Machine Learning Optimization

KAI RU

Keio university

ABSTRACT: Convex optimization like Mean Variance Optimization solutions tend to be unstable, to the point of entirely offsetting the benefits of optimization. For example, in the context of financial applications, it is known that portfolios optimized in-sample often underperform the naïve (equal weights) allocation out-of-sample. This instability can be traced back to two sources: (i) noise in the input variables; and (ii) signal structure that magnifies the estimation errors in the input variables. A first innovation of this paper is to introduce the nested clustered optimization algorithm (NCO), a method that tackles both sources of instability.

Over the past 60 years, various approaches have been developed to address these two sources of instability. These approaches are flawed in the sense that different methods may be appropriate for different input variables, and it is unrealistic to expect that one method will dominate all the rest under all circumstances. Accordingly, a second innovation of this paper is to introduce MCOS, a Monte Carlo approach that estimates the allocation error produced by various optimization methods on a particular set of input variables. The result is a precise determination of what method is most robust to a particular case. Thus, rather than relying always on one

particular approach, MCOS allows users to apply opportunistically whatever optimization method is best suited in a particular setting.

We will compare all three methods of optimizatio, Machine Learning Optimization, Black Litterman and Mean Variance Optimization in this paper by optimizing 20 stocks in US market.

1. The goal: Build a portfolio from the US stock market, simulate a three-month short-term investment, and evaluate the actual return by comparing the two models, the mean variance model and the Black Litterman model. The training period will be from October 1, 2012 to September 24, 2018. The simulation period is from September 25, 2018 to September 24, 2019. Set the brand to 20.

(1) As external information, it is first necessary to know the risk-free interest rate and market price. Measured using the 52 Week Treasury Bill as a risk-free interest rate.

In [1]:
```
#Simulation period
import datetime
datetime.datetime(2018, 9, 24)-datetime.datetime(2012, 10, 1)
```

Out[1]:
```
datetime.timedelta(days=2184)
```

In [2]:
```
import datetime
datetime.datetime(2019, 9, 24)-datetime.datetime(2018, 9, 25)
```

Out[2]:
```
datetime.timedelta(days=364)
```

In [3]:
```
import quandl
quandl.ApiConfig.api_key = 'DxKMsvF36hXo5BAMpeDK'
Wk_Bank_Discount_Rate_52=quandl.get("USTREASURY/BILLRATES" ,
                          start_date=datetime.datetime(2012, 10, 1),
                          end_date=datetime.datetime(2019, 9, 12))
```

In [4]:
```
2184/(52*7)
```

Out[4]:
```
6.0
```

In [5]:
```
#Downloading bond price
yield_list=[]
for i in range(6):
    yield_list.append(Wk_Bank_Discount_Rate_52[datetime.datetime(2012, 10, 1)+datetime.timedelta(days=364*i):]\
                      ["52 Wk Bank Discount Rate"][0])
```

In [6]:
```
yield_list
```

Out[6]:
```
[0.16, 0.09, 0.1, 0.32, 0.56, 1.27]
```

In [ ]:

Simulation period Yield from October 1, 2012 to September 12, 2019 S = (1 + S0) x (1 + S1) x (1 + S2) x (1 + S3) x (1 + S4) x(1 + S5) -1

In [7]:
```
S=(1+yield_list[0]/100)*(1+yield_list[1]/100)*(1+yield_list[2]/100)*\
(1+yield_list[3]/100)*(1+yield_list[4]/100)*(1+yield_list[5]/100)-1
```

In [8]:
```
S
```

Out[8]:
```
0.025209638953526792
```

If you invest $1 in the bond on October 1, 2012, you will have an asset of 1.025 on September 12, 2019. This is defined as a safe asset, and the interest rate of this safe asset is a risk-free interest rate.

In [9]:
```
risk_free=S
```

In [10]:
```
risk_free
```

Out[10]:
```
0.025209638953526792
```

In [11]:
```
risk_free_annual=risk_free/6
```

In [12]:
```
risk_free_annual
```

Out[12]:
```
0.004201606492254466
```

(3) Download the selected brand as Training Datasets

In [13]:

```
import pandas_datareader as pdr
import numpy as np
import pandas as pd
from scipy import stats
dateparse = lambda dates: pd.datetime.strptime(dates, '%Y-%m-%d')

from matplotlib import pylab as plt
import seaborn as sns
%matplotlib inline


from matplotlib.pylab import rcParams
rcParams['figure.figsize'] = 15, 6


data=pd.DataFrame([])
name=["AAPL","GOOGL","MCD","GM","XOM","BRK-A","MSFT","WFC","AMZN","FB","JPM","V",
             "WMT","MA","PG","BAC","T","INTC","UNH","DIS"]

columns=["APPLE","GOOGLE","McDonalds","GM","XOM","BRK","MSFT","WFC","AMZN","FB","JPM","VISA",
             "WMT","MA","PG","BAC","ATT","Intel","UnitedHealth_Group","The_Walt_Disney"]

for idx,stock in enumerate(name):
    names = pdr.get_data_yahoo(stock, start=datetime.datetime(2012, 10, 1),
                                  end=datetime.datetime(2018, 9, 24))
    j=columns[idx]
    data[j]=names["Adj Close"]
```

```
/usr/local/anaconda3/lib/python3.7/site-packages/pandas_datareader/compat/__init__.py:7: FutureWarning: panda
s.util.testing is deprecated. Use the functions in the public API at pandas.testing instead.
  from pandas.util.testing import assert_frame_equal
```

