SOLUTIONS TO THE LAB 4 ASSIGNMENT

Question 1

As the Steel B sample has tunqstoid added to it (applying a treatment), the overall design would indicate an experiment. Since random sampling is present, population inferences can be applied to the population of interest. The population of interest could be described as all pieces of steel constructed by the manufacturer. With random assignment not mentioned, causal inferences cannot apply. Thus, it is uncertain whether steel type causes change in conductivity.

Note: No treatment is applied to the Steel A sample, so part of the design could arguably be only an observational study. Students may note this but the presence of a treatment permits "experiment" to apply to the whole design.

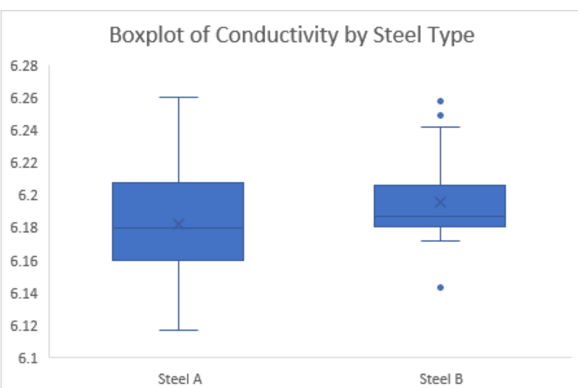
Question 2

(a) The summaries with the margin of error (at the very bottom) of a 98% two-sided confidence interval for each type are provided below.

Steel A		*Steel B*	
Mean	6.1824	Mean	6.1960
Standard Error	0.0071	Standard Error	0.0048
Median	6.1800	Median	6.1870
Mode	6.1630	Mode	6.1810
Standard Deviation	0.0388	Standard Deviation	0.0263
Sample Variance	0.0015	Sample Variance	0.0007
Kurtosis	-0.6902	Kurtosis	0.4334
Skewness	0.1191	Skewness	0.8924
Range	0.1430	Range	0.1150
Minimum	6.1170	Minimum	6.1430
Maximum	6.2600	Maximum	6.2580
Sum	185.4720	Sum	185.8790
Count	30	Count	30
Confidence Level(98.0%)	0.0175	Confidence Level(98.0%)	0.0118

The mean conductivity of Steel A is 6.1824×10^6 S/m, which is smaller than the mean conductivity of Steel B of 6.1960×10^6 S/m, but not by much given the magnitude of the standard deviations. There is also a minor difference in the standard deviations, with 0.0388 for Steel A and 0.0263 for Steel B. Overall, Steel B has the better conductivity with the higher value as well as lower variation.

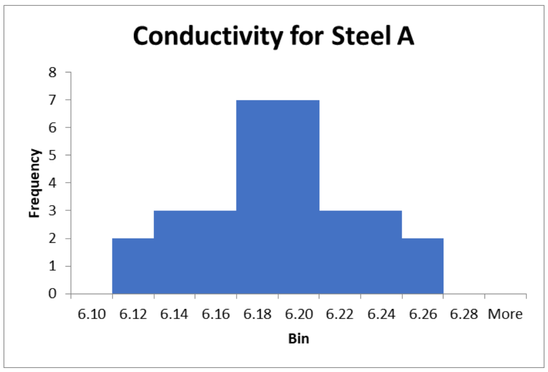
(b) The side-by-side boxplot of conductivity is displayed below.

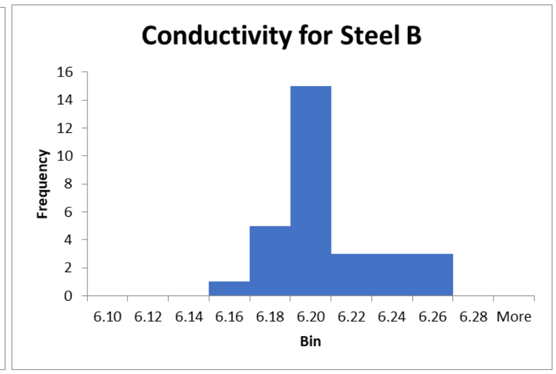


The Steel A distribution has whiskers of reasonably similar length (0.0430 for the lower whisker and 0.0527 for the upper whisker) yet the median is relatively closer to Q_1 than Q_3, indicating slight to moderate right-skewness. The Steel B distribution is more clearly right-skewed with the median notably closer to Q_1 than Q_3 as well as a longer upper whisker. The Steel A distribution has no outliers while the Steel B distribution has two upper outliers and one lower outlier. (The lower outlier has some impact on the shape, yet the forthcoming histogram reveals the overall shape.)

