Machine Learning Optimization

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ABSTRACT:

We compared all two tranditional methods of optimization, Black Litterman as maximum returns of a portfolio and Mean Variance Optimization with new methods of Machine Learning Optimization Nested Clustered Optimization (NCO) and Convex Optimization(CVO) in this paper by optimizing 92 stocks in US market.

1. The goal: Build a portfolio from the US stock market, simulate a six-month short-term investment, and evaluate the actual return by comparing the all 5 models. We use SP100 dataset which contain 92 stocks without missing data.

(1) As external information, it is first necessary to know the risk-free interest rate and market price. Measured using the 52 Week Treasury Bill as a risk-free interest rate.

In [1]:
```python
#Simulation period
import datetime
datetime.datetime(2019, 7, 4)-datetime.datetime(2010, 6, 16)
```

Out[1]:
```
datetime.timedelta(days=3305)
```

In [2]:
```python
import datetime
datetime.datetime(2020, 1, 1)-datetime.datetime(2019, 7, 5)
```

Out[2]:
```
datetime.timedelta(days=180)
```

In [3]:
```python
3305/(52*7)
```

Out[3]:
```
9.07967032967033
```

In [4]:
```python
import quandl
quandl.ApiConfig.api_key = 'DxKMsvF36hXo5BAMpeDK'
Wk_Bank_Discount_Rate_52=quandl.get("USTREASURY/BILLRATES" ,
                          start_date=datetime.datetime(2010, 6, 16),
                          end_date=datetime.datetime(2019, 7, 4))
```

In [5]:
```python
#Downloading bond price
yield_list=[]
for i in range(10):
    yield_list.append(Wk_Bank_Discount_Rate_52[datetime.datetime(2010, 6, 16)+datetime.timedelta(days=364*i):]\
                      ["52 Wk Bank Discount Rate"][0])
```

In [6]:
```python
yield_list
```

Out[6]:
```
[0.28, 0.18, 0.17, 0.13, 0.1, 0.26, 0.58, 1.14, 2.24, 1.99]
```

In [ ]:
```python

```

Simulation period Yield from October 1, 2012 to September 12, 2019 S = (1 + S0) x (1 + S1) x (1 + S2) x (1 + S3) x (1 + S4) x(1 + S5)x(1 + S6)x(1 + S7)x(1 + S8)x(1 + S9) -1

In [7]:
```
S=(1+yield_list[0]/100)*(1+yield_list[1]/100)*(1+yield_list[2]/100)*\
(1+yield_list[3]/100)*(1+yield_list[4]/100)*(1+yield_list[5]/100)*(1+yield_list[6]/100)\
*(1+yield_list[7]/100)*(1+yield_list[8]/100)*(1+yield_list[9]*(3305/((52*7)*10))/100)-1
```

In [8]:
```
S
```

Out[8]:
```
0.07075832744810984
```

In [ ]:

If you invest $1 in the bond on June 16, 2010, you will have an asset of 1.072 on March 17, 2020. This is defined as a safe asset, and the interest rate of this safe asset is a risk-free interest rate.

In [9]:
```
risk_free=S
```

In [10]:
```
risk_free
```

Out[10]:
```
0.07075832744810984
```

In [11]:
```
risk_free_annual=risk_free/9.07967032967033
```

In [12]:
```
risk_free_annual
```

Out[12]:
```
0.007793050284754003
```

Risk-free interest rate for simulation period

In [13]:
```
import quandl
quandl.ApiConfig.api_key = 'DxKMsvF36hXo5BAMpeDK'
Wk_Bank_Discount_Rate_8=quandl.get("USTREASURY/BILLRATES" ,
                          start_date=datetime.datetime(2019, 7, 5),
                          end_date=datetime.datetime(2020, 1, 1))
```

In [14]:
```
datetime.datetime(2020, 1, 1)-datetime.datetime(2019, 10, 3)
```

Out[14]:
```
datetime.timedelta(days=90)
```

In [15]:
```
#Downloading bond price
rate_free_simulation=(Wk_Bank_Discount_Rate_8["8 Wk Bank Discount Rate"][0]/100+1)\
      *(Wk_Bank_Discount_Rate_8["8 Wk Bank Discount Rate"][datetime.datetime(2019, 10, 3)]/100+1)-1

rate_free_simulation
```

Out[15]:
```
0.03937496000000018
```

(3) Download the selected stocks as Training Datasets

In [16]:
```
import pandas as pd
import numpy as np
data=pd.read_excel("S&P 100 constituents Aktienkurse.xlsx",encoding="SHIFT-JIS",header=3)
data=data.drop(labels=0)
data=data.reset_index(drop= True)
data.index=data["Name"]
data=data.drop(["Name"],axis=1)
data=data.dropna(axis=1)
symbols = data.columns
df = data[symbols]
df=df.astype("float")
data=df[datetime.datetime(2010, 6, 16):datetime.datetime(2019, 12, 18)]
```

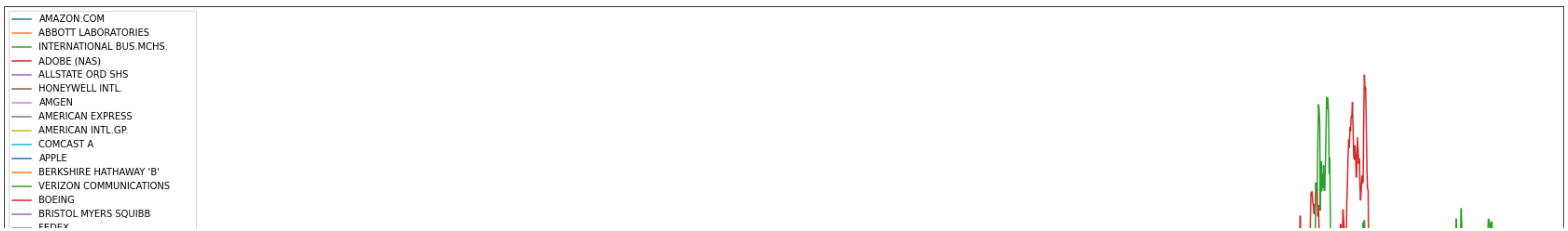
In [17]:
```
data.head()
```
Out[17]:

