STAT235 FINAL EXAM VERSION – Long Answer Questions

Instructions

1. In the second part, there are **two** long-answer problems. Show all your work because partial credit may be given in this part. Final numerical answers should have **THREE** significant decimal places (such as 0.00235) unless related to the z-table (probabilities are 4 decimal places, z-scores are 2).

2. When asked to "carry out a full hypothesis test ", set up the hypotheses, briefly discuss and check assumptions, calculate the test statistic, state the distribution of the test statistic (like t_9 or z), approximate the p-value (or its range), and state your conclusion in plain English.

3. When asked for a "confidence interval/bound", briefly discuss and check assumptions, state the estimate, the standard error, and the critical value. Then, calculate and interpret the interval.

LONG ANSWER QUESTIONS

1. **(13 total marks)** A manufacturing company of exhaust ports for intergalactic space stations receives customer complaints, claiming the ports are too wide. To be certain, the engineer randomly samples 45 ports from the warehouse and measures the diameter of each one (in metres). From the sample, they found a mean of 1.977 m and a standard deviation of 0.11 m.

(a) (6 marks) Is there enough evidence to conclude that the average diameter less than two metres? Carry out a full hypothesis test. (See instructions on page 1.)

H_0: $\mu \geq 2$ H_A: $\mu < 2$

Assumptions: Random sample? Yes, it is a simple random sample.
Independent observations? Yes, random sample should justify independence.
Normality? Yes. Normal population not mentioned yet $n = 40 > 30$.
→ Since σ is unknown, we use a one-mean sample t-test.

$$t_0 = \frac{\bar{x} - \mu_0}{s / \sqrt{n}} = \frac{1.977 - 2}{0.11 / \sqrt{45}} = \frac{-0.023}{0.164} = -1.403$$

Distribution: $t_0 \sim t_{n-1} = t_{44} \approx t_{40}$

$-1.684 < t_0 = -1.403 < -1.303$
$0.05 < p\text{-value} < 0.10$ → The p-value range is (0.05, 0.10).

Conclusion: Using the judgment approach, we have moderate to suggestive evidence against H_0.
→ Do not reject H_0.
→ There is insufficient evidence that the average diameter is less than 2 m.

(b) (7 marks) As time went on, the company received more complaints, so one of the head scientists (Galen Erso) decided to investigate to see if the cause was a particular employee. He decided to randomly select two exhaust ports each from 10 random employees, yielding the data below. Previous studies determined that diameter (in metres) follows a normal distribution.

Summary statistic	Port 1	Port 2	Difference
Average	1.980	2.015	-0.035
Standard Deviation	0.580	0.230	0.520

Is there any evidence that the average difference in diameter indicates a change? Construct the best corresponding 90% confidence interval/bound to answer the question. (See instructions on page 1.)

Assumptions: Paired samples? Yes, samples are paired because there are 2 ports per employee.
Random sample? Yes, there is random sample of differences.
Independent observations? Yes, random sample of differences should justify independence.
Normality? Yes. Though $n = 10 < 30$, population is said to be normal.

Let $d = x_1 - x_2$

$$\bar{d} \pm t_{\alpha/2,\, n-1} \times \left(\frac{s_d}{\sqrt{n}}\right) \rightarrow -0.035 \pm (1.833) \times \left(\frac{0.520}{\sqrt{10}}\right) \rightarrow -0.035 \pm 0.301 \rightarrow (-0.336, 0.266)$$

With 90% confidence, the average difference of diameter is between -0.336 m and 0.266 m.

Since zero is in the interval, there is insufficient evidence of a difference in average diameters within employees at $\alpha = 0.10$.

(Note that Erso did not quite investigate the issue, so he's either a bad scientist or doesn't like the people he's working for and they don't understand statistics well enough to question him.)

2. **(12 total marks)** In a hardness test, a steel ball is pressed into the material being tested at a standard load. The diameter of the indentation is measured (in mm), relating to hardness. Two types of steel balls are available and their performance is compared on 20 random specimens: 10 specimens for the first ball and another 10 specimens for the second ball. It is known that indentation diameter follows a normal distribution. The results are given in the following table:

Summary statistic	Ball 1	Ball 2	Difference
Average	58.500	55.000	3.500
Standard Deviation	3.467	6.167	5.595

(a) (8 marks) Is there enough evidence to conclude a difference in the average hardness measurements of the two ball types? Carry out a full hypothesis test. (See instructions on page 1.)

H_0: $\mu_1 - \mu_2 = 0$
H_A: $\mu_1 - \mu_2 \neq 0$

Assumptions:

Random sample? Yes. Random sampling is mentioned.
Independent observations? Yes, random sampling should justify independence.
Independent samples? Independence is not mentioned yet data are made using two different balls; independence might be a reasonable assumption. Failure of this is an issue.
Normality? Both $n_1 = 10$ & $n_2 = 10 < 30$, yet indentation diameter is known to be normal.
Equal variance? Sample sizes are equal ($n_1 = n_2 = 10$) and ratio is $6.167/3.467 < 2$, but sample sizes are both too small (< 15), so we cannot assume equal variances.

→ We use the two independent sample t-test not assuming equal variances (general procedure).

$$t_0 = \frac{\bar{x}_1 - \bar{x}_2 - \delta_0}{\sqrt{\frac{s_1^2}{n_1} + \frac{s_2^2}{n_2}}} = \frac{58.500 - 55.000 - 0}{\sqrt{\frac{3.467^2}{10} + \frac{6.167^2}{10}}} = 1.564$$

Distribution: $df \geq \min\{n_1 - 1, n_2 - 1\} = \min\{10 - 1, 10 - 1\} = 9$

The distribution of test statistic when the null hypothesis is true is t_9.

Since $1.383 < t_0 = 1.564 < 1.833$, we conclude that $2(0.05) = 0.10 < p\text{-value} < 2(0.10) = 0.20$.
Therefore, the p-value range is (0.10, 0.20).

Conclusion: There is little to no evidence against H_0. There is insufficient evidence to conclude that the average hardness measurements of two balls are different.

(b) (4 marks) Construct the best corresponding 95% confidence interval/bound for the difference in the average hardness measurements of two balls. (See instructions on page 1.)

Assumptions: Same as above. All hold such that the general procedure is used.

Estimate: $\bar{x}_1 - \bar{x}_2 = 58.500 - 55.000 = 3.500$

Standard Error: $S.E.(\bar{x}_1 - \bar{x}_2) = \sqrt{\frac{s_1^2}{n_1} + \frac{s_2^2}{n_2}} = \sqrt{\frac{3.467^2}{10} + \frac{6.167^2}{10}} = 2.237$

Degrees of freedom: $df \geq \min\{n_1 - 1, n_2 - 1\} = \min\{10 - 1, 10 - 1\} = 9$

Critical Value: $t_{0.025,\,9} = 2.262$

$$\bar{x}_1 - \bar{x}_2 \pm t_{0.025, df=14}\sqrt{\frac{s_1^2}{n_1} + \frac{s_2^2}{n_2}} = 3.500 \pm (2.262)(2.237) \Rightarrow 3.500 \pm 5.061 \Rightarrow (-1.561, 8.561)$$

Conclusion: It is estimated with 95% confidence that the average hardness measurements for the first ball is between 1.561 mm smaller to 8.561 mm larger than the average hardness measurements for the second ball. (Note that zero is inside the interval, which would coincide with the result of the test, using α = 0.05.)