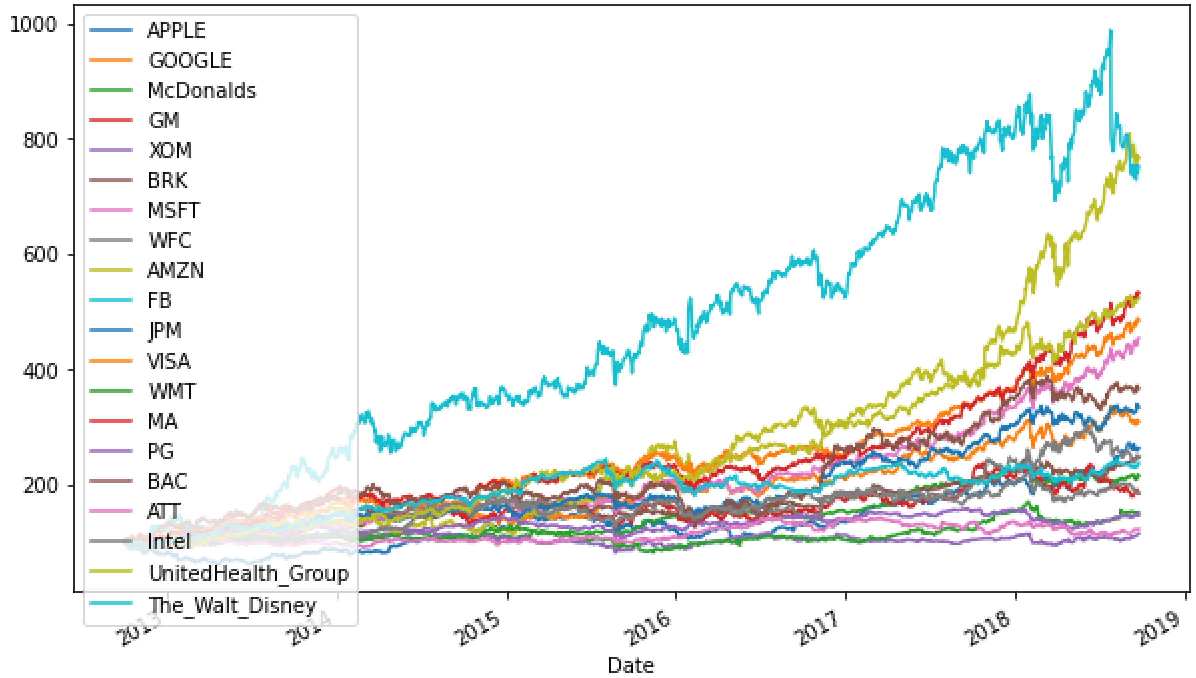
(4) Plot time series transition and rate of return

In [ ]:

```
```

In [23]:
```
(data / data.iloc[0] * 100).plot(figsize=(10, 6))
```

Out[23]: <matplotlib.axes._subplots.AxesSubplot at 0x7faced449d50>



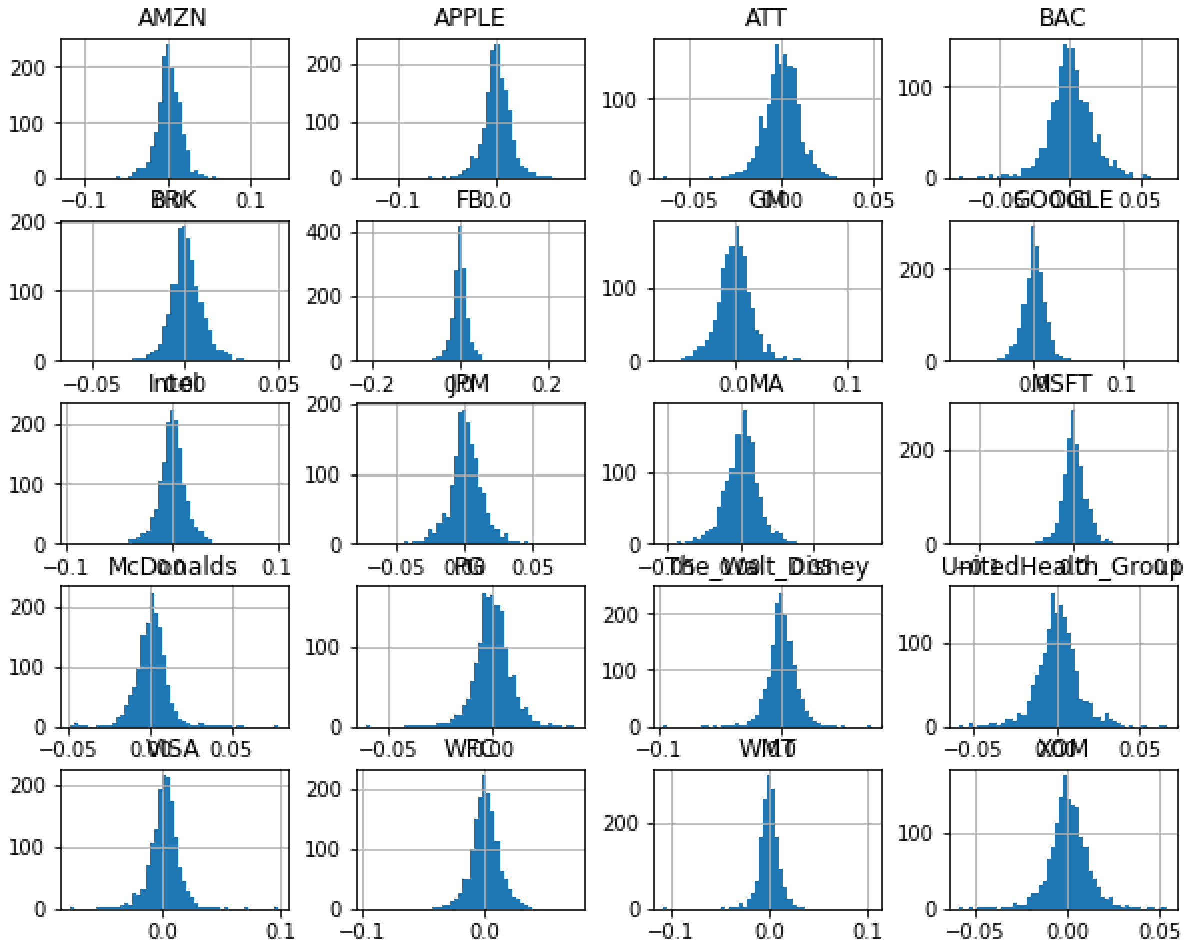
In [24]:
```
log_returns = np.log(data / data.shift(1))
log_returns.head()
```