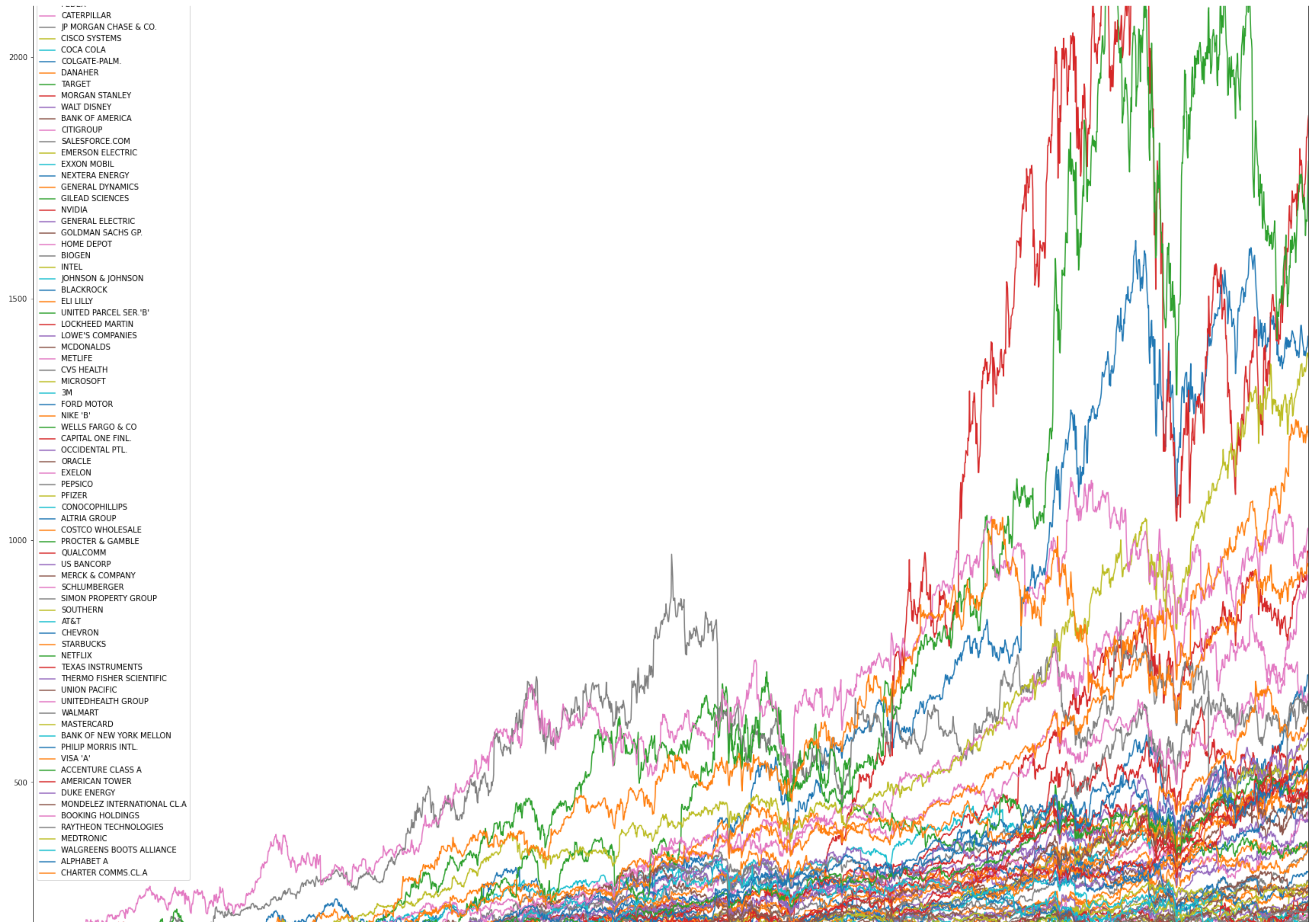
Name	AMAZON.COM	ABBOTT LABORATORIES	INTERNATIONAL BUS.MCHS.	ADOBE (NAS)	ALLSTATE ORD SHS	HONEYWELL INTL.	AMGEN	AMERICAN EXPRESS	AMERICAN INTL.GP.	COMCAST A	...	ACCENTURE CLASS A	AMI
2010-06-17	125.890	23.2682	130.98	33.1200	30.06	40.6034	55.44	42.06	31.6433	9.290	...	38.49	
2010-06-18	125.830	23.3351	130.15	33.5200	30.54	40.8605	55.20	42.03	31.7606	9.255	...	38.93	
2010-06-21	122.550	23.1103	130.65	33.1300	30.27	40.8700	56.52	42.60	32.4727	9.175	...	38.81	
2010-06-22	122.307	22.9619	129.30	32.7625	30.55	40.1750	56.12	41.94	32.0622	9.120	...	38.24	
2010-06-23	121.450	22.7562	130.11	30.3800	30.45	39.8323	56.20	42.17	31.6768	9.090	...	38.17	

