STAT235 FINAL EXAM VERSION 111 SOLUTION

Complete the following (please **print**):

Section Number: ___________ Last name, First name: ______________________________

Instructions

1. Enter your lecture section number, your last name (the same one that is illustrated on your One Card), and first name (the same one that is illustrated on your One Card) into the space provided above. Enter your student ID # in the upper right hand corner on all the *other* pages of the exam.

2. Make sure you use ONLY PENCIL to put and mark your name, student ID #, and your exam version number on the Scantron sheet. Your name (last name and first name) should be entered into the NAME block, your student ID # into the IDENTIFICATION NUMBER block, and finally, the three-digit exam version into the SPECIAL CODES block. The exam version is specified at the top of the exam. Make sure to shade in the circle that corresponds to the letter, digit, or empty space in the box at the top of each column.

3. This is a multiple-choice closed book exam. There are **50** questions in the exam. For each question, carry out the appropriate analysis and put your answer on the Scantron sheet by shading the letter A, B, C, D, or E that corresponds to your chosen answer. Make sure your answers are clearly marked with ONLY PENCIL. Otherwise, no marks will be given. For each question exactly one of the five answers is correct. If you fill in more than one answer to a question, the question will be scored incorrect. Each question is worth 1 mark.

4. Please note that only the answers in the Scantron sheet will be considered. If you initially mark your answers in the exam sheet, make sure that you copy them correctly into the Scantron sheet. If you change an answer on the Scantron sheet, be sure that you erase your first mark completely and then blacken the circle of the answer choice you prefer. Your score will be based on the number of questions you answer correctly. No credit will be given for omitted answers and no credit will be lost for wrong answers. You must mark all your answers on the Scantron sheet during the allotted time. No additional time will be allowed at the end of the session for this purpose.

5. You are permitted to use a **non-programmable** calculator approved by the Faculty of Engineering. During the lab exam you are forbidden to use any devices with communication capabilities including cell phones and pagers. You are also forbidden to use any photographically capable devices in the exam room. Copying questions or answers on paper to take from the exam room is prohibited.

6. Note that the formula sheet and the table of the cumulative distribution function of the standard normal distribution are attached to the exam. There are **11** pages in the exam. The exam is graded out of **50** points.

7. You must return your Scantron and exam booklet when you finish the exam. You have 180 minutes to complete the exam.

8. Sign the exam booklet in the space provided below.

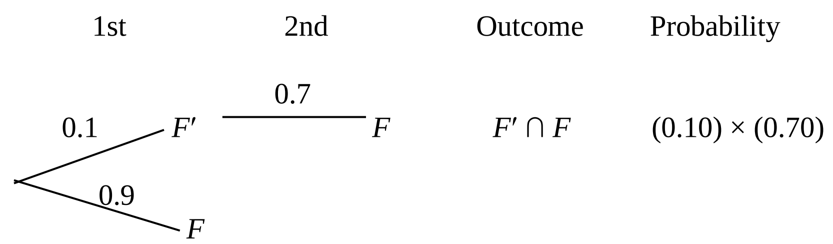
SIGNATURE: ________________________________

MULTIPLE-CHOICE QUESTIONS, 1 POINT EACH

1. Items are inspected for flaws by two quality inspectors. If a flaw is present, it will be detected by the first inspector with probability 0.90 and by the second inspector with probability 0.70. Assume the second inspector examines only those items that have been passed by the first inspector. If an item has a flaw, what is the probability that the second inspector will find it?

(a) 0.03 (b) 0.07 (c) 0.13 (d) 0.17 (e) 0.63

Solution: Let F denote the event that an inspected item has a flaw. The following tree diagram can be useful:



2. The reading given by a thermometer calibrated in ice water (actual temperature: $0^o C$) is a random variable with the following cumulative distribution function:

$$F(x) = \begin{cases} 0 & x < -1 \\ \frac{3}{4}x - \frac{1}{4}x^3 + \frac{1}{2} & -1 \le x < 1 \\ 1 & 1 \le x \end{cases}$$

What is the probability that the reading is within $0.25^o C$ of the actual temperature?

(a) 0.181 (b) 0.222 (c) 0.254 (d) 0.289 (e) 0.367

Solution: Let X be the reading given by the thermometer. Since X is continuous,

$P(-0.25 \le X \le 0.25) = F(0.25) - F(-0.25) = 0.367$

3. Of the items manufactured by a certain process, 20% are defective. Of the defective items, 60% can be repaired. What is the probability that exactly 2 of 20 randomly selected items are defective and cannot be repaired?

(a) 0.080 (b) 0.137 (c) 0.200 (d) 0.271 (e) 0.400

Solution: Let D denote a defective item and R an item that can be repaired. Then, $P(D) = 0.2$ and $P(R \mid D) = 0.6$, so $P(R' \mid D) = 0.4$. Thus, $P(R' \cap D) = P(D) \times P(R'|D) = (0.2) \times (0.4) = 0.08$.

Let X be the number of defective items among the 20 items. Then X follows a binomial distribution with $n = 20$ and $p = 0.08$ and

$$P(X = 2) = \binom{20}{2}(0.08)^2(1-0.08)^{20-2} = 190(0.08)^2(0.92)^{18} \approx 0.271$$

4. The number of large cracks in a length of pavement along a certain street has a Poisson distribution with a mean of 2 cracks per 100 m. What is the probability that the distance between two successive cracks will be more than 50 m?

(a) e^{-1} (b) $1-e^{-1}$ (c) e^{-2} (d) $1-e^{-2}$ (e) $e^{-1.5}$

Solution: Let X be the distance between two successive cracks. Then, X follows an exponential distribution with $\lambda = 1/50$ cracks per meter. Thus,

$$P(X > 50) = 1 - F(50) = 1 - (1 - e^{-(1/50)(50)}) = e^{-1}$$

5. A process that fills packages is stopped whenever four packages have weights falling outside specifications. Assume that each package has probability 0.01 of falling outside specifications and that weights of the packages are independent. What is (approximately) the standard deviation of the number of packages that will be filled before the process is stopped?