Out[24]:

Date	APPLE	GOOGLE	McDonalds	GM	XOM	BRK	MSFT	WFC	AMZN	FB	JPM	VISA	WMT
2012-10-01	NaN	NaN	NaN	NaN	NaN	NaN	NaN	NaN	NaN	NaN	NaN	NaN	NaN
2012-10-02	0.002908	-0.006308	-0.011590	0.025231	-0.000872	0.001705	0.005748	0.003452	-0.005611	0.012653	-0.001221	-0.005354	-0.004060
2012-10-03	0.015217	0.007252	-0.006399	0.029542	-0.000218	0.006270	0.006721	0.017649	0.021007	-0.019955	0.005891	0.018360	0.006083
2012-10-04	-0.006950	0.007252	0.007498	0.010604	0.005655	0.009029	0.005677	0.014844	0.017623	0.005482	0.023223	0.008268	0.006983
2012-10-05	-0.021541	-0.000521	-0.000330	0.006067	0.003572	0.002023	-0.006012	-0.003621	-0.007553	-0.048540	-0.002634	0.004215	0.005472

In [25]:
```
log_returns.hist(bins=50, figsize=(10, 8))
```

Out[25]:
```
array([[<matplotlib.axes._subplots.AxesSubplot object at 0x7faced70eb90>,
        <matplotlib.axes._subplots.AxesSubplot object at 0x7faced726150>,
        <matplotlib.axes._subplots.AxesSubplot object at 0x7facec1ef790>,
        <matplotlib.axes._subplots.AxesSubplot object at 0x7facec225d50>],
       [<matplotlib.axes._subplots.AxesSubplot object at 0x7facec268410>,
        <matplotlib.axes._subplots.AxesSubplot object at 0x7faced741a90>,
        <matplotlib.axes._subplots.AxesSubplot object at 0x7faced783150>,
        <matplotlib.axes._subplots.AxesSubplot object at 0x7faced7ba7d0>],
       [<matplotlib.axes._subplots.AxesSubplot object at 0x7faced7c2a50>,
        <matplotlib.axes._subplots.AxesSubplot object at 0x7facee015250>,
        <matplotlib.axes._subplots.AxesSubplot object at 0x7facee077a50>,
        <matplotlib.axes._subplots.AxesSubplot object at 0x7facee0ba110>],
       [<matplotlib.axes._subplots.AxesSubplot object at 0x7facee0ef790>,
        <matplotlib.axes._subplots.AxesSubplot object at 0x7facee126e10>,
        <matplotlib.axes._subplots.AxesSubplot object at 0x7facee1674d0>,
        <matplotlib.axes._subplots.AxesSubplot object at 0x7facee19cb50>],
       [<matplotlib.axes._subplots.AxesSubplot object at 0x7facee1e0210>,
        <matplotlib.axes._subplots.AxesSubplot object at 0x7facee216890>,
        <matplotlib.axes._subplots.AxesSubplot object at 0x7facee24cf10>,
        <matplotlib.axes._subplots.AxesSubplot object at 0x7facee28f5d0>]],
      dtype=object)
```



2, mean variance model

(1) Model optimization

In [26]:
```
from pypfopt.efficient_frontier import EfficientFrontier
from pypfopt import risk_models
from pypfopt import expected_returns

mu = expected_returns.mean_historical_return(data)
S = risk_models.sample_cov(data,frequency=252)

#mean variance model optimization
EF_min = EfficientFrontier(mu, S)

EF_min.min_volatility()
#portfolio performance
EF_min.portfolio_performance(verbose=True)
```

```
Expected annual return: 11.6%
Annual volatility: 10.6%
Sharpe Ratio: 0.90
```

Out[26]:
```
(0.1155764576157817, 0.10646529100156261, 0.8977241006590485)
```

In [ ]:

In [ ]:

In [27]:
```
#CAPM理論に基づき、平均分散モデルを最適化
#無リスク金利を入れる
EF = EfficientFrontier(mu, S)
weights = EF.max_sharpe(risk_free_rate=risk_free_annual)
#ポートフォリオの年リターン、ボラティリティ、シャープ・レシオを求める
EF.portfolio_performance(verbose=True)
```

```
Expected annual return: 31.3%
Annual volatility: 16.0%
Sharpe Ratio: 1.83
```

Out[27]:
```
(0.31330291556754775, 0.16004365290494513, 1.832643221045132)
```

In [28]:
```
#各ウェイトをプリントする
EF.clean_weights()
```

Out[28]:
```
OrderedDict([('APPLE', 0.0),
             ('GOOGLE', 0.0),
             ('McDonalds', 0.02215),
             ('GM', 0.0),
             ('XOM', 0.0),
             ('BRK', 0.0),
             ('MSFT', 0.09304),
             ('WFC', 0.0),
             ('AMZN', 0.13933),
             ('FB', 0.10216),
             ('JPM', 0.0),
             ('VISA', 0.10452),
             ('WMT', 0.0),
             ('MA', 0.16295),
             ('PG', 0.0),
             ('BAC', 0.0),
             ('ATT', 0.0),
             ('Intel', 0.0),
             ('UnitedHealth_Group', 0.37584),
             ('The_Walt_Disney', 0.0)])
```

In [ ]:

(2) Download the data of each stock from September 13, 2019 to December 13, 2019 will be collected for simulation.

In [29]:
```
data2=pd.DataFrame([])
name2=["AAPL","GOOGL","MCD","GM","XOM","BRK-A","MSFT","WFC","AMZN","FB","JPM","V",
             "WMT","MA","PG","BAC","T","INTC","UNH","DIS"]

columns2=["APPLE","GOOGLE","McDonalds","GM","XOM","BRK","MSFT","WFC","AMZN","FB","JPM","VISA",
             "WMT","MA","PG","BAC","ATT","Intel","UnitedHealth_Group","The_Walt_Disney"]

for idx,stock in enumerate(name2):
    name = pdr.get_data_yahoo(stock, start=datetime.datetime(2018, 9, 25),
                              end=datetime.datetime(2019, 9, 24))
    j=columns[idx]
    data2[j]=name["Adj Close"]
```

(3) If managed from September 13, 2019 to December 13, 2019, the average return of the portfolio will be

R = 1r1 + w2r2 + ... + wn * rn

ri = Return of individual stock

wi = weight of individual stock

R = average revenue of the portfolio

In [ ]:

In [30]:
```
Mean_variance_return=np.sum(np.array(EF.weights)*np.array(expected_returns.mean_historical_return(data2, freque
```

In [31]:
```
Mean_variance_return
```

Out[31]:
```
0.052360018732317506
```

(4) Volatility of the mean variance model portfolio

In [32]:
```python
from pypfopt import objective_functions
objective_functions.portfolio_variance(EF.weights, risk_models.sample_cov(data2))
```

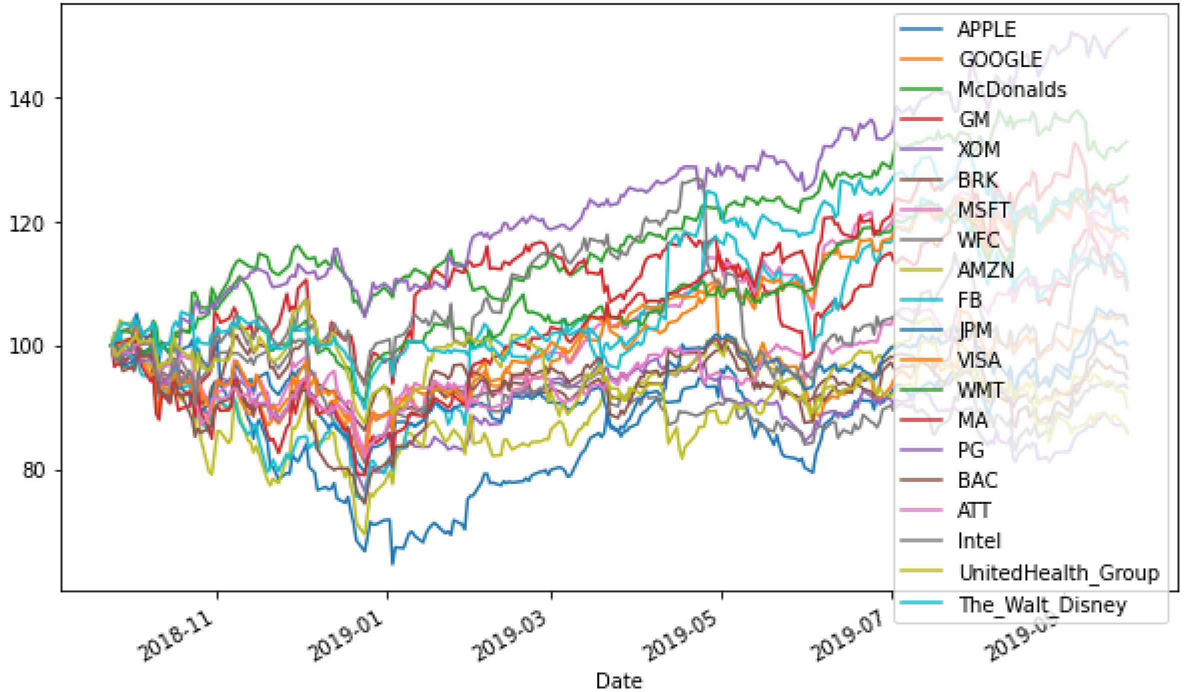
Out[32]:
```
0.05157591285429528
```

3. Black – Litterman model

(1) For the simulation period, calculate the return of each issue from September 13, 2019 to December 13, 2019

