Intelligent Systems Engineering

EC7 Particle Swarm Optimization

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Outline

1. Swarm Intelligence

2. Particle Swarm Optimization
   - Introduction
   - Global Best PSO
   - Local Best PSO
   - Velocity Components

These notes are based on [Engelbrecht 2007], chapter 15.

EC

- population Based
- Random stochastic
- Evolu iteration

Treasure Hunt

- You are with a **group** of friends
- Your goal is to find a treasure (or at least a part of it)
- You know the approximate area of the treasure (but not exactly where it is located)
- You have a **metal detector**
- But you also have **your friends** (can share position/strength of signal of their detectors)
- Possible agreed upon **sharing mechanism**
  - all who have taken part in the search will be rewarded, but with the person who found the treasure getting a higher reward than all others
  - the rest are rewarded based on distance from the treasure at the time when the first one finds the treasure
- **What will you do?**
  - Ignore your friends, or
  - Use the information from your neighboring friends?

What is a Swarm?

- A loosely structured collection of interacting agents
- Agents:
  - Individuals that belong to a group (but are not necessarily identical)
  - They contribute to and benefit from the group
  - They can recognize, communicate, and/or interact with each other
- A swarm is better understood if thought of as agents exhibiting a collective behavior

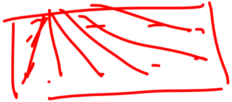
School of fish

Swarm of bees

Flock of birds

Swarm Intelligence (SI)

- Computational intelligence (CI) technique based on the collective behavior in decentralized, self-organized systems
- Generally made up of agents who interact with each other and the environment
- No centralized control structures
- No need to have very complex and superior intelligent agents
- Based on group behavior found in nature



Swarm of jelly fish

Swarm of crabs

Swarm of butterflies

Swarms and Swarm Intelligence

Swarm

A group of (generally mobile) agents that communicate with each other (directly or indirectly), by acting on their local environment.

- The interactions between agents result in distributed, collective problem-solving strategies
- Swarm intelligence (SI) refers to the problem-solving behavior that emerges from the interaction of such agents
- More formally, swarm intelligence is the property of a system whereby the collective behaviors of unsophisticated agents interacting locally with their environment cause coherent functional global patterns to emerge
- Computational swarm intelligence (CSI) refers to algorithmic models of such behavior

Emergent Bahavior

- Within swarms, individuals are relatively simple in structure, but their collective behavior is usually very complex
- The complex behavior of a swarm is a result of the pattern of interactions between the individuals of the swarm over time
- This complex behavior is not a property of any single individual, and is usually not easily predicted or deduced from the simple behaviors of the individuals
- The complex behavior is referred to as emergent behavior

Examples of Emergent Behavior

- Termites build large nest structures with a complexity far beyond the comprehension and ability of a single termite.
- Tasks are dynamically allocated within an ant colony, without any central manager or task coordinator.

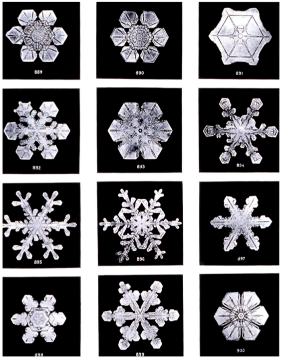
Examples of Emergent Behavior (cont)

- Recruitment via waggle dances in bee species, which results in optimal foraging behavior.
- Foraging behavior in ant colonies as a result of simple trail-following behaviors.
- Birds in a flock and fish in a school self-organize in optimal spatial patterns.
- Predators, for example a group of lionesses, exhibit hunting strategies to outsmart their prey.
- Bacteria communicate using molecules (comparable to pheromones) to collectively keep track of changes in their environment.
- Slime moulds consist of very simple cellular organisms with limited abilities. However, in times of food shortage they aggregate to form a mobile slug with the ability to transport the assembled individuals to new feeding areas.

Examples of Emergent Behavior (cont)

The whole is more than the sum of its parts.

Fractal



single atomes arranged due to the laws of physics form a geometric structure which is not related to the features of the single atomes in any obvious way

single termites aggregate pieces of clay, forming a giant nest whose structure is not obviously related to the behavioral patterns of a single termite

Particle swarm optimization (PSO)

What is PSO?

