Intelligent Systems Engineering

EC8 Ant Colony Optimization

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Outline

1. And Colony Optimization
   - Ant Colony Optimization Meta-Heuristic
   - Simple Ant Colony Optimization (SACO)
   - Ant System (AS)
   - Advanced Topics
2. Summary

These notes are based on [Engelbrech 2007], chapter 17.

Ant colony

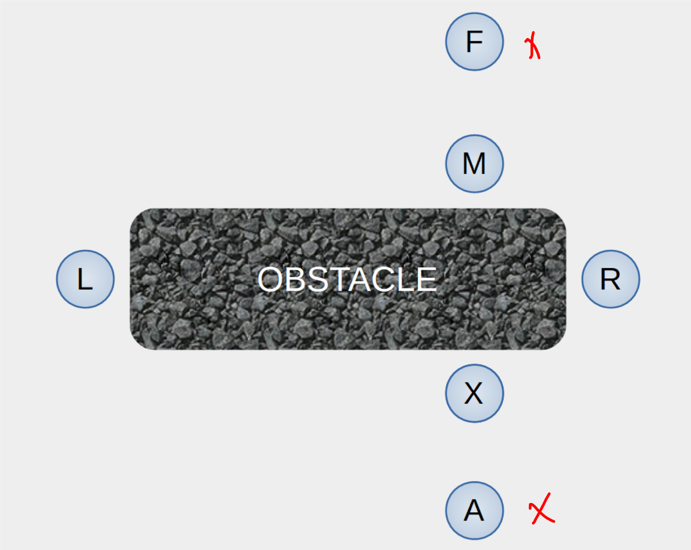
Some Facts

- Ants appeared on earth some 100 million years ago
- The total ant population is estimated at 10^{16} individuals
- The total weight of ants is in the same order of magnitude as the total weight of human beings
- Most of these ants are social insects, living in colonies of 30 to millions of individuals
- Many studies of ant colonies have been done, to better understand their collective behaviors:
  - foraging behavior,
  - division of labour,
  - cemetery organization and brood care, and
  - nest construction
- The first algorithmic models of foraging behavior were implemented in 1992 [Dorigo]

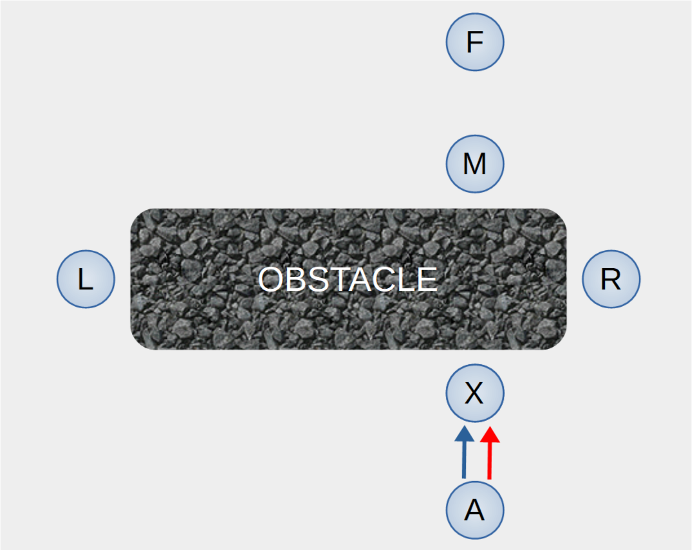
Ant Path Finding

- How do ants find the shortest path between their nest and food source, without any visible, central, active coordination mechanisms?
- Initial studies of foraging behavior showed:
  - Initial random or chaotic activity pattern in the search for food
  - When food source is located, activity patterns become more organized
  - More and more ants follow the same path to the food source
  - Auto-magically, most ants follow the same, shortest path.
- This emergent behavior is the result of a recruitment mechanism, via pheromone trail following