In [34]:
```python
(data2 / data2.iloc[0] * 100).plot(figsize=(10, 6))
```

Out[34]:
```
<matplotlib.axes._subplots.AxesSubplot at 0x7facef902450>
```



In [35]:
```
expected_returns.mean_historical_return(data2, frequency=252)
```

Out[35]:
```
APPLE                  0.052789
GOOGLE                 0.073778
McDonalds              0.299691
GM                     0.137557
XOM                   -0.132187
BRK                   -0.035906
MSFT                   0.232971
WFC                   -0.047611
AMZN                  -0.043119
FB                     0.147280
JPM                    0.057230
VISA                   0.194010
WMT                    0.258941
MA                     0.250547
PG                     0.434399
BAC                   -0.003515
ATT                    0.183480
Intel                  0.134533
UnitedHealth_Group    -0.122655
The_Walt_Disney        0.199716
dtype: float64
```

(3) Setting critic reviews for each brand

Referring to the above figures and data, For example, one critic predicts that after three months, Apple, United Health Group, Google, Microsoft, Facebook, McDonalds, Procter & Gamble ,will rise by 0.05, -0.12, 0.07, 0.23, 0.15, 0.3,0.43 and that other stocks are unknown. Then, use the Black Litterman model and set as follows

In [36]:
```
from pypfopt.black_litterman import BlackLittermanModel
S = risk_models.sample_cov(data)

viewdict = {"APPLE":  0.05,
           "UnitedHealth_Group": -0.12, "GOOGLE": \
            0.07,"MSFT": 0.23,"FB": 0.15,"McDonalds": 0.3,"PG": 0.43}
```

In [37]:
```
bl = BlackLittermanModel(S, absolute_views=viewdict)
rets = bl.bl_returns()
```

```
/usr/local/anaconda3/lib/python3.7/site-packages/pypfopt/black_litterman.py:252: UserWarning: Running Black-L
itterman with no prior.
  warnings.warn("Running Black-Litterman with no prior.")
```

(4) Calculate the return of each brand

In [38]:
```
rets
```

Out[38]:
```
APPLE                   0.081179
GOOGLE                  0.102669
McDonalds               0.179195
GM                      0.092806
XOM                     0.111946
BRK                     0.102497
MSFT                    0.165690
WFC                     0.091560
AMZN                    0.109526
FB                      0.114165
JPM                     0.104841
VISA                    0.125748
WMT                     0.113014
MA                      0.128785
PG                      0.224452
BAC                     0.089426
ATT                     0.107138
Intel                   0.126267
UnitedHealth_Group      0.009140
The_Walt_Disney         0.102901
dtype: float64
```

In [ ]:

(5) Introduce SP500 as market price

In [39]:
```
SP500 = pdr.get_data_yahoo('^GSPC',
                           start=datetime.datetime(2012, 10, 1),
                           end=datetime.datetime(2018, 9, 24))
```

In [40]:
```
market_prices=SP500["Close"]
```

In [41]:
```
# (market_prices.pct_change().values[1:].sum()/len(market_prices.pct_change().values[1:]))*252*8
```

In [ ]:
```

```

(6) The study period will be from October 1, 2012 to September 12, 2019.

In [42]:
```
from pypfopt import black_litterman

delta = black_litterman.market_implied_risk_aversion(market_prices,risk_free_rate=risk_free_annual)

ef = EfficientFrontier(rets, S)

bl.bl_weights(delta)
weights = bl.clean_weights()
```

In [43]:
```
bl.portfolio_performance(verbose=True)
```

```
Expected annual return: 26.5%
Annual volatility: 14.8%
Sharpe Ratio: 1.66
```

Out[43]:
```
(0.26542973424544986, 0.14759656473150295, 1.6628417788172642)
```

In [44]:
```
weights
```

Out[44]:
```
OrderedDict([('APPLE', -0.04205),
             ('GOOGLE', -0.0521),
             ('McDonalds', 0.4077),
             ('GM', 0.0),
             ('XOM', 0.0),
             ('BRK', 0.0),
             ('MSFT', 0.10176),
             ('WFC', 0.0),
             ('AMZN', 0.0),
             ('FB', 0.02402),
             ('JPM', 0.0),
             ('VISA', 0.0),
             ('WMT', 0.0),
             ('MA', 0.0),
             ('PG', 0.8193),
             ('BAC', 0.0),
             ('ATT', 0.0),
             ('Intel', 0.0),
             ('UnitedHealth_Group', -0.25864),
             ('The_Walt_Disney', 0.0)])
```

In [45]:
```
sum(weights.values())
```

Out[45]:
```
0.9999900000000002
```

In [ ]:
```

```

(7) If managed from September 13, 2019 to December 13, 2019, the average return of the portfolio will be