The median of Steel A (6.1800) is slightly smaller than the median of Steel B (6.1870). There is a reasonable difference in the spread of the two distributions. The spread of Steel A (IQR is 0.0473) is considerably higher than the spread of Steel B (IQR is 0.0252), but not more than double the lower value.

(c) The histograms for conductivity for the two steel types are displayed below.





Both graphs are unimodal and do not show any outliers visually. The Steel A histogram appears perfectly symmetric (and reasonably bell-shaped) while the Steel B histogram displays prominent right-skewness. Thus, the Steel B histogram displays evidence that the normality assumption is moderately unsatisfied while the Steel A histogram may indicate agreement with normality.

Question 3

(a) Based on the output in Question 2, the 98% two-sided confidence intervals for Steel A and Steel B types are respectively equal to $6.1824 \pm 0.0175 = (6.1649, 6.1999)$ and $6.1960 \pm 0.0118 = (6.1842, 6.2078)$.

With 98% confidence, the population mean conductivity of Steel A pieces is between 6.1649×10^6 and 6.1999×10^6 S/m. With 98% confidence, the population mean conductivity of Steel B pieces is between 6.1842×10^6 and 6.2078×10^6 S/m.

(b) The 98% two-sided confidence interval for Steel A is entirely below 6.200×10^6 S/m (though just barely), so there is sufficient evidence for this type that the target mean is not met. The 98% two-sided confidence interval for Steel B captures the target value of 6.200×10^6 S/m, however, so there is insufficient evidence the mean target is not met (in other words, the target mean value *may* have been met).

Question 4

(a) Since Question 3 used 98% confidence ($1 - \alpha = 0.98$) for a two-sided interval, the appropriate significance level for a corresponding two-sided test would be $\alpha = 0.02$.

Let μ_1 be the population mean conductivity of Steel A pieces and μ_2 be the population mean conductivity of Steel B pieces. To see whether the population mean of the Steel A pieces does not meet the mean target value, define the null and alternative hypotheses as follows.

H_0: $\mu_1 = 6.200$ vs. H_A: $\mu_1 \neq 6.200$

As the standard deviation σ is unknown, the t-test should be used. The test statistic seen below follows a t-distribution with $df = n - 1 = 30 - 1 = 29$ degrees of freedom. Note that SE stands for standard error.

$$t_0 = \frac{\overline{x} - \mu_0}{s / \sqrt{n}} = \frac{\overline{x} - \mu_0}{SE} \sim t_{n-1} = t_{29}$$

Using the value of the sample mean and standard error provided by the output in Question 2, the value of the test statistic, to four decimal places, is as follows.

$$t_0 = \frac{\overline{x} - \mu_0}{s / \sqrt{n}} = \frac{6.1824 - 6.200}{0.0388 / \sqrt{30}} = -2.4823$$

The two-sided *p*-value can be obtained with the TDIST() function in Excel. Using the unrounded test statistic value, the *p*-value is 0.0191 (to four decimal places). Since $\alpha = 0.02 > p$-value, reject H_0. There is sufficient evidence that the mean conductivity of the Steel A pieces does not meet the target value.

Now repeat the same procedure for the Steel B pieces with the following hypotheses.

H_0: $\mu_2 = 6.200$ vs. H_A: $\mu_2 \neq 6.200$

As in case of the Steel A pieces, the test statistic follows a *t*-distribution with $df = n - 1 = 30 - 1 = 29$ degrees of freedom. Using the value of the sample mean and standard error provided by the output in Question 2, the value of the test statistic, to four decimal places, is as follows.

$$t_0 = \frac{\overline{x} - \mu_0}{s / \sqrt{n}} = \frac{6.1960 - 6.200}{0.0263 / \sqrt{30}} = -0.8415$$

Using the unrounded test statistic value, the *p*-value is 0.4069 (to four decimal places). Since $\alpha = 0.02 < p$-value, do not reject H_0. There is insufficient evidence that the mean conductivity of the Steel B pieces does not meet the target value of 6.200×10^6 S/m.

(b) In order to make the tests in part (a) valid, we need to assume that the mean conductivity for each type approximately follows a normal distribution, simple random samples, and the observations are independent. Given the sample sizes (30), the Central Limit Theorem applies, so the distribution of each sample mean is at least approximately normal. This is sufficient for approximate normality, despite both histograms having right-skewness, Steel B having higher skewness. The data collection indicates that each sample is random and though it does not explicitly mention independent observations, having random samples makes this assumption highly plausible.

These are the same assumptions as the intervals in Question 3 because the hypothesis test and confidence interval are intentionally opposite procedures of each other that need the same assumptions to hold to carry out the proper statistical inference.

Question 5

(a) As before, we let μ_1 be the mean conductivity of Steel A pieces and μ_2 be the mean conductivity of Steel B pieces. To test that the mean conductivity of steel B type is higher than the steel A type, define the null and alternative hypotheses as follows:

H_0: $\mu_1 - \mu_2 \geq 0$ (no difference in mean conductivity of A and B steel types)
H_A: $\mu_1 - \mu_2 < 0$ (mean conductivity of steel B type is higher)

As the observations in the two groups are independent and the population standard deviations are unknown, a two independent sample *t*-test should be applied. According to material discussed in class, if the sample sizes are approximately the same, both sizes are at least 15, and the larger standard deviation is no more than twice as large as the smaller standard deviation, the *t*-test assuming equal yet unknown population variances (or pooled *t*-test) is recommended.

The sample sizes for the two groups (30) are the same, larger than 15, and the larger standard deviation (0.0388) is less than twice as large as the smaller standard deviation (0.0263). Thus, all 3 conditions are met and the *t*-test assuming equal yet unknown population variances (or pooled variance procedure) should be used in this case.

The Excel output is below.

t-Test: Two-Sample Assuming Equal Variances

	Steel A	*Steel B*
Mean	6.1824	6.195967
Variance	0.001508	0.000689
Observations	30	30
Pooled Variance	0.001099	
Hypothesized Mean Difference	0	
df	58	
t Stat	-1.58524	
P(T<=t) one-tail	0.059175	
t Critical one-tail	1.671553	
P(T<=t) two-tail	0.11835	
t Critical two-tail	2.001717	

The test statistic is –1.5852 (to four decimal places). The test statistic follows a *t*-distribution with 58 degrees of freedom and has a corresponding one-sided *p*-value of 0.0592 (to four decimal places). Since the *p*-value is between 0.05 and 0.10 (defined JA range), there is suggestive to moderate (and, thus, inconclusive) evidence against H_0, so do not reject H_0. There is insufficient evidence that the mean conductivity of steel B type is higher than the steel A type.

(b) Beyond assuming the population variances are unknown yet equal (as checked in part (a)), we assume that the data are two independent simple random samples, each drawn from a normally distributed population or have sample sizes of at least 30. Given the sample sizes of $n_1 = 30$ and $n_2 = 30$, the distributions of the respective sample means are approximately normal. The data collection method suggests the samples are random and independent (both between and within the samples).

(c) Since the alternative hypothesis above is a lower-tailed test, than a one-sided 95% upper confidence bound is the most appropriate 95% confidence interval/bound. Lastly, since the test in part (a) did not reject the claimed value of zero, the one-sided 95% upper confidence bound should be more than the value of zero. In other words, the one-sided 95% upper confidence interval should be $(-\infty, b)$, where $b > 0$.

Note: Though not required for marks, the 95% upper confidence bound is $(-\infty, 0.000739)$.

Question 6

(a) Let μ_T be the mean conductivity of Steel B pieces after tunqstoid combination and μ_B be the mean conductivity of Steel B pieces before tunqstoid combination. As the conductivity of the same pieces were measured twice, the paired *t*-test is appropriate to see whether there is evidence of any combination change. Define the null and alternative hypotheses as follows with $\mu_D = \mu_T - \mu_B$.