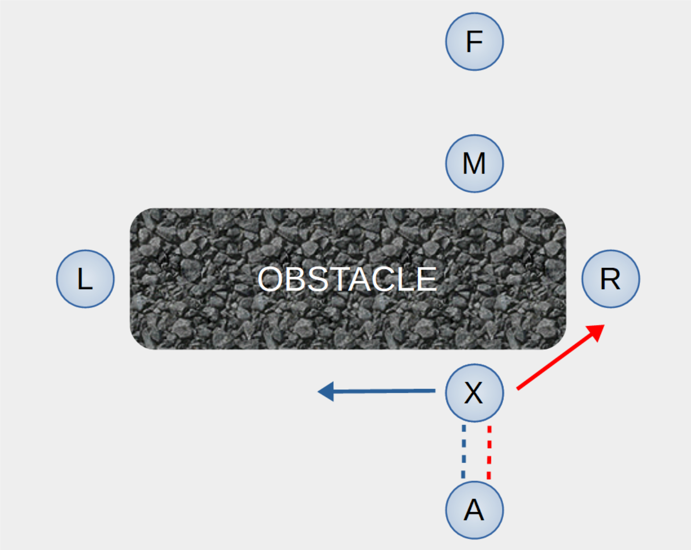
Ant Path Finding



Ant Path Finding



Ant Path Finding



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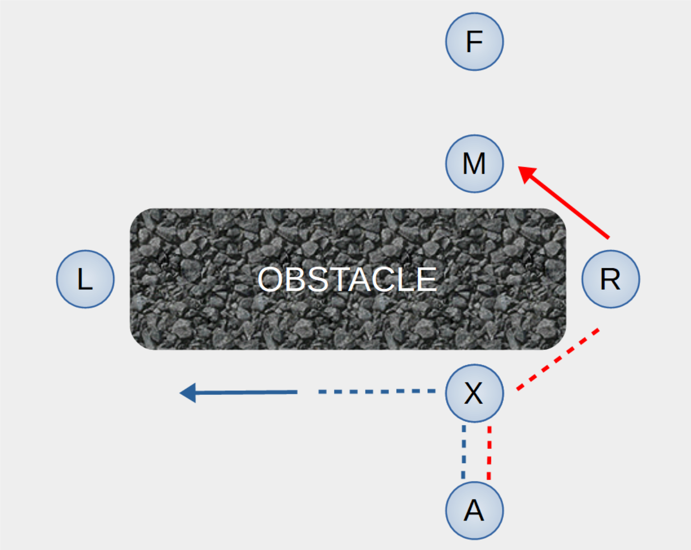

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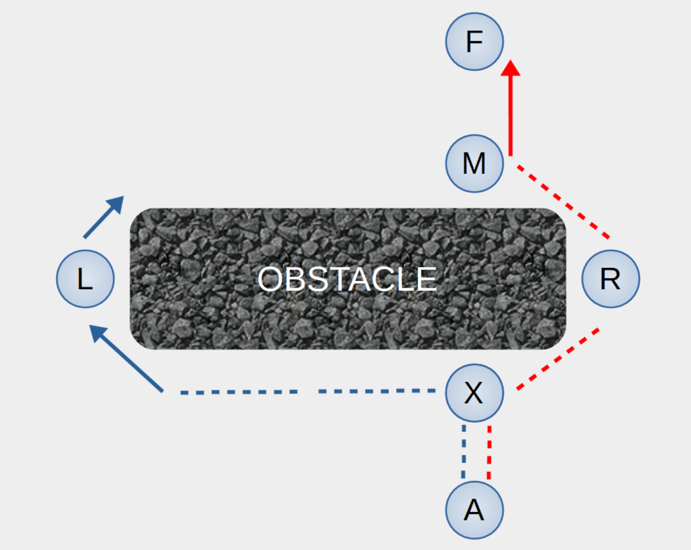
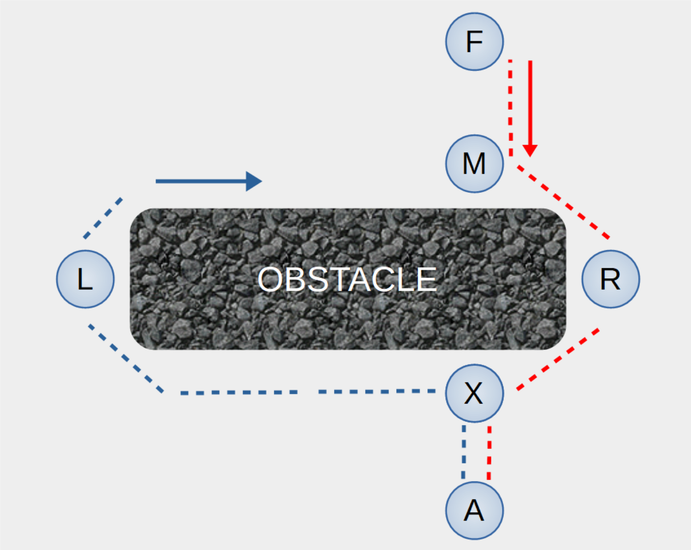
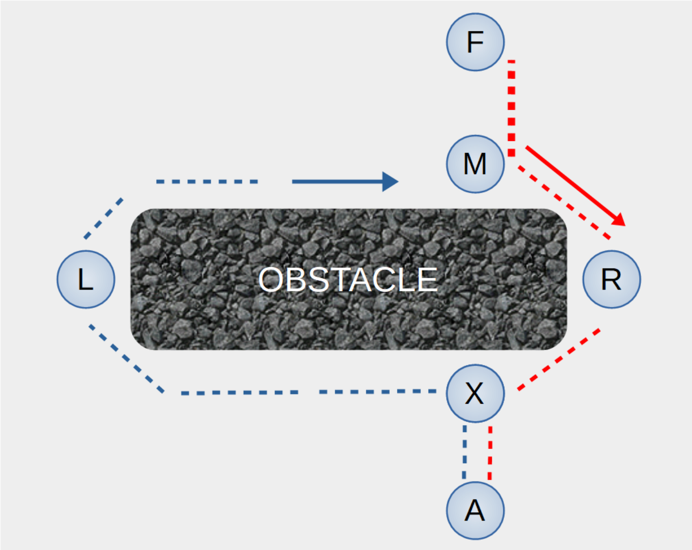


Ant Path Finding



Ant Path Finding



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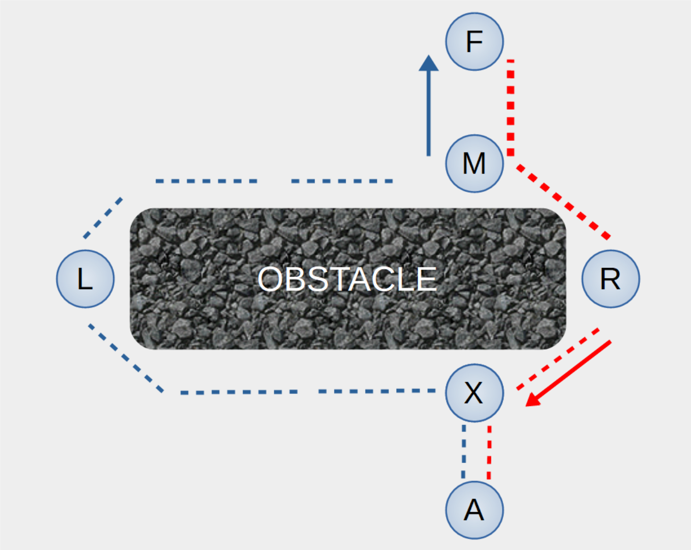

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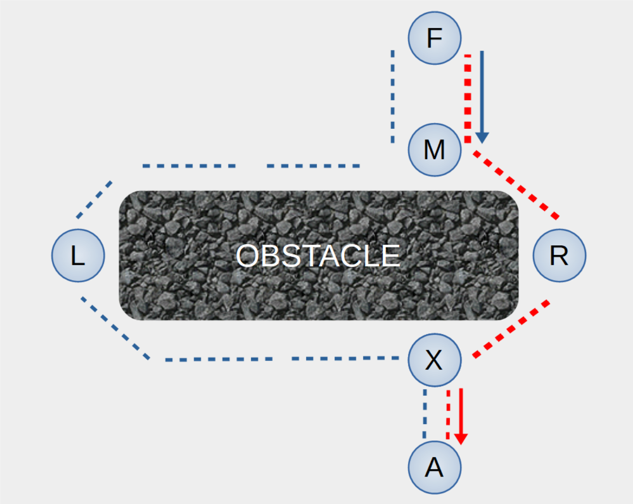

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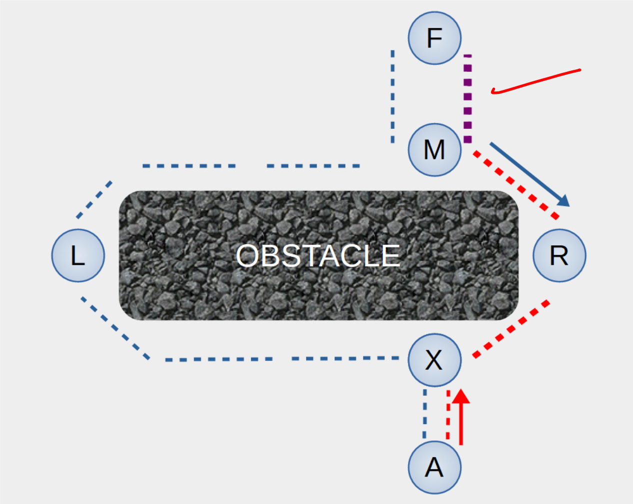

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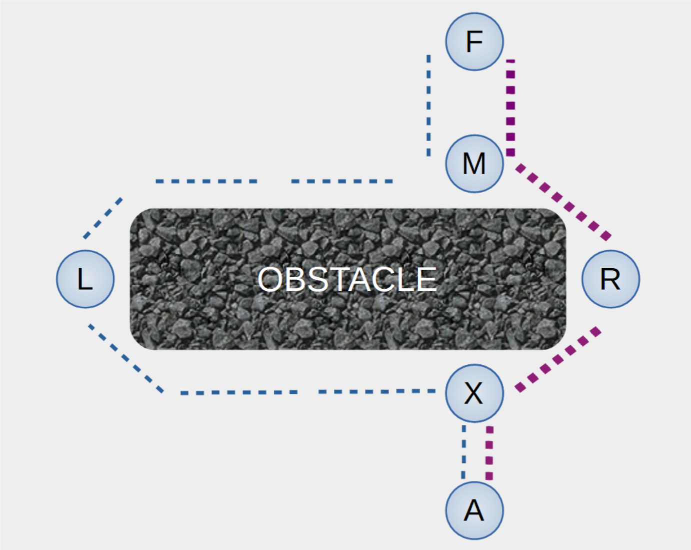

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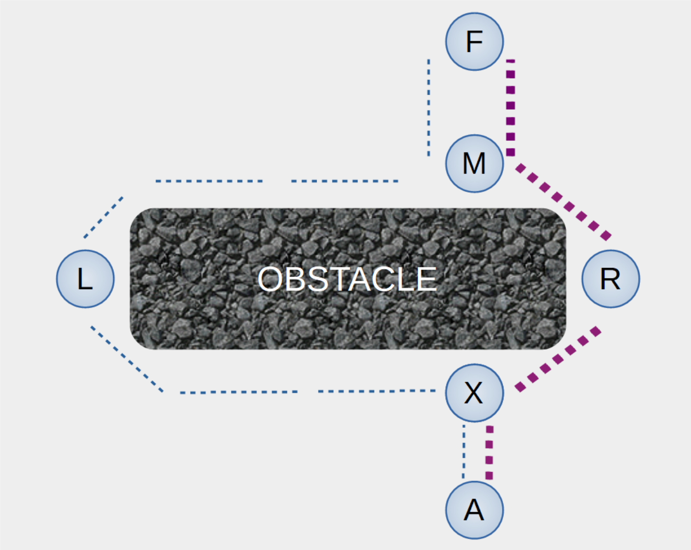
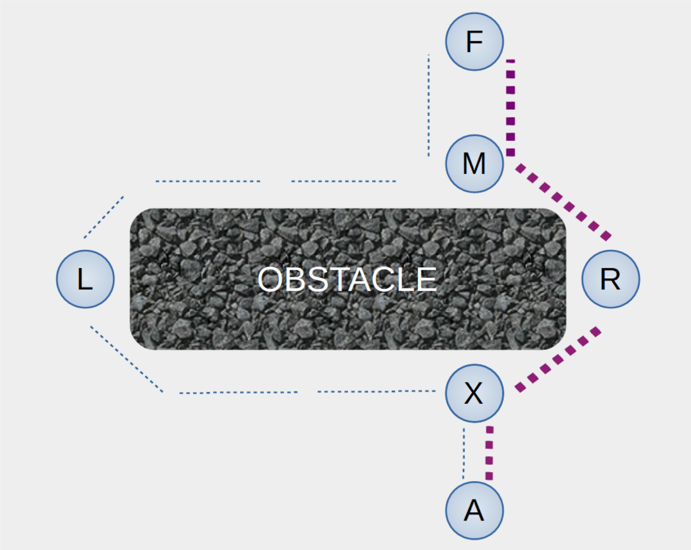
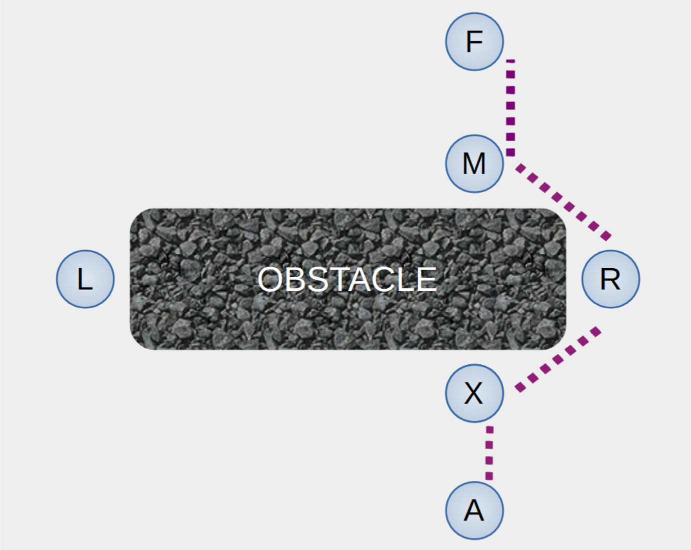


Ant Path Finding

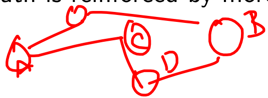


Ant Path Finding



Foraging Behavior of Ants

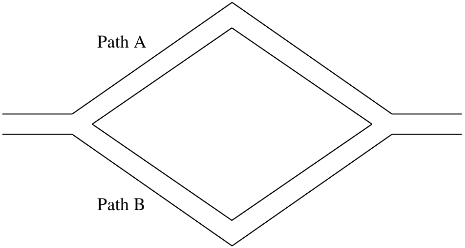
- **Autocatalytic behavior** - positive feedback:
  - Forager ants lay pheromones along followed trails
  - Paths with a larger pheromone concentration have a higher probability of being selected
  - As more ants follow a specific trail, the desirability of that path is reinforced by more pheromone being deposited
  - This attracts more ants to follow that path



- **Stigmergy** - indirect communication where ants modify their environment by laying pheromones to influence the behavior of other ants
- **Artificial stigmergy** - the indirect communication mediated by numeric modifications of environmental states which are only locally accessible by the communicating agents
  - The essence of modeling ant behavior is to find a mathematical model that accurately describes the stigmergic characteristics of the corresponding ant individuals
  - Define stimergic variables which encapsulate the information used by artificial ants to communicate indirectl: (for foraging behavior - artificial pheromone)

Foraging: Bridge experiment

$\Sigma P_A + P_B = 1$



Probability of the next ant to choose path A:

$$P_A(t+1) = \frac{(c+n_A(t))^\alpha}{(c+n_A(t))^\alpha + (c+n_B(t))^\alpha} = 1 - P_B(t+1)$$

- $n_A(t)$ and $n_B(t)$ are the numbers of ants on paths A and B, respectively, at time step t

Foraging: Bridge experiment

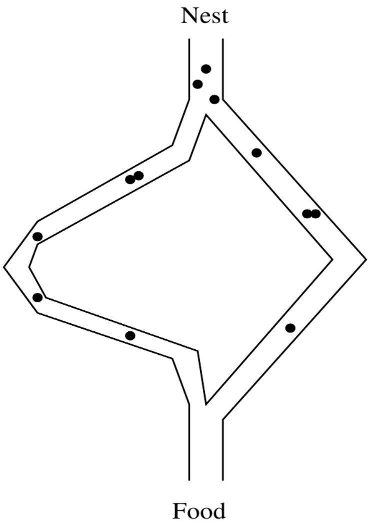
$$P_A(t+1) = \frac{(c + n_A(t))^\alpha}{(c + n_A(t))^\alpha + (c + n_B(t))^\alpha} = 1 - P_B(t+1)$$

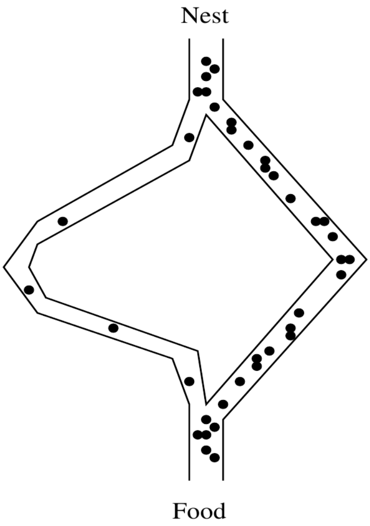
- α biases towards pheromone deposits in the decision process; the larger it is, the higher the probability that the next ant will follow the path with a higher pheromone concentration
- c quantifies the degree of attraction of an unexplored branch; the larger it is, the more pheromone deposits are required to make the choice of path non-random

- After determining the probability, the following decision rule is applied

$$\text{Follow path} \begin{cases} A & \text{if } U(0,1) \leq P_A(t+1) \\ B & \text{otherwise} \end{cases}$$

Foraging: Extended binary bridge





Foraging Behavior of Ants (cont)

Each ant is a stimulus-response agent, following simple production rules :

Let $r \sim U(0, 1)$
for each potential path A **do**
    Calculate P_A;
    if $r \leq P_A$ **then**
        Follow path A;
        Break;
    end if
end for

Pheromone marking in Argentine ants: `http://www.youtube.com/watch?v=tAe3PQdSqzg`

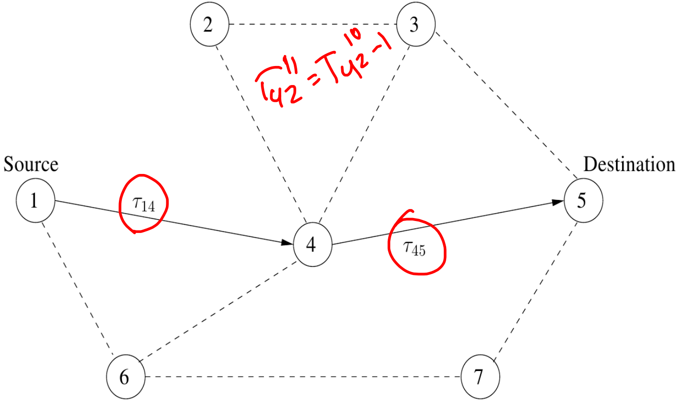
Ant Algorithms

- Ant algorithms are population-based systems inspired by observations of real ant colonies
- Cooperation among individuals in an ant algorithm is achieved by exploiting the stigmergic communication mechanisms observed in real ant colonies
- Foraging-based ant algorithms are generally referred to as

Ant Colony Optimization Meta-Heuristics

Simple Ant Colony Optimization (SACO)

- An algorithmic implementation of the binary bridge experiment
- Consider the general problem of finding a shortest path between two nodes (τ_{ij} is *pheromone intensity*)

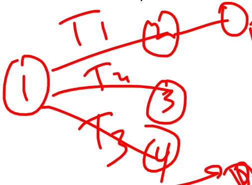


Simple Ant Colony Optimization Algorithm

```
Initialize τ_ij(0) to small random values and let t = 0;
Place n_k ants on the origin node;
while stopping conditions are not true do
    for each ant k = 1, ..., n_k do
        Construct a path x^k(t);
    end for
    for each link (i, j) of the graph do
        Pheromone evaporation;
    end for
    for each ant k = 1, ..., n_k do
        for each link (i, j) of x^k(t) do
            Update τ_ij ;
        end for
    end fort = t + 1;
end while
```

SACO: Path Construction

- For each iteration, each ant incrementally constructs a path (solution)
- At node i, ant k selects the next node $j \in \mathcal{N}_i^k$, based on the transition probability



$P_{12} = \frac{T_1}{T_1+T_2+T_3}$

$$P_{ij}^k(t) = \begin{cases} \dfrac{\tau_{ij}^{\alpha}(t)}{\sum_{j \in \mathcal{N}_i^k} \tau_{ij}^{\alpha}} & \text{if } j \in \mathcal{N}_i^k \\ 0 & \text{if } j \notin \mathcal{N}_i^k \end{cases}$$

Direct conn.

where $\mathcal{N}_i^k$ is the set of feasible nodes connected to node i, with respect to ant k

- If $\mathcal{N}_i^k = \emptyset$, the predecessor to node i is included in $\mathcal{N}_i^k$
- Loops are removed once the destination node has been reached

Pheromones in SACO

Pheromone evaporation to improve exploration and prevent premature convergence

$\rho = 1$ Random Blind Search.

$$\tau_{ij}(t) \leftarrow (1-\rho)\tau_{ij}(t) \text{ with } \rho \in [0,1]$$

- ρ specifies the rate at which pheromones evaporate, causing ants to "forget" previous decisions; it controls the influence of search history
  - large values of ρ: pheromone evaporates rapidly (more exploration, more random search),
  - small values of ρ: slower evaporation rates (more exploitation)

Pheromone update

$$\tau_{ij}(t+1) = \tau_{ij}(t) + \sum_{k=1}^{n_k} \Delta\tau_{ij}^k(t) \text{ where } \Delta\tau_{ij}^k(t) = \frac{1}{L^k(t)}$$

$L^k(t)$ is the length of the path constructed by ant k at time step t, n_k is the number of ants

SACO: Important Notes

- Solution construction is the result of cooperative behavior that emerges from the simple behaviors of individual ants
- Each ant chooses the next link of its path based on information provided by other ants, in the form of pheromone deposits, referring to the autocatalytic behavior exhibited by forager ants
- The information used to aid in the decision making process is limited to the local environment of the ant

Ant System (AS)

- First ant colony optimization (ACO) algorithm developed by Dorigo
- Transition probability:

$\alpha = \beta = 1$

$$p_{ij}^k(t) = \begin{cases} \dfrac{\tau_{ij}^{\alpha}(t)\eta_{ij}^{\beta}(t)}{\sum_{u \in \mathcal{N}_i^k(t)} \tau_{iu}^{\alpha}(t)\eta_{iu}^{\beta}(t)} & \text{if } j \in \mathcal{N}_i^k \\ 0 & \text{if } j \notin \mathcal{N}_i^k(t) \end{cases}$$

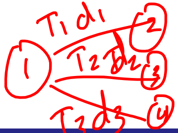
1/d.

Direct connect.

where

τ_{ij} represents the *a posteriori* effectiveness of the move from node i to node j (pheromone concentration on the link)

η_{ij} represents the *a priori* effectiveness of the move from i to j, i.e. the desirability of the move (desirability of the link)



$\eta = \frac{1}{d}$

$$P_{12} = \frac{T_1/d_1}{\frac{T_1}{d_1} + \frac{T_2}{d_2} + \frac{T_3}{d_3}}$$

Ant System: Exploration-Explotation Trade-off

- A balance between pheromone intensity, τ_{ij} , and heuristic information, η_{ij}
- If $\alpha = 0$:
  - no pheromone information is used, i.e. previous search experience is neglected
  - the search then degrades to a stochastic greedy search
- If $\beta = 0$:
  - the attractiveness of moves is neglected
  - the search algorithm is similar to SACO
- Heuristic information adds an explicit bias towards the most attractive solutions, e.g.

$$\eta_{ij} = \frac{1}{d_{ij}}$$

- Pheromone updates are based on path length (global), ant count on each link (semi-global) or a priori link preference (local)

Advanced Topics

Many different approaches to improve the performance of ACO heursitics

- Ant Colony System (yet another modification),
  - a different transition rule
  - a different pheromone update rule is defined,
  - local pheromone updates are introduced, and
  - candidate lists are used to favor specific nodes
- Max-min AS (pheromone clamping)
- FANT (Fast Ant System - simplified, reduced computational complexity)
- hybrid approaches
  - Ant-Q (combination with Q-learning)
  - Antabu (combination with tabu search),
- other extensions (constrained and multi-objective optimization, dynamic environments, continuous problems)

Summary

Ant colonies as inspiration for solving optimization problems

- Long history of ants on earth and scientists studying them

Ant colony optimization as inspiration for solving optimization problems

- Inspired by path-finding behavior of ants
- Probability of choosing a path depends on its recent use by others
- The us of a path marked using stigmergy
- Simple Ant Colony Optimization (SACO) and a number of variations
    - Ant system, Ant colony system, Max-min ant system, Ant-Q, ...

Applying ant colony metaheuristic

- Problems that involve *paths through graphs* and *permutations* (TSP)
- can be applied to dynamic problems

Groups of simple agents can achieve great success → **Swarm Intelligence**

- PSO: Copy flocking behavior
- ACO: Copy ground-based movements / stigmergy
- Other algorithms: bee colony, fireflies , bats, wolf packs, ...