R = 1r1 + w2r2 + ... + wn * rn

ri = Return of individual stock

wi = weight of individual stock

R = average revenue of the portfolio

In [ ]:
```
BL_return=np.sum(np.array(bl.weights)*np.array(expected_returns.mean_historical_return(data2, frequency=252)))
```

In [185]:
```
BL_return
```

Out[185]:
```
0.5309927605199798
```

(8) Portfolio volatility

In [186]:
```
from pypfopt import objective_functions
objective_functions.portfolio_variance(bl.weights, risk_models.sample_cov(data2))
```

Out[186]:
```
0.03780342465637397
```

In [ ]:

4. Machine Learning Optimization, Nested Clustered Optimization algorithm(NCO), Convex Optimization Solution(CVO) and Monte Carlo approach(MCOS)

(1)Calculate the Return of Data

In [291]:
```
data_return=data.pct_change().fillna(0)
```

(2)Optimization

In [292]:

```
import pandas as pd
from mlfinlab.portfolio_optimization import NCO
max_num_clusters = 19

# Import dataframe of returns for assets in a portfolio


# Calculate empirical covariance of assets
assets_cov = np.array(data_return.cov())

# Calculate empirical means of assets
assets_mean = np.array(data_return.mean()).reshape(-1, 1)

# Class that contains needed functions
nco = NCO()

# Find optimal weights using the NCO algorithm
w_nco = nco.allocate_nco(assets_cov, assets_mean,max_num_clusters)

# Find optimal weights using the CVO algorithm
w_cvo = nco.allocate_cvo(assets_cov, assets_mean)

# Compare the NCO solutions to the CVO ones using MCOS
# Parameters are: 10 simulations, 100 observations in a simulation
# goal of minimum variance, no LW shrinkage
```

In [ ]:

In [293]:
```
w_nco/sum(w_nco)
```

Out[293]:
```
array([[ 0.04219501],
       [ 0.08730925],
       [ 0.0991364 ],
       [-0.03645715],
       [-0.28615967],
       [ 0.18754904],
       [ 0.10727043],
       [-0.22232391],
       [ 0.10917067],
       [ 0.07619004],
       [ 0.24198512],
       [ 0.09368157],
       [ 0.00318379],
       [ 0.18014768],
       [-0.04459596],
       [ 0.04120791],
       [-0.08884568],
       [ 0.02804965],
       [ 0.29556298],
       [ 0.08574281]])
```

In [294]:
```
NCO_return=np.sum((w_nco/sum(w_nco)).flatten()*np.array(expected_returns.mean_historical_return(data2, frequency
```

In [295]:
```
NCO_return
```

Out[295]:
```
0.13335703253042905
```

In [296]:
```
CVO_return=np.sum((w_cvo/sum(w_cvo)).flatten()*np.array(expected_returns.mean_historical_return(data2, frequenc
```

In [297]:
```
CVO_return
```

Out[297]:
```
0.15772837375294288
```

In [298]:
```
NCO_variance=objective_functions.portfolio_variance((w_nco/sum(w_nco)).flatten(), risk_models.sample_cov(data2)
```

In [299]:
```
NCO_variance
```

Out[299]:
```
0.060426056421350044
```

In [300]:
```
CVO_variance=objective_functions.portfolio_variance((w_cvo/sum(w_cvo)).flatten(), risk_models.sample_cov(data2)
```

In [301]:
```
CVO_variance
```

Out[301]:
```
0.09947296315335558
```

In [302]:
```
w_cvo, w_nco = nco.allocate_mcos(assets_mean, assets_cov, 100, 10, 0.01, True, False)

# Find the errors in estimations of NCO and CVO in simulations
err_cvo, err_nco = nco.estim_errors_mcos(w_cvo, w_nco, assets_mean, assets_cov, True)
```

In [ ]:

In [303]:
```
w_nco.T
```

Out[303]:

	0	1	2	3	4	5	6	7	8	9
0	0.030823	0.000356	-0.024801	-0.006986	-0.023305	0.074871	0.021925	0.023359	0.054369	-0.029993
1	-0.021236	-0.004521	-0.006298	0.009236	0.021095	0.043717	0.013141	-0.014436	0.008106	-0.044245
2	0.139488	0.203016	0.090932	0.136616	0.160450	0.187716	0.046144	0.290947	0.176557	0.273012
3	0.022455	-0.047175	-0.000176	-0.006267	0.006740	-0.011677	0.013166	0.031066	-0.005650	-0.096755
4	0.119858	0.242006	0.085192	0.089590	0.126343	0.110217	0.023717	0.068910	0.126969	0.058068
5	0.172263	0.152173	0.135389	0.146763	0.057844	0.269641	0.281878	0.082681	0.283852	0.219984
6	-0.024670	-0.038302	0.006225	0.014142	-0.018958	0.003662	0.061724	-0.000917	-0.068139	-0.068619
7	0.062433	0.066449	0.000351	0.051479	0.024682	-0.050318	0.024171	0.013363	0.000375	-0.051899
8	-0.040138	-0.029141	0.011316	-0.013625	-0.016979	-0.036181	0.041042	0.009284	0.031622	0.011002
9	-0.036458	-0.021178	0.006683	0.007404	-0.025814	-0.018741	-0.000680	-0.018280	-0.027082	-0.022359
10	-0.001269	0.025083	-0.008355	0.036401	-0.007433	-0.063589	0.011408	-0.098206	-0.042790	-0.038619
11	-0.021744	0.002174	0.018154	0.037629	-0.000368	0.055771	0.049518	0.041741	-0.040422	0.015523
12	0.112789	0.086931	0.193481	0.078649	0.189896	0.093451	0.042849	0.064634	0.103276	0.113329
13	-0.015693	0.001743	0.012245	0.011812	0.028097	-0.058372	-0.027164	0.034435	-0.023977	0.019037
14	0.242338	0.236010	0.232990	0.205659	0.343254	0.368970	0.290758	0.191748	0.216285	0.308210
15	-0.025637	-0.025607	-0.035849	-0.014705	-0.043013	-0.118560	0.000521	-0.117063	-0.050177	-0.087454
16	0.152898	0.079523	0.154190	0.175971	0.109042	0.091515	0.044060	0.252278	0.128490	0.316853
17	-0.001192	-0.007060	-0.025586	-0.020623	-0.019486	-0.016644	0.007546	0.046506	-0.022581	-0.036561
18	0.025628	0.028836	0.002250	0.026676	0.017487	0.048439	0.014431	0.086087	0.042653	0.043803
19	0.107064	0.048684	0.151667	0.034180	0.070425	0.026110	0.039847	0.011861	0.108262	0.097683