H_0: $\mu_D = 0$ (no change in the mean conductivity of Steel B pieces after the combination)
H_A: $\mu_D > 0$ (the combination increased the mean conductivity of Steel B pieces)

The paired *t*-test output is below. The test statistic is 4.6406 (to four decimal places). The test statistic follows a *t*-distribution with 29 degrees of freedom and has a corresponding one-sided *p*-value of 3.4399×10^{-5} in scientific notation (with four decimal places). Since the *p*-value is between 0 and 0.01

(defined JA range), there is strong to convincing evidence against H_0, so we reject H_0. There is sufficient evidence that the combination increased the mean conductivity of Steel B pieces.

t-Test: Paired Two Sample for Means

	Tunqstoid	*Steel B*
Mean	6.2172667	6.195967
Variance	0.000122	0.000689
Observations	30	30
Pearson Correlation	0.3088831	
Hypothesized Mean Difference	0	
df	29	
t Stat	4.6406428	
P(T<=t) one-tail	3.44E-05	
t Critical one-tail	1.699127	
P(T<=t) two-tail	6.88E-05	
t Critical two-tail	2.0452296	

(b) To obtain the 95% confidence interval, create a new variable, *Change*, defined as the difference in conductivity between *Tunqstoid* and *Steel B*. Enter the formula (= C2 – B2) into cell D2 and copy the formula down for the remaining 29 observations.

Then use the *Descriptive Statistics* feature for the *Change* variable. The *Descriptive Statistics* output is shown below (note that 90% confidence must be used to get a 95% one-sided interval):

Change	
Mean	0.0213
Standard Error	0.00459
Median	0.027
Mode	0.033
Standard Deviation	0.02514
Sample Variance	0.000632
Kurtosis	1.492034
Skewness	-1.00091
Range	0.114
Minimum	-0.037
Maximum	0.077
Sum	0.639
Count	30
Confidence Level(90.0%)	0.007799

According to the above output, the estimate of mean change in conductivity of Steel B pieces after combination is equal to 0.0213 and a 95% one-sided confidence interval/bound for the change is 0.0213 – 0.0078 = (0.0135, ∞).

With 95% confidence, the mean change in conductivity is between 0.0135 and ∞. As the interval does not contain zero and is entirely positive, there is sufficient evidence that the combination has increased the mean conductivity. This is consistent with the outcome of the test in part (a).

(c) The assumptions are that the samples are paired and that the 30 differences are a simple random sample of independent observations from a normal distribution or the sample size is at least 30. Given the sample size (n = 30), the Central Limit Theorem applies and the distribution of the sample mean difference is approximately normal. This is sufficient for approximate normality, despite the histogram of *Change* being left-skewed. The data collection also suggests the sample is random.

MARKING SCHEMA TO THE LAB 4 ASSIGNMENT

Question 1 (6)

Type of study: 2 points
Population inferences: 2 points
Causal inferences: 2 points

Question 2 (37)

(a) Summary statistics: 3 points
Comparison of means and standard deviations: 2 points each (4 points total)
Better type: 2 points
(b) Correctly formatted side-by-side boxplots: 4 points
Analysis of the shape of each boxplot: 2 points each (4 points total)
Comparing the centers and spreads: 2 points each (4 points total)
(c) Correctly formatted histogram: 4 points each (8 points total)
Analysis of the shape of each histogram: 3 points each (6 points total)
Comments about normality: 2 points

Question 3 (12)

(a) 98% two-sided confidence interval for each type: 2 points each (4 points total)
Interpretation of each confidence interval: 2 points each (4 points total)
(b) Relating to target mean value: 2 points each (4 points total)

Question 4 (23)

(a) Appropriate significance level: 1 point
Test for Steel A: 8 points
(hypotheses: 2, distribution: 1, test statistic: 2, *p*-value: 1, conclusion: 2)
Test for Steel B: 8 points
(b) Assumptions: 2 points
Checking assumptions: 2 points
Relating assumptions to Question 3: 2 points

Question 5 (21)

(a) Choosing appropriate test: 2 points
Output: 3 points
Test to compare Steel A and B types: 8 points
(hypotheses: 2, distribution: 1, test statistic: 2, *p*-value: 1, conclusion: 2)
(b) Assumptions: 2 points
Checking assumptions: 2 points
(c) Appropriate confidence interval/bound: 2 points
Conclusion about claimed value in part (a) in relation to interval/bound: 2 points

Question 6 (27)

(a) Choosing appropriate test: 2 points
Output: 3 points
Test for the combination change: 8 points
(hypotheses: 2, distribution: 1, test statistic: 2, *p*-value: 1, conclusion: 2)

(b) Output: 3 points
Confidence interval for the combination change: 3 points
Interpretation of confidence interval: 2 points
Consistency of the interval with the outcome of the test in part (a): 2 points

(c) Assumptions: 2 points
Checking assumptions: 2 points

TOTAL = 126