(a) 1.99 (b) 19.9 (c) 100 (d) 199 (e) 396

Solution: Let X be the number of packages that will be filled before the process is stopped. Then X follows a negative binomial distribution with $r = 4$ and $p = 0.01$. Thus,

$$Var(X) = \frac{r(1-p)}{p^2} = \frac{4(1-0.01)}{0.01^2} = 39600,\ \sigma = \sqrt{Var(X)} = \sqrt{39600} = 199$$

6. Two independent observations are selected randomly from a standard normal distribution. The probability that their total is smaller than 2 is

(a) 0.6115 (b) 0.6923 (c) 0.7645 (d) 0.8413 (e) 0.9207

Solution: Let $Y = Z_1 + Z_2$, where $Z_i \sim N(0, 1)$. Thus, $E(Y) = 0 + 0 = 0$; $Var(Y) = 1 + 1 = 2$.

$$P(Y < 2) = P\left(\frac{Y-\mu}{\sigma} < \frac{2-0}{\sqrt{2}}\right) = P(Z < 1.41) = 0.9207$$

7. From a random sample of 100 components, a 95% confidence interval for the proportion of defectives is computed. The confidence interval specifies that the margin of error is 0.6. If the sample size increases to 400, what will be the approximate margin of error of the new confidence interval?

(a) 0.015 (b) 0.30 (c) 0.06 (d) 0.12 (e) 0.24

Solution: The margin of error of a 95% confidence interval for the proportion is

$$E = z_{0.025}\sqrt{\frac{\hat{p}(1-\hat{p})}{100}} = 0.6 \rightarrow z_{0.025}\sqrt{\frac{\hat{p}(1-\hat{p})}{4\cdot 100}} = \frac{0.6}{\sqrt{4}} = 0.30.$$

8. In order to estimate the mean of a population with a standard deviation of 1, a simple random sample of size $n = 9$ was taken and the following confidence interval was obtained 2 ± 0.7753. The confidence level of the confidence interval is

(a) 90% (b) 95% (c) 98% (d) 99% (e) 99.7%

Solution: Since $\sigma = 1$ is given, use z. In this case, $E = z_{\alpha/2}\frac{\sigma}{\sqrt{n}} = z_{\alpha/2}\frac{1}{\sqrt{9}} = 0.7753$,

so $z_{\alpha/2} = 3\cdot 0.7753 = 2.326$, which corresponds to $\alpha/2 = 0.01$ in the z-table.
Thus, $\alpha = 0.02$ and $1 - \alpha = 0.98$.

Use the following information for question 9:

A new catalyst is being investigated for use in the production of a plastic chemical. Ten batches of the chemical are produced. The mean yield of the 10 batches is 72.5% and the standard deviation is 5.8%. Assume the yields are independent and approximately normally distributed.

9. Which among the following is the 99% confidence interval for the mean yield when a new catalyst is used?

(a) (67.775, 77.225)
(b) (69.138, 75.862)
(c) (57.559, 87.441)
(d) (66.539, 78.461)
(e) (63.731, 81.269)

Solution: Since s is given, use t. Here $n = 10$, $\bar{x} = 72.5$, and $s = 5.8$. Thus,

$$\bar{x} \pm t_{\alpha/2,n-1}\times\left(\frac{s}{\sqrt{n}}\right) = 72.5 \pm t_{0.005,9}\times\left(\frac{5.8}{\sqrt{10}}\right) = 72.5 \pm 3.25\times\left(\frac{5.8}{\sqrt{10}}\right) = (66.539, 78.461)$$

10. A soft-drink manufacturer purchases aluminum cans from an outside vendor. A random sample of 70 cans is selected from a large shipment, and each is tested for strength by applying an increasing load to the side of the can until it punctures. Of the 70 cans, 62 meet the specifications for puncture resistance. What is the 90% confidence interval for the proportion of cans in the shipment that meet the specifications?

(a) (0.823, 0.948) (b) (0.808, 0.964) (c) (0.883, 0.888)
(d) (0.811, 0.960) (e) (0.788, 0.984)

Solution: The 90% confidence interval is

$$\hat{p} \pm z_{\alpha/2} \times \sqrt{\frac{\hat{p}(1-\hat{p})}{n}} \rightarrow \frac{62}{70} \pm 1.645 \times \sqrt{\frac{\frac{62}{70}\left(1-\frac{62}{70}\right)}{70}} \rightarrow 0.8857 \pm 0.0626 \rightarrow (0.8232,\ 0.9483)$$

11. In a poll of 484 voters in a city, 64% said that they backed a bill which would limit further construction in their city. The margin of error in the poll was reported as exactly 4 percentage points with a 95% confidence level. Which among the following statements is TRUE about the adequacy of the sample size for the given margin of error?

(a) The reported margin of error is consistent with the sample size.
(b) The sample size is too small to achieve the stated margin of error.
(c) For the given sample size, the margin of error should be smaller than stated.
(d) The sample size is too large to achieve the stated margin of error.
(e) For the given sample size, the margin of error is correct for a 90% confidence level.

Solution: The margin of error is

$$E = z_{\alpha/2}\sqrt{\frac{\hat{p}(1-\hat{p})}{n}} = 1.96\sqrt{\frac{0.64(1-0.64)}{484}} = 0.0428$$

Since 0.0428 > 0.04, the sample of n = 484 is too small; it needs to be larger for a smaller E.

12. A manufacturer claims that the mean volume of juice in its 16-ounce bottle is 16.1 ounces. A consumer advocacy group wants to perform a hypothesis test to determine whether the mean amount is actually less than this claim. The mean volume of juice for a random sample of 70 bottles was 15.94 ounces. Does the data provide sufficient evidence to conclude that the mean volume of juice for all 16-ounce bottles is less than 16.1 ounces? Assume that σ = 0.9 ounces. The CORRECT conclusion is that:

(a) There is sufficient evidence to conclude that the mean volume of juice is less than 16.1 oz at the 0.10 level of significance but not at the 0.05 level.

(b) There is insufficient evidence to conclude that the mean volume of juice is less than 16.1 oz at both the 0.10 and 0.05 levels of significance.

(c) There is sufficient evidence to conclude that the mean volume of juice is 15.94 at the 0.10 level of significance but not at the 0.05 level.

(d) There is insufficient evidence to conclude that the mean volume of juice is less than 16.1 oz at the 0.10 level of significance.

(e) There is sufficient evidence to conclude that the mean volume of juice is less than 16.1 oz at both the 0.05 and 0.025 levels of significance.