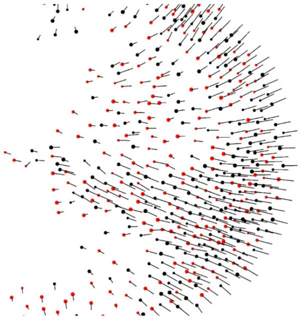
- a simple, computationally efficient optimization method
- population-based, stochastic search
- based on a social-psychological model of social influence and social learning
- individuals follow a very simple behavior: emulate the success of neighboring individuals
- emergent behavior: discovery of optimal regions in high dimensional search spaces

PSO algorithm

- developed by Kennedy & Eberhart,
- first published in 1995,
- exponential increase in the number of publications since then.

Introduction: What are the origins of PSO?

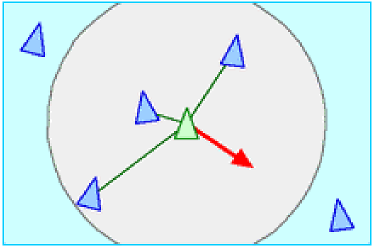
- In the work of Reynolds on "boids": Flocking is an emergent behavior which arises from the interaction of simple rules:
  - Collision avoidance
  - Velocity matching
  - Flock centering



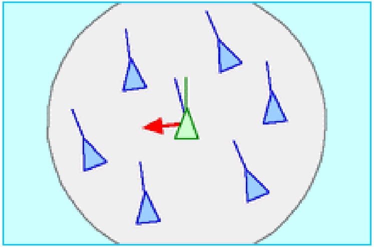
- The work of Heppner and Grenander on using a "roost" as an attractor for a bird flock
- Simplified social model of determining nearest neighbors and velocity matching

Origins of PSO (cont)

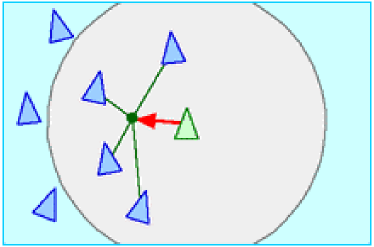
Boids: flocks, herds, and schools



Separation: steer to avoid crowding local flockmates



Alignment: steer towards the average heading of local flockmates



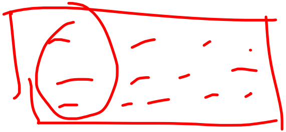
Cohesion: steer to move toward the average position of local flockmates

(http://www.red3d.com/cwr/boids/ ©Craig Reynolds)

Origins of PSO (cont)

- Initial objective: to simulate the graceful, unpredictable choreography of collision-proof birds in a flock
- At each iteration, each individual determines its nearest neighbor and replaces its velocity with that of its neighbor
- Resulted in synchronous movement of the flock
- Random adjustments to velocities prevented individuals to settle too quickly on an unchanging direction
- Adding roosts as attractors:
  - personal best
  - neighborhood best

  → particle swarm optimization



Overview of basic PSO

What are the main components?

- A swarm of particles — possible solutions pop-
- Each particle represents a candidate solution
- Elements of a particle represent parameters to be optimized

The search process:

- Position updates

$$x_i(t+1) = x_i(t) + v_i(t+1)$$

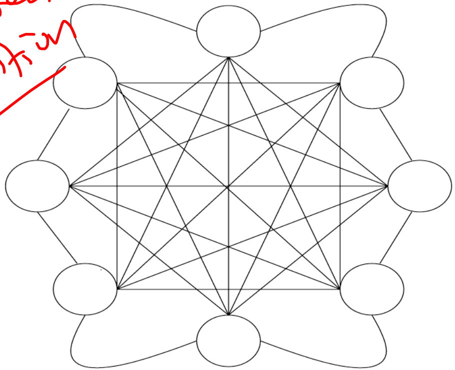
  where

$$x_{ij}(0) \sim U(x_j^{\min}, x_j^{\max})$$

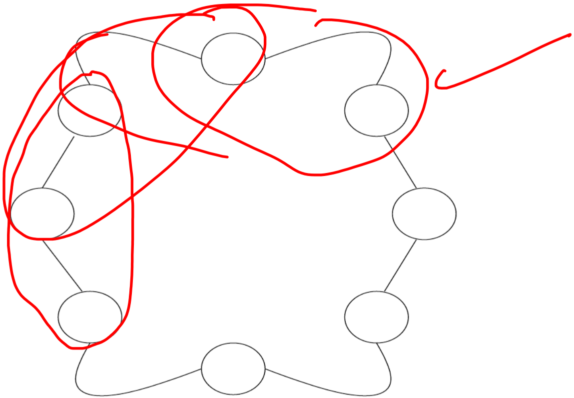
- Velocity
  - drives the optimization process
  - step size
  - reflects experiential knowledge and socially exchanged information

Social interaction based on neighborhoods - topologies

global best position



Star



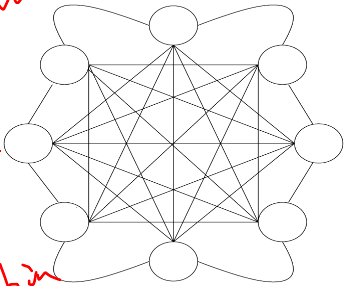
Ring

(A. P. Engelbrecht, Computational Intelligence, ©2007 Wiley)

Global Best (gbest) PSO

Uses the **star** social network

Pairwise connection



$x(t+1) = x(t) + v(t+1)$

Velocity update per dimension

best position so far

current position

$$v_{ij}(t+1) = v_{ij}(t) + c_1 r_{1j}(t)[y_{ij}(t) - x_{ij}(t)] + c_2 r_{2j}(t)[\hat{y}_j(t) - x_{ij}(t)]$$

experience

social

group global best position.

- $v_{ij}(0) = 0$ (usually, but can be random)
- c_1, c_2 are positive acceleration coefficients
- $r_{1j}(t), r_{2j}(t) \sim U(0,1)$

Global Best PSO (cont)

$\boldsymbol{y}_i$ (t) is the **personal best** position (for minimization):

$$\boldsymbol{y}_i(t+1) = \begin{cases} \boldsymbol{y}_i(t) & \text{if } f\big(\boldsymbol{x}_i(t+1)\big) \geq f\big(\boldsymbol{y}_i(t)\big) \\ \boldsymbol{x}_i(t+1) & \text{if } f\big(\boldsymbol{x}_i(t+1)\big) < f\big(\boldsymbol{y}_i(t)\big) \end{cases}$$

$\hat{\boldsymbol{y}}(t)$ is the **global best** position calculated as

$$\hat{\boldsymbol{y}}(t) \in \{\boldsymbol{y}_0(t), ..., \boldsymbol{y}_{n_s}(t)\} | f(\hat{\boldsymbol{y}}(t)) = \min f\big(\boldsymbol{y}_0(t)\big), ..., f\big(\boldsymbol{y}_{n_s}(t)\big)$$

or

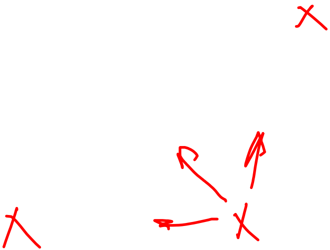
$$\hat{\boldsymbol{y}}(t) \in \{\boldsymbol{x}_0(t), ..., \boldsymbol{x}_{n_s}(t)\} | f(\hat{\boldsymbol{y}}(t)) = \min f\big(\boldsymbol{x}_0(t)\big), ..., f\big(\boldsymbol{x}_{n_s}(t)\big)$$

where n_s is the number of particles in the swarm.

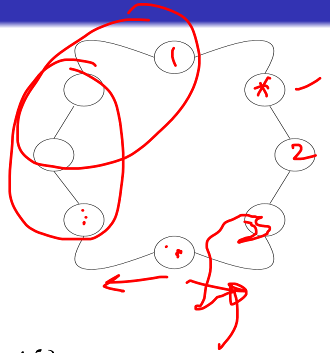
gbest PSO Algorithm

$f(x)$

```
Create and initialize an n_x-dimensional swarm S;
while stopping condition is not true do
    for each particle i = ..., n_s do
        if f(x_i) < f(y_i) then
            y_i = x_i;                self Best
        end if
        if f(y_i) < f(ŷ) then
            ŷ = y_i;                  global Best.
        end if
    end for
    for each particle i = 1, ..., n_s do
        update the velocity and then the position;
    end for
end while
```



Local Best (lbest) PSO



Uses the **ring** social network

Velocity update per dimension (same as gbest)

Self best

$$v_{ij}(t+1) = v_{ij}(t) + c_1 r_{1j}(t)[y_{ij}(t) - x_{ij}(t)] + c_2 r_{2j}(t)[\hat{y}_{ij}(t) - x_{ij}(t)]$$

To change Route (N)

but $\hat{\boldsymbol{y}}_i$ is now the **neighborhood best**, defined as

$$\hat{\boldsymbol{y}}_i(t+1) \in \{\mathcal{N}_i | f(\hat{\boldsymbol{y}}_i(t+1)) = \min\{f(\boldsymbol{x})\}, \forall \boldsymbol{x} \in \mathcal{N}_i\}$$

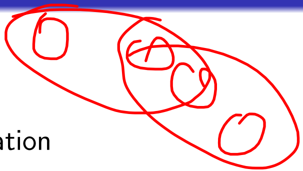
with the neighborhood defined as

Direct Connection.

$$\mathcal{N}_i = \{\boldsymbol{y}_{i-n_{\mathcal{N}_i}}(t), \boldsymbol{y}_{i-n_{\mathcal{N}_i}+1}(t), \ldots, \boldsymbol{y}_{i-1}(t), \boldsymbol{y}_i(t), \boldsymbol{y}_{i+1}(t), \ldots, \boldsymbol{y}_{i+n_{\mathcal{N}_i}}(t)\}$$

where $n_{\mathcal{N}_i}$ is the neighborhood size

Local Best PSO (cont...)



Neighborhoods:

- Neighborhoods are based on particle indices, not spatial information
- Spatial approach is possible, but computationally expensive and not that effective in spreading information (indices-based neighborhoods allow leaps in search space)
- Neighborhoods overlap to facilitate information exchange

`gbest` PSO vs `lbest` PSO:

- Speed of convergence: due to greater inter-connectivity of `gbest` PSO, it converges faster than the `lbest` PSO (but has less diversity)
- Susceptibility to local minima: larger diversity of `lbest` PSO (i.e. it covers larger parts of the search space) makes it less susceptible to being trapped

* lbest faster less complex.

Velocity Components

v(t+1) =

Previous velocity, $\boldsymbol{v}_i(t)$

- inertia component
- memory of previous flight direction
- prevents particle from drastically changing direction

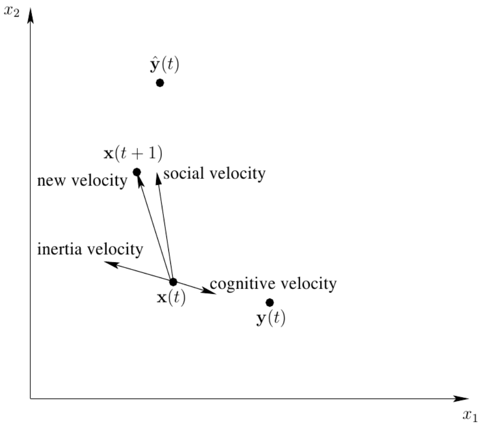
Cognitive component, $c_1\boldsymbol{r}_1(\boldsymbol{y}_i - \boldsymbol{x}_i)$

- quantifies performance relative to past performances
- memory of previous best position
- a.k.a. nostalgia

