STAT 235 MIDTERM EXAM FORMULA SHEET

Summaries:

$$\bar{x} = \frac{x_1 + x_2 + \ldots + x_n}{n}, \quad s^2 = \frac{\sum_{i=1}^{n}(x_i - \bar{x})^2}{n-1} = \frac{1}{n-1}\left[\sum x_i^2 - \frac{(\sum x_i)^2}{n}\right]$$

IQR = $Q_3 - Q_1$, outliers are observations 1.5*IQR below Q_1 or 1.5*IQR above Q_3

Probability:

Conditional Probability: $P(A \mid B) = \dfrac{P(A \cap B)}{P(B)}$

Complement Law: $P(A') = 1 - P(A)$

Multiplication Law: $P(A \cap B) = P(A) \times P(B \mid A) = P(B) \times P(A \mid B)$

If A and B are independent, $P(A \cap B) = P(A) \times P(B)$ since $P(A \mid B) = P(A)$.

Addition Law: $P(A \cup B) = P(A) + P(B) - P(A \cap B)$

If A and B are mutually exclusive, $P(A \cup B) = P(A) + P(B)$

For any two events A and B: $P(A) = P(A \cap B) + P(A \cap B')$

Permutations: $P_k^n = n \times (n-1) \times (n-2) \times \ldots \times (n-k+1) = \dfrac{n!}{(n-k)!}$

Combinations: $C_k^n = \dbinom{n}{k} = \dfrac{n!}{k!(n-k)!}$, where $n! = n \times (n-1) \times (n-2) \times \ldots \times 2 \times 1$

Linear Combinations of Random Variables:

If $Y = a_1X_1 + a_2X_2 + \ldots + a_nX_n + b$, $E(Y) = a_1E(X_1) + a_2E(X_2) + \ldots + a_nE(X_n) + b$

If $X_1, X_2, \ldots, X_n$ are independent, $Var(Y) = a_1^2 Var(X_1) + a_2^2 Var(X_2) + \ldots + a_n^2 Var(X_n)$

Discrete Variables:

$\mu = E(X) = \sum_{i=1} x_i p_i$, $F(x) = P(X \le x) =$ Sum of probabilities p_i for $x_i \le x$,

$$\sigma^2 = Var(X) = E(X^2) - [E(X)]^2 = \sum_{i=1}(x_i - \mu)^2 p_i = \sum_{i=1} x_i^2 p_i - \mu^2$$

Distribution	Probability mass function	Mean	Variance
Binomial	$f(x) = P(X = x) = \binom{n}{x} p^x (1-p)^{n-x}$	$E(X) = np$	$Var(X) = np(1-p)$
Geometric	$f(x) = P(X = x) = (1-p)^{x-1} p$	$E(X) = \frac{1}{p}$	$Var(X) = \frac{(1-p)}{p^2}$
Negative Binomial	$f(x) = P(X = x) = \binom{x-1}{r-1}(1-p)^{x-r} p^r$	$E(X) = \frac{r}{p}$	$Var(X) = \frac{r(1-p)}{p^2}$
Poisson	$f(x) = P(X = x) = \frac{e^{-\lambda}\lambda^x}{x!}$	$E(X) = \lambda$	$Var(X) = \lambda$

Continuous Variables:

$\int_{-\infty}^{\infty} f(x)dx = 1$, $P(a \le X \le b) = \int_a^b f(x)dx$, $F(x) = P(X \le x) = \int_{-\infty}^{x} f(t)dt$

$\mu = E(X) = \int_{-\infty}^{\infty} xf(x)dx$, $\sigma^2 = Var(X) = E(X^2) - [E(X)]^2 = \int_{-\infty}^{\infty} x^2 f(x)dx - \mu^2$

Distribution	**Density function**	**Mean**	**Variance**	***F*(*x*)**
Uniform	$f(x) = \frac{1}{b-a}$	$E(X) = \frac{b+a}{2}$	$Var(X) = \frac{(b-a)^2}{12}$	$F(x) = \frac{x-a}{b-a}$
Exponential	$f(x) = \lambda e^{-\lambda x}$	$E(X) = \frac{1}{\lambda}$	$Var(X) = \frac{1}{\lambda^2}$	$F(x) = 1 - e^{-\lambda x}$

Normal: If X has a normal distribution with mean μ and variance σ^2, then $X \sim N(\mu, \sigma^2)$.

Standard normal: If $X \sim N(\mu, \sigma^2)$, then $Z = \frac{X-\mu}{\sigma} \sim N(0, 1)$ and $\Phi(z) = P(Z \le z)$.

Normal approximation to binomial: If X follows a binomial distribution with parameters n and p, then

$$P(a \le X \le b) \approx P(a - 0.5 \le Y \le b + 0.5),$$

where Y is normal with the mean np and the variance $np(1-p)$, under the assumption that $np \ge 15$ and $n(1-p) \ge 15$.

Sampling Distributions

Sample mean	Sample proportion
$E\left(\bar{X}\right) = \mu$	$E(\hat{p}) = p$
$Z = \frac{\bar{X} - \mu_{\bar{x}}}{\sigma_{\bar{x}}} = \frac{\bar{X} - \mu}{\sigma / \sqrt{n}} \sim N(0,1)$	$Z = \frac{\hat{p} - \mu_{\hat{p}}}{\sigma_{\hat{p}}} = \frac{\hat{p} \pm \frac{0.5}{n} - p}{\sqrt{\frac{p(1-p)}{n}}} \sim N(0,1)$
Normal if $n \ge 30$ OR population is normal	Normal if $np \ge 15$ and $n(1-p) \ge 15$