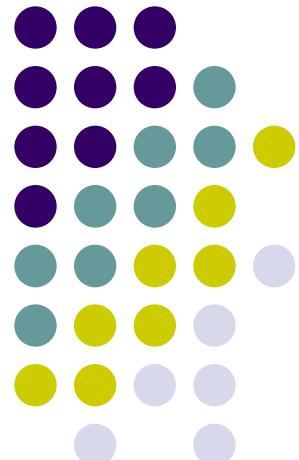


Angewandte Mathematik

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Lösungsvorschlag für die Aufgabe 8.1 & 8.2



Aufgabe 8.1 (vergangene Klausur Aufgabe (10 Punkte))

Lösen Sie die Differentialgleichung erster Ordnung durch geeignete Separation:

$$y'(x) = xy(x), \quad x \in \mathbb{R}$$

Im Fall $y \neq 0$:

$$y'(x) = xy(x) \Rightarrow \int \frac{y'}{y} dx = \int x dx \Rightarrow \ln|y| = \frac{x^2}{2} + c$$

$$\Rightarrow |y| = e^{\frac{x^2}{2} + c} \Rightarrow y = \pm e^c e^{\frac{x^2}{2}} = d e^{\frac{x^2}{2}} \quad (d \in \mathbb{R} \setminus \{0\})$$

$y \equiv 0$ ist auch eine Lösung.

$$\Rightarrow y = d e^{\frac{x^2}{2}} \quad (d \in \mathbb{R})$$



Aufgabe 8.2

Bestimmen Sie die allgemeine Lösung der linearen Differenzialgleichung mit konstanten Koeffizienten

$$1) y'''(x) - 3y''(x) + 3y'(x) - y(x) = 0, \quad x \in \mathbb{R}$$

$$2) y'''(x) - y''(x) + y'(x) - y(x) = 0, \quad x \in \mathbb{R}$$

$$3) y''(x) - 3y'(x) + 2y(x) = 0, \quad x \in \mathbb{R}$$

Zu 3): Lösen Sie das Anfangswertproblem mit

$$y(0) = 1, \quad y'(0) = 0.$$

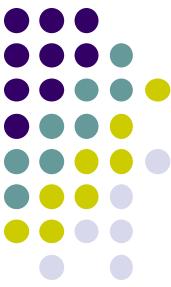
(vergangene Klausur Aufgabe (20 Punkte))

$$1) \lambda^3 - 3\lambda^2 + 3\lambda - 1 = (\lambda - 1)^3 = 0 \Rightarrow \lambda = 1$$

$$\Rightarrow y(x) = c_1 e^x + c_2 x e^x + c_3 x^2 e^x, \quad c_1, c_2, c_3 \in \mathbb{R}$$

$$2) \lambda^3 - \lambda^2 + \lambda - 1 = (\lambda - 1)(\lambda^2 + 1) = 0 \Rightarrow \lambda = 1, \pm i$$

$$\Rightarrow y(x) = c_1 e^x + c_2 \cos(x) + c_3 \sin(x), \quad c_1, c_2, c_3 \in \mathbb{R}$$



$$3) \lambda^2 - 3\lambda + 2 = (\lambda - 1)(\lambda - 2) = 0 \Rightarrow \lambda = 1, 2$$

$$\Rightarrow y(x) = c_1 e^x + c_2 e^{2x}, \quad c_1, c_2 \in \mathbb{R}$$

$$\Rightarrow y'(x) = c_1 e^x + 2c_2 e^{2x}$$

$$\begin{cases} y(0) = c_1 + c_2 = 1 \\ y'(0) = c_1 + 2c_2 = 0 \end{cases} \Rightarrow c_1 = 2, c_2 = -1$$

$$\Rightarrow y(x) = 2e^x - e^{2x}$$

$$4) y''(x) - 2y'(x) + y(x) = e^{2x}, \quad x \in \mathbb{R}$$

$$\lambda^2 - 2\lambda + 1 = (\lambda - 1)^2 = 0 \Rightarrow \lambda = 1$$

$$\Rightarrow y_1(x) = e^x, \quad y_2(x) = xe^x$$

Nach der Formel auf S. 24 in der letzten Vorlesung

$$c'_1(x) = -\frac{f(x)y_2(x)}{a_2(y_1(x)y'_2(x) - y_2(x)y'_1(x))} = -xe^x, \quad c'_2(x) = \frac{f(x)y_1(x)}{a_2(y_1(x)y'_2(x) - y_2(x)y'_1(x))} = e^x$$

$$\Rightarrow c_1(x) = -xe^x + e^x, \quad c_2(x) = e^x$$

$$\Rightarrow y(x) = c_1 e^x + c_2 x e^x + (-xe^x + e^x)e^x + e^x(xe^x) = \textcolor{red}{c_1 e^x + c_2 x e^x + e^{2x}}, \quad c_1, c_2 \in \mathbb{R}$$



Heute

Differenzialgleichungen

- Numerische Lösungsmethoden

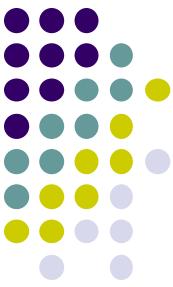


Numerische Lösungsmethoden

Anfangswertaufgabe
(Differentialgleichung erster Ordnung)

$$(1) \quad \begin{cases} \frac{dx}{dt} = f(t, x) \\ x(t_0) = x_0 \end{cases}$$

mit $f(t, x) : \Omega \rightarrow \mathbb{R}, (t_0, x_0) \in \Omega$.



Beispiel

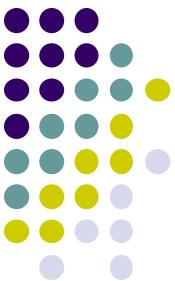
$$\begin{cases} \frac{dx}{dt} = x \\ x(0) = 1 \end{cases}$$

$$\longrightarrow x(t) = e^t \text{ (exakte Lösung, geschlossene Lösung)}$$

Wenn im allgemeinen keine geschlossene Lösung bestimmt werden kann, müssen wir generelle numerische Zugänge verwenden.

