital signature certifies that the public key belongs to Bob.

# Digital signature

Digital signatures verify the authenticity

- Detect the identity of the sender/signer

Digital signatures check the integrity

- Verify that the message was not changed

Digital signatures ensure non-repudiation

- Verify that the signature is not fake

# Question on cyclic codes



Suppose  $g(x) = 1101$  for a  $(7, 4)$  cyclic code. Bob receives a codeword 0010111 from Alice.

**Question:** Is there an error? If yes, justify why. If not, what is the original data? (show your steps)

[2 points]



# Question on cyclic codes

Suppose  $g(x) = 1101$  for a  $(7, 4)$  cyclic code. Bob receives a codeword 0010111 from Alice.

**Question:** Is there an error? If yes, justify why. If not, what is the original data? (show your steps)

**Thought process:** We have the values  $g(x)$  and  $c(x)$ , asked whether there is a remainder in  $c(x)/g(x)$ . If yes, then there is a error.

- Solve  $d(x) = c(x)/g(x)$ 
  - Step 1: convert  $c(x)$  and  $g(x)$  into polynomials
  - Step 2: calculate polynomial division  $c(x)/g(x)$ , check for remainder

# Question on cyclic codes

Suppose  $g(x) = 1101$  for a  $(7, 4)$  cyclic code. Bob receives a codeword 0010111 from Alice.

**Question:** Is there an error? If yes, justify why. If not, what is the original data? (show your steps)

**Solution:** Solve  $d(x) = c(x)/g(x)$

Step 1: convert  $c(x)$  and  $g(x)$  into polynomials

- $c(x) = x^2 + x^4 + x^5 + x^6$  (0.5 pts)
- $g(x) = 1 + x + x^3$  (0.5 pts)

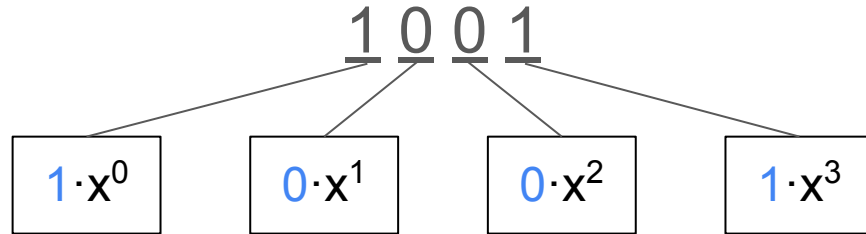
# Lecture 9: Step 1: data polynomial

Data = {1001}

The data can be represented as a polynomial:

$$a(x) = a_0 \cdot x^0 + a_1 \cdot x^1 + \dots + a_{n-1} \cdot x^{n-1}$$

The data can also be visualized as:



So we represent 1001 as:

$$d(x) = 1 \cdot x^0 + 0 \cdot x^1 + 0 \cdot x^2 + 1 \cdot x^3 = 1 + x^3$$

# Question on cyclic codes

## Solution (cont.):

Step 2: calculate polynomial division  $c(x)/g(x)$ , check for remainder

- $(x^2 + x^4 + x^5 + x^6)/(1 + x + x^3) = x^2 + x^3$ , no remainder

$$\begin{array}{r|l} x^6 + x^5 + x^4 + x^2 & x^3 + x + 1 \\ x^6 + x^4 + x^3 & \hline \hline x^5 + x^3 + x^2 & x^3 + x^2 \\ x^5 + x^3 + x^2 & \hline \hline 0 & \end{array}$$

No remainder  
No error

# Lecture 10: Polynomial division (1)

$$\begin{array}{r|l} x^6 + x^5 + x^4 + x^3 + x^2 + x + 1 & x^3 + x + 1 \\ x^6 + x^4 + x^3 & \hline \hline x^5 + x^2 + x + 1 & x^3 + x^2 + 1 \\ x^5 + x^3 + x^2 & \hline \hline x^3 + x + 1 & \\ x^3 + x + 1 & \\ \hline 0 & \end{array}$$

No error

Lecture 10  
Slide 34

# Question on cyclic codes

Suppose  $g(x) = 1101$  for a  $(7, 4)$  cyclic code. Bob receives a codeword 0010111 from Alice.

**Question:** Is there an error? If yes, justify why. **If not, what is the original data?** (show your steps)

**Solution:** The original data is 0011 (or 0011000) (1 pt)

- Partial mark:  $x^3 + x^2$  (0.5 pts)

# Question on extended Hamming code



**Question:** Calculate the final code after encoding the code 11110100100 into 16-bit even parity extended Hamming code. (show your steps, e.g., P1: {0000001}, odd parity, P1 = 1) (Hint: 1 111 010 0100)

[3 points]

# Question on extended Hamming code



**Question:** Calculate the final code after encoding the code 11110100100 into 16-bit even parity extended Hamming code. (show your steps, e.g., P1: {0000001}, odd parity, P1 = 1) (Hint: 1 111 010 0100)

[3 points]

(16, 11) Extended Hamming code



			1
	1	1	1
	0	1	0
0	1	0	0



# Lecture 8: Extended Hamming codes

Extended Hamming code is a linear code that can detect and correct single-bit errors, and also detect double-bit errors.

- Uses an extra parity check for the whole block of bits
- For example, parity check on {1100 111 0010 0100}

1	1	1	0
0	1	1	1
0	0	1	0
0	1	0	0

Extended Hamming code (even parity)

# Question on extended Hamming code

			1
	1	1	1
	0	1	0
0	1	0	0

Parity: ?

Parity bit at  
position 1

			1
	1	1	1
	0	1	0
0	1	0	0

Parity: ?

Parity bit at  
position 2

			1
	1	1	1
	0	1	0
0	1	0	0

Parity: ?

Parity bit at  
position 4

			1
	1	1	1
	0	1	0
0	1	0	0

Parity: ?

Parity bit at  
position 8

# Question on extended Hamming code

	0		1
	1	1	1
	0	1	0
0	1	0	0

Parity: 0

Parity bit at  
position 1

$P_1: \{1110010\}$   
even parity

$P_1 = 0$

(0.6 pts)

	0	0	1
	1	1	1
	0	1	0
0	1	0	0

Parity: 0

Parity bit at  
position 2

$P_2: \{1111000\}$   
even parity

$P_2 = 0$

(0.6 pts)

	0	0	1
0	1	1	1
	0	1	0
0	1	0	0

Parity: 0

Parity bit at  
position 4

$P_3: \{1110100\}$   
even parity

$P_3 = 0$

(0.6 pts)

	0	0	1
0	1	1	1
0	0	1	0
0	1	0	0

Parity: 0

Parity bit at  
position 8

$P_4: \{0100100\}$   
even parity

$P_4 = 0$

(0.6 pts)

# Question on extended Hamming code

	0	0	1
0	1	1	1
0	0	1	0
0	1	0	0

Parity: 0

Parity bit at  
position 0

$P_5$ : {001011100100100}

even parity

$P_5 = 0$

(0.6 pts)

**Question:** Calculate the final code after encoding the code 11110100100 into 16-bit even parity extended Hamming code. (show your steps, e.g.,  $P_1$ : {0000001}, odd parity,  $P_1 = 1$ ) (Hint: 1 111 010 0100)

**Solution:** The final code is:

0001 0111 0010 0100

# Secure File System Project

Project description and marking guide available on eClass

- Week 6: February 12, 2024
- Same groups of 3 people
- Programming language of your choice (e.g., Python, Java, and C++)
- Following the agile methodology

Final report (6-10 pages)

- Expands on the deliverable, based on the finished product

Demonstration (10-15 minutes)

- After the final report submission, scheduled with the TAs.

# Course projects

## Project 2: Secure File System

A secure file system that allows its internal users to store data on an untrusted file server.

### Project deliverable (10%)

- Due Friday, March 15
- More than two weeks from now

### Final report and demo (15%)

- Due Monday, April 8
- Three weeks from the submission of the deliverable

# Project deliverable

Project 2: Secure File System Deliverable (3-5 pages)

- Due Friday, March 15

Design

- Class diagram

Tools and technologies

- What technologies you plan to use? Why?

User stories

- Three user stories (persona + need + purpose)
- Each user story should be broken down into sub-tasks

Planning

- Timeline of the subtasks