Solution: Since σ is given, use z. We test H_0: μ = 16.1 vs. H_A: μ < 16.1.

$$z_0 = \frac{\bar{x} - \mu_0}{\sigma / \sqrt{n}} = \frac{15.94 - 16.1}{0.9 / \sqrt{70}} = -1.49$$

P-value = $P(Z < -1.49)$ = 0.0681, which is less than α = 0.10 and greater than α = 0.05.

13. In the past, the mean running time for a certain type of flashlight battery has been 8.0 hours. The manufacturer has introduced a change in the production method and wants to perform a hypothesis test to determine whether the mean running time has increased as a result. Which among the following pairs of hypotheses is CORRECT?

(a) H_0: $\mu = 8.0$ and H_A: $\mu \neq 8.0$.
(b) H_0: $\mu < 8.0$ and H_A: $\mu > 8.0$.
(c) H_0: $\mu \neq 8.0$ and H_A: $\mu = 8.0$.
(d) H_0: $\mu \geq 8.0$ and H_A: $\mu \neq 8.0$.
(e) H_0: $\mu = 8.0$ and H_A: $\mu > 8.0$.

Use the following information for questions 14 – 15:

A supplier of 3.5″ disks claims that no more than 1% of the disks are defective. In a random sample of 600 disks, it is found that 3% are defective, but the supplier claims that this is only a sample fluctuation. At the 0.01 level of significance, test the supplier's claim that no more than 1% are defective.

14. What is the numerical value of the test statistic?

(a) 0.2010 (b) 2.8735 (c) -2.8375 (d) 4.926 (e) -4.926

Solution: The value of the test statistic

$$z_0 = \frac{\hat{p} - p_0}{\sqrt{\frac{p_0(1-p_0)}{n}}} = \frac{0.03 - 0.01}{\sqrt{\frac{0.01(1-0.01)}{600}}} = 4.926$$

15. Which among the following is the CORRECT conclusion?

(a) Do not reject the null hypothesis and conclude that there is evidence to support the claim that more than 1% of the disks are defective.

(b) Reject the null hypothesis and conclude that there is insufficient evidence to support that more than 1% of the disks are defective.

(c) Do not reject the null hypothesis and conclude that there is insufficient evidence to support the claim that more than 1% of the disks are defective.

(d) Reject the null hypothesis and conclude that there is sufficient evidence to support the claim that more than 1% of the disks are defective.

(e) There is not enough information to make a decision.

Solution: We test H_0: $p \leq 0.01$ vs. H_A: $p > 0.01$. Using the z-table and $z = 3.89 < z_0 = 4.926$, P-value = $P(Z > 4.93)$ has a range of (0, 0.0001), which is less than $\alpha = 0.01$.

16. A trucking firm suspects that the mean life of a certain tire it uses is less than 35,000 miles. To check the claim, the firm randomly selects and tests 18 of these tires and gets a mean lifetime of 34,400 miles with a standard deviation of 1,200 miles. Respectively, which among the following are the numerical value of the test statistic and the range of the P-value?

(a) 1.768 and (0.025, 0.05).
(b) -2.121 and (0.01, 0.025).
(c) -1.718 and (0.05, 0.10).
(d) 1.718 and (0.05, 0.10).
(e) 1.800 and (0.025, 0.05).

Solution: Since s is given, use t. We test H_0: $\mu \geq 35{,}000$ vs. H_A: $\mu < 35{,}000$.

$$t_0 = \frac{\bar{x} - \mu_0}{s/\sqrt{n}} = \frac{34{,}400 - 35{,}000}{1200/\sqrt{18}} = -2.121$$

The test statistic follows a t-distribution with $df = n - 1 = 17$. Based on the t-table, the P-value is between 0.01 and 0.025.

17. Photocopiers at the Primatech Paper Company are always malfunctioning. One on the sixth floor only seems to be malfunctioning during the second half of the week. To be certain, an employee takes a random sample of photocopying jobs in the first half of the week and a random sample of photocopying jobs in the second half of the week. The employee measures the proportion of jobs containing mistakes and wants to test if the proportion of jobs containing mistakes in the second half of the week is significantly higher. If the employee found a *P*-value of 0.1515 for the test, what could they conclude?

(a) The proportion of jobs containing mistakes is higher in the second half of the week.

(b) The proportion of jobs containing mistakes is higher in the first half of the week.

(c) The proportion of jobs containing mistakes in the first half of the week is different from the proportion from the second half of the week.

(d) The proportion of jobs containing mistakes may be higher in the second half of the week.

(e) The proportion of jobs containing mistakes is lower in the first half of the week.

Solution: The question describes a one-tailed test, so the possible hypotheses are:

$$H_0: p_1 - p_2 \geq 0 \text{ and } H_A: p_1 - p_2 < 0$$

where p_1 is the population proportion of jobs containing mistakes in the first half of the week and p_2 is the population proportion of jobs containing mistakes in the second half. The *P*-value is too high to reject H_0, so either H_0 or H_A may be correct.

18. If a 95% confidence interval for the difference between two sample proportions does not contain zero but a 99% confidence interval does, which of the following statements is TRUE?

(a) A two-sided test will reject H_0: $p_1 - p_2 = 0$ at $\alpha = 0.01$, but not $\alpha = 0.05$.

(b) A two-sided test will reject H_0: $p_1 - p_2 = 0$ at $\alpha = 0.05$, but not $\alpha = 0.01$.

(c) A two-sided test will reject H_0: $p_1 - p_2 = 0$ at $\alpha = 0.01$ and $\alpha = 0.05$.

(d) A two-sided test will not reject H_0: $p_1 - p_2 = 0$ at $\alpha = 0.01$ and $\alpha = 0.05$.

(e) A two-sided test will not reject H_0: $p_1 - p_2 = 0$ at $\alpha = 0.05$ and $\alpha = 0.1$.

Solution: Since 95% confidence relates to $\alpha = 0.05$ and 99% relates to $\alpha = 0.01$, the provided information about the intervals implies at which levels of α that zero will be rejected.

19. If a 90% confidence interval for the difference between two sample proportions is (0.043, 0.425), what is the 99% confidence interval?

(a) (-0.112, 0.581) (b) (-0.150, 0.618) (c) (-0.036, 0.504)
(d) (-0.017, 0.485) (e) (-0.065, 0.533)

Solution: Find the three major components of the first interval to then find the second.
The critical value for 90% confidence is $z_{0.05} = 1.645$.