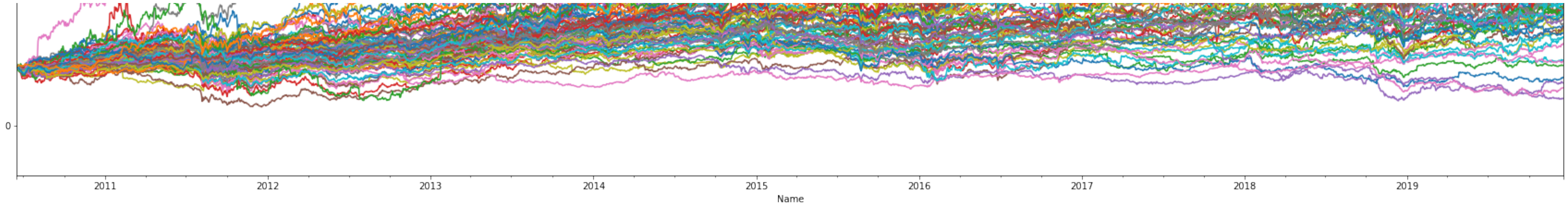
5 rows × 92 columns

(4) Plot time series transition and rate of return

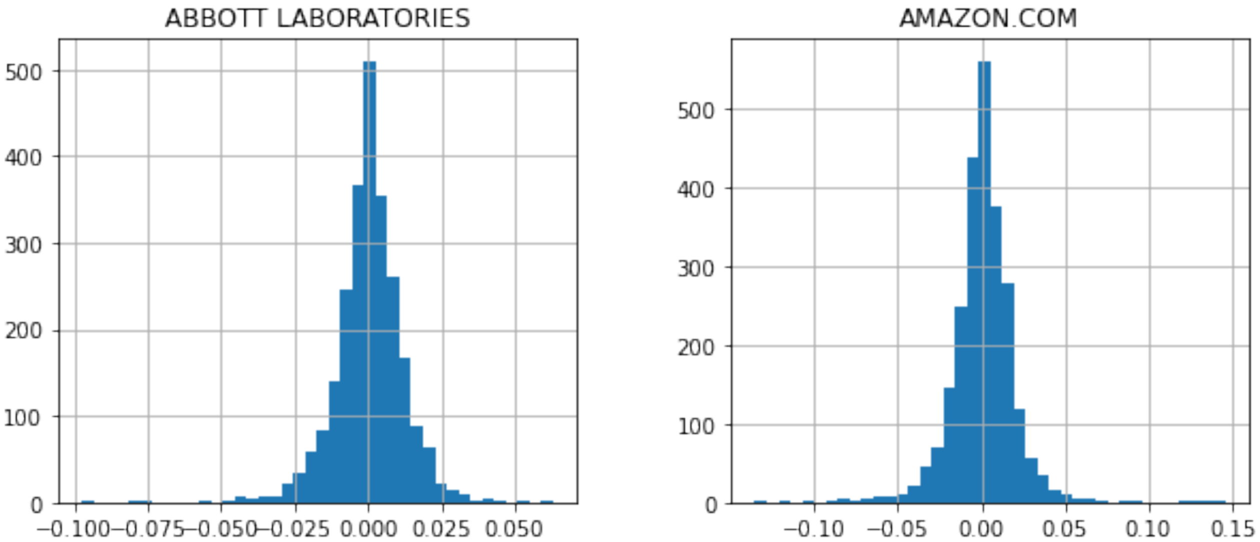
In [18]:
```
import matplotlib.pyplot as plt
%matplotlib inline
(data / data.iloc[0] * 100).plot(figsize=(30, 30))
plt.savefig('stat_01.png')
```




2000
1500
1000
500
CATERPILLAR
JP MORGAN CHASE & CO.
CISCO SYSTEMS
COCA COLA
COLGATE-PALM.
DANAHER
TARGET
MORGAN STANLEY
WALT DISNEY
BANK OF AMERICA
CITIGROUP
SALESFORCE.COM
EMERSON ELECTRIC
EXXON MOBIL
NEXTERA ENERGY
GENERAL DYNAMICS
GILEAD SCIENCES
NVIDIA
GENERAL ELECTRIC
GOLDMAN SACHS GP.
HOME DEPOT
BIOGEN
INTEL
JOHNSON & JOHNSON
BLACKROCK
ELI LILLY
UNITED PARCEL SER.'B'
LOCKHEED MARTIN
LOWE'S COMPANIES
MCDONALDS
METLIFE
CVS HEALTH
MICROSOFT
3M
FORD MOTOR
NIKE 'B'
WELLS FARGO & CO
CAPITAL ONE FINL.
OCCIDENTAL PTL.
ORACLE
EXELON
PEPSICO
PFIZER
CONOCOPHILLIPS
ALTRIA GROUP
COSTCO WHOLESALE
PROCTER & GAMBLE
QUALCOMM
US BANCORP
MERCK & COMPANY
SCHLUMBERGER
SIMON PROPERTY GROUP
SOUTHERN
AT&T
CHEVRON
STARBUCKS
NETFLIX
TEXAS INSTRUMENTS
THERMO FISHER SCIENTIFIC
UNION PACIFIC
UNITEDHEALTH GROUP
WALMART
MASTERCARD
BANK OF NEW YORK MELLON
PHILIP MORRIS INTL.
VISA 'A'
ACCENTURE CLASS A
AMERICAN TOWER
DUKE ENERGY
MONDELEZ INTERNATIONAL CL.A
BOOKING HOLDINGS
RAYTHEON TECHNOLOGIES
MEDTRONIC
WALGREENS BOOTS ALLIANCE
ALPHABET A
CHARTER COMMS.CL.A




```
In [19]: noa = len(symbols)
         data = data[symbols]
         rets = np.log(data / data.shift(1))
         rets[symbols[:2]].hist(bins=40, figsize=(10, 4))
         plt.savefig('stat_2.png')
```



2, mean variance model

(1) Model optimization

In [20]:
```
from pypfopt.efficient_frontier import EfficientFrontier
from pypfopt import risk_models
from pypfopt import expected_returns

mu = expected_returns.mean_historical_return(data)
S = risk_models.sample_cov(data,frequency=252)

#mean variance model optimization
EF_min = EfficientFrontier(mu, S)

EF_min.min_volatility()
#portfolio performance
EF_min.portfolio_performance(verbose=True)
```

```
Expected annual return: 9.1%
Annual volatility: 10.0%
Sharpe Ratio: 0.71
```

Out[20]:
```
(0.09073917348377833, 0.10031802484173862, 0.7051491852573475)
```

In [21]:
```
#CAPM optimization
#Enter non-risky asset
EF = EfficientFrontier(mu, S,weight_bounds=(-1, 1))
weights = EF.max_sharpe(risk_free_rate=risk_free_annual)
#Portfolio ratio
EF.portfolio_performance(verbose=True)
```

```
Expected annual return: 121.0%
Annual volatility: 36.7%
Sharpe Ratio: 3.27
```

Out[21]:
```
(1.2095219312190972, 0.3669526775015833, 3.2748878932193053)
```

In [22]:
```
#Weights in each stock
EF_clean_weights=EF.weights
```

In [ ]:
```
```

(2) Download the data of each stock from March 18, 2020 to June 16, 2020 will be collected for simulation.

In [23]:
```
import pandas as pd
import numpy as np
data2=pd.read_excel("S&P 100 constituents Aktienkurse.xlsx",encoding="SHIFT-JIS",header=3)
data2=data2.drop(labels=0)
data2=data2.reset_index(drop= True)
data2.index=data2["Name"]
data2=data2.drop(["Name"],axis=1)
data2=data2.dropna(axis=1)
symbols2 = data2.columns
df2 = data2[symbols]
df2=df2.astype("float")
data2=df2[datetime.datetime(2019, 7, 5):datetime.datetime(2020, 1, 1)]
```

(2) -2 Covid Download the data of each stock from March 18, 2020 to June 16, 2020 will be collected for simulation.

In [24]:
```
data3=pd.read_excel("S&P 100 constituents Aktienkurse.xlsx",encoding="SHIFT-JIS",header=3)
data3=data3.drop(labels=0)
data3=data3.reset_index(drop= True)
data3.index=data3["Name"]
data3=data3.drop(["Name"],axis=1)
data3=data3.dropna(axis=1)
symbols3 = data3.columns
data3=data3[datetime.datetime(2019, 12, 19):datetime.datetime(2020, 6, 16)]
```

(3) If managed from March 18, 2020 to June 16, 2020, the average return of the portfolio will be

R = 1r1 + w2r2 + ... + wn * rn

ri = Return of individual stock

wi = weight of individual stock

R = average revenue of the portfolio

In [65]:
```
Mean_variance_return=np.sum(EF.weights*np.array(expected_returns.mean_historical_return(data2)))
Mean_variance_return
data3_Mean_variance_return=np.sum(EF.weights*np.array(expected_returns.mean_historical_return(data3)))
data3_Mean_variance_return

data4_Mean_variance_return=np.sum(np.maximum(EF.weights,0)*np.array(expected_returns.mean_historical_return(data3)))
data4_Mean_variance_return
```

Out[65]:
```
-0.1001645952798076
```

In [ ]:
```

```

(4) Volatility of the mean variance model portfolio

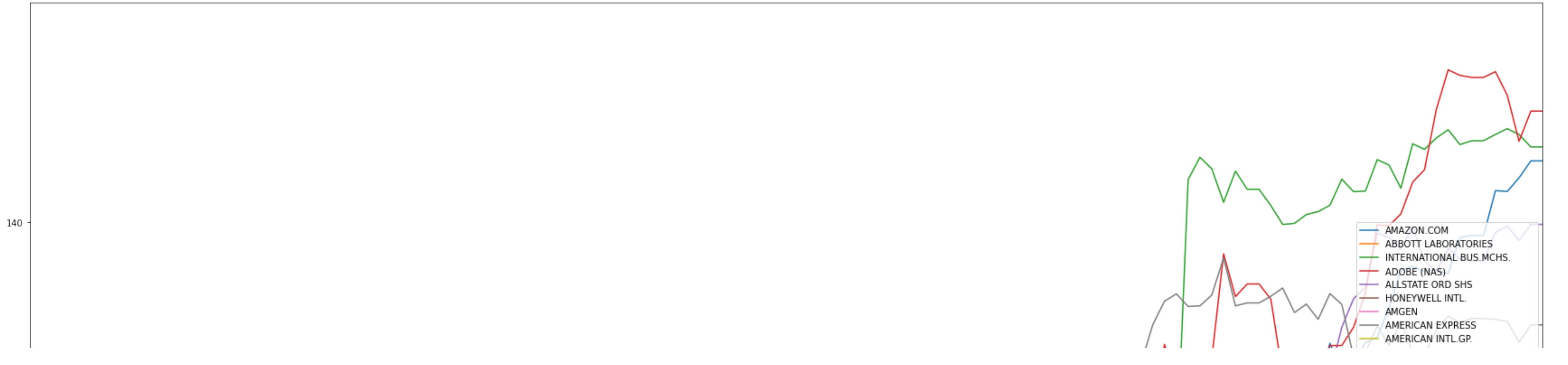
In [66]:
```python
from pypfopt import objective_functions
Mean_variance_Volatility=np.sqrt(objective_functions.portfolio_variance(EF.weights, risk_models.sample_cov(data2, freq
Mean_variance_Volatility
data3_Mean_variance_Volatility=np.sqrt(objective_functions.portfolio_variance(EF.weights, risk_models.sample_cov(data3
data3_Mean_variance_Volatility

data4_Mean_variance_Volatility=np.sqrt(objective_functions.portfolio_variance(np.maximum(EF.weights,0), risk_models.sa
data4_Mean_variance_Volatility
```

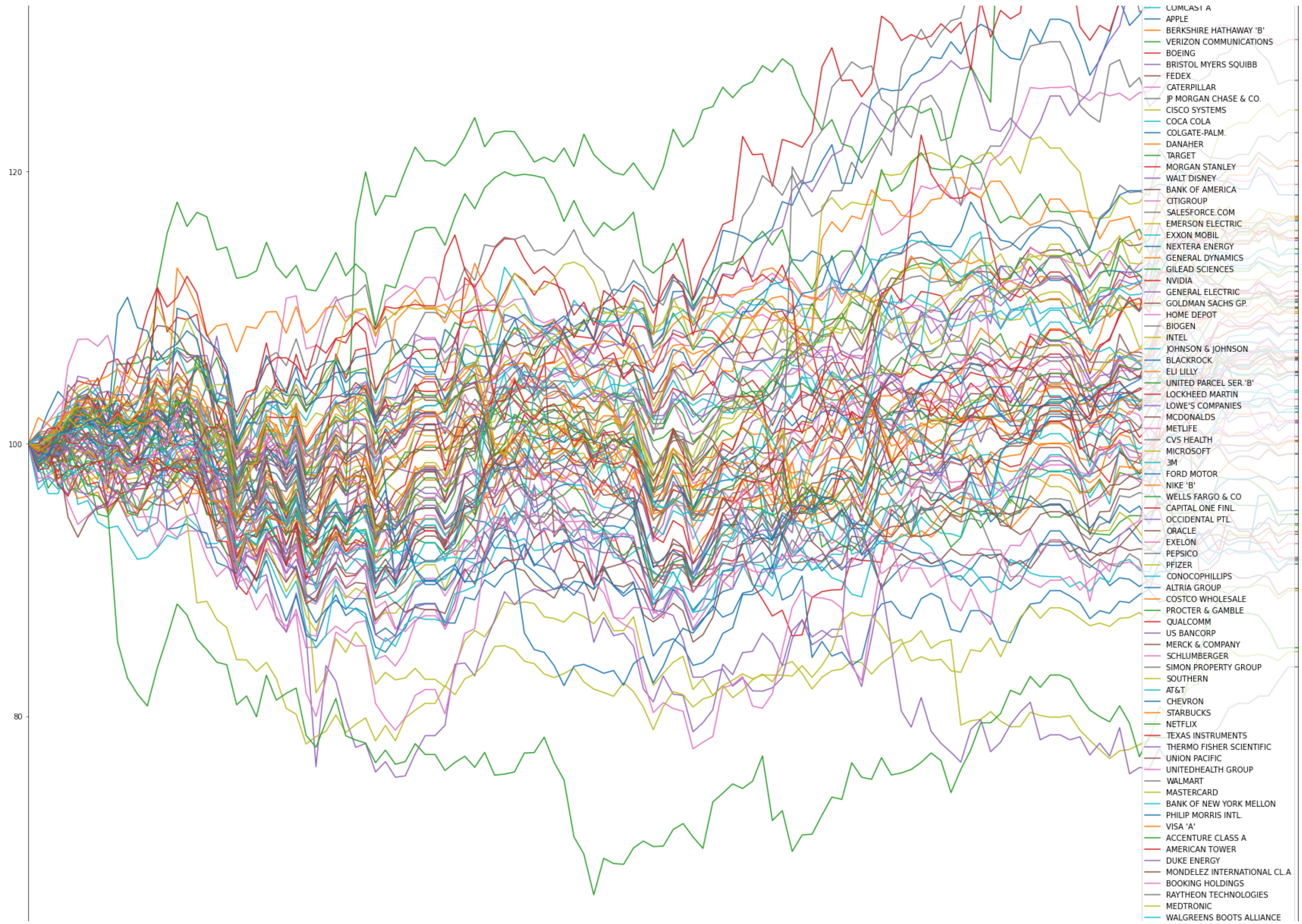
Out[66]: 2.1314431454775473

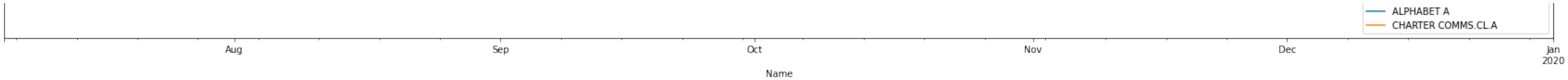
3. Black – Litterman model

(1) For the maxmize the Return of portfolio, calculate the actual return of each stock from March 18, 2020 to June 16, 2020

In [27]:
```python
import matplotlib.pyplot as plt
%matplotlib inline
(data2 / data2.iloc[0] * 100).plot(figsize=(30, 30))
plt.savefig('stat_02.png')
```




120
100
80
COMCAST A
APPLE
BERKSHIRE HATHAWAY 'B'
VERIZON COMMUNICATIONS
BOEING
BRISTOL MYERS SQUIBB
FEDEX
CATERPILLAR
JP MORGAN CHASE & CO.
CISCO SYSTEMS
COCA COLA
COLGATE-PALM.
DANAHER
TARGET
MORGAN STANLEY
WALT DISNEY
BANK OF AMERICA
CITIGROUP
SALESFORCE.COM
EMERSON ELECTRIC
EXXON MOBIL
NEXTERA ENERGY
GENERAL DYNAMICS
GILEAD SCIENCES
NVIDIA
GENERAL ELECTRIC
GOLDMAN SACHS GP.
HOME DEPOT
BIOGEN
INTEL
JOHNSON & JOHNSON
BLACKROCK
ELI LILLY
UNITED PARCEL SER.'B'
LOCKHEED MARTIN
LOWE'S COMPANIES
MCDONALDS
METLIFE
CVS HEALTH
MICROSOFT
3M
FORD MOTOR
NIKE 'B'
WELLS FARGO & CO
CAPITAL ONE FINL.
OCCIDENTAL PTL.
ORACLE
EXELON
PEPSICO
PFIZER
CONOCOPHILLIPS
ALTRIA GROUP
COSTCO WHOLESALE
PROCTER & GAMBLE
QUALCOMM
US BANCORP
MERCK & COMPANY
SCHLUMBERGER
SIMON PROPERTY GROUP
SOUTHERN
AT&T
CHEVRON
STARBUCKS
NETFLIX
TEXAS INSTRUMENTS
THERMO FISHER SCIENTIFIC
UNION PACIFIC
UNITEDHEALTH GROUP
WALMART
MASTERCARD
BANK OF NEW YORK MELLON
PHILIP MORRIS INTL.
VISA 'A'
ACCENTURE CLASS A
AMERICAN TOWER
DUKE ENERGY
MONDELEZ INTERNATIONAL CL.A
BOOKING HOLDINGS
RAYTHEON TECHNOLOGIES
MEDTRONIC
WALGREENS BOOTS ALLIANCE



In [28]:
```
expected_returns.mean_historical_return(data2)
```

Out[28]:
```
AMAZON.COM                  -0.094050
ABBOTT LABORATORIES          0.033460
INTERNATIONAL BUS.MCHS.     -0.099640
ADOBE (NAS)                  0.174170
ALLSTATE ORD SHS             0.155508
                               ...
RAYTHEON TECHNOLOGIES        0.271859
MEDTRONIC                    0.331101
WALGREENS BOOTS ALLIANCE     0.138931
ALPHABET A                   0.391028
CHARTER COMMS.CL.A           0.449794
Length: 92, dtype: float64
```

(3) Setting critic reviews for each brand

Referring to the above figures and data supposed that I could correctly predict the future returns after six months, exactly same as the actual returns above.

In [29]:
```
from pypfopt.black_litterman import BlackLittermanModel
S = risk_models.sample_cov(data)

viewdict = expected_returns.mean_historical_return(data2, frequency=180*252/365)
```

In [30]:
```
bl = BlackLittermanModel(S, absolute_views=viewdict)
rets = bl.bl_returns()
```

```
/usr/local/anaconda3/lib/python3.7/site-packages/pypfopt/black_litterman.py:257: UserWarning: Running Black-Litterman
with no prior.
  warnings.warn("Running Black-Litterman with no prior.")
```

In [31]:
```
bl.weights
```

(4) Calculate the return of each brand

In [32]:
```
rets
```

Out[32]:
```
AMAZON.COM                   0.030890
ABBOTT LABORATORIES          0.064932
INTERNATIONAL BUS.MCHS.      0.013261
ADOBE (NAS)                  0.083895
ALLSTATE ORD SHS             0.057072
                               ...
RAYTHEON TECHNOLOGIES        0.066101
MEDTRONIC                    0.097625
WALGREENS BOOTS ALLIANCE     0.093773
ALPHABET A                   0.113111
CHARTER COMMS.CL.A           0.120316
Length: 92, dtype: float64
```

In [ ]:

(5) Introduce SP500 as market price

In [33]:
```
import pandas_datareader as pdr
SP500 = pdr.get_data_yahoo('^GSPC',
                          start=datetime.datetime(2010, 6, 16),
                          end=datetime.datetime(2020, 3, 17))
```

```
/usr/local/anaconda3/lib/python3.7/site-packages/pandas_datareader/compat/__init__.py:7: FutureWarning: pandas.util.t
esting is deprecated. Use the functions in the public API at pandas.testing instead.
  from pandas.util.testing import assert_frame_equal
```

In [34]:
```
market_prices=SP500["Close"]
```

In [ ]:

(6) Black – Litterman model simulation

In [35]:
```
from pypfopt import black_litterman

delta = black_litterman.market_implied_risk_aversion(market_prices,risk_free_rate=risk_free_annual)



bl.bl_weights(delta)
weights = bl.clean_weights()
```

In [36]:
```
bl.portfolio_performance(verbose=True)
```

```
Expected annual return: 84.2%
Annual volatility: 35.9%
Sharpe Ratio: 2.29
```

Out[36]:
```
(0.8416819673775235, 0.35946554224782523, 2.2858434837435233)
```

In [37]:
```
Black_Litterman_weights=weights
```

In [38]:
```
sum(weights.values())
```

Out[38]:
```
0.9999799999999996
```

In [ ]:

(7) If managed from March 18, 2020 to June 16, 2020, the average return of the portfolio will be

R = 1r1 + w2r2 + ... + wn * rn

ri = Return of individual stock

wi = weight of individual stock

R = average revenue of the portfolio

```
In [39]: BL_return=np.sum(np.array(bl.weights)*np.array(expected_returns.mean_historical_return(data2)))
```

```
In [40]: BL_return
```

```
Out[40]: 5.038821238318581
```

(8) Portfolio volatility

```
In [41]: from pypfopt import objective_functions
         Black_Litterman_volatility=np.sqrt(objective_functions.portfolio_variance(bl.weights, risk_models.sample_cov(data2)))
```

```
In [42]: Black_Litterman_volatility
```

```
Out[42]: 0.42542507748072506
```

4. Machine Learning Optimization, Nested Clustered Optimization algorithm(NCO), Convex Optimization Solution(CVO) and Monte Carlo approach(MCOS)

(1)Calculate the Return and Variance of Data

In [43]:
```
from pypfopt.expected_returns import mean_historical_return
from pypfopt.risk_models import sample_cov
assets_mean=mean_historical_return(data,frequency=252)
assets_cov=sample_cov(data,frequency=252)
```

(2)Optimization of NCO & CVO

In [ ]:
```

```

In [44]:
```
import pandas as pd
from portfoliolab.clustering import NestedClusteredOptimisation


# Import dataframe of returns for assets in a portfolio
max_num_clusters=91

# Calculate empirical covariance of assets
assets_cov = np.array(assets_cov)

# Calculate empirical means of assets
assets_mean =  np.array(assets_mean).reshape(-1, 1)

# Class that contains needed functions
nco = NestedClusteredOptimisation()

# Find optimal weights using the NCO algorithm
w_nco = nco.allocate_nco(assets_cov, assets_mean,max_num_clusters,n_init=10)

# Find optimal weights using the CVO algorithm
w_cvo = nco.allocate_cvo(assets_cov, assets_mean)
```

In [45]:
```
nco_weights= w_nco/sum(w_nco)
```

In [46]:
```
cvo_weights=(w_cvo/sum(w_cvo))
```

(3)Return of NCO method

In [70]:
```
NCO_return=np.sum(nco_weights.flatten()*np.array(expected_returns.mean_historical_return(data2)))
NCO_return
data3_NCO_return=np.sum(nco_weights.flatten()*np.array(expected_returns.mean_historical_return(data3)))
data3_NCO_return

data4_NCO_return=np.sum(np.maximum(nco_weights.flatten(),0)*np.array(expected_returns.mean_historical_return(data3)))
data4_NCO_return
```

Out[70]: 0.04430053254540261

In [ ]:

(4)Return of CVO method

In [71]:
```
CVO_return=np.sum(cvo_weights.flatten()*np.array(expected_returns.mean_historical_return(data2)))
CVO_return
data3_CVO_return=np.sum(cvo_weights.flatten()*np.array(expected_returns.mean_historical_return(data3)))
data3_CVO_return

data4_CVO_return=np.sum(np.maximum(cvo_weights.flatten(),0)*np.array(expected_returns.mean_historical_return(data3)))
data4_CVO_return
```

Out[71]: -0.09585451173965258

In [ ]:

(5)Volatility of NCO method

In [75]:

```
NCO_volatility=np.sqrt(objective_functions.portfolio_variance(nco_weights.flatten(), risk_models.sample_cov(data2)))
NCO_volatility
data3_NCO_volatility=np.sqrt(objective_functions.portfolio_variance(nco_weights.flatten(), risk_models.sample_cov(data
data3_NCO_volatility


data4_NCO_volatility=np.sqrt(objective_functions.portfolio_variance(np.maximum(nco_weights.flatten(),0), risk_models.s
data4_NCO_volatility
```

Out[75]: 2.4262606780950344

(6)Volatility of CVO method

In [76]:

```
CVO_volatility=np.sqrt(objective_functions.portfolio_variance(cvo_weights.flatten(), risk_models.sample_cov(data2)))
CVO_volatility
data3_CVO_volatility=np.sqrt(objective_functions.portfolio_variance(cvo_weights.flatten(), risk_models.sample_cov(data
data3_CVO_volatility


data4_CVO_volatility=np.sqrt(objective_functions.portfolio_variance(np.maximum(cvo_weights.flatten(),0), risk_models.s
data4_CVO_volatility
```

Out[76]: 4.2278979801565235

In [51]:
```
# Compare the NCO solutions to the CVO ones using MCOS
# Parameters are: 1 simulations, 2545 observations in a simulation
# goal of maximum sharpe ratio, using LW shrinkage


w_cvo_mcos, w_nco_mcos = nco.allocate_mcos(assets_mean, assets_cov, 2545, 10, 0.01, False, False)

# Find the errors in estimations of NCO and CVO in simulations
err_cvo_mcos, err_nco_mcos = nco.estim_errors_mcos(w_cvo, w_nco, assets_mean, assets_cov, False)
```

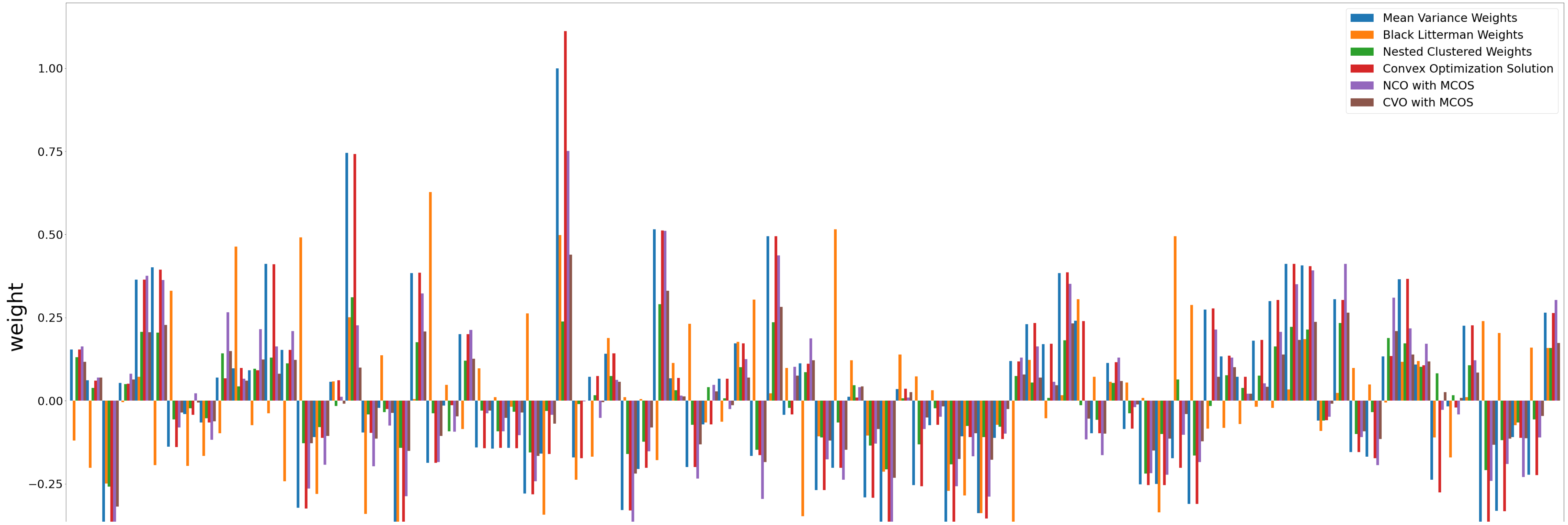
In [77]:
```
weight_nco_mcos=(w_cvo_mcos.sum(axis=0)/10)/sum((w_cvo_mcos.sum(axis=0)/10))
weight_cvo_mcos=(w_nco_mcos.sum(axis=0)/10)/sum((w_nco_mcos.sum(axis=0)/10))

nco_mcos_performance=sum(np.array(weight_nco_mcos)*np.array(expected_returns.mean_historical_return(data)))
cvo_mcos_performance=sum(np.array(weight_cvo_mcos)*np.array(expected_returns.mean_historical_return(data)))
nco_mcos_performance2=np.sqrt(objective_functions.portfolio_variance(np.array(weight_nco_mcos), (risk_models.sample_co
cvo_mcos_performance2=np.sqrt(objective_functions.portfolio_variance(np.array(weight_cvo_mcos), risk_models.sample_cov


nco_mcos_return=sum(np.array(weight_nco_mcos)*np.array(expected_returns.mean_historical_return(data2)))
cvo_mcos_return=sum(np.array(weight_cvo_mcos)*np.array(expected_returns.mean_historical_return(data2)))
nco_mcos_volatility=np.sqrt(objective_functions.portfolio_variance(np.array(weight_nco_mcos), (risk_models.sample_cov(
cvo_mcos_volatility=np.sqrt(objective_functions.portfolio_variance(np.array(weight_cvo_mcos), risk_models.sample_cov(d


data3_nco_mcos_return=sum(np.array(weight_nco_mcos)*np.array(expected_returns.mean_historical_return(data3)))
data3_cvo_mcos_return=sum(np.array(weight_cvo_mcos)*np.array(expected_returns.mean_historical_return(data3)))
data3_nco_mcos_volatility=np.sqrt(objective_functions.portfolio_variance(np.array(weight_nco_mcos), (risk_models.sampl
data3_cvo_mcos_volatility=np.sqrt(objective_functions.portfolio_variance(np.array(weight_cvo_mcos), risk_models.sample



data4_nco_mcos_return=sum(np.maximum(np.array(weight_nco_mcos),0)*np.array(expected_returns.mean_historical_return(dat
data4_cvo_mcos_return=sum(np.maximum(np.array(weight_cvo_mcos),0)*np.array(expected_returns.mean_historical_return(dat
data4_nco_mcos_volatility=np.sqrt(objective_functions.portfolio_variance(np.maximum(np.array(weight_nco_mcos),0), (ris
data4_cvo_mcos_volatility=np.sqrt(objective_functions.portfolio_variance(np.maximum(np.array(weight_cvo_mcos),0), risk
```

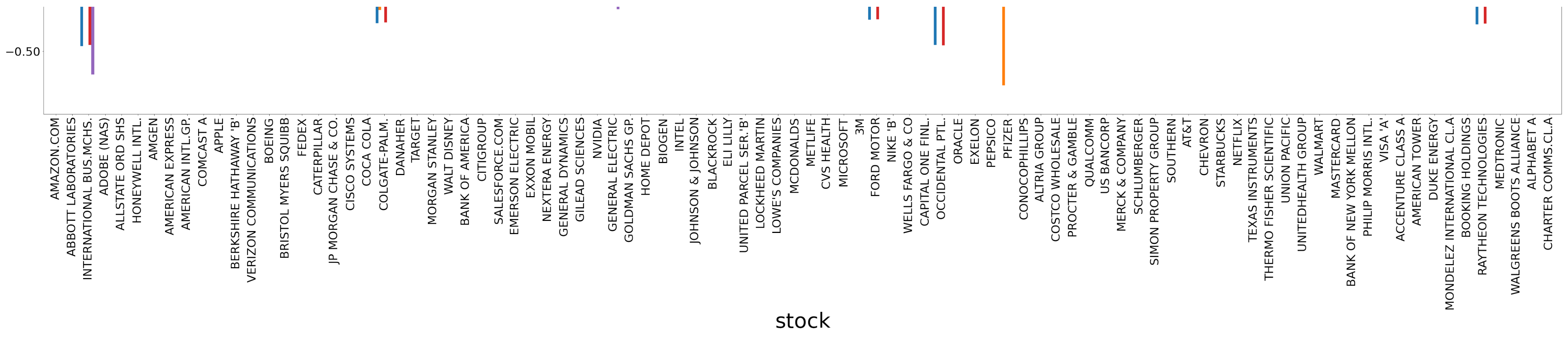
```
In [ ]:
```

4, Portfolio comparison

(1) Portfolio weight comparison

```
In [500]: df = pd.DataFrame([EF.weights.reshape(92),np.array(list(bl.weights)).reshape(92),(w_nco/sum(w_nco)).reshape(92),\
                              (w_cvo/sum(w_cvo)).reshape(92),np.array(weight_nco_mcos),np.array(weight_cvo_mcos)],
                             columns=data.columns,
                             index=['Mean Variance Weights','Black Litterman Weights','Nested Clustered Weights','Convex Optimiza
          plot=df.T.plot(kind='bar',figsize=(70, 30),legend=True, fontsize=30,width=1)
          plot.legend(loc=1,fontsize=30)
          plot.set_xlabel('stock',fontdict={'fontsize':54})
          plot.set_ylabel('weight',fontdict={'fontsize':54})

Out[500]: Text(0, 0.5, 'weight')
```





(2)Comparison of trained portfolios average return and volatility

In [501]:

```
nco_por_ret=np.sum(nco_weights.flatten()*np.array(expected_returns.mean_historical_return(data, frequency=252)))
cvo_por_ret=np.sum(cvo_weights.flatten()*np.array(expected_returns.mean_historical_return(data, frequency=252)))



nco_por_var=np.sqrt(objective_functions.portfolio_variance(nco_weights.flatten(), (risk_models.sample_cov(data, freque
cvo_por_var=np.sqrt(objective_functions.portfolio_variance(cvo_weights.flatten(), risk_models.sample_cov(data, frequer
```

In [502]:
```
df = pd.DataFrame([[EF.portfolio_performance(verbose=True)[0],bl.portfolio_performance(verbose=True)[0],nco_por_ret,cv
                    [EF.portfolio_performance(verbose=True)[1],\
                    bl.portfolio_performance(verbose=True)[1]
                    ,nco_por_var,cvo_por_var,nco_mcos_performance2,cvo_mcos_performance2],\
                    [EF.portfolio_performance(verbose=True)[2],\
                    bl.portfolio_performance(verbose=True)[2]\
                     ,(nco_por_ret-risk_free_annual)/nco_por_var,(cvo_por_ret-risk_free_annual)/cvo_por_var,(nco_mcos_
                        'NCO with MCOS simulated value','COV with MCOS simulated value'],index=["Return","Volatility'

df
```

```
Expected annual return: 131.8%
Annual volatility: 39.1%
Sharpe Ratio: 3.32
Expected annual return: 79.9%
Annual volatility: 34.2%
Sharpe Ratio: 2.28
Expected annual return: 131.8%
Annual volatility: 39.1%
Sharpe Ratio: 3.32
Expected annual return: 79.9%
Annual volatility: 34.2%
Sharpe Ratio: 2.28
Expected annual return: 131.8%
Annual volatility: 39.1%
Sharpe Ratio: 3.32
Expected annual return: 79.9%
Annual volatility: 34.2%
Sharpe Ratio: 2.28
```

Out[502]:

	Mean Variance simulated value	Black Litterman simulated value	Nested Clustered simulated value	Convex Optimization simulated value	NCO with MCOS simulated value	COV with MCOS simulated value
Return	1.317799	0.799154	0.789909	1.327950	1.237470	0.837969
Volatility	0.391099	0.341644	0.253938	0.393990	0.397157	0.270803
Portfolio Sharpe Ratio	3.318338	2.280601	3.079948	3.350733	3.096195	3.065601

(3) Comparison of simulated portfolios average return and volatility

In [ ]:

In [ ]:

In [ ]:

In [568]:

```
df2 = pd.DataFrame([[Mean_variance_return,BL_return,NCO_return,CVO_return,\
                     nco_mcos_return,cvo_mcos_return],\
                     [Mean_variance_Volatility,\
                     Black_Litterman_volatility
                     ,NCO_volatility,CVO_volatility,nco_mcos_volatility,cvo_mcos_volatility],\
                     [(Mean_variance_return-risk_free_annual)/Mean_variance_Volatility,\
                      (BL_return-risk_free_annual)/Black_Litterman_volatility\
                      ,(NCO_return-risk_free_annual)/NCO_volatility,(CVO_return-risk_free_annual)/CVO_volatility
                     ,(nco_mcos_return-risk_free_annual)/nco_mcos_volatility,(cvo_mcos_return-risk_free_annual)/cvo_mcc
                          'NCO with MCOS simulated value','COV with MCOS simulated value'],index=["Return","Volatility"

df2
```

Out[568]:

	Mean Variance simulated value	Black Litterman simulated value	Nested Clustered simulated value	Convex Optimization simulated value	NCO with MCOS simulated value	COV with MCOS simulated value
Return	0.671215	4.934892	0.367944	0.700696	0.310424	0.295841
Volatility	0.287613	0.433405	0.359631	0.586424	0.602040	0.398368
Portfolio Sharpe Ratio	2.306650	11.368343	1.001448	1.181573	0.502677	0.723068

In [ ]:

(4) Comparison of Covid simulated portfolios average return and volatility

In [ ]:

In [86]:
```
df3 = pd.DataFrame([[data3_Mean_variance_return,data3_NCO_return,data3_CVO_return,\
                    data3_nco_mcos_return,data3_cvo_mcos_return],\
                    [data3_Mean_variance_Volatility
                    ,data3_NCO_volatility,data3_CVO_volatility,data3_nco_mcos_volatility,data3_cvo_mcos_volatility],\
                    [(data3_Mean_variance_return-risk_free_annual)/data3_Mean_variance_Volatility,\
                     (data3_NCO_return-risk_free_annual)/data3_NCO_volatility,(data3_CVO_return-risk_free_annual)/data
                    ,(data3_nco_mcos_return-risk_free_annual)/data3_nco_mcos_volatility,(data3_cvo_mcos_return-risk_fr
                         'NCO with MCOS simulated value','COV with MCOS simulated value'],index=["Return","Volatility"

df3
```

Out[86]:

	Mean Variance simulated value	Nested Clustered simulated value	Convex Optimization simulated value	NCO with MCOS simulated value	COV with MCOS simulated value
Return	2.272084	1.408660	2.233404	1.996221	1.344646
Volatility	0.564385	0.692469	1.118165	0.939512	0.667084
Portfolio Sharpe Ratio	4.011962	2.023003	1.990413	2.116447	2.004024

In [ ]:

In [85]:
```
df4 = pd.DataFrame([[data4_Mean_variance_return,data4_NCO_return,data4_CVO_return,\
                    data4_nco_mcos_return,data4_cvo_mcos_return],\
                    [data4_Mean_variance_Volatility
                    ,data4_NCO_volatility,data4_CVO_volatility,data4_nco_mcos_volatility,data4_cvo_mcos_volatility],\
                    [(data4_Mean_variance_return-risk_free_annual)/data4_Mean_variance_Volatility,\
                     (data4_NCO_return-risk_free_annual)/data4_NCO_volatility,(data4_CVO_return-risk_free_annual)/data
                    ,(data4_nco_mcos_return-risk_free_annual)/data4_nco_mcos_volatility,(data4_cvo_mcos_return-risk_fr
                         'NCO with MCOS simulated value','COV with MCOS simulated value'],index=["Return","Volatility"

df4
```

Out[85]:

	Mean Variance simulated value	Nested Clustered simulated value	Convex Optimization simulated value	NCO with MCOS simulated value	COV with MCOS simulated value
Return	-0.100165	0.044301	-0.095855	0.117714	0.202540
Volatility	2.131443	2.426261	4.227898	3.854024	2.342933
Portfolio Sharpe Ratio	-0.050650	0.015047	-0.024515	0.028521	0.083121

In [ ]:

5, conclusion

6, reference list

translated by David G. Ruenberger, Hiroshi Konno, Kenichi Suzuki, Norio Bibiki, "Introduction to Financial Engineering: Second Edition," Nihon Keizai Shimbun (2015)

Takahiro Komatsu “Optimal Investment Strategy” Asakura Shoten (2018)

PyPortfolioOpt, https://pyportfolioopt.readthedocs.io/en/latest/ (https://pyportfolioopt.readthedocs.io/en/latest/)

Machine Learning Financial Laboratory (mlfinlab) https://mlfinlab.readthedocs.io/en/latest/index.html (https://mlfinlab.readthedocs.io/en/latest/index.html)

A ROBUST ESTIMATOR OF THE EFFICIENT FRONTIER https://papers.ssrn.com/sol3/papers.cfm?abstract_id=3469961 (https://papers.ssrn.com/sol3/papers.cfm?abstract_id=3469961)

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In [ ]:

In [ ]: