Certifiable Controller Synthesis for Underactuated Robotic Systems

From Convex Optimization to Learning-based Control

Lujie Yang

Northeast Systems and Control Workshop

05/04/2024

Agenda

HJB equation

Lyapunov stability

SOS

NN verification

optimality & stability

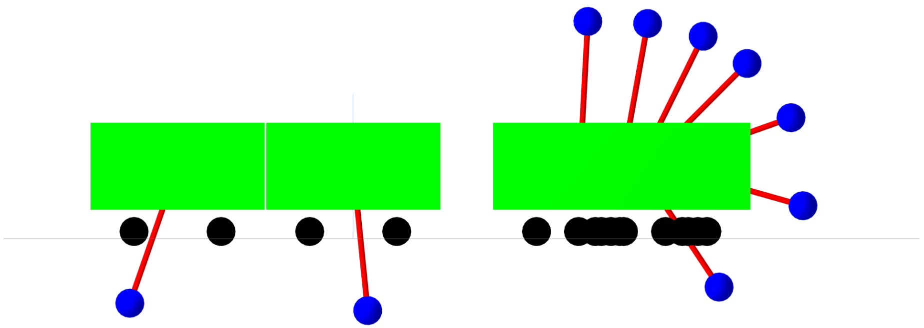
state & output feedback

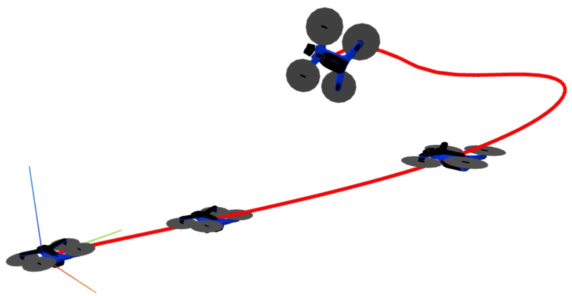
certifiable

Approximate Optimal Controller Synthesis for CartPoles and Quadrotors via Sums-of-Squares

Lujie Yang, Hongkai Dai, Alexandre Amice, Russ Tedrake

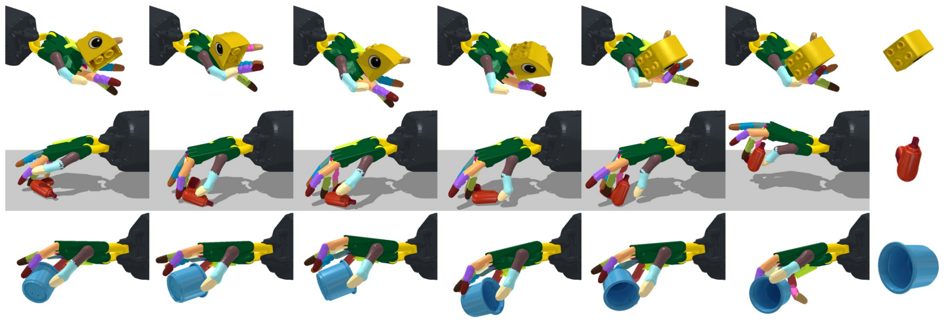
RA-L 2023





Motivation

- Huge empirical success of RL
  - No formal guarantees
- Some approximate dynamic programming works provide theoretical guarantees
  - Can not scale to complicated robotics system yet
- Synthesize controllers with certifiable optimality and stability guarantees for underactuated robotics systems



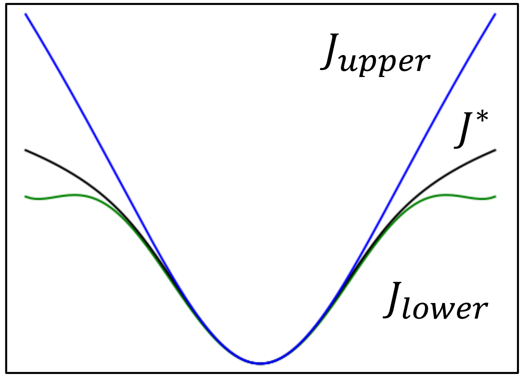
SIAM J. CONTROL OPTIM.
Vol. 47, No. 4, pp. 1643–1666

NONLINEAR OPTIMAL CONTROL VIA OCCUPATION MEASURES AND LMI-RELAXATIONS*

JEAN B. LASSERRE†, DIDIER HENRION‡, CHRISTOPHE PRIEUR§, AND EMMANUEL TRÉLAT¶

Method – HJB Inequalities

- Intractable to solve exactly
- “Curse of dimensionality”
- Approximately solve with guarantees



$$\forall x, \min_{u \in U} l(x,u) + \frac{\partial J^*}{\partial x} f(x,u) = 0$$

$$\forall x, \min_{u \in U} l(x,u) + \frac{\partial J_{lower}}{\partial x} f(x,u) \geq 0$$

global J_{lower}

$$\forall x, \min_{u \in U} l(x,u) + \frac{\partial J_{upper}}{\partial x} f(x,u) \leq 0$$

global J_{upper}

Sums of Squares

Lasserre, Jean B., et al. "Nonlinear optimal control via occupation measures and LMI-relaxations." SIAM journal on control and optimization.
Jiang, Y., & Jiang, Z. P. (2015). Global adaptive dynamic programming for continuous-time nonlinear systems. IEEE Transactions on Automatic Control.

Method – Under-Approximation

$$\forall x, \min_{u \in U} l(x,u) + \frac{\partial J_{lower}}{\partial x} f(x,u) \geq 0 \quad \Longleftrightarrow \quad \forall x, u \in U, l(x,u) + \frac{\partial J_{lower}}{\partial x} f(x,u) \geq 0$$

Integral to push up under-approximation

$$\max \int_{X_{int}} J_{lower}(x)\, dx$$

$$\text{s.t. } l(x,u) + \frac{\partial J_{lower}}{\partial x} f(x,u) \geq 0 \text{ for } u \in U, x \in X$$

$$J_{lower}(x) \succcurlyeq 0$$

Value-function-like

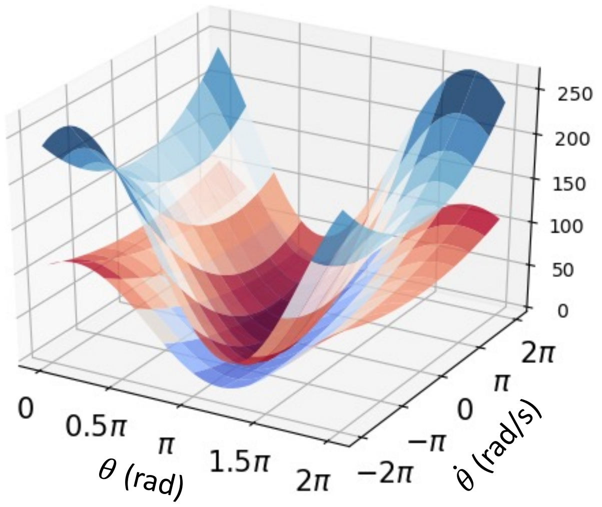
SOS conditions to enforce regional HJB inequality

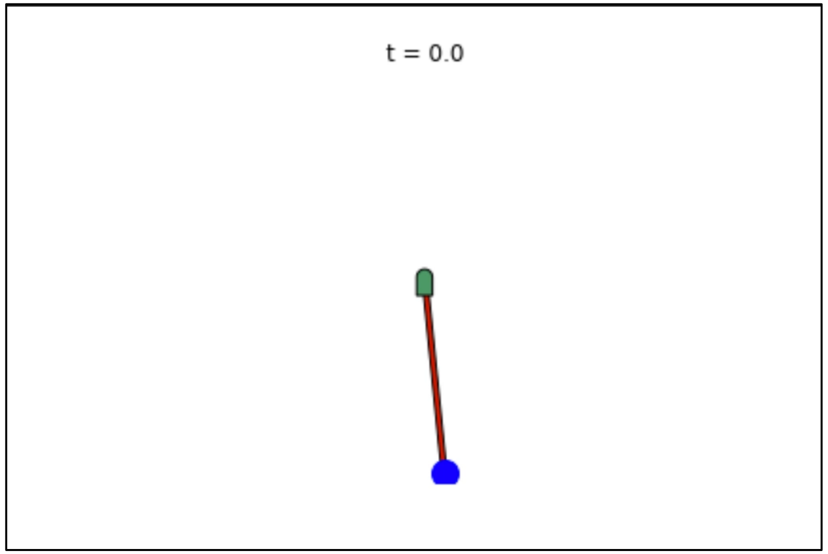
$$l(x,u) = q(x) + u^T R u$$

$$\pi_{lower}(x) = \mathrm{clamp}\left(-\frac{1}{2} R^{-1} f_2(x)^T \frac{\partial J_{lower}}{\partial x}^T, u_{\min}, u_{\max}\right)$$

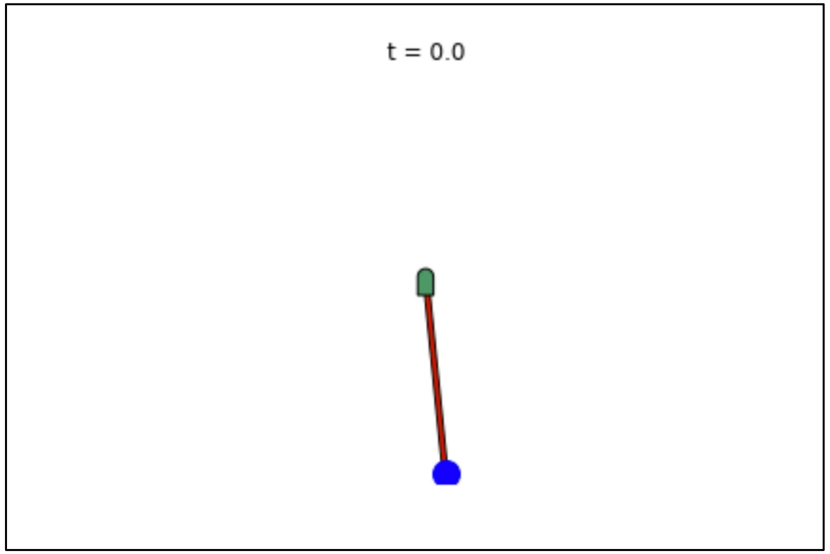
Inverted Pendulum

- Input limits = $0.37mgl$
- Nontrivial pumping





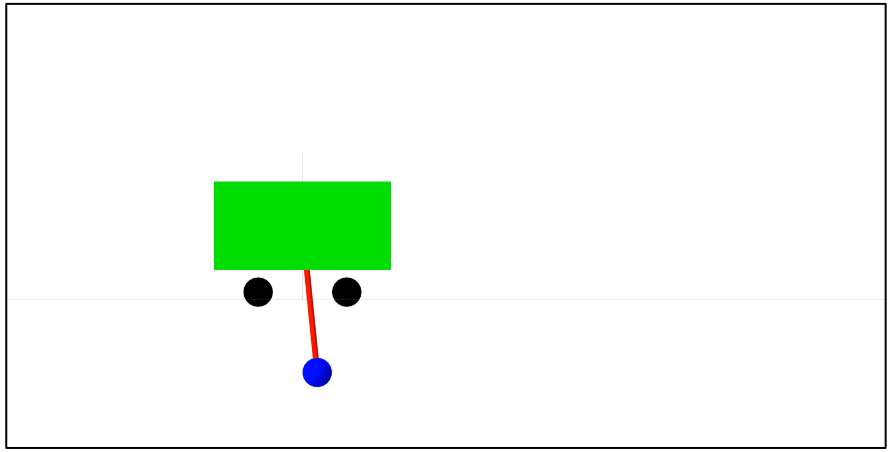
3-deg J_{upper}



3-deg J_{lower}

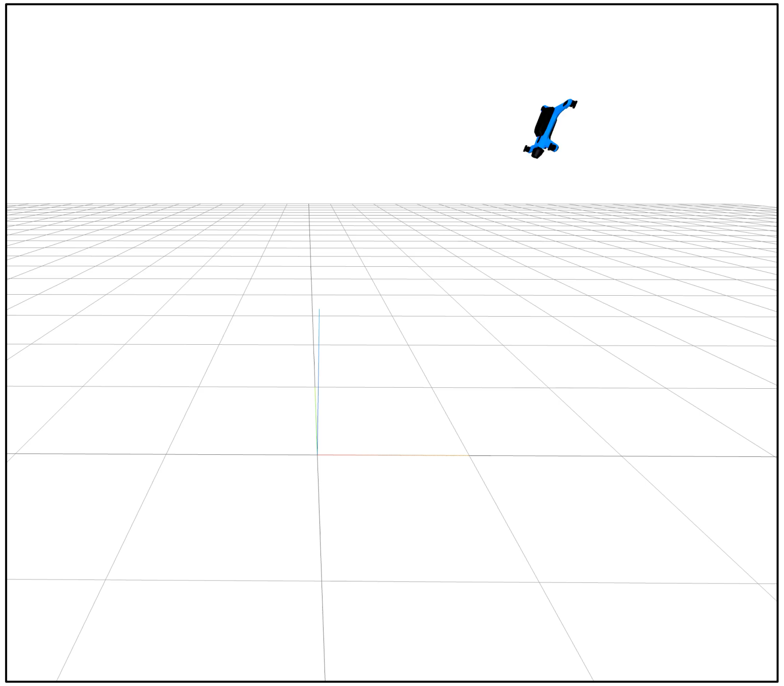
Cartpole

2-deg J_{upper}

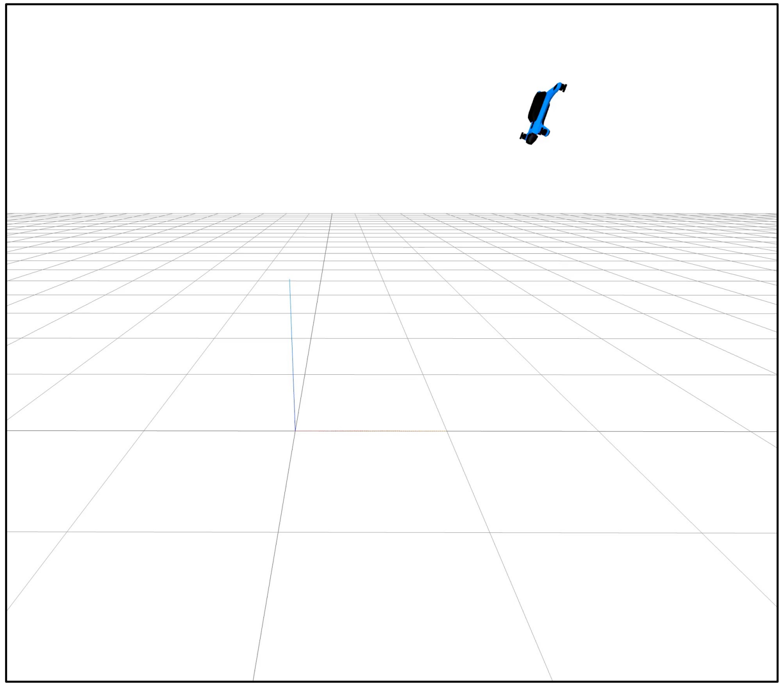


6-deg J_{lower}

3D Quadrotor



2-deg J_{upper}

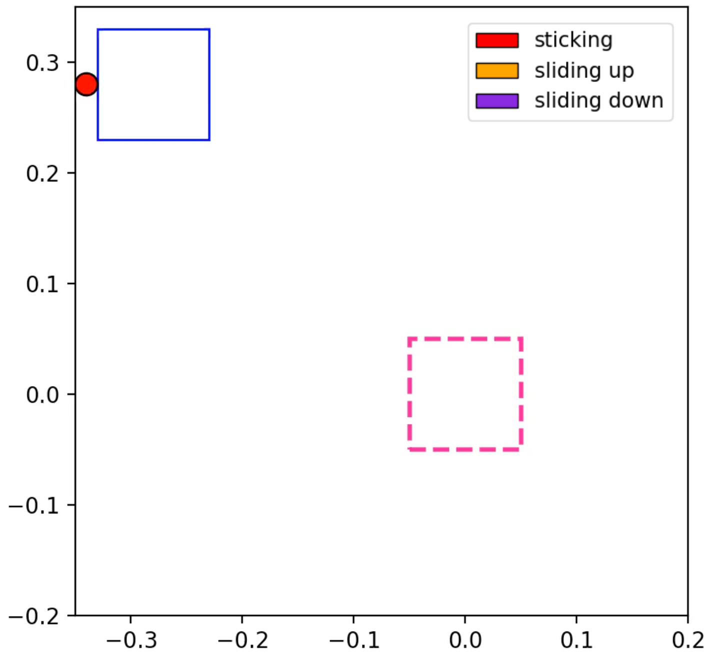


2-deg J_{lower}

$$x_{init} = [1, 1, 1, \pi, 0.4\pi, \pi, 1, 1, 1, 1, 1, 1]$$

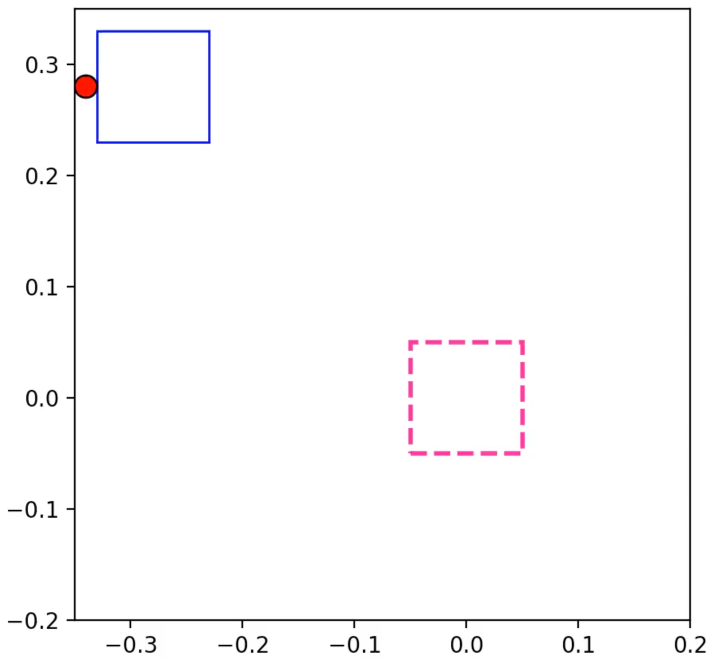
Planar Pusher

2-deg J_{lower}



sticking, sliding up and down

face switch



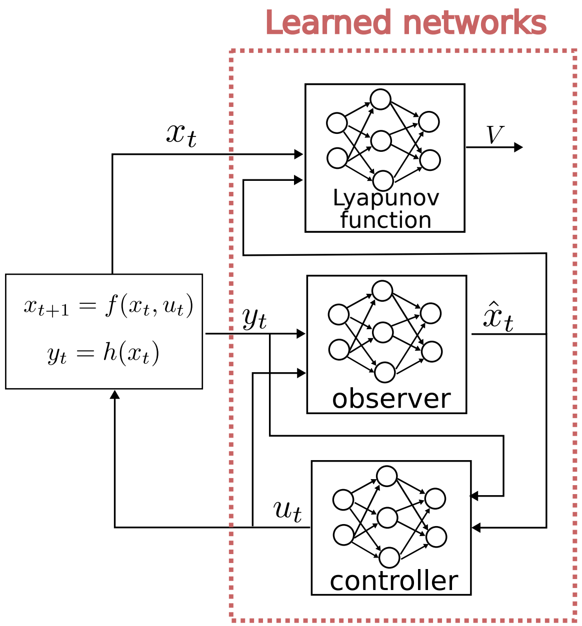
teleportation

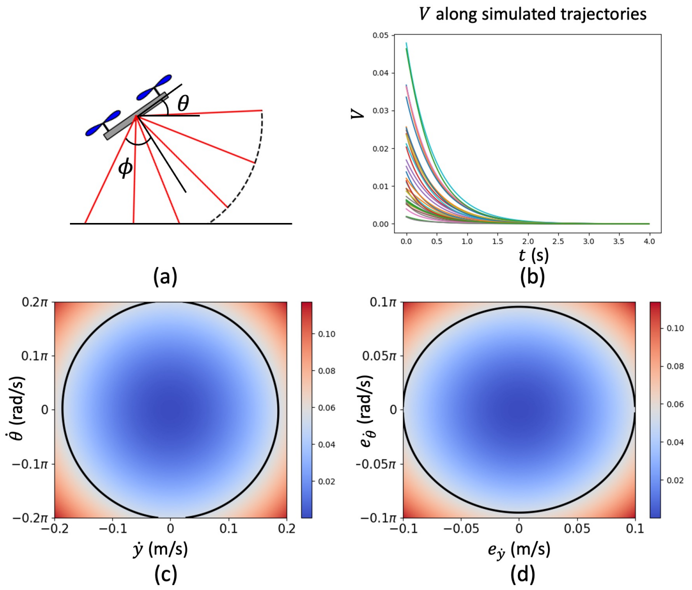
Lyapunov-stable Neural Control for State and Output Feedback

A Novel Formulation for Efficient Synthesis and Verification

Lujie Yang*, Hongkai Dai*, Zhouxing Shi, Cho-Jui Hsieh, Russ Tedrake, Huan Zhang

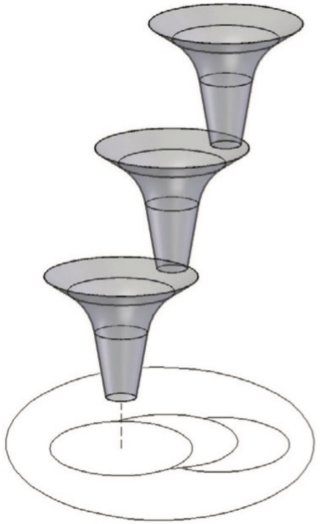
ICML 2024

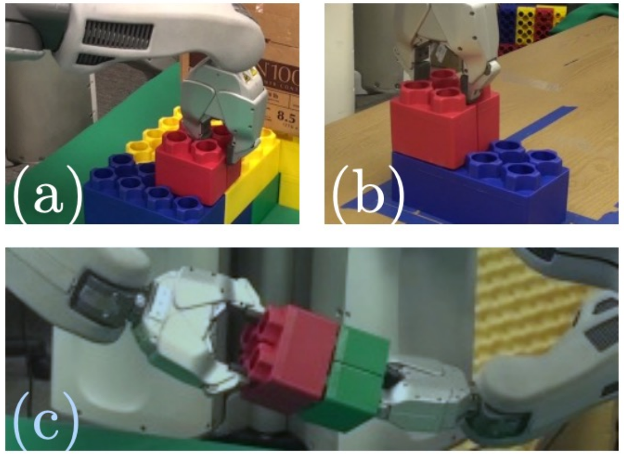


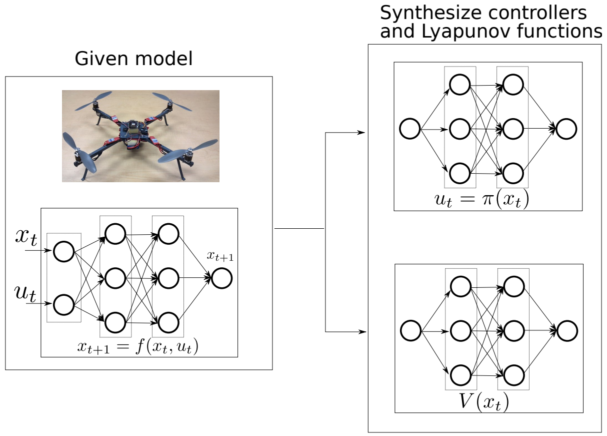


Motivation

- Synthesize complex stabilizing controllers with certifiable region of attraction
- Verification for output feedback control







<u>SOS control synthesis</u>

[Prajna et al. '04, Tedrake et al. '10]

- State feedback
- Can't handle complicated observation function

<u>Visuomotor policy</u>

[Levine et al. '15, Florence et al. '19]

- Impressive empirical performance
- Brittle, no formal guarantees

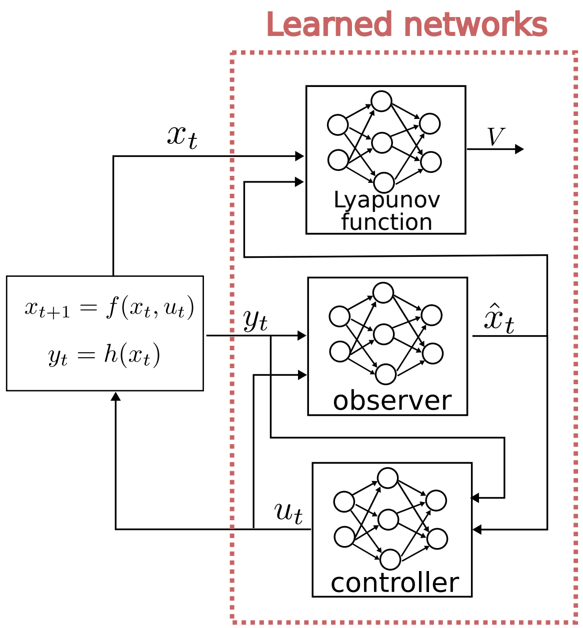
<u>Lyapunov-stable NN control</u>

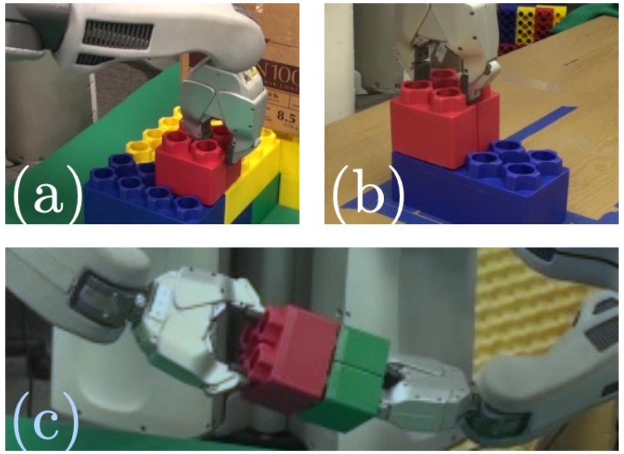
[Chang et al. '19, Dai et al. '21]

- NN Verification
- Require expensive solvers: MIP, SMT, SDP

Contribution

- Lyapunov-stable NN controller synthesis for both state and output feedback
- Post hoc rigorous NN verification
- Expensive complete solvers → fast empirical falsification + regularization





Visuomotor policy
[Levine et al '15, Pete et al '19]

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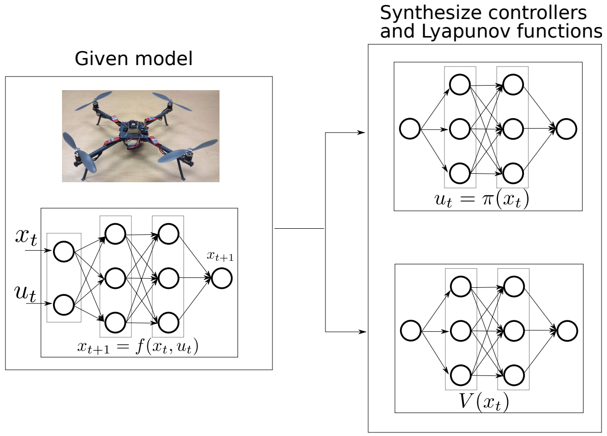

Lyapunov-stable NN control
[Ya-Chien et al '19, Hongkai et al '21]

- NN Verification
- Require expensive solvers: MIP, SMT

Contribution

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- Post hoc rigorous NN verification
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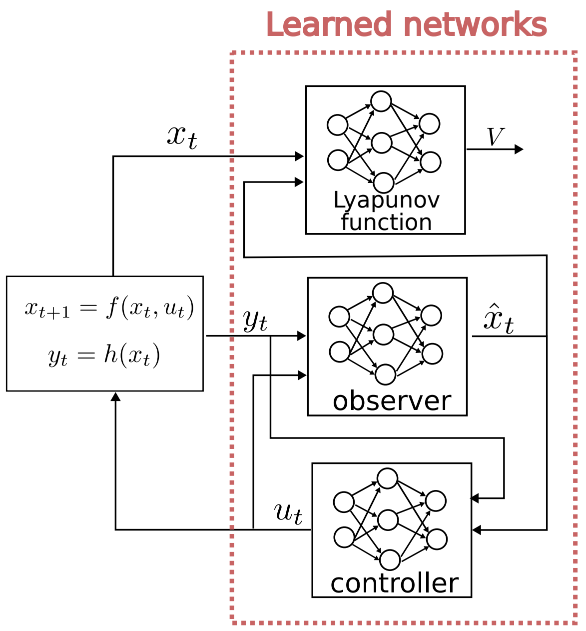


Novel Formulation

- Easier to train & certify
- Afford control over ROA growth during training

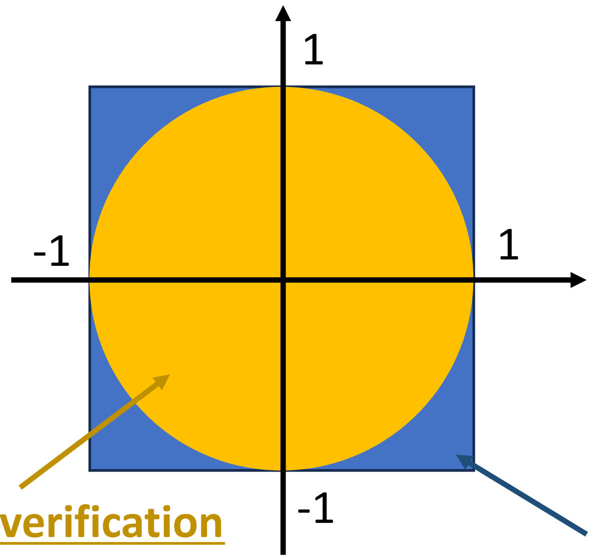
Verification Formulation

Theorem:

- Defines larger ROA inner-approximation
- Removes previous unnecessarily restrictive conditions in uncertified regions

2D Double Integrator

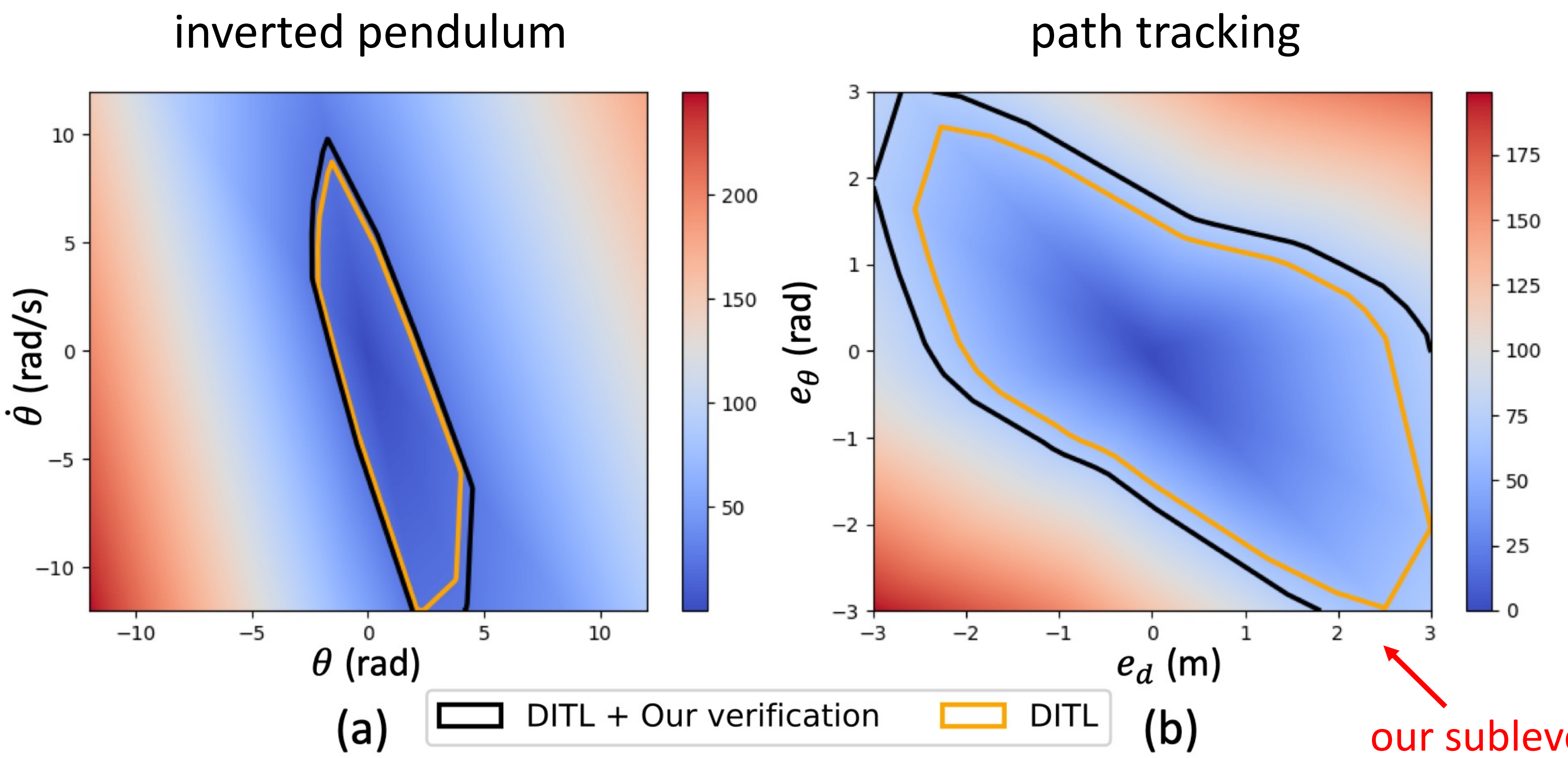
$$x_{t+1} = x_t + 0.1u_t$$
$$\pi(x_t) = -x_t$$
$$V(x_t) = x_t^T x_t$$
$$V(x_{t+1}) - (1 - 0.1)V(x_t) = -0.09x_t^T x_t \leq 0$$



previous NN verification
largest sublevel set within square

ours
entire invariant square

Verifying Existing Neural Lyapunov Models



our sublevel set expands beyond $\mathcal{B}$

- DITL uses MIP to verify $\tilde{\mathcal{S}} = \{\xi_t | V(\xi_t) < \min_{\bar{\xi}_t \in \partial\mathcal{B}} V(\bar{\xi}_t)\}$

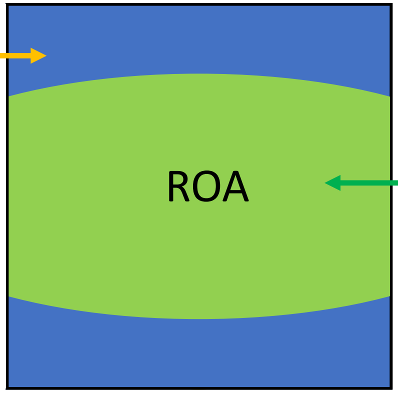
Wu, Junlin, et al. "Neural Lyapunov Control for Discrete-Time Systems." Neurips (2023).

Training Formulation

- Jointly optimize controller, observer and Lyapunov function, all parameterized with NNs
- Explicitly reason about ROA during training

previous NN verification

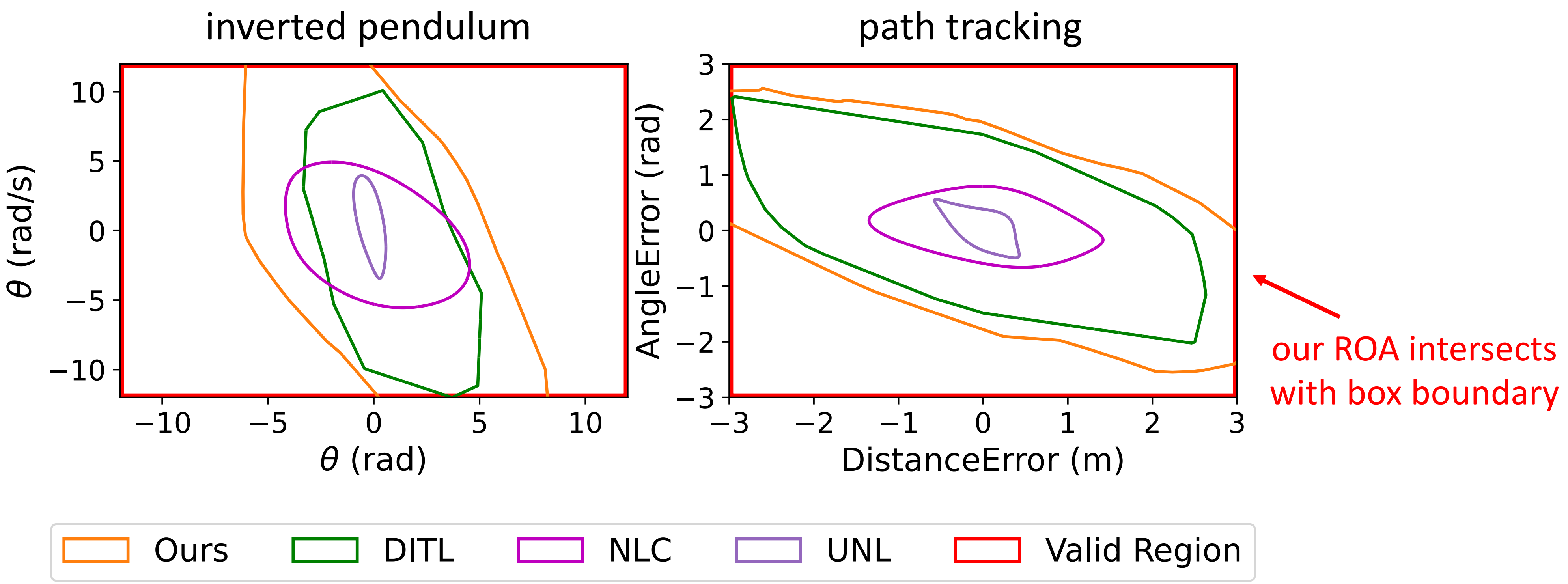
- train Lyapunov time derivative condition for the entire square
- expensive solvers: MIP, SMT, SDP…



ours

- train only within certifiable ROA
- cheap projected gradient descent attack
- regularization to ease post-training verification
- GPU-accelerated complete α, β-CROWN NN Verifier (https://abcrown.org)

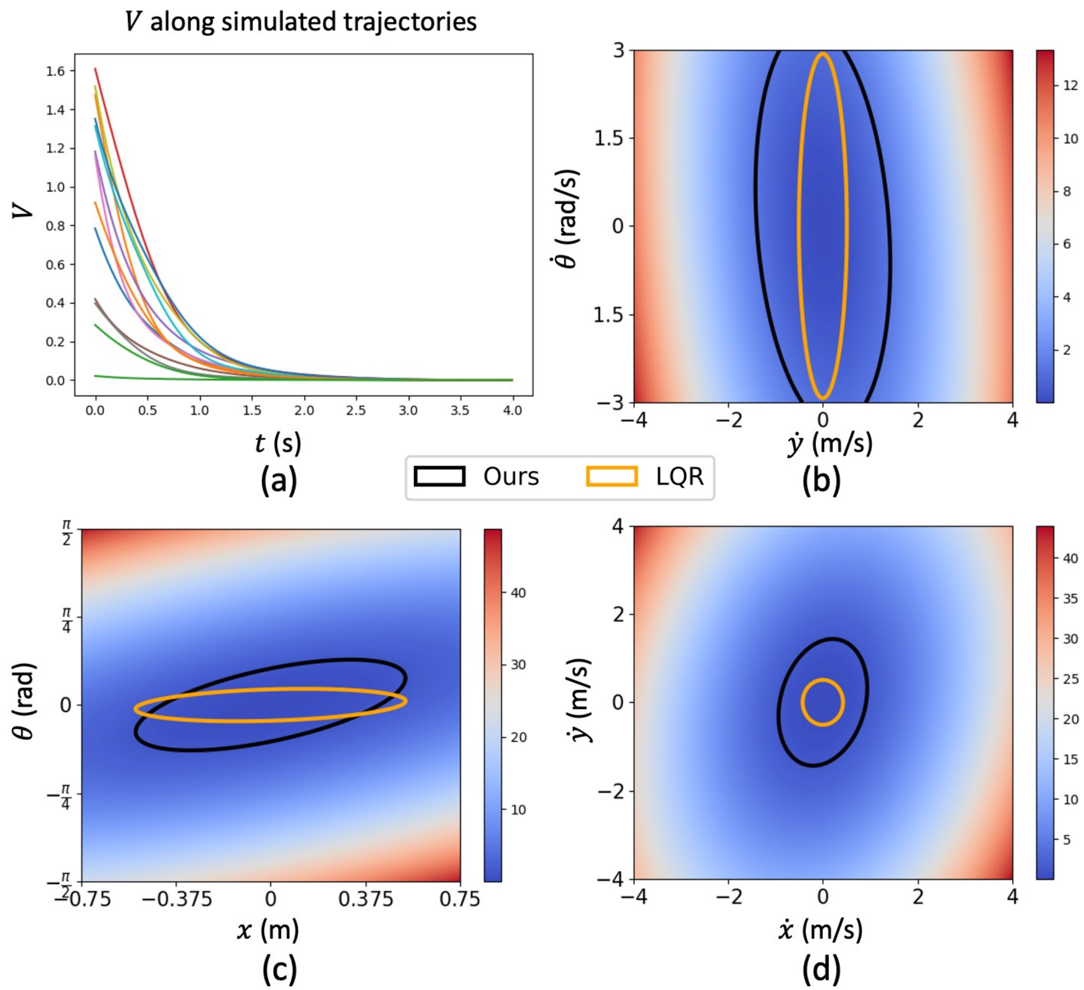
Training + Verification with New Formulation



Wu, Junlin, et al. "Neural Lyapunov Control for Discrete-Time Systems." Neurips (2023).
Chang, Ya-Chien, Nima Roohi, and Sicun Gao. "Neural lyapunov control." Neurips (2019).
Zhou, Ruikun, et al. "Neural Lyapunov control of unknown nonlinear systems with stability guarantees." Neurips (2022).

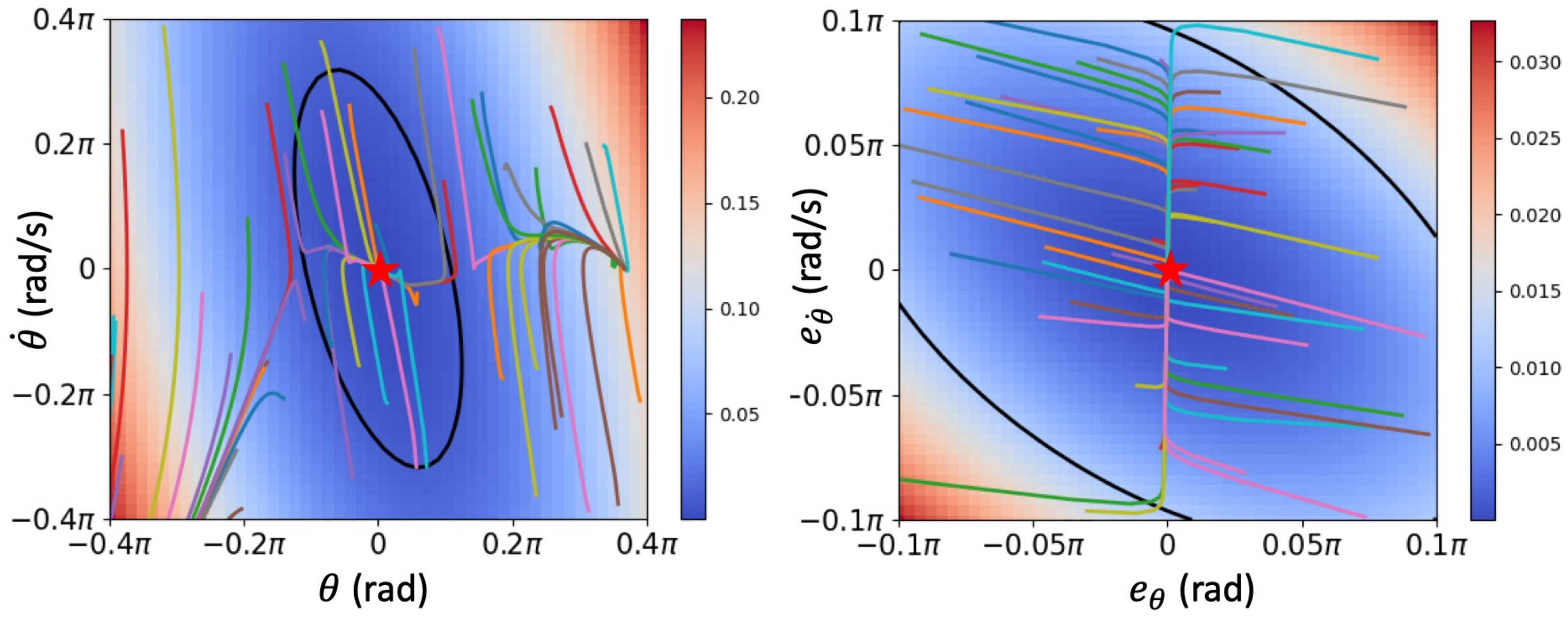
2D Quadrotor

- Previous methods fail to train
- Our method succeeds for large ROA



Inverted Pendulum with Angle observation

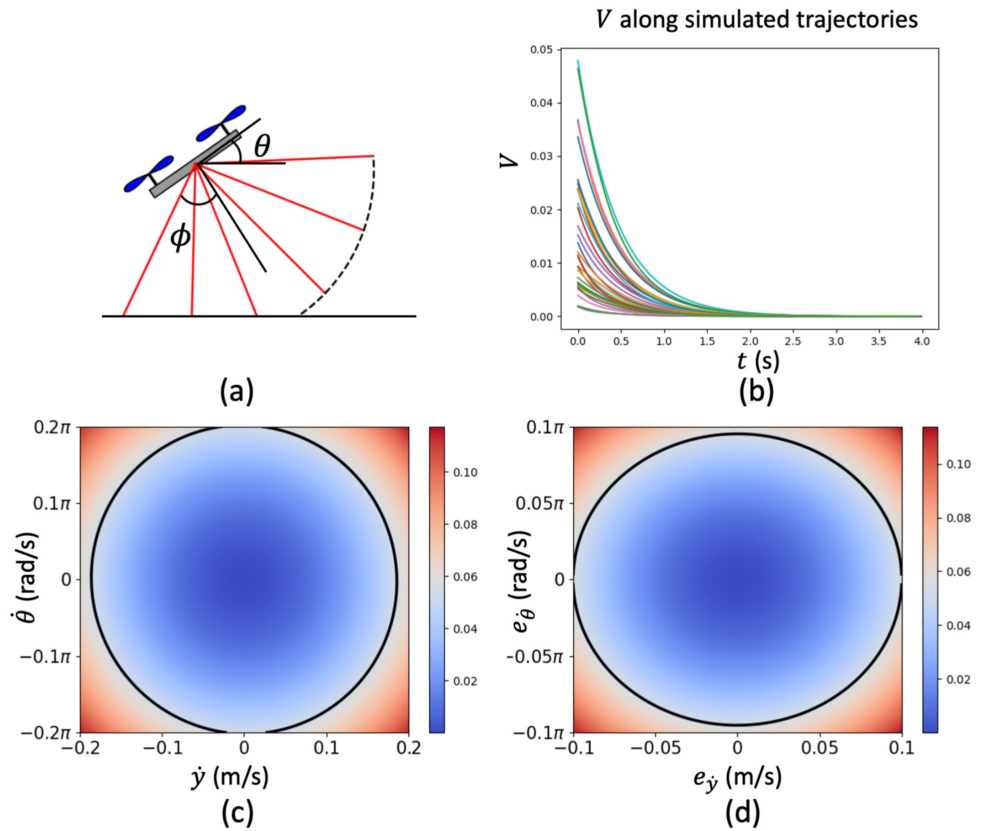
- Previous work: $|u| \leq 8.15\, mgl$ for state feedback
- Our work: succeed under challenging torque limit $|u| \leq \frac{mgl}{3}$

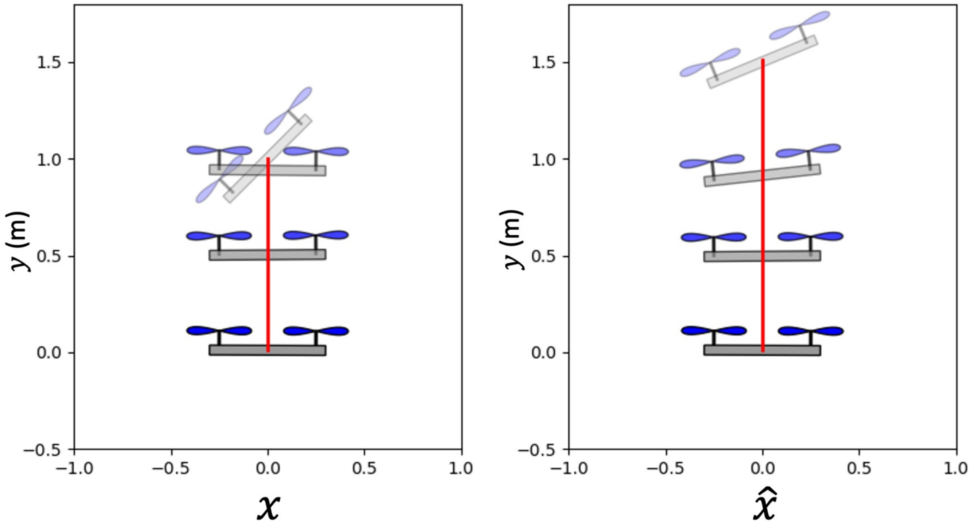


(a) (b)

2D Quadrotor with Lidar Sensor

- Output feedback control
- Generalize well outside of ROA





$$e = [0.5, -\frac{\pi}{8}, 0.1, \frac{\pi}{8}]$$

Conclusion

- Solve HJB inequalities for value function under-/over-approximation via SOS
    - Optimality & stability
    - Planar pushing task
- Lyapunov-stable neural control via NN verification
    - Novel formulation: efficient synthesis & verification
    - Output feedback control

Thank you!

- "Approximate Optimal Controller Synthesis for Cart-Poles and Quadrotors via Sums-of-Squares."
  Lujie Yang, Hongkai Dai, Alexandre Amice, Russ Tedrake
  RA-L 2023

- "Lyapunov-stable Neural Control for State and Output Feedback: A Novel Formulation for Efficient Synthesis and Verification."
  Lujie Yang*, Hongkai Dai*, Zhouxing Shi, Cho-Jui Hsieh, Russ Tedrake, Huan Zhang
  ICML 2024