By averaging the endpoints, $\hat{p}_1 - \hat{p}_2 = \dfrac{(0.043 + 0.425)}{2} = 0.234$.

$0.234 + (1.645) \times S.E.(\hat{p}_1 - \hat{p}_2) = 0.425 \rightarrow S.E.(\hat{p}_1 - \hat{p}_2) = 0.116$.

For 99% confidence, $z_{0.005} = 2.576$. Thus, $0.234 \pm (2.576) \times (0.116) \rightarrow (-0.065, 0.533)$.

20. A population follows a normal distribution with mean μ and standard deviation $\sigma = 1.2$. You construct a 95% confidence interval for μ to be 1.1 ± 0.8. Which of the following is true?

(a) H_0: $\mu = 1.2$, H_A: $\mu \neq 1.2$ would be rejected at the 0.05 level
(b) H_0: $\mu = 1.1$, H_A: $\mu \neq 1.1$ would be rejected at the 0.05 level
(c) H_0: $\mu = 1.0$, H_A: $\mu \neq 1.0$ would be rejected at the 0.05 level
(d) H_0: $\mu = 0.8$, H_A: $\mu \neq 0.8$ would be rejected at the 0.05 level
(e) H_0: $\mu = 0$, H_A: $\mu \neq 0$ would be rejected at the 0.05 level

Solution: Since 95% confidence relates to $\alpha = 0.05$, the provided interval of (0.3, 1.9) allows for only one of the claimed values to be rejected at $\alpha = 0.05$.

Use the following information for questions 21 – 22:

A materials engineer tries to design the perfect acoustic guitar. Working for the large company, Strings Attached, the engineer focuses on measuring axial tension (in Newtons) before and after applying the company's patented technique to see if it increases the axial tension. By measuring 35 randomly selected guitars, she obtained the following summary statistics.

Summary statistic	Before	After	Difference
Average	67.8	71.2	-3.40
Standard Deviation	5.42	8.42	10.01

21. If the test statistic for the technique evaluation mentioned above is -2.009, what is the approximate range of the *P*-value?

(a) (0.025, 0.05) (b) (0.01, 0.025) (c) (0.02, 0.05)
(d) (0.975, 0.99) (e) (0.05, 0.10)

Solution: The situation above indicates a one-tailed paired test, so $n - 1 = 35 - 1 = 34 \approx 30$ and the test statistic of -2.009 fits between -1.697 and -2.042, so the *P*-value range is (0.025, 0.05). Note that the negative test statistic coincides with the provided order of $d = x_{bef} - x_{aft}$ and corresponding H_A: $\mu_d < 0$.

22. If the standard error of the appropriate estimate is 1.692, what is the corresponding 99% confidence interval for the parameter related to this estimate?

(a) (-∞, 0.757) (b) (-7.759, 0.959) (c) (-∞, 0.536)
(d) (-8.053, 1.253) (e) (-∞, 0.644)

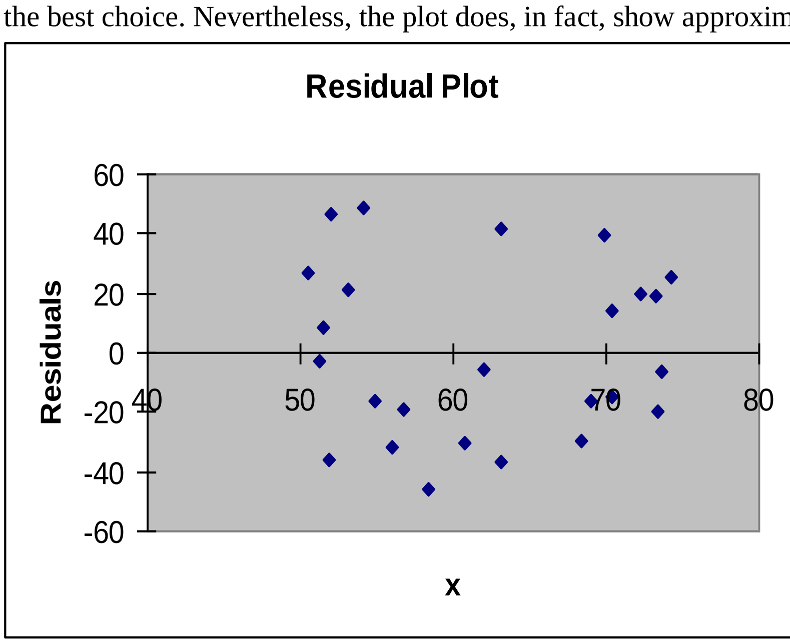
Solution: Again, the data are paired and the one-tailed question requires a one-sided upper confidence bound (since lower tail used in previous question), so the critical value for 99% confidence is $t_{0.01,\,34} \approx t_{0.01,\,30} = 2.457$.

$$\bar{d} + t_{\alpha,\,n-1} \times \left(\frac{s_d}{\sqrt{n}}\right) \rightarrow -3.40 + (2.457)(1.692) \rightarrow -3.40 + 4.157 \rightarrow (-\infty, 0.757)$$

23. Using the following residual plot, what can you conclude about the corresponding simple linear regression model?

(a) All the errors are independent of each other.
(b) The slope of the estimated regression line is positive.
(c) The standard deviation of the errors at any x is approximately constant.
(d) The explanatory variable is causing the response variable.
(e) The errors follow a normal distribution.

Solution: The residual plot is best at verifying the constant variance assumption, so this is already the best choice. Nevertheless, the plot does, in fact, show approximately constant variance.



Use the following information for questions 24 – 26:

A company, Biff Tannen's Finest Fertilizers, wants to introduce a new compound of fertilizer that contains small concentrations of sulfur to help reduce the pH of soil in dry climates. Randomly sampling nine different soil collections, a chemical engineer measures the exact concentration of sulfur in each of the fertilizer compounds before he applies one to each soil collection. A week later, the engineer measures the soil pH. He obtained the following output for sulfur concentration (measured in %) and soil pH. Assumptions for regression inference were satisfied.

SUMMARY OUTPUT

Regression Statistics	
Multiple R	0.720832
R Square	[redacted]
[redacted]	[redacted]
Standard Error	0.412651
Observations	9

ANOVA

	df	*SS*	*MS*	*F*	*Significance F*
Regression	1	1.289217	1.289217	7.571137	0.02844
Residual	7	1.191964	0.170281		
Total	8	2.481181			

	Coefficients	*Standard Error*	*t Stat*	*P-value*
Intercept	5.571639	0.307616	18.11234	3.87E-07
sulfurConc	123.9813	45.0584		

24. A 99% confidence interval for the estimated intercept is approximately:

(a) 5.572 ± 0.308 (b) 5.572 ± 0.922 (c) 5.572 ± 0.847
(d) 5.572 ± 1.076 (e) 5.572 ± 1.032

Solution: Due to the output, two of three components are provided and the calculations are similar to that of the slope, so the critical value for 99% confidence is $t_{0.005,\,9-2} = t_{0.005,\,7} = 3.499$.

$\hat{\beta}_0 \pm t_{\alpha/2,\,v} \times s.e.(\hat{\beta}_0)$ → 5.572 ± (3.499)(0.307616) → 5.572 ± 1.076

25. Given the above output, which of the following statements is CORRECT?

(a) A sulfur concentration of 0.0088% has a predicted pH of 6.663.
(b) The *P*-value for a significantly positive slope is 0.02844.
(c) The coefficient of determination is 0.721.
(d) The test statistic for a significantly positive slope is 1.376.
(e) The estimated regression line likely goes through the origin.

Solution:
$\hat{\beta}_0 + \hat{\beta}_1 x^* = 5.571639 + 123.9813(0.0088) = 6.663$. → (a) is correct.
The *P*-value for a positive slope is 0.02844/2 = 0.01422. → (b) is wrong.
The coefficient of determination (R^2) is $(0.721)^2 = 0.520$. → (c) is wrong.
The test statistic for a positive slope is 123.9813/45.0584 = 2.752. → (d) is wrong.
The *P*-value for H_0: $\beta_0 = 0$ is approximately zero, so H_0 is rejected. → (e) is wrong.

26. The estimate s of the model's standard deviation σ is

(a) 0.4127 (b) 0.4879 (c) 0.6884 (d) 0.8261 (e) 1.344

Solution: $s = \hat{\sigma} = \sqrt{MSE} = \sqrt{SSE/(n-2)} = \sqrt{1.191964/(9-2)} = 0.4127$

27. Sean Everuke is a civil engineer new at his job and he's concerned about speeding buses. Drivers going too fast will result in unhappy and, therefore, fewer passengers. By attaching a device to randomly-selected buses, Sean was able to observe the average speed of the bus (x) during the course of the day while a video camera helped him measure the number of people (y) who entered the bus over the same day. The explanatory variable has units of km/h. Assumptions for regression were satisfied. The estimated regression equation is: $\hat{y} = 982.468 - 5.699x$. If the proportion of variation in the number of people explained by average bus speed is 74.3%, what can you conclude about R?

(a) R = 0.862 and indicates a strong, positive relationship.
(b) R = 0.743 and indicates a strong, positive relationship.
(c) R = -0.862 and indicates a strong, negative relationship.
(d) R = -0.743 and indicates a strong, negative relationship.
(e) R = -0.862 and indicates a strong, positive relationship.

Solution: Since R^2 = 0.743 and slope is -5.699, $R = -\sqrt{0.743} = -0.862$, which is a large, negative number for R.

28. A waste water treatment program is designed to produce treated water with a pH of 7. Let μ represent the mean pH of water treated by this process. The pH of 60 water specimens are measured to correspond to a test of H_0: $\mu = 7$ versus H_A: $\mu \neq 7$. The sample mean was found to be 6.85. Assume it is known from previous experiments that the standard deviation is approximately 0.5. What are the respective test statistic and P-value?

(a) z_0 = -2.32 and 0.9898
(b) t_0 = -2.32 and (0.01, 0.025)
(c) z_0 = -2.32 and 0.0204
(d) t_0 = -2.32 and (0.02, 0.05)
(e) z_0 = -2.32 and 0.0102

Solution: Since σ is given, use z. Question also mentions test is two-tailed.

$$z_0 = \frac{\bar{x} - \mu_0}{\sigma / \sqrt{n}} = \frac{6.85 - 7}{0.5 / \sqrt{60}} = -2.32$$

P-value = 2*$P(Z < -2.32)$ = 2*0.0102 = 0.0204

29. Refer to previous question. If the rejection region only consisted of all sample mean values greater than 7.2 (but still using the two-tailed test above), what is the level of significance of the corresponding test?

(a) $\alpha = 0.6892$
(b) $\alpha = 0.3446$
(c) $\alpha = 0.0010$
(d) $\alpha = 0.9990$
(e) $\alpha = 0.0020$

Solution: All $\bar{x}$ > 7.2 are in the rejection region.

$$P(\bar{x} > 7.2) = P\left(\frac{\bar{x} - \mu}{\sigma / \sqrt{n}} > \frac{7.2 - 7}{0.5 / \sqrt{60}}\right) = P(Z > 3.10) = 0.0010$$

This is a two-sided test, hence level of significance α = 2*0.0010 = 0.002.

30. To reduce the loss of human life in war zones, Cyberdyne Systems Corporation is manufacturing cybernetic organisms for "peacekeeping" during international conflicts. The company is concerned with the machines having a certain range of volume to be heard easily. Suppose the mean sound volume of *ALL* machines is 85 decibels (dB) with a standard deviation of 4.5 dB. A scientist (Miles Dyson) takes a random sample of 81 of the machines. What is the probability that the average volume is less than 84.1 dB or greater than 86.2 dB?

(a) 0.0359
(b) 0.0441
(c) 0.8143
(d) 0.0082
(e) 0.0883

Solution: $P(\bar{X} < 84.1) = P\left(\frac{\bar{X}-\mu}{\sigma/\sqrt{n}} < \frac{84.1-85}{4.5/\sqrt{81}}\right) = P(Z < -1.80) = 0.0359$

$$P(\bar{X} > 86.2) = P\left(\frac{\bar{X}-\mu}{\sigma/\sqrt{n}} > \frac{86.2-85}{4.5/\sqrt{81}}\right) = P(Z > 2.40) = 1 - 0.9918 = 0.0082$$

$$P\big((\bar{X} < 84.1) \cup (\bar{X} > 86.2)\big) = 0.0359 + 0.0082 = 0.0441$$

31. Refer to the previous question. Suppose the true proportion outside the range of 84.15 dB and 86.2 dB is 0.0521. If a new sample consists of 400 machines, what is the probability of the sample containing at least 5% of machines outside the range?

(a) 0.0111
(b) 0.4247
(c) 0.1890
(d) 0.5753
(e) 0.0521

Solution: A true proportion and sample size are provided, looking for a probability for a sample proportion. Thus, the solution before the Fall 2021 term is:

$$P(\hat{p} \geq 0.05) = P\left(\frac{\hat{p}-p}{\sqrt{\frac{p(1-p)}{n}}} \geq \frac{0.05-0.0521}{\sqrt{\frac{0.0521(1-0.0521)}{400}}}\right) = P\left(\frac{\hat{p}-p}{\sqrt{\frac{p(1-p)}{n}}} \geq \frac{-0.0021}{0.0111}\right)$$

$= P(Z \geq -0.19) = 1 - 0.4247 = 0.5753$

<u>For the Fall 2021 term and after:</u>

$$P(\hat{p} \geq 0.05) = P(0.05 \leq \hat{p}) \approx P\left(\frac{0.05 - 0.5/400 - 0.0521}{\sqrt{\frac{0.0521(1-0.0521)}{400}}} \leq \frac{\hat{p} - \frac{0.5}{n} - p}{\sqrt{\frac{p(1-p)}{n}}}\right) = P\left(\frac{-0.0034}{0.0111} \leq \frac{\hat{p} - \frac{0.5}{n} - p}{\sqrt{\frac{p(1-p)}{n}}}\right)$$

$= P(-0.30 \leq Z) = P(Z \geq -0.30) = 1 - P(Z < -0.30) = 1 - 0.3821 = 0.6179$

Use the following information for questions 32 – 37:

Suppose a manufacturer of steel rods is experimenting with a new technology that is supposed to increase the mean strength of the rods. A random sample of 6 rods was obtained with the standard technology and another sample of 6 was obtained with the new technology. The breaking strength of each rod in the two groups was recorded. The data are provided in the first two columns below. The new technology rods were subjected to high pressure and the strength of each of them was determined again. These data are provided in the third column. All rods come from normal populations.

STANDARD (S)	NEW (N)	HIGH PRESSURE (HP)
57	65	66
58	63	62
53	55	56
62	71	72
58	68	69
63	65	66

The following summaries were obtained for each of the three columns:

	STANDARD	NEW	HIGH PRESSURE
MEAN	58.5	64.5	65.17
VARIANCE	13.1	29.5	31.37

The following summaries were obtained for the differences between each pair of columns:

	S – N	S – HP	N – HP
MEAN	-6.00	-6.67	-0.67
VARIANCE	12.40	13.87	0.67

32. Test that the new technology rods are stronger on average than the old (standard) technology rods. Let μ_1 be the mean breaking strength of rods obtained with the old technology and μ_2 be the mean breaking strength of rods obtained with the new technology. What are the appropriate hypotheses to answer this test?

(a) H_0: $\mu_1 - \mu_2 = 0$ H_A: $\mu_1 - \mu_2 < 0$
(b) H_0: $\mu_1 - \mu_2 = 0$ H_A: $\mu_1 - \mu_2 > 0$
(c) H_0: $\mu_1 - \mu_2 = 0$ H_A: $\mu_1 - \mu_2 \neq 0$
(d) H_0: $\mu_1 - \mu_2 < 0$ H_A: $\mu_1 - \mu_2 = 0$
(e) H_0: $\mu_1 - \mu_2 > 0$ H_A: $\mu_1 - \mu_2 = 0$

Solution: The question ("stronger") suggests a one-tailed test where mean breaking strength of new technology rods should be higher. Based off the defined symbols, μ_2 should be higher, so the defined difference should be negative in the alternative.

33. If the standard error of the appropriate estimate is 2.665, what are the respective test statistic and degrees of freedom for the test?

(a) t_0 = -5.514 and df = 5
(b) t_0 = -2.252 and df = 10
(c) z_0 = -2.252 and df = 10
(d) t_0 = -5.514 and df = 10
(e) t_0 = -2.252 and df = 5

Solution: This situation involves independent samples where the standard deviation ratio is less than 2 (or 29.5/13.1 = 2.25 < 4 for variance), but since the sample sizes are too small (< 15), the independent sample t-test not assuming equal variance applies. Thus,

$$t_0 = \frac{\bar{x}_1 - \bar{x}_2 - \delta}{S.E.(\bar{x}_1 - \bar{x}_2)} = \frac{58.5 - 64.5 - 0}{2.665} = \frac{-6.0}{2.665} = -2.252$$

$df = \min\{n_1, n_2\} - 1 = \min\{6, 6\} - 1 = 5$

34. Refer to the previous question. With a 5% significance level, what is the appropriate conclusion?

(a) At α = 0.05, the new technology rods are stronger on average than the old technology rods.
(b) At α = 0.05, the new technology rods are weaker on average than the old technology rods.
(c) At α = 0.05, the new technology rods may be stronger on average than the old technology rods.
(d) At α = 0.05, the new technology rods may be weaker on average than the old technology rods.
(e) At α = 0.05, the new technology rods are different on average than the old technology rods.

Solution: Since -2.015 < t_0 = -2.252 < -2.571, the p-value range is (0.025, 0.05); thus, reject H_0 since the p-value < α = 0.05. In conclusion, the new technology rods are stronger on average than the old technology rods.

35. Is there any evidence that high pressure increased the strength of new technology rods? Calculate a 95% confidence interval.

(a) ($-\infty$, 0.00334) (b) (-1.324, -0.0150) (c) ($-\infty$, 0.120)
(d) ($-\infty$, -0.119) (e) ($-\infty$, 0.979)

Solution: A 95% confidence upper bound (confidence interval with the lower bound equal to -∞) for the mean $\mu_D = \mu_N - \mu_{HP}$ is best to answer the question.

The 95% upper bound for μ_D has the form

$$\mu_D = \mu_N - \mu_{HP} \le -0.67 + t_{0.05,5} \cdot \frac{\sqrt{0.67}}{\sqrt{6}} = -0.67 + 2.015 \cdot \frac{0.819}{\sqrt{6}} = 0.00334 \rightarrow (-\infty, 0.00334)$$

36. Refer to the previous question. Interpret the 95% confidence interval.

(a) Since zero is included in the interval, there is enough evidence that high pressure increased the strength of new technology rods.
(b) Since zero is included in the interval, there is not enough evidence that high pressure increased the strength of new technology rods.
(c) Since zero is not included in the interval, there is not enough evidence that high pressure increased the strength of new technology rods.
(d) Since zero is not included in the interval, there is enough evidence that high pressure increased the strength of new technology rods.
(e) Since zero is included in the interval, there is not enough evidence that high pressure decreased the strength of new technology rods.