Grundtechnik

$$x(t_i) : \text{gegeben} \longrightarrow \text{Näherungswert für } x(t_i + \delta)$$



Anfangswertaufgabe

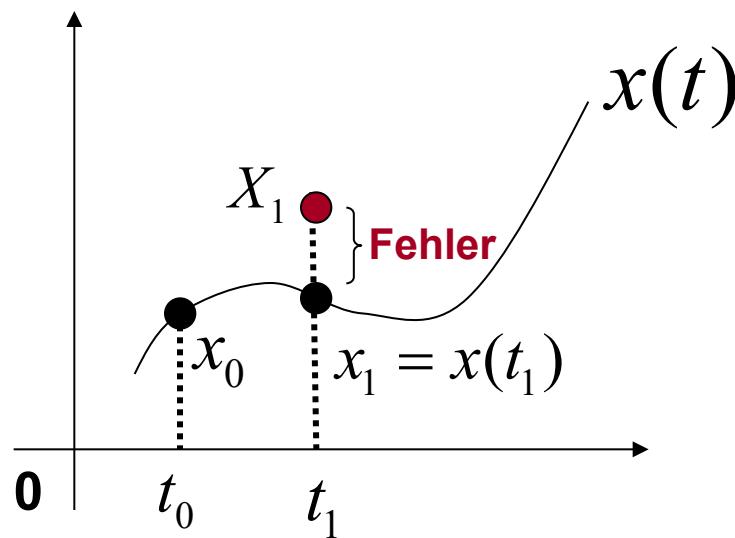
$$\frac{dx}{dt} = f(t, x), \quad t_0 \leq t \leq T, \quad x(t_0) = x_0$$

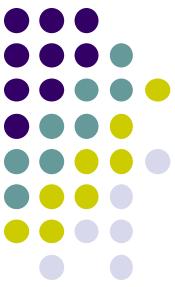
Für ausgewählte t_1, t_2, \dots berechnen wir die Näherungswerte X_1, X_2, \dots für $x(t_1), x(t_2), \dots$.

$$x(t_1) \approx X_1$$

$$x(t_2) \approx X_2$$

...





Euler-Verfahren

$$\frac{dx}{dt} = f(t, x)$$



$$\frac{x(t+h) - x(t)}{h}$$

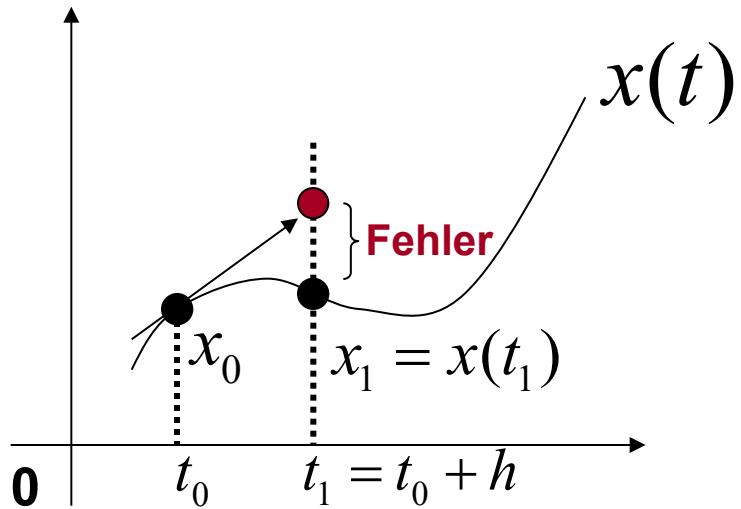
$$\Rightarrow x(t+h) \approx x(t) + h \cdot f(t, x(t)) \quad \cdots (2)$$

$x_0 = x(t_0)$: gegeben

$\longrightarrow x_1 = x(t_0 + h), x_2 = x(t_0 + 2h), \dots$

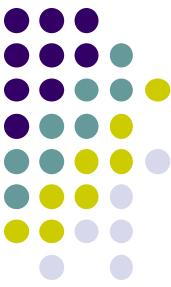


Bedeutung vom Euler-Verfahren



Taylor-Entwicklung (Taylorreihe)

$$x(t+h) = x(t) + \frac{h}{1!} x'(t) + \frac{h^2}{2!} x''(t) + \dots$$



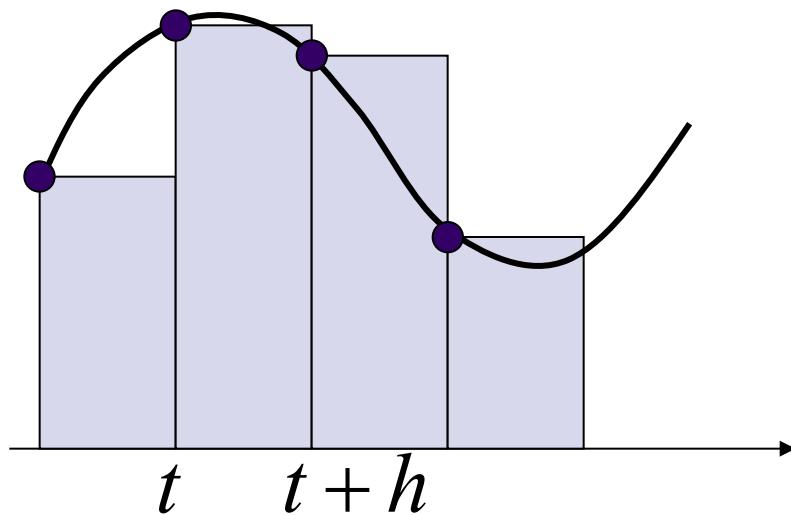
Bedeutung vom Euler-Verfahren

Nach dem 2. Hauptsatz der Differenzial - und Integralrechnung gilt

$$x(t+h) = x(t) + \int_t^{t+h} f(t, x(t)) dt$$

Euler-Verfahren:

$$\int_t^{t+h} f(t, x(t)) dt \approx h \cdot f(t, x(t))$$



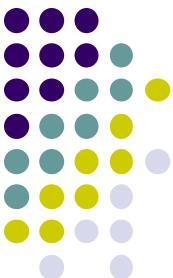


Heun-Verfahren

Taylor-Entwicklung

$$\begin{aligned}x_{i+1} &= x_i + h \boxed{x'(t_i)} + \frac{h^2}{2} \boxed{x''(t_i)} + O(h^3) \\&\quad \downarrow \qquad \searrow \\f(t_i, x_i) &\quad \frac{f(t_i + h, x_i + hf(t_i, x_i)) - f(t_i, x_i) - O(h^2)}{h} \\&= x_i + \frac{h}{2} (f(t_i, x_i) + f(t_{i+1}, x_i + hf(t_i, x_i))) + O(h^3)\end{aligned}$$

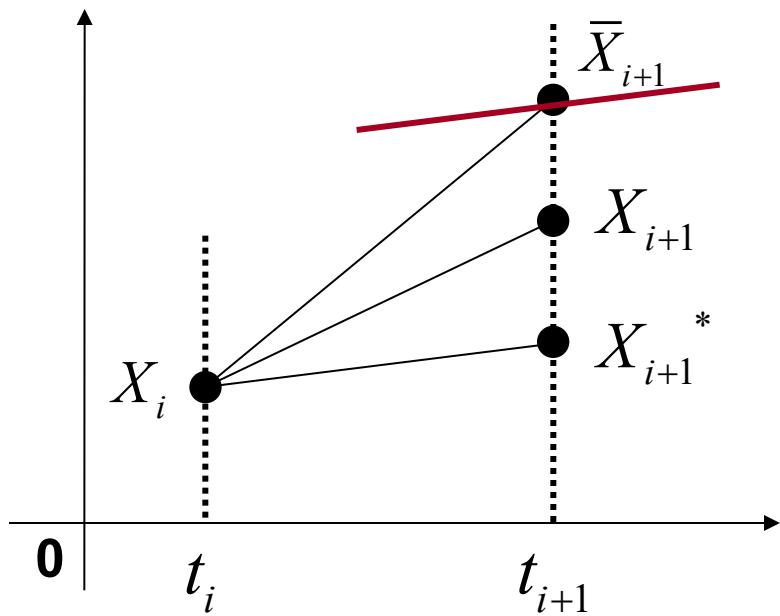
$$X_{i+1} = X_i + \frac{h}{2} (f(t_i, X_i) + f(t_{i+1}, X_i + hf(t_i, X_i)))$$



$$\bar{X}_{i+1} = X_i + h f(t_i, X_i) \quad \longleftarrow \text{Euler-Verfahren}$$

$$X_{i+1}^* = X_i + h f(t_{i+1}, \bar{X}_{i+1})$$

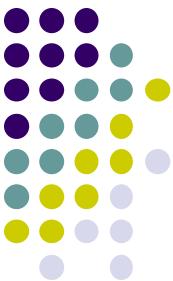
$$X_{i+1} = \frac{1}{2} (\bar{X}_{i+1} + X_{i+1}^*) \quad \longleftarrow \text{Mittelwert}$$



$$k_1 = h f(t_i, X_i)$$

$$k_2 = h f(t_i + h, X_i + k_1)$$

$$X_{i+1} = X_i + \frac{k_1 + k_2}{2}$$



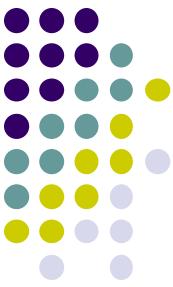
4-stufige Runge-Kutta- Verfahren

$$x(t+h) = x(t) + \int_t^{t+h} f(t, x(t)) dt$$

Simpson

$$\frac{h}{6} \left\{ f(t, x(t)) + 4f\left(t + \frac{h}{2}, x\left(t + \frac{h}{2}\right)\right) + f(t + h, x(t + h)) \right\}$$

approximieren



4-stufige Runge-Kutta- Verfahren

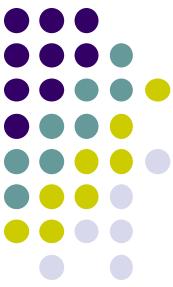
$$k_1 = h \cdot f(t_i, X_i)$$

$$k_2 = h \cdot f\left(t_i + \frac{h}{2}, X_i + \frac{k_1}{2}\right)$$

$$k_3 = h \cdot f\left(t_i + \frac{h}{2}, X_i + \frac{k_2}{2}\right)$$

$$k_4 = h \cdot f(t_i + h, X_i + k_3)$$

$$X_{i+1} = X_i + \frac{1}{6} \{k_1 + 2k_2 + 2k_3 + k_4\}$$



Beispiel

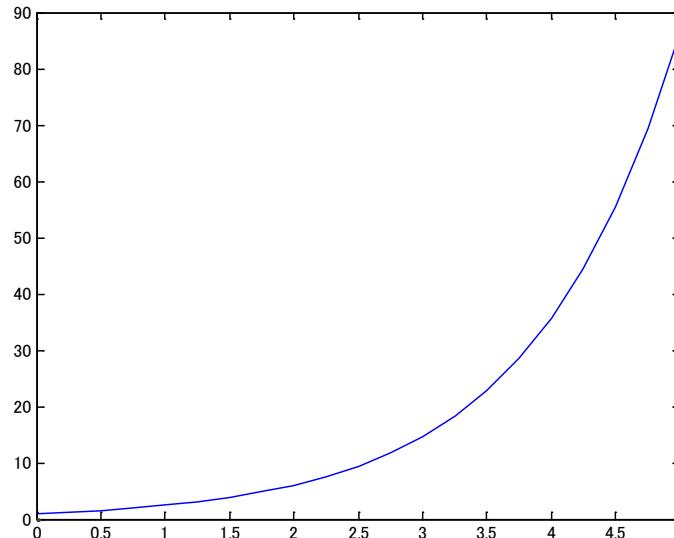
$$\begin{cases} \frac{dx}{dt} = x \\ x(0) = 1 \end{cases}$$

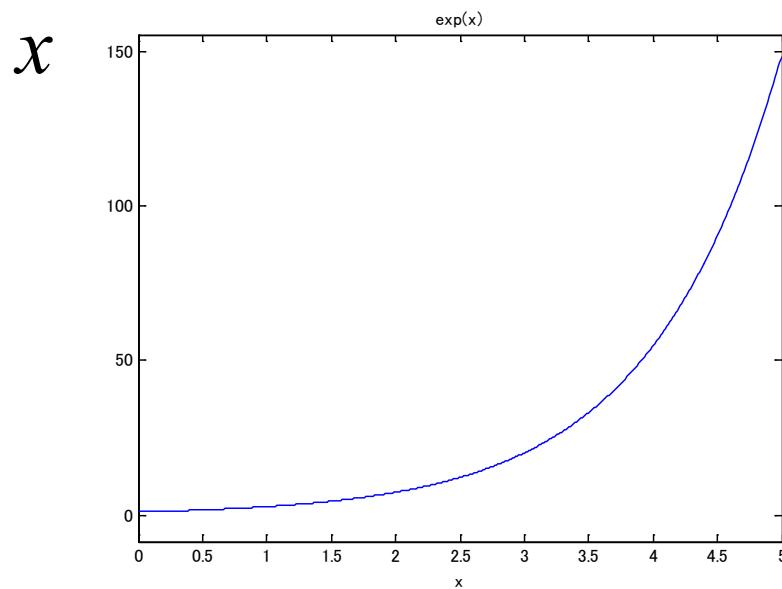
euler.m

```
N=20  
T=5  
h=T/N  
  
t=zeros(N+1); x=zeros(N+1);  
  
t(1)=0; x(1)=1;  
  
for i=1:N  
    x(i+1)=x(i)+h*f(t(i),x(i));  
    t(i+1)=t(i)+h;  
end  
  
plot(t,x)
```

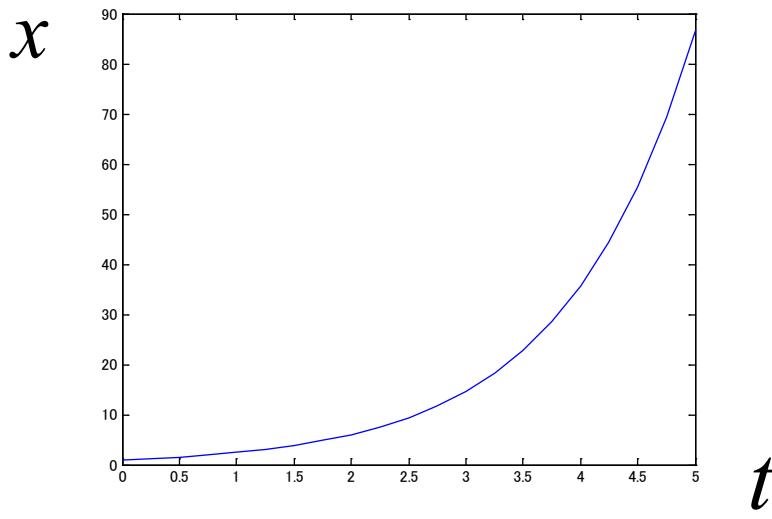
f.m

```
function [a] = f(t,x)  
a=x;  
end
```

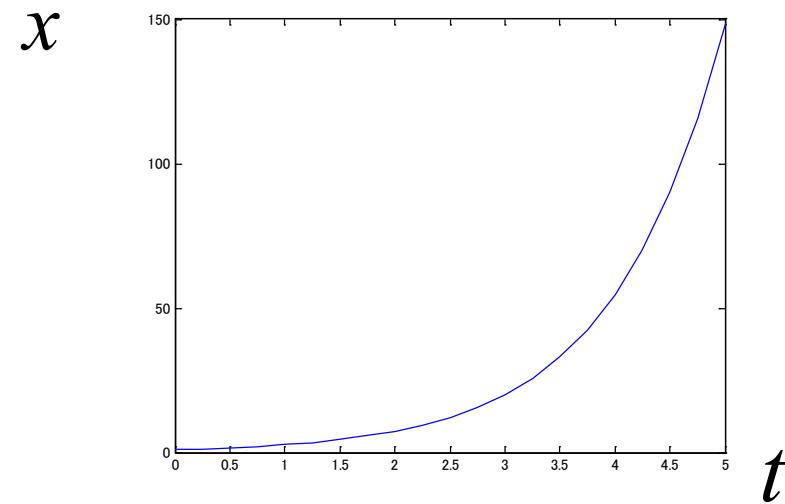




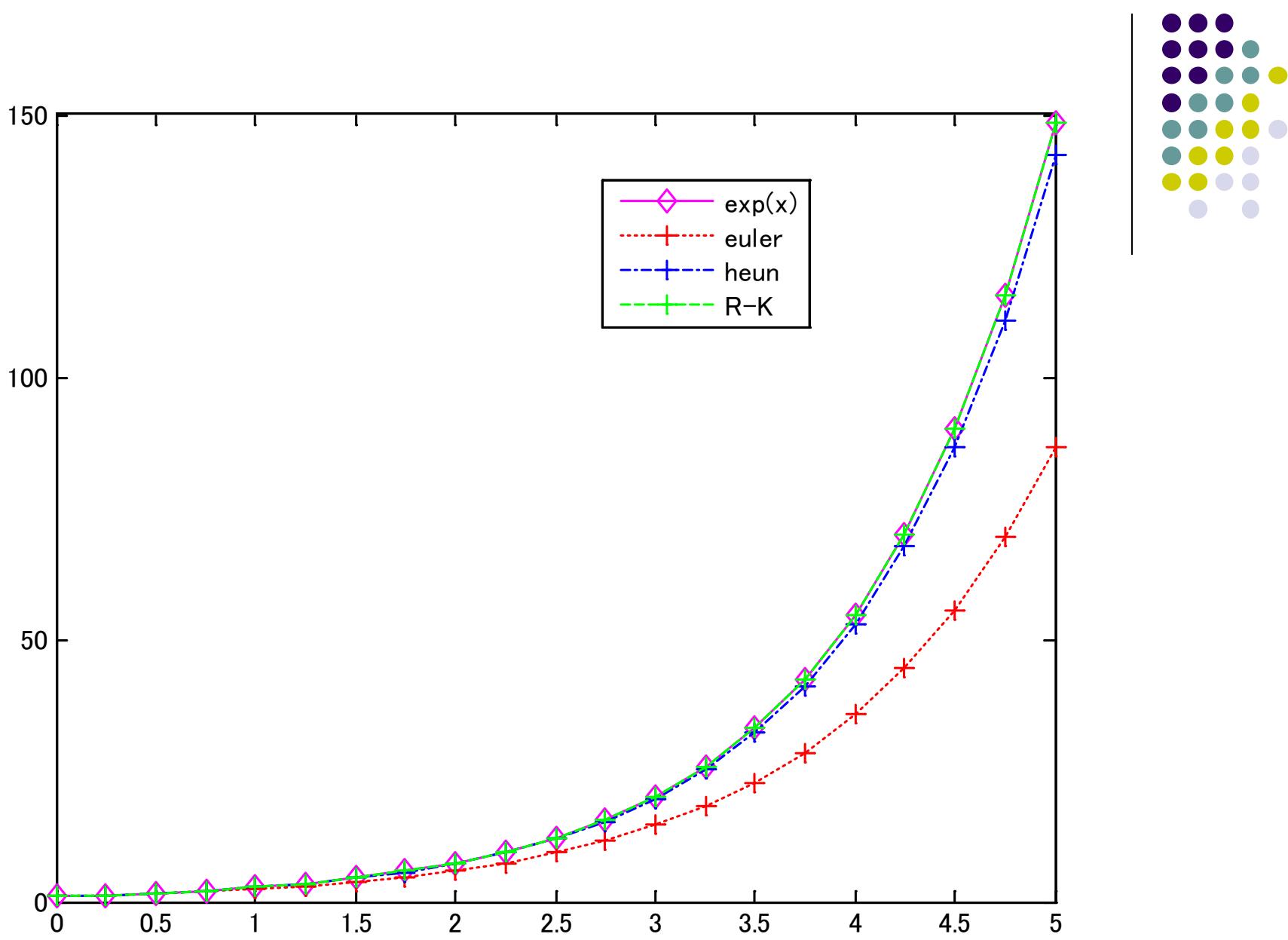
$$x = e^t$$

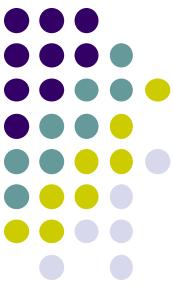


Euler-Verfahren



Runge - Kutta - Verfahren



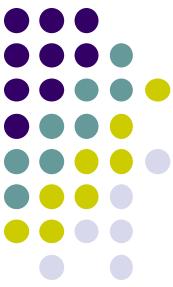


Differentialgleichungssystem

$$\begin{cases} \frac{dx}{dt} = f_1(t, x, y) \\ \frac{dy}{dt} = f_2(t, x, y) \\ x(t_0) = x_0, y(t_0) = y_0 \end{cases}$$

$$\mathbf{x}(t) = \begin{pmatrix} x(t) \\ y(t) \end{pmatrix}, \quad \mathbf{f}(t, \mathbf{x}) = \begin{pmatrix} f_1(t, x, y) \\ f_2(t, x, y) \end{pmatrix}, \quad \mathbf{x}_0 = \begin{pmatrix} x_0 \\ y_0 \end{pmatrix}$$

$$\Rightarrow \begin{cases} \frac{d\mathbf{x}}{dt} = \mathbf{f}(t, \mathbf{x}) \\ \mathbf{x}(t_0) = \mathbf{x}_0 \end{cases}$$



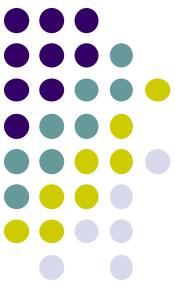
Differentialgleichung zweiter Ordnung

$$\begin{cases} \frac{d^2x}{dt^2} = f(t, x, x') \\ x(t_0) = a_0, x'(t_0) = a_1 \end{cases}$$

$$y(t) := x'(t)$$

$$\Rightarrow \begin{cases} \frac{dx}{dt} = y & \xleftarrow{\hspace{1cm}} f_1(t, x, y) \\ \frac{dy}{dt} = f(t, x, y) & \xleftarrow{\hspace{1cm}} f_2(t, x, y) \\ x(t_0) = a_0, y(t_0) = a_1 \end{cases}$$

Differentialgleichungssystem

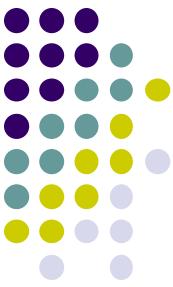


Differentialgleichung n-ter Ordnung

$$\begin{cases} x^{(n)} = f(t, x, x', \dots, x^{(n-1)}) \\ x(t_0) = \eta_0, x'(t_0) = \eta_1, \dots, x^{(n-1)}(t_0) = \eta_{n-1} \end{cases}$$

$$u_1 := x, u_2 := x', u_3 := x'', \dots, u_n := x^{(n-1)}$$

$$\Rightarrow u_1' = u_2, \dots, u_{n-1}' = u_n, u_n' = f(t, u_1, \dots, u_n), \\ u_1(t_0) = \eta_0, \dots, u_n(t_0) = \eta_{n-1}$$

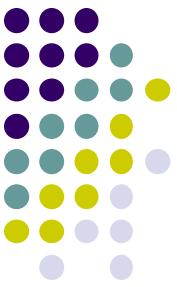


Differentialgleichungssystem

$$\begin{cases} u_1' = f_1(t, u_1, \dots, u_n) \\ \vdots \\ u_n' = f_n(t, u_1, \dots, u_n) \\ u_1(t_0) = \eta_0, \dots, u_n(t_0) = \eta_{n-1} \end{cases}$$

$$\mathbf{u}(t) = \begin{pmatrix} u_1(t) \\ \vdots \\ u_n(t) \end{pmatrix}, \quad \mathbf{f}(t, \mathbf{u}) = \begin{pmatrix} f_1(t, \mathbf{u}) \\ \vdots \\ f_n(t, \mathbf{u}) \end{pmatrix}, \quad \boldsymbol{\eta}_0 = \begin{pmatrix} \eta_0 \\ \vdots \\ \eta_{n-1} \end{pmatrix}$$

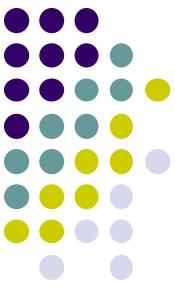
$$\Rightarrow \begin{cases} \frac{d\mathbf{u}}{dt} = \mathbf{f}(t, \mathbf{u}) \\ \mathbf{u}(t_0) = \boldsymbol{\eta}_0 \end{cases}$$



Beispiel (n=2)

$$\begin{cases} \frac{d\mathbf{x}}{dt} = \mathbf{f}(t, \mathbf{x}) \\ \mathbf{x}(t_0) = \mathbf{x}_0 \end{cases}$$

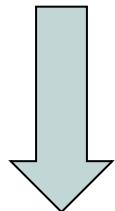
$$\mathbf{x}(t) = \begin{pmatrix} x(t) \\ y(t) \end{pmatrix}, \quad \mathbf{f}(t, \mathbf{x}) = \begin{pmatrix} f_1(t, x, y) \\ f_2(t, x, y) \end{pmatrix}, \quad \mathbf{x}_0 = \begin{pmatrix} x_0 \\ y_0 \end{pmatrix}$$



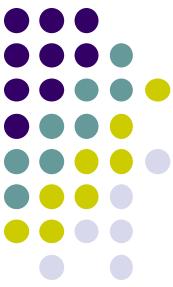
Euler-Verfahren

Vektorform

$$\mathbf{X}_{i+1} = \mathbf{X}_i + h \cdot \mathbf{f}(t_i, \mathbf{X}_i)$$



$$\begin{cases} X_{i+1} = X_i + h \cdot f(t_i, X_i, Y_i) \\ Y_{i+1} = Y_i + h \cdot g(t_i, X_i, Y_i) \end{cases}$$



Runge-Kutta-Verfahren

Vektorform

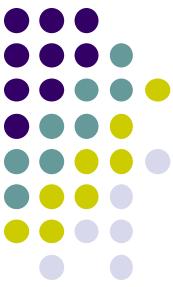
$$\mathbf{k}_1 = h \cdot \mathbf{f}(t_i, \mathbf{X}_i)$$

$$\mathbf{k}_2 = h \cdot \mathbf{f}\left(t_i + \frac{h}{2}, \mathbf{X}_i + \frac{\mathbf{k}_1}{2}\right)$$

$$\mathbf{k}_3 = h \cdot \mathbf{f}\left(t_i + \frac{h}{2}, \mathbf{X}_i + \frac{\mathbf{k}_2}{2}\right)$$

$$\mathbf{k}_4 = h \cdot \mathbf{f}(t_i + h, \mathbf{X}_i + \mathbf{k}_3)$$

$$\mathbf{X}_{i+1} = \mathbf{X}_i + \frac{1}{6} \left\{ \mathbf{k}_1 + 2\mathbf{k}_2 + 2\mathbf{k}_3 + \mathbf{k}_4 \right\}$$



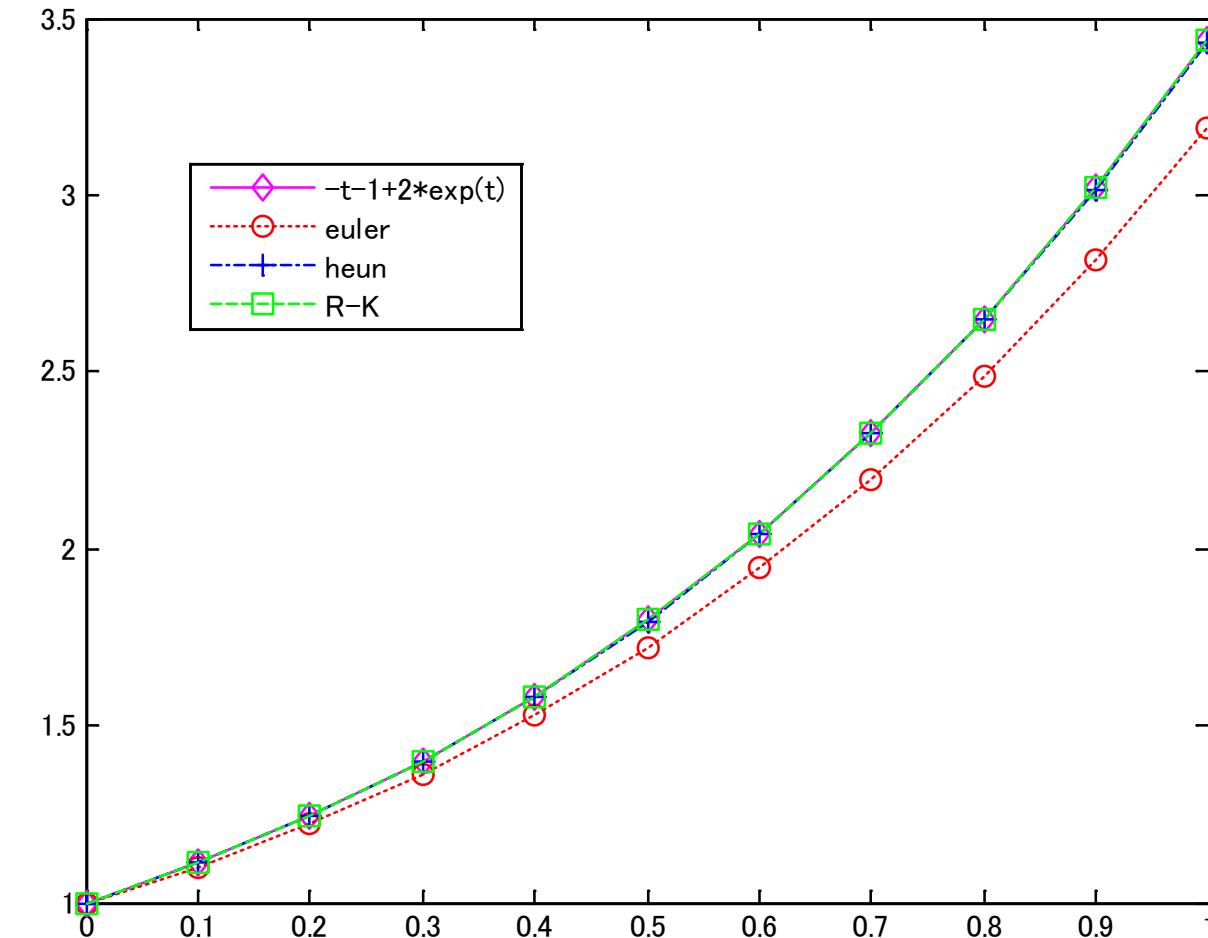
Beispiel

$$\begin{cases} \frac{dx}{dt} = t + x, & 0 \leq t \leq 1 \\ x(0) = 1 \end{cases}$$

Wir rechnen die Näherungswerte von $x = x(t)$ mit Euler-Verfahren, Heun-Verfahren und Runge – Kutta-Verfahren für $h = 0.1, 0.05, 0.025$ und die jeweilige Fehler.

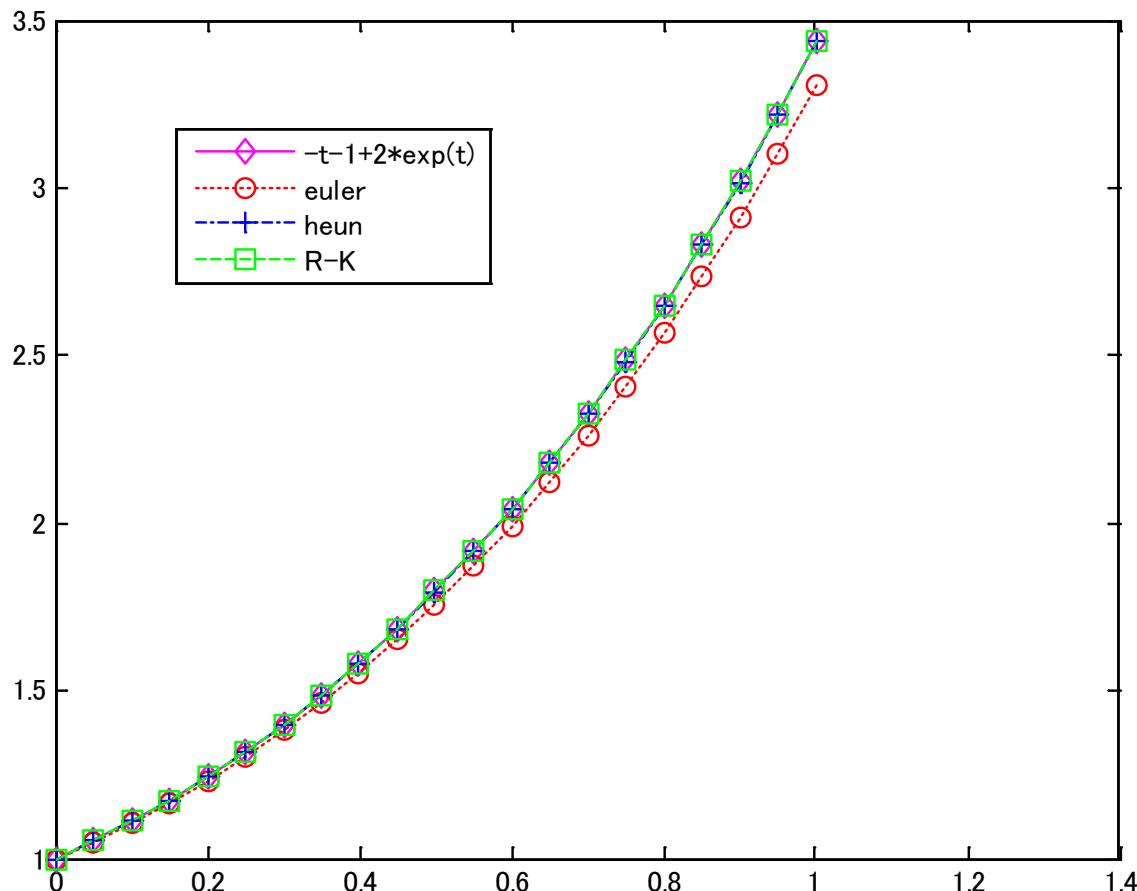
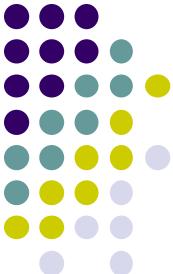
Exakte Lösung : $x(t) = -t - 1 + 2e^t$

$h = 0.1$

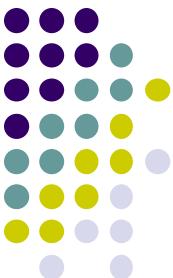


Runge-Kutta: $\max_{1 \leq i \leq N+1} |x(t_i) - X_i| = 4.168647758096000\text{e-}006$

