P1. (a) With the increase of the mean temperature, shape of the density curve remains the same due to the unchanged value of σ. The position of the whole density curve shifts right-wards as the mean temperature μ increases from 205°F to 215°F.

The proportion of manufactured unacceptable engines remains the same as μ increases from 205°F to 210°F, and increases as μ increases from 210°F to 215°F. Because for the increase in μ from 210°F to 215°F, μ gets closer to the boundary ‘225°F’ of unacceptable engines, it results in a larger area the under the curve exceeds the boundary.

(b) With the increase of the σ of temperature and unchanged μ of temperature, the density curve gets wider. Since the probability in total under a normal distribution curve is 1, the density curve gets flatter as it gets wider.

P2.

|  |  |  |  |
| --- | --- | --- | --- |
| **PART** | **PARAMETERS** | **PROBLEM** | **ANSWER** |
| (a) | m = 208 and s = 7.25 | Proportion of unacceptable |  |
| m = 215 and s = 7.25 | Proportion of unacceptable |  |
| (b) | m = 208 and s = 4.75 | Proportion of unacceptable |  |
| (c) | m = 208 and s = 6.25 | Within 1 standard deviation |  |
| Within 2 standard deviations |  |
| Within 3 standard deviations |  |
| (d) | m = 208 and s = 6.25 | Temp exceeded by 90% |  |
| Temp exceeded by 95% |  |

P3. (a)

n=20, # of unacceptable engines for sample = 1, relative frequency = 0.0500

n=60, # of unacceptable engines for sample = 3, relative frequency = 0.0500

n=400, # of unacceptable engines for sample = 7, relative frequency = 0.0175

population, ideal relative frequency = 0.0160 (P2(a))

For all 3 samples, relative frequencies are not consistent with the theoretical one in P2(a).

The largest sample of n=400 produces the value closest to the predicted value.

(b)

|  |  |  |
| --- | --- | --- |
| n=20 | Count | Proportion |
| 1σ | 13 | 0.6500 |
| 2σ | 18 | 0.9000 |
| 3σ | 20 | 1.0000 |
| n=60 |  |  |
| 1σ | 35 | 0.5833 |
| 2σ | 53 | 0.8833 |
| 3σ | 60 | 1.0000 |
| n=400 |  |  |
| 1σ | 271 | 0.6775 |
| 2σ | 377 | 0.9425 |
| 3σ | 400 | 1.0000 |

The Empirical rule is the 68-95-99.7 rule, 68% of frequency shall be within 1SD, 95% of frequency shall be within 2SD, 99.7% of frequency shall be within 3SD. The frequencies of sample n=20, n=60 are not consistent with the empirical rule since 0.65 and 0.583 have a significant difference comparing to the ideal 68%, 0.9000 and 0.8833 have a significant difference comparing to the ideal 95% as well. The sample of n=400 is the closest approach to the empirical rule, they are basically the same.

(c)

The plotted points (observations) fall approximately along a straight line.

The straight, diagonal line without skewness means that we have a set of normally distributed data. All the data fit with a normal distribution without any exception that falls away from the straight line.

If there is any data that doesn’t not follow the normal distribution, it will reflect on the normal probability plot that we will have a (some) point(s) far away from the fitting curve.

P4. (a) By the theory,

Consider sample size does not satisfy the condition n>=30, we cannot apply the central limit theorem on the random sample of 20. Also, in practice, the larger the sample size, the closer the approximation to a normal distribution.

(b)

The shape of the histogram is bimodal, left-skewed, outliers exist in between temperature 203°F to 204°F, and 205°F to 206°F, in total, 1+5=6 outliers.

(c) Mean of the averages = 208.0307

Standard deviation of the averages = 1.3655

μ and σ of the averages should be basically the same as the values predicted by theory.

The mean is approximately the same as the theoretical mean 208. The standard deviations have a significant difference between practical and theoretical values, may due to the unstable small amount of sample size n=20 applied.

P5. (a) By the theory,

Consider sample size satisfies the condition n>=30, and n>>30 we apply the central limit theorem on the random sample of 400. Also, in practice, the larger the sample size, the closer the approximation to a normal distribution.

(b)

The shape of the histogram is unimodal, a little right-skewed, no outliers.

(c) Mean of the averages = 207.9278

Standard deviation of the averages = 0.2483

μ and σ of the averages should be close but not identical to the values predicted by theory.

The spread of the sample means (the standard deviation of the sample means) gets smaller with the increase in the sample size. Also, the standard deviation for sample size n=400 is obviously closer to zero compared to n=20 in P4.