In [ ]:
```

```

In [304]:
```
sum(np.sum(w_nco*np.array(expected_returns.mean_historical_return(data2, frequency=252)),axis=1))/10
```

Out[304]: 0.20443248582042087

In [305]:
```
sum(np.sum(w_cvo*np.array(expected_returns.mean_historical_return(data2, frequency=252)),axis=1))/10
```

Out[305]: 0.2140528608347716

In [ ]:

In [ ]:

In [ ]:

In [ ]:

In [ ]:

In [ ]:

In [218]:
```
objective_functions.portfolio_variance(w_nco.T, risk_models.sample_cov(data2))
```

Out[218]: Expression(CONSTANT, UNKNOWN, (10, 10))

In [ ]:

In [ ]:

In [137]:
```
x=np.random.randn(20)
```

In [138]:
```
x
```

Out[138]:
```
array([ 0.6510935 , -0.73616444, -1.49256906, -0.93492217, -0.71702539,
        0.15959525, -0.32990774,  0.34082243,  0.14204804,  1.51414753,
       -0.61458907,  2.32480477, -1.4643581 ,  2.0359321 ,  0.21990283,
        0.3419911 ,  0.50044433,  0.67161612,  1.25184107, -0.43293365])
```

In [139]:
```
x/(sum(x))
```

Out[139]:
```
array([ 0.1897253 , -0.21451454, -0.43492696, -0.27243152, -0.20893752,
        0.04650524, -0.09613342,  0.09931391,  0.04139207,  0.44121482,
       -0.1790881 ,  0.67743617, -0.42670643,  0.59326016,  0.06407856,
        0.09965445,  0.14582691,  0.19570549,  0.36478006, -0.12615464])
```

In [140]:
```
np.sum(x/(sum(x)).flatten()*np.array(expected_returns.mean_historical_return(data2, frequency=252)))
```

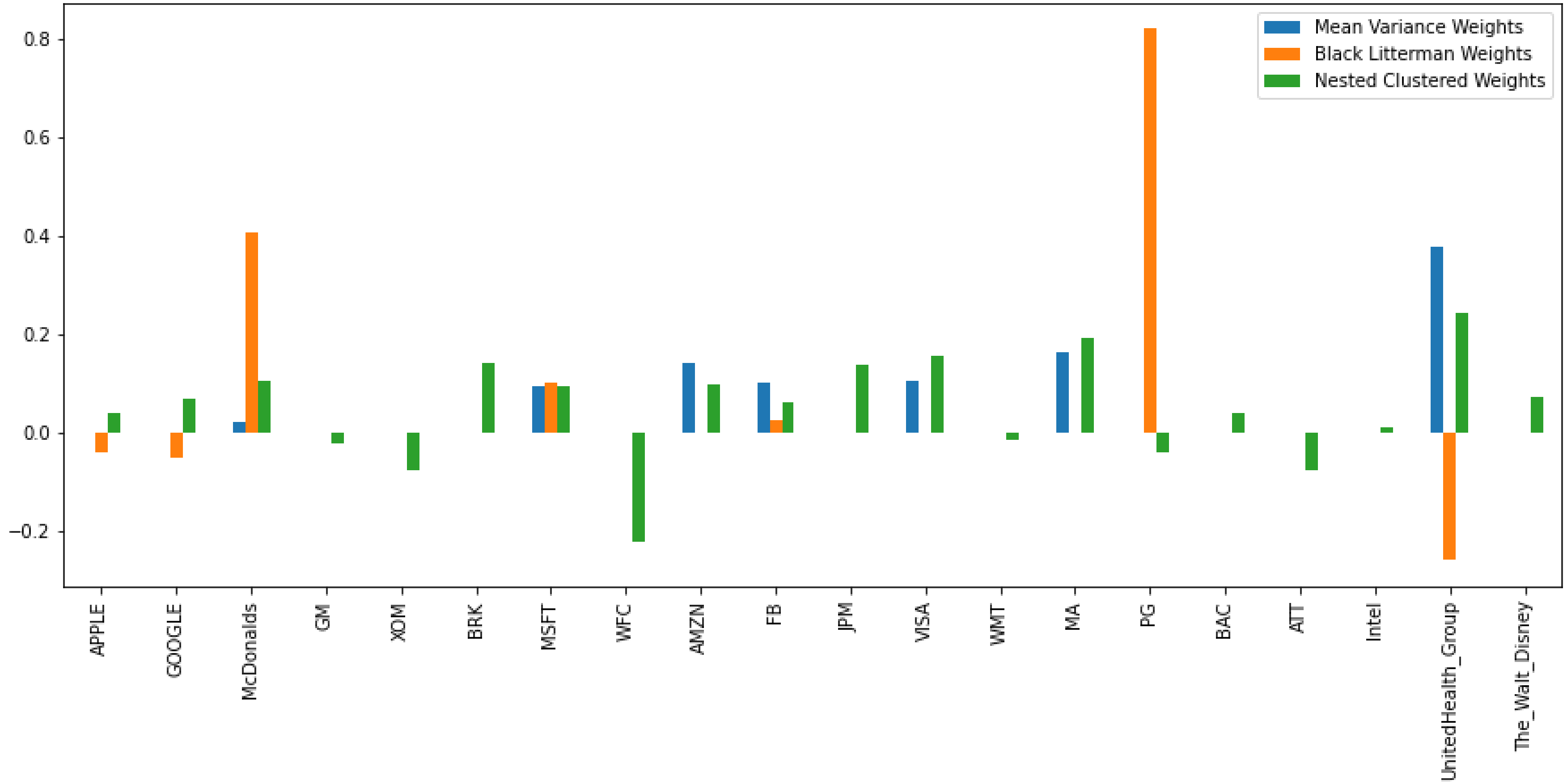
Out[140]:
```
0.058354099868544625
```

4, Portfolio comparison

(1) Portfolio weight comparison

In [82]:
```
df = pd.DataFrame([EF.weights.reshape(20),np.array(list(bl.weights)).reshape(20),(w_nco/sum(w_nco)).reshape(20)
                    columns=["APPLE","GOOGLE","McDonalds","GM","XOM","BRK","MSFT","WFC","AMZN","FB","JPM","VISA",
               "WMT","MA","PG","BAC","ATT","Intel","UnitedHealth_Group","The_Walt_Disney"],
                    index=['Mean Variance Weights','Black Litterman Weights','Nested Clustered Weights'])
df.T.plot(kind='bar',figsize=(15, 6))
```

Out[82]: <matplotlib.axes._subplots.AxesSubplot at 0x7facd4e9dfd0>



(2) Analysis

Blue represents the original stock weight and red represents the newly calculated portfolio weight incorporating the investor's view. Weights have been newly calculated for the Black Litterman model, given information that stocks from Apple, Ghoull, JP Morgan, United Health Group and others will rise.

(3) Comparison of simulated portfolio returns and volatility

In [219]:

```
df2 = pd.DataFrame([[EF.portfolio_performance()[0],bl.portfolio_performance()[0],Mean_variance_return,BL_return
                    [EF.portfolio_performance()[1],\
                     bl.portfolio_performance()[1],\
                    objective_functions.portfolio_variance(EF.weights, risk_models.sample_cov(data2)),\
                    objective_functions.portfolio_variance(bl.weights, risk_models.sample_cov(data2))
                    ],[EF.portfolio_performance()[2],bl.portfolio_performance()[2],"N/A","N/A"]],columns=["Mean

df2
```

Out[219]:

	Mean Variance expected value	Black Litterman expected value	Mean Variance simulated value	Black Litterman simulated value
Return	0.313303	0.265430	0.0523601	0.530993
Variance	0.160044	0.147597	0.0515759	0.0378034
Portfolio Sharpe Ratio	1.832643	1.662842	N/A	N/A

5, conclusion

From September 25, 2018 to September 24, 2019, the mean variance model and the Black Litterman model were compared, the average revenue of the portfolio was calculated, and the Black Litterman model was adopted. In simulation period the volatility increased slightly by 0.013%, resulting in a Black Litterman model with a return to 53.1% much more higher than the mean variance model.

The average return of Black Litterman simulated value in 252 days is much higher than the expected annual return of Black Litterman expected value and the annual return of Mean Variance expected value.

The Portfolio Sharpe Ratio did not exactly assume the performance of portfolio, because it only calculate based on the historic Return data which is not correct.

Reading related research papers may seem obvious, but those who believe they have better information than others suggest that they perform better than market-average portfolios. In portfolio management, it is important to perform not only algorithms but also critic information, market information, and most importantly, corporate analysis.

6, reference list

References, translated by David G. Ruenberger, Hiroshi Konno, Kenichi Suzuki, Norio Bibiki, "Introduction to Financial Engineering: Second Edition," Nihon Keizai Shimbun (2015)

References, Takahiro Komatsu “Optimal Investment Strategy” Asakura Shoten (2018)

References, PyPortfolioOpt, [https://pyportfolioopt.readthedocs.io/en/latest/ (https://pyportfolioopt.readthedocs.io/en/latest/)](https://pyportfolioopt.readthedocs.io/en/latest/)

In [ ]:

In [ ]:

In [ ]: