Site percolation on the square lattice

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1 Introduction

The site percolation model was written in Python 3.6.9 using Visual Studio Code 1.55.1. The tests were performed on PC with Intel Core i5 4690K CPU and 16GB of RAM.

The simulations were conducted on square lattices of size $L \times L$ where $L = 10, 50, 100$. The goal was to find the probability of occurance of spanning cluster P_{flow}, average size of maximal cluster $< s_{max} >$ depending on function p (probability of each site being occupied) and distribution of clusters $n(s, p, L)$ where s is a size of a cluster.

To find out if there is a spanning cluster on the lattice, the DFS (Depth First Search) algorithm was used instead of the Burning Method. It does not iterate through all sites of the lattice therefore it is faster, yet it still can find out if there is the path between the top and the bottom side of the lattice.

2 Results

L	t[s]
10	0.563
50	10.860
100	44,171

Table 1: Times needed to create a single output file of type "Ave_L{L}T{T}.txt" for given L where $T = 100$ and p ranges from $p_0 = 0.01$ to $p_k = 1$ with step $dp = 0.01$.

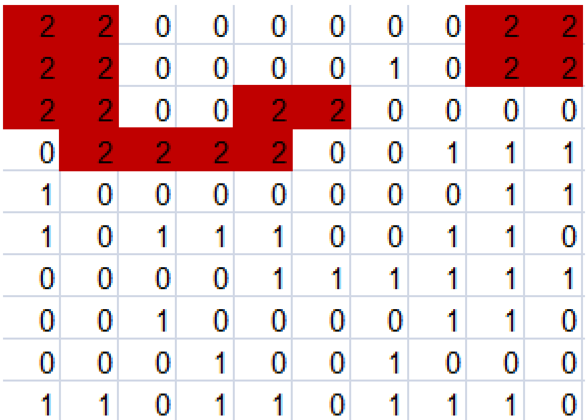
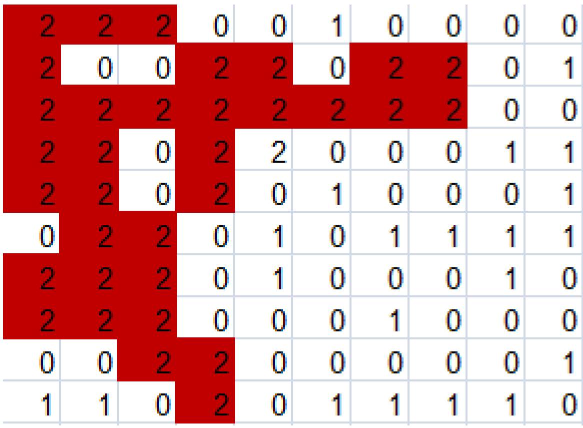


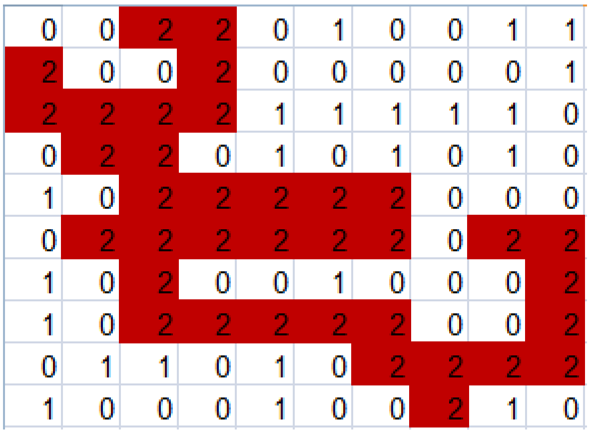
Figure 1: Visualisation of DFS for $L = 10$ and $p = 0.4$.

For large values of p we can observe some peaks in the number of large clusters. This is the result of p being above the critical point which allows larger clusters to group more often than the smaller ones.



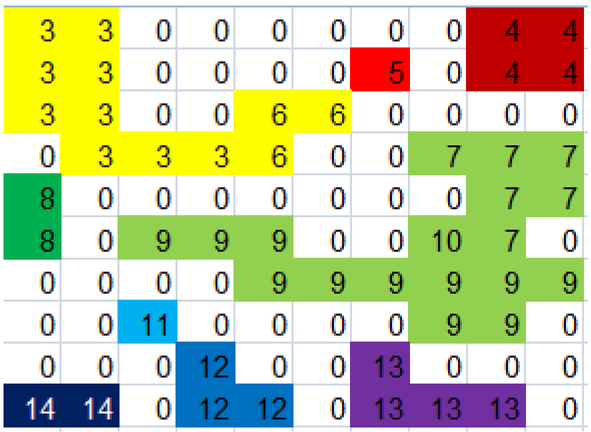
2	2	2	0	0	1	0	0	0	0
2	0	0	2	2	0	2	2	0	1
2	2	2	2	2	2	2	2	0	0
2	2	0	2	2	0	0	0	1	1
2	2	0	2	0	1	0	0	0	1
0	2	2	0	1	0	1	1	1	1
2	2	2	0	1	0	0	0	1	0
2	2	2	0	0	0	1	0	0	0
0	0	2	2	0	0	0	0	0	1
1	1	0	2	0	1	1	1	1	0

Figure 2: Visualisation of DFS for $L = 10$ and $p = 0.5$.



0	0	2	2	0	1	0	0	1	1
2	0	0	2	0	0	0	0	0	1
2	2	2	2	1	1	1	1	1	0
0	2	2	0	1	0	1	0	1	0
1	0	2	2	2	2	2	0	0	0
0	2	2	2	2	2	2	0	2	2
1	0	2	0	0	1	0	0	0	2
1	0	2	2	2	2	2	0	0	2
0	1	1	0	1	0	2	2	2	2
1	0	0	0	1	0	0	2	1	0

Figure 3: Visualisation of DFS for $L = 10$ and $p = 0.6$.



3	3	0	0	0	0	0	0	4	4
3	3	0	0	0	0	5	0	4	4
3	3	0	0	6	6	0	0	0	0
0	3	3	3	6	0	0	7	7	7
8	0	0	0	0	0	0	0	7	7
8	0	9	9	9	0	0	10	7	0
0	0	0	0	9	9	9	9	9	9
0	0	11	0	0	0	0	9	9	0
0	0	0	12	0	0	13	0	0	0
14	14	0	12	12	0	13	13	13	0

Figure 4: Visualisation of clusterization for $L = 10$ and $p = 0.4$.

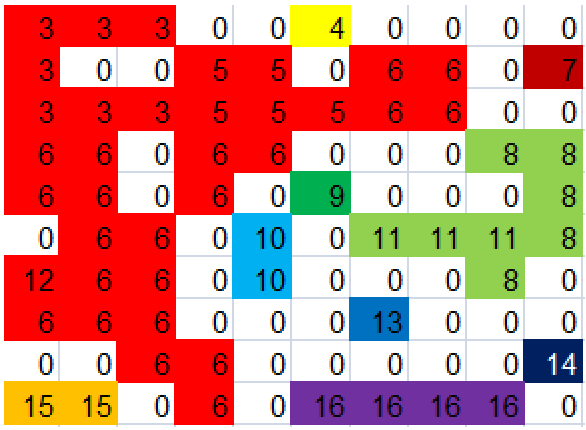


Figure 5: Visualisation of clusterization for $L = 10$ and $p = 0.5$.

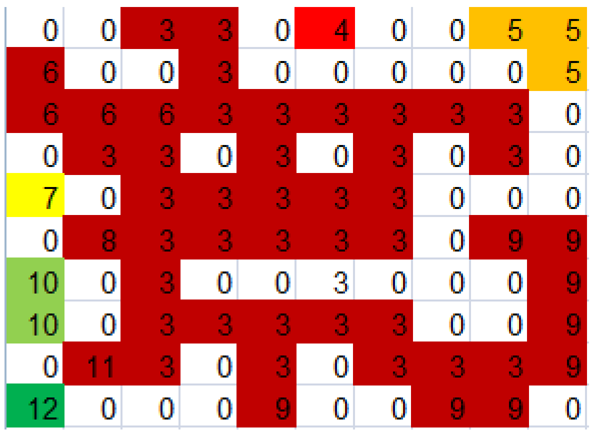


Figure 6: Visualisation of clusterization for $L = 10$ and $p = 0.6$.

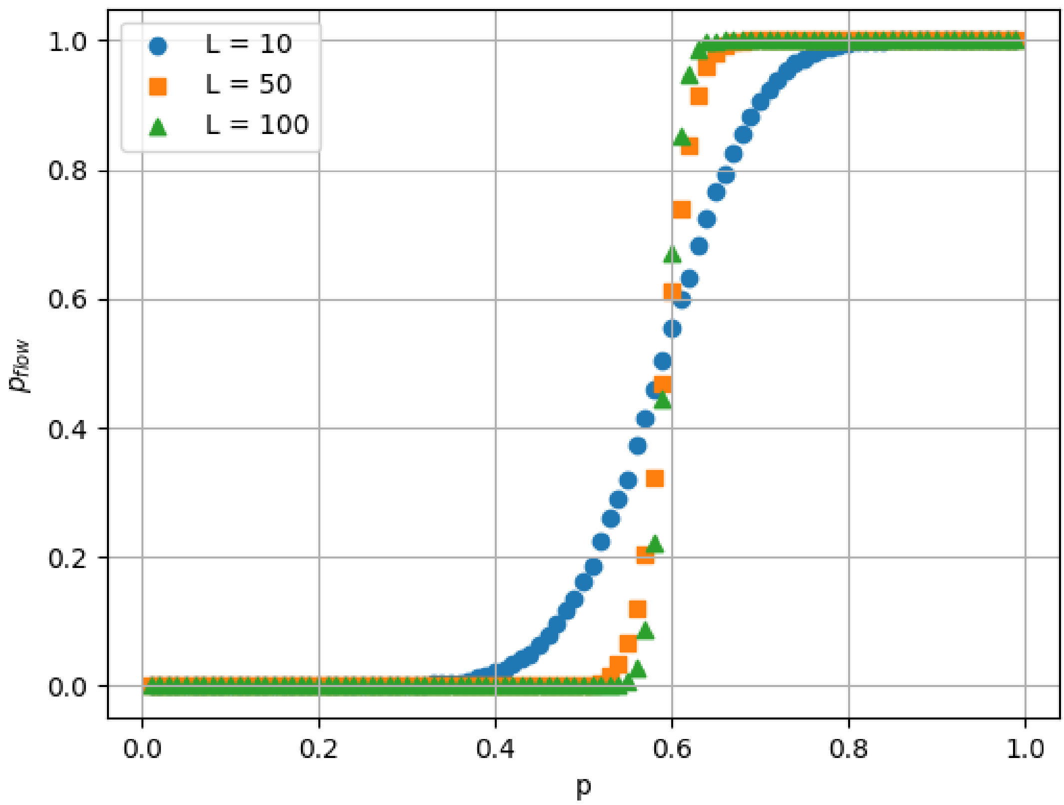


Figure 7: Probability P_{flow} that the path connecting the first and the last row exists as a function of p for $L = 10, 50, 100$.

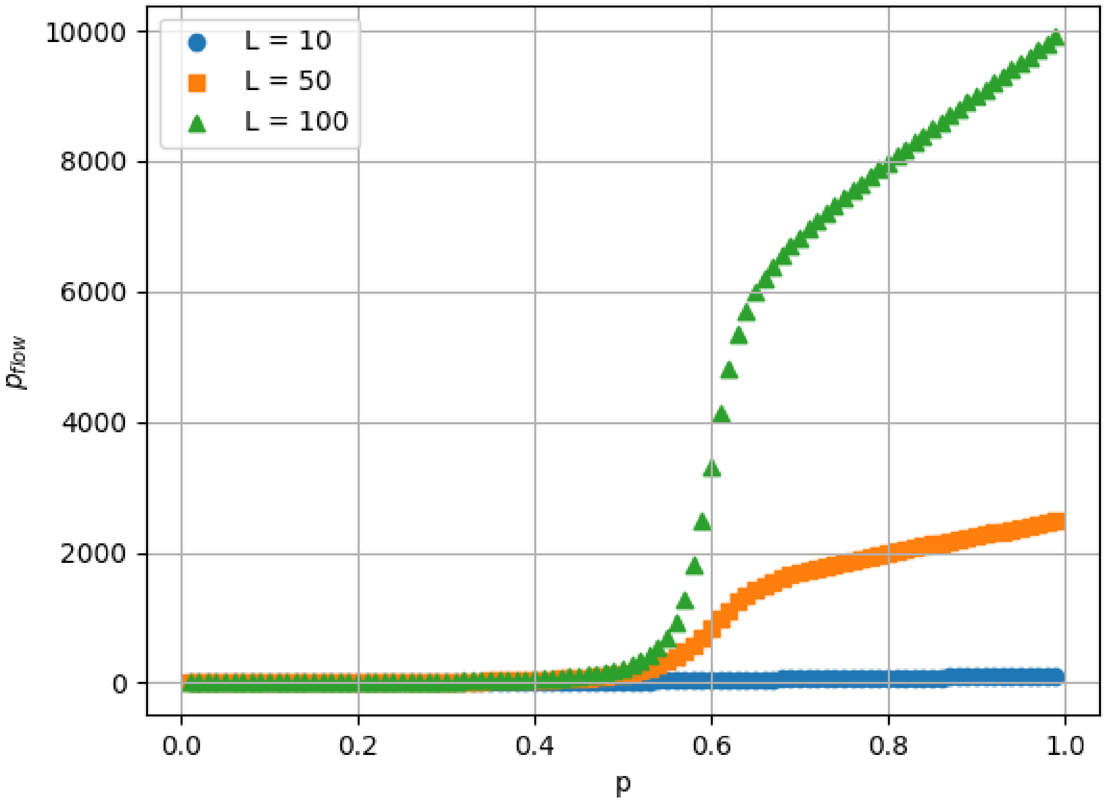


Figure 8: The average size of the maximum cluster $< s_{max} >$ as a function of p for $L = 10, 50, 100$.

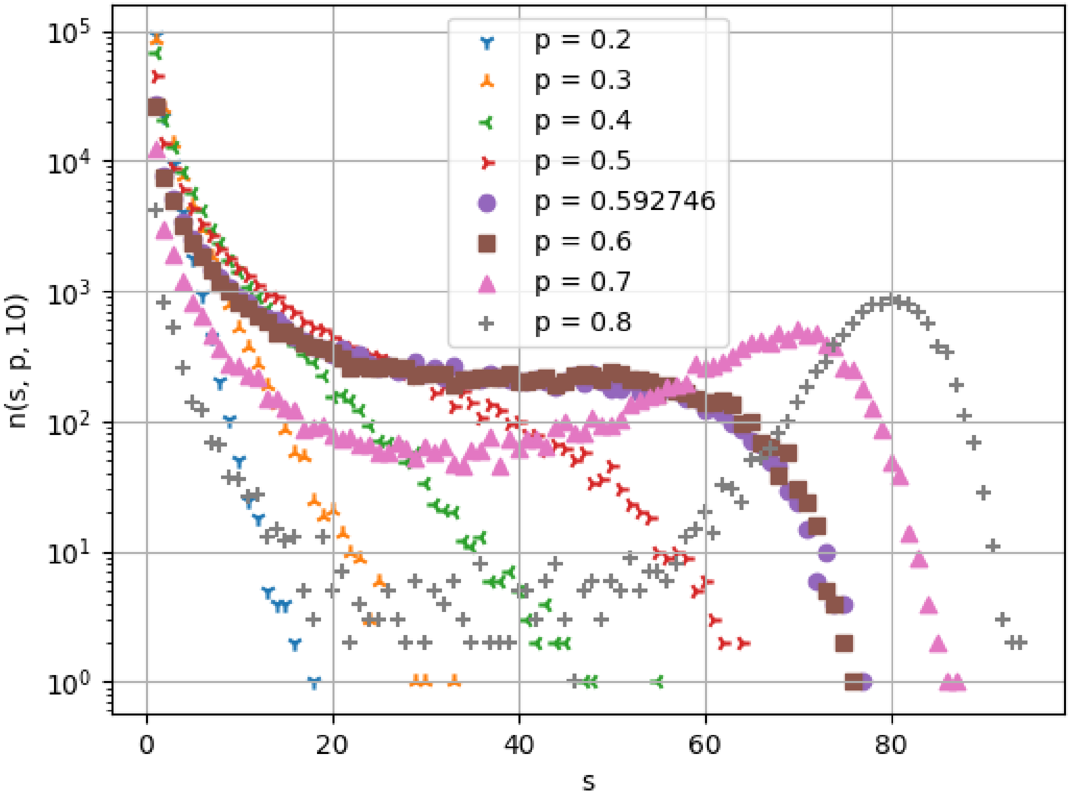


Figure 9: Distribution of clusters $n(s, p, L)$ for a given p for $L = 10$.

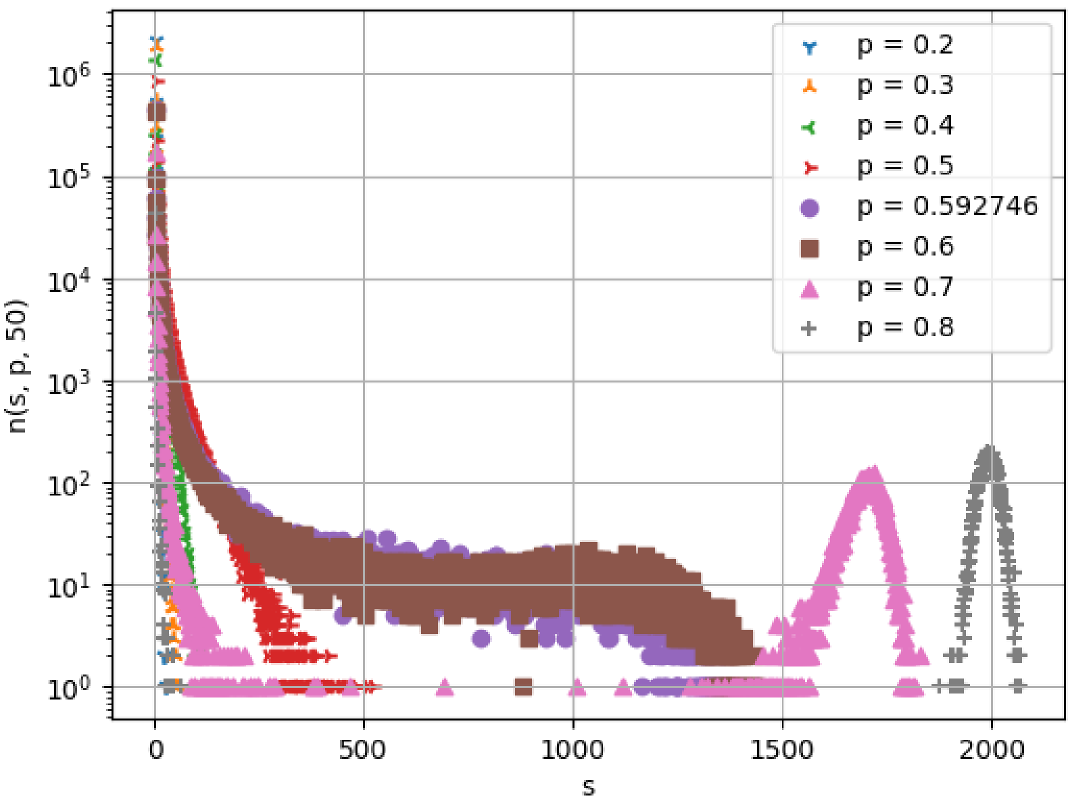


Figure 10: Distribution of clusters $n(s, p, L)$ for a given p for $L = 50$.

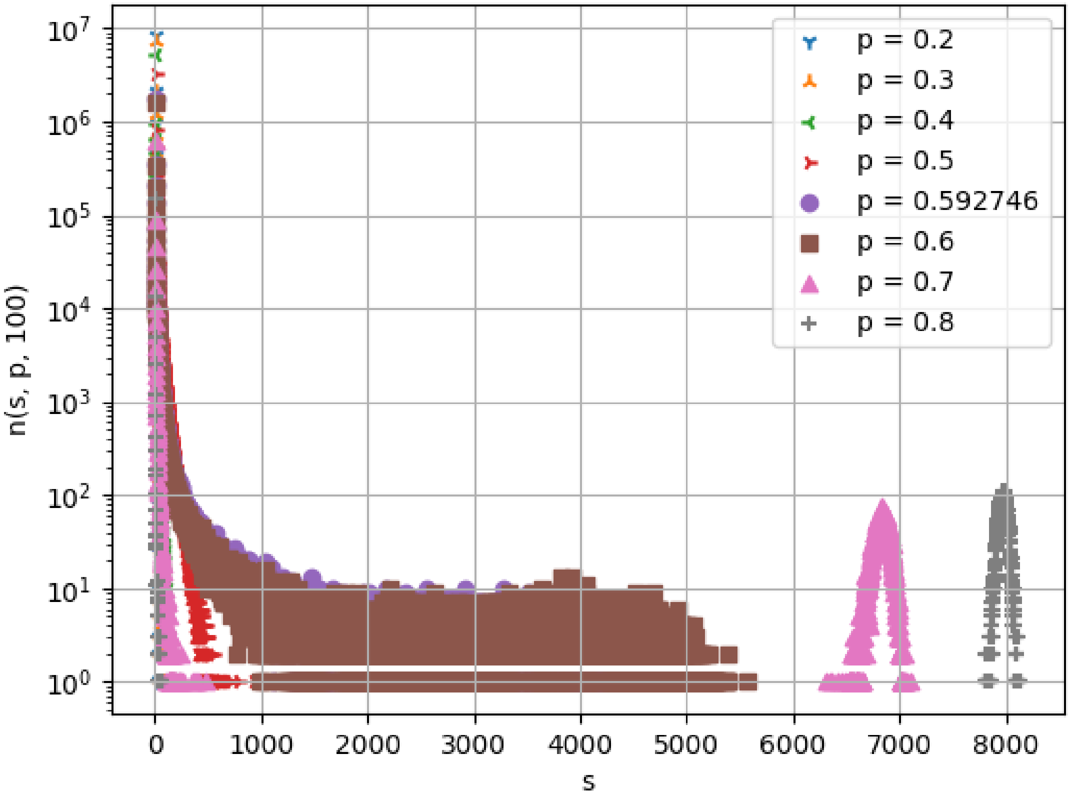


Figure 11: Distribution of clusters $n(s, p, L)$ for a given p for $L = 100$.

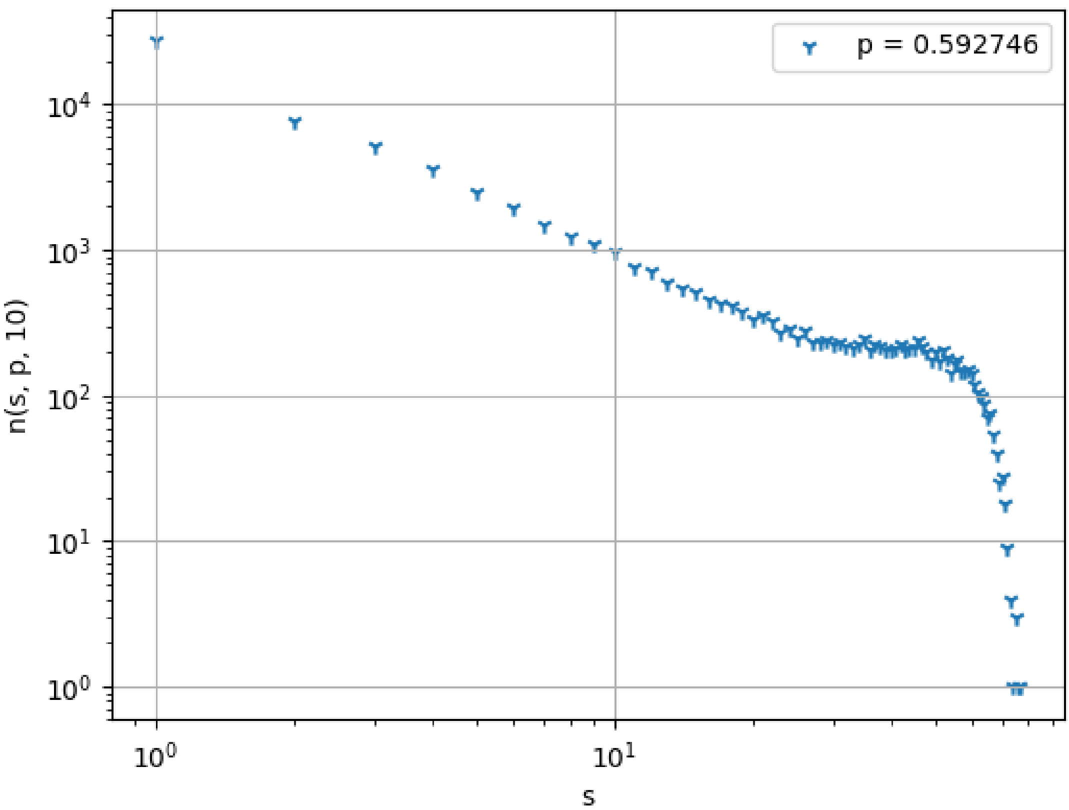


Figure 12: Distribution of clusters $n(s, p, L)$ for a $p = p_c$ for $L = 10$ in logarithmic scale.

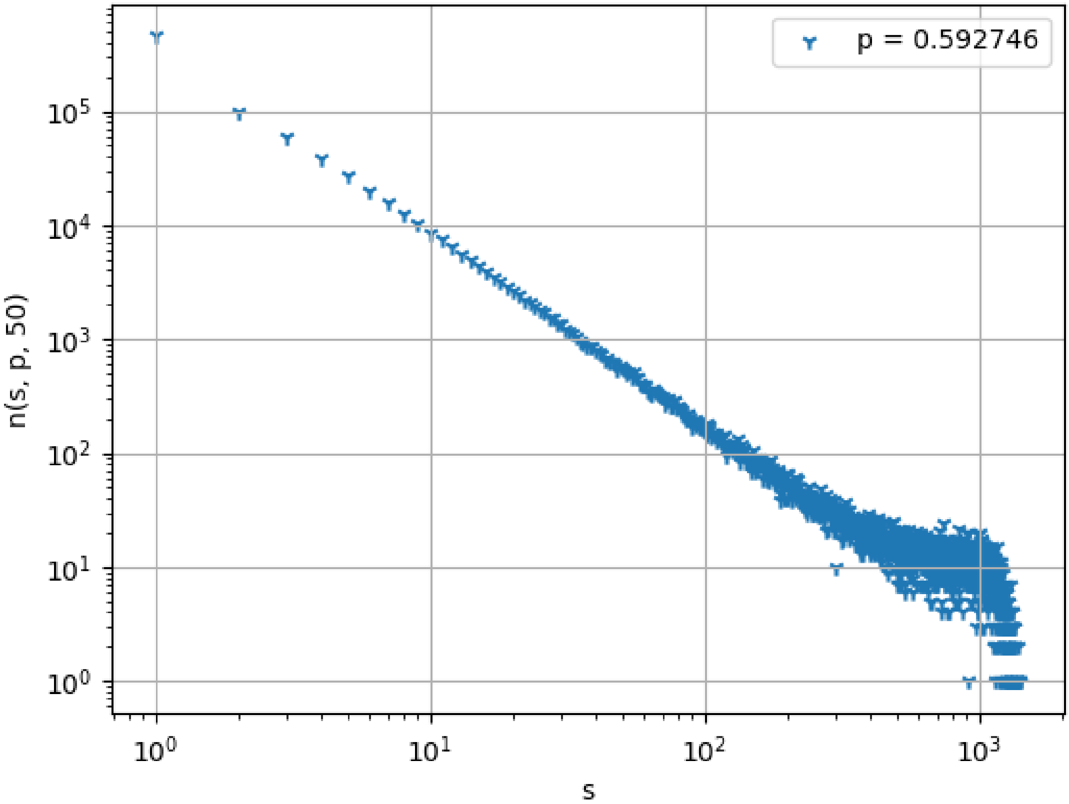


Figure 13: Distribution of clusters $n(s, p, L)$ for a $p = p_c$ for $L = 50$ in logarithmic scale.

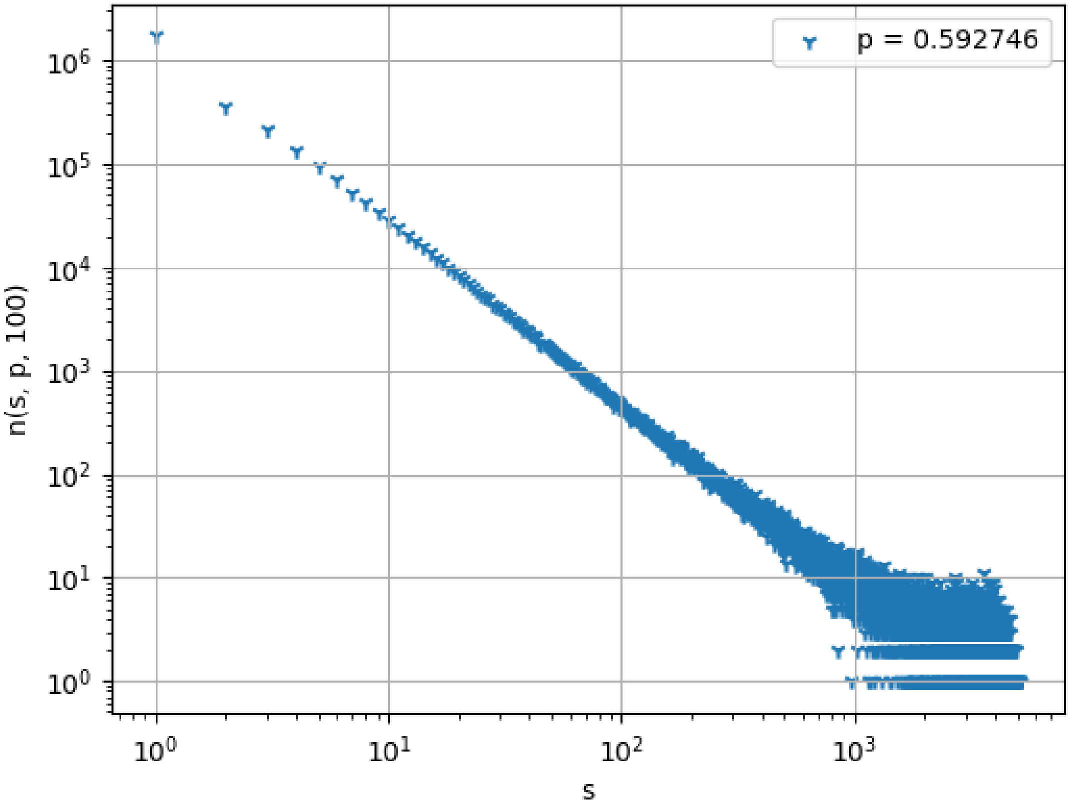


Figure 14: Distribution of clusters $n(s, p, L)$ for a $p = p_c$ for $L = 100$ in logarithmic scale.