Solution: There is a zero in the interval, so there is not enough evidence to reject it as a value. There is not enough evidence that high pressure increased the strength of new technology rods.

37. Refer to the previous two questions. What assumptions are required to make the inferences valid?

I. The samples are paired.
II. The samples are independent.
III. The samples are random.
IV. The samples are large enough.
V. The samples come from a normal population.
VI. The samples have unknown yet equal population variances.

(a) I, III, and V
(b) II, III, IV, and VI
(c) II, III, V, and VI
(d) II, III, and V
(e) I, IV, and V

Solution: For the interval, the data are paired. The original paragraph mentions random sampling and a normal population.

38. In a 2012 report, 18% of all U.S. airplanes required repairs. In the same year, a random sample of airplanes in Wyoming resulted in obtaining a 95% confidence interval for airplanes requiring repairs of (0.09, 0.17). Which statement below is CORRECT?

(a) Since 0.18 does not fall within this confidence interval, the proportion of airplanes requiring repairs in Wyoming may not be different from the countrywide proportion.
(b) Since 0.18 does not fall within this confidence interval, the proportion of airplanes requiring repairs in Wyoming is different from the countrywide proportion.
(c) Since 0 does not fall within this confidence interval, the proportion of airplanes requiring repairs in Wyoming may not be different from the countrywide proportion.
(d) Since 0 does not fall within this confidence interval, the proportion of airplanes requiring repairs in Wyoming is different from the countrywide proportion.
(e) If we took many random samples of U.S. airplanes, about 95% of the samples will produce this confidence interval.

Solution: The value of 0 is more useful in two-sample situations. The country-wide proportion value of 18% falls outside the interval, suggesting it is different enough from the sample data obtained in Wyoming.

39. In 1882, Michelson measured the speed of light (usually denoted c in Einstein's famous equation, $E = mc^2$). His values are in km/sec and have 299,000 subtracted from them. He reported the results of 23 trials with a mean of 756.220 and a standard deviation of 107.120. What is a 90% confidence interval for the true speed of light from these statistics?

(a) (726.714, 785.726)
(b) (717.869, 794.571)
(c) (719.477, 792.963)
(d) (572.295, 940.145)
(e) (733.884, 778.556)

Solution: Since s is given, use t. Here $n = 23$, $\overline{x} = 756.220$, and $s = 107.120$. Thus,

$$\overline{x} \pm t_{\alpha/2,n-1} \times \left(\frac{s}{\sqrt{n}}\right) = 756.220 \pm t_{0.05,22} \times \left(\frac{107.120}{\sqrt{23}}\right) = 756.220 \pm 1.717 \times \left(\frac{107.120}{\sqrt{23}}\right) = (717.869, 794.571)$$

40. The estimate of the difference, $\mu_1 - \mu_2$, in vehicle miles travelled per week between Ford trucks in Midwest (μ_1) and Southern (μ_2) U.S. states was calculated to be -1.46 and a 95% confidence interval for the difference was (-4.87, 1.95). Which statement below is CORRECT?

(a) Since the estimate of the difference (-1.46) falls within the confidence interval, it is estimated with 95% confidence that there is no difference in mean miles travelled between Ford trucks in the Midwest and Southern states.
(b) Since the estimate of the difference (-1.46) falls within the confidence interval, it is estimated with 95% confidence that there is a difference in mean miles travelled between Ford trucks in the Midwest and Southern states.
(c) Based on this sample, we can say with 95% confidence that the mean household vehicle miles travelled by Ford trucks living in Midwest states is between 4.87 less and 1.95 more miles per week than Ford trucks living in Southern states.
(d) We are 95% sure that the difference in distance travelled by household vehicles between Ford trucks living in Midwest and Southern states is between -4.87 and 1.95 miles per week.
(e) Since the lower endpoint is farther from 0 than the upper endpoint, it is estimated with 95% confidence that the mean miles travelled by Ford trucks in the Midwest states is lower than in the Southern states.

Solution: Review interpretation of the confidence interval for two independent samples.

Use the following information to answer questions 41 – 42:

An engineer wanted to measure the monthly amount of alcohol they purchased (by individual receipts) while on industrial internship in Fort McMurray. From three different months, they randomly sampled the same number of receipts. Assume all populations are normal with some common variance. Unfortunately, some excess alcohol washed away some of their work, leaving only the information in the table below.

Source	**df**	**SS**	**MS**	***F***
Treatments		7450		
Error				
Total	**53**	**10000**		

41. What are the appropriate hypotheses relating to this table?

(a) H_0: $\mu_1 = \mu_2 = \mu_3$ and H_A: at least one μ is different
(b) H_0: $\mu_1 = \mu_2 = \mu_3$ and H_A: each μ is different from the others
(c) H_0: $\mu_1 = \mu_2 = 0$ and H_A: each μ is different from the others
(d) H_0: $\mu_1 = \mu_2 = \mu_3 = 0$ and H_A: at least one μ is different
(e) H_0: $\mu_1 = \mu_2$ and H_A: each μ is different from the others

Solution: There are three months (or three groups) in this question and with the null hypothesis representing equality of the three means, it is sufficient for the alternative to have at least one mean different to reject the null hypothesis.

42. What are valid conclusions to draw from the analysis?

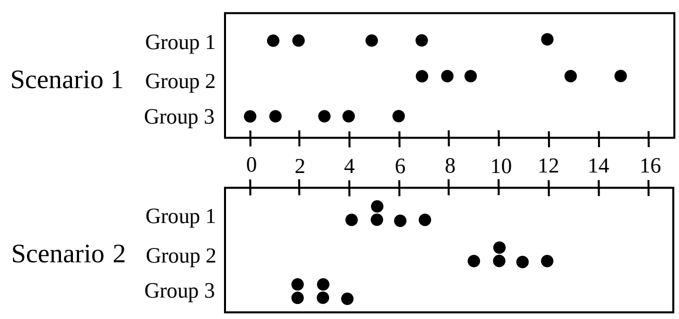
(a) There are 3 observations per group and the engineer will likely reject H_0.
(b) There are 18 observations per group and the engineer will likely reject H_0.
(c) There are 3 observations per group and the engineer will likely fail to reject H_0.
(d) There are 18 observations per group and the engineer will likely fail to reject H_0.
(e) There are 54 observations per group and the engineer will likely reject H_0.