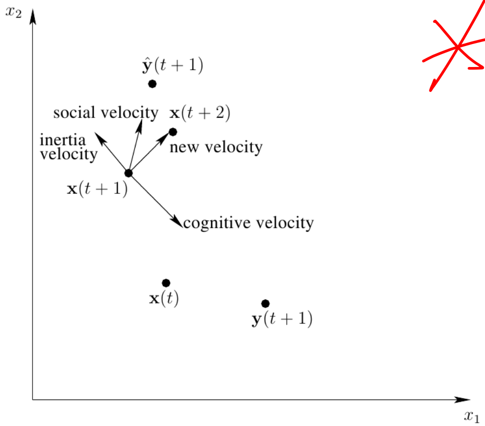
Social component, $c_2\boldsymbol{r}_2(\hat{\boldsymbol{y}}_i - \boldsymbol{x}_i)$

- quantifies performance relative to neighbors
- a.k.a. envy

Geometric Illustration for a Single Two-Dimensional Particle



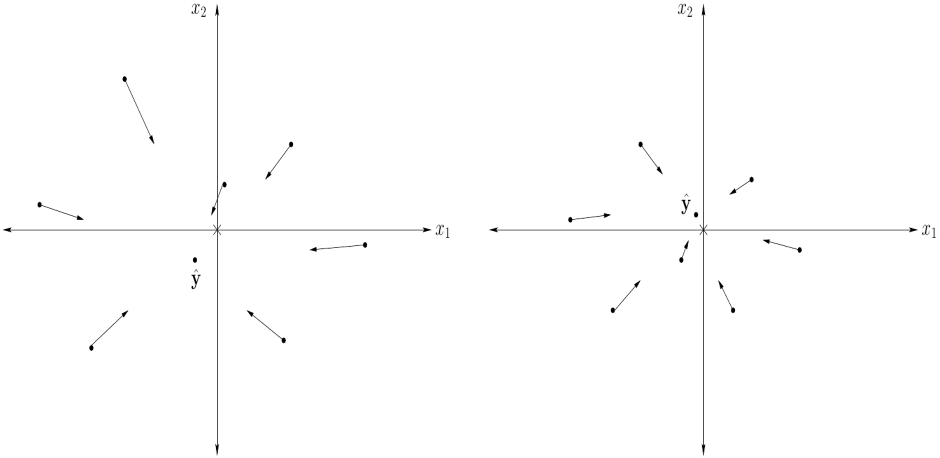
Time step t



Time step $t+1$

(A. P. Engelbrecht, Computational Intelligence, ©2007 Wiley)

Cumulative Effect of Position Updates (gbest PSO)



Time step t Time step $t + 1$

(A. P. Engelbrecht, Computational Intelligence, ©2007 Wiley)

Stopping Conditions

- Terminate when a maximum number of iterations, or function evaluations (FEs), has been exceeded
- Terminate when an acceptable solution has been found, i.e. when (assuming minimization)
$$f(\boldsymbol{x}_i) \leq |f(\boldsymbol{x}^*) - \epsilon|$$
- Terminate when no improvement is observed over a number of iterations
- Terminate when the normalized swarm radius is close to zero, i.e.
$$R_{\text{norm}} = \frac{R_{\text{max}}}{\text{diameter}\big(S(0)\big)}$$
where $R_{\text{max}} = ||\boldsymbol{x}_m - \hat{\boldsymbol{y}}||, m = 1, \ldots, n_s$ with $||\boldsymbol{x}_m - \hat{\boldsymbol{y}}|| \geq ||\boldsymbol{x}_i - \hat{\boldsymbol{y}}||, \forall i = 1, \ldots, n_s$.
- Terminate when the objective function slope is approximately zero, i.e.
$$f^{'}(t) = \frac{f(\hat{\boldsymbol{y}}(t)) - f(\hat{\boldsymbol{y}}(t-1))}{f(\hat{\boldsymbol{y}}(t))} \approx 0$$

Summary

Swarm Intelligence

- Swarm is a loosely structured collection of interacting agents exhibiting emergent collective behavior

Particle Swarm Optimization is a simple, population-based, computationally efficient optimization method

- Originated in work on group movement (boids)
- Particles represent candidate solutions
- Search process updates their position and velocity
  - `gbest` - all particles interact
  - `lbest` - only particles in predefined neighborhood are considered
- Stopping conditions must be defined
- There are PSO variations to improve convergence and quality of solutions