Solution: $k = 3$, $N = 54$, $SSE = 10{,}000 - 7450 = 2550$, so 54/3 = 18 observations per group and

$$F_0 = \frac{SSTr/(k-1)}{SSE/(N-k)} = \frac{7450/(3-1)}{2550/(54-3)} = \frac{3725}{50} = 74.500$$

The F_0 value is quite high, so the p-value will be quite low, leading to the rejection of H_0.

43. Consider the following two scenarios:



Note that for each of the two scenarios, $\bar{x}_1 = 5.4$, $\bar{x}_2 = 10.4$, $\bar{x}_3 = 2.8$.

Which of the following statements is TRUE?

(a) The variance within groups is smaller in the first scenario.
(b) The variance within groups is larger in the first scenario.
(c) The variance between groups is smaller in the first scenario.
(d) The variance between groups is larger in the first scenario.
(e) The F-statistic must equal 1.

Solution: Since the two scenarios have the same sample means, *MSTr* (variance between groups) is the same for both scenarios. The data being more spread out in Scenario 1, however, indicates there is more variance within groups.

44. Suppose students went to RATT every Friday, intent on forgetting the woes and strife of student life. The bill will often vary from week to week. Assume that the mean value of all bills is \$30 and the standard deviation is \$7.50. What is the probability that a random sample of 50 bills will give an average less than \$32 and greater than \$28?

(a) 0.0588
(b) 0.9706
(c) 0.2128
(d) 0.9412
(e) 0.2667

Solution: $P\left((\bar{X} < 32) \cap (\bar{X} > 28)\right) = P\left(28 < \bar{X} < 32\right) = P\left(\frac{28-30}{7.5/\sqrt{50}} < \frac{\bar{X}-\mu}{\sigma/\sqrt{n}} < \frac{32-30}{7.5/\sqrt{50}}\right)$

$P(-1.89 < Z < 1.89) = 1 - 2P(Z \le -1.89) = 1 - 2(0.0294) = 0.9412$

45. Suppose the true proportion of defective computers that cross the Canadian-American border is 0.08. (Maybe they're also *defecting*! HA HA! … anyway, back to the exam…) If a random sample consists of 60 computers, then what can be said about the corresponding sampling distribution of the sample proportion?

(a) It may not be normal with a standard error of 0.00123.
(b) It is approximately normal with a standard error of 0.00123.
(c) It may not be normal with a standard error of 0.2713.
(d) It may not be normal with a standard error of 0.0350.
(e) It is approximately normal with a standard error of 0.0350.

Solution: Although $n(1 - p) = 60(1 - 0.08) = 55.2 > 5$ (or 10 or 15), $np = 60(0.08) = 4.8$ is too small to satisfy the success/failure condition, so there may not be normality.

$$\sigma_{\hat{p}} = \sqrt{\frac{p(1-p)}{n}} = \sqrt{\frac{0.08(1-0.08)}{60}} = 0.0350$$

46. In comparing probability distributions, which of the following statements is CORRECT?

(a) The geometric and Poisson distributions have the lack of memory property.
(b) The uniform and normal distributions have the same distribution shapes.
(c) The Poisson distribution is the limiting form of the negative binomial distribution.
(d) The exponential and Poisson distributions are interchangeable because they have the exact same random variable.
(e) The geometric and negative binomial distributions both have a random number of trials.

Solution: See definitions of distributions.

47. The password to a computer system consists of five digits with the first digit different from zero and the last two digits different from nine. How many different possible passwords are there?

(a) 59 049
(b) 81 000
(c) 90 000
(d) 72 900
(e) 100 000

Solution: From the provided restrictions, the first digit and last two digits have 9 options, but the second and third digits still have 10, so $9\times10\times10\times9\times9 = 72\,900$.

48. A first-year geologist is completing his project about relating two variables: ground composition and depth. For the three basic rock types (igneous, metamorphic, and sedimentary), the geologist found coefficients of determination of 71.1%, 1.4%, 93.5%, respectively. What can you conclude about the corresponding correlations of each pair?

(a) Two of the correlations suggest no linear relationship.
(b) All three correlations are strong.
(c) Two of the correlations suggest weak linear relationships.
(d) All three correlations are positive.
(e) Two of the correlations suggest strong linear relationships.

Solution: Coefficients of determination are also known as R^2 values, which can provide the strength of a relationship but not a direction, so there is enough information to interpret strength of the three relationships (two are strong), but not if they are positive or negative.

49. In an experiment, gypsum was added in four different amounts to soil samples before leaching. Three soil samples received each amount added. The pH measurements of the samples are presented in the following incomplete ANOVA table. Assume groups are normally distributed with a common variance. What is the numerical value of the test statistic?

Source	**df**	**SS**	**MS**	***F***
Treatments				
Error			0.01509	
Total		**0.13383**		

(a) 0.290
(b) 0.121
(c) 2.906
(d) 2.623
(e) 0.0437

Solution: $k = 4$, $N = 3\times4 = 12$, $SSE = 0.01509(8) = 0.12074$,
$SSTr = 0.13383 - 0.12074 = 0.01309$,

$$F_0 = \frac{SSTr/(k-1)}{SSE/(N-k)} = \frac{0.01309/(4-1)}{0.12074/(12-4)} = \frac{0.00437}{0.01509} = 0.290$$

50. Refer to the previous question. If the corresponding *p*-value for the test statistic is 0.832, what is an appropriate conclusion at a 5% significance level?

(a) Since $\alpha > p$-value, there is enough evidence of at least one difference between the gypsum amounts.
(b) Since $\alpha > p$-value, there is not enough evidence of any differences between the gypsum amounts.
(c) Since $\alpha < p$-value, there is not enough evidence of any differences between the gypsum amounts.
(a) Since $\alpha < p$-value, there is enough evidence of at least one difference between the gypsum amounts.
(e) Since $\alpha > p$-value, there is not enough evidence of any differences between soil samples.

Solution: The *p*-value is larger than alpha, so do not reject H_0 and there is not enough evidence.