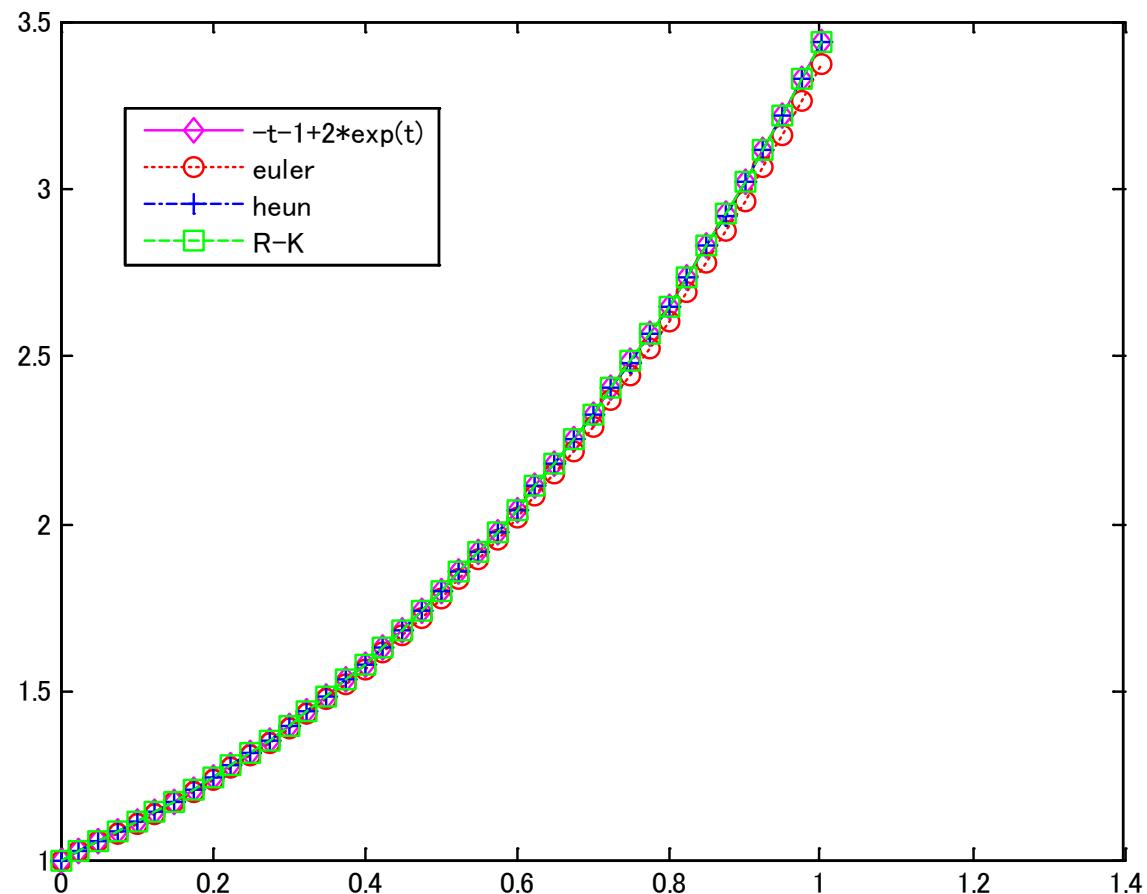
$h = 0.05$



Runge-Kutta: $\max_{1 \leq i \leq N+1} |x(t_i) - X_i| = 2.716054234852550e-007$



$h = 0.025$



Runge-Kutta: $\max_{1 \leq i \leq N+1} |x(t_i) - X_i| = 1.733237908752017\text{e-}008$



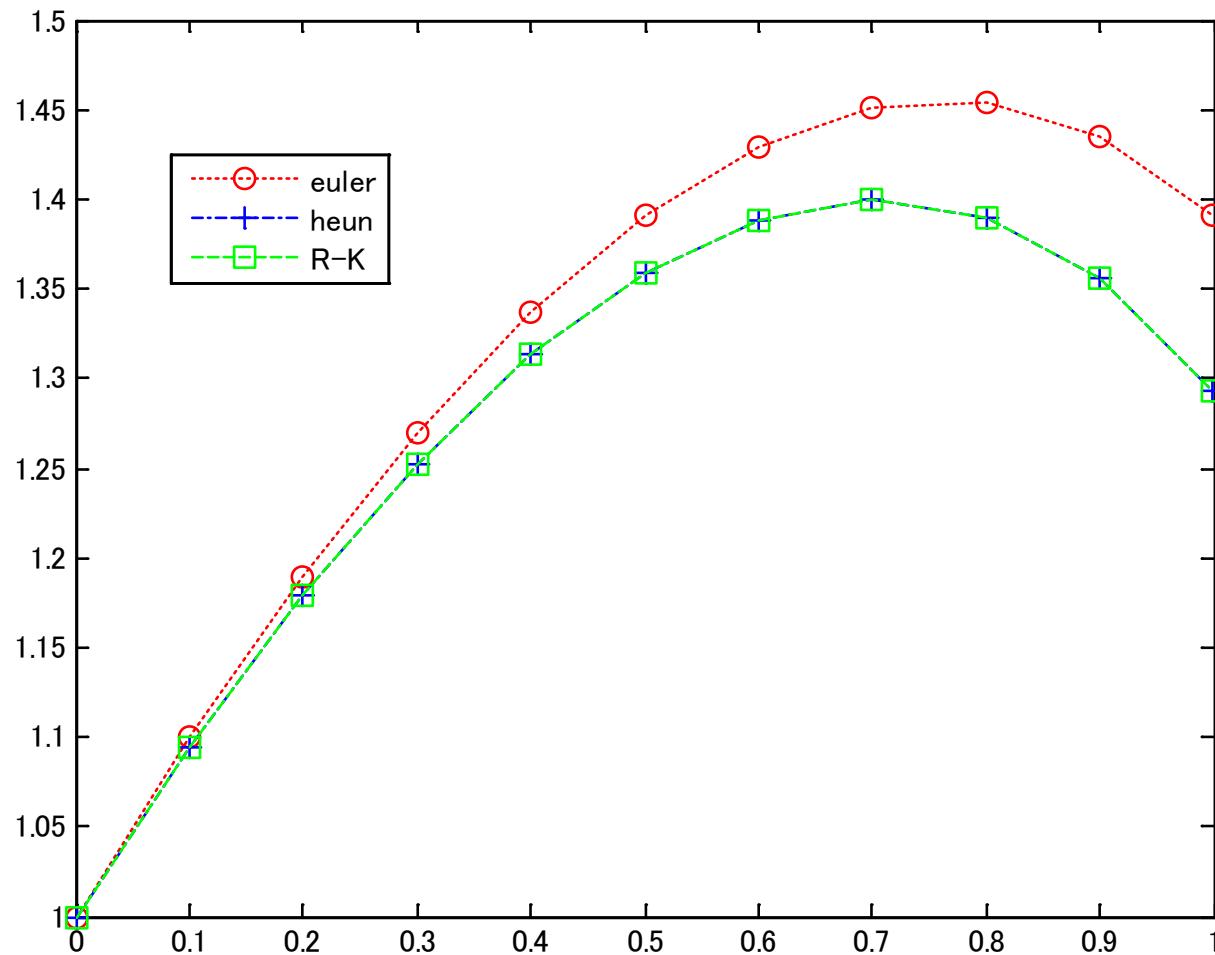
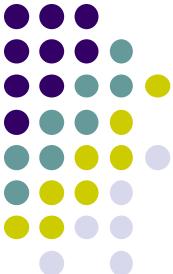
Beispiel

$$\begin{cases} \frac{dx}{dt} = 1 - t^2 - \frac{t}{x(t)}, & 0 \leq t \leq 1 \\ x(0) = 1 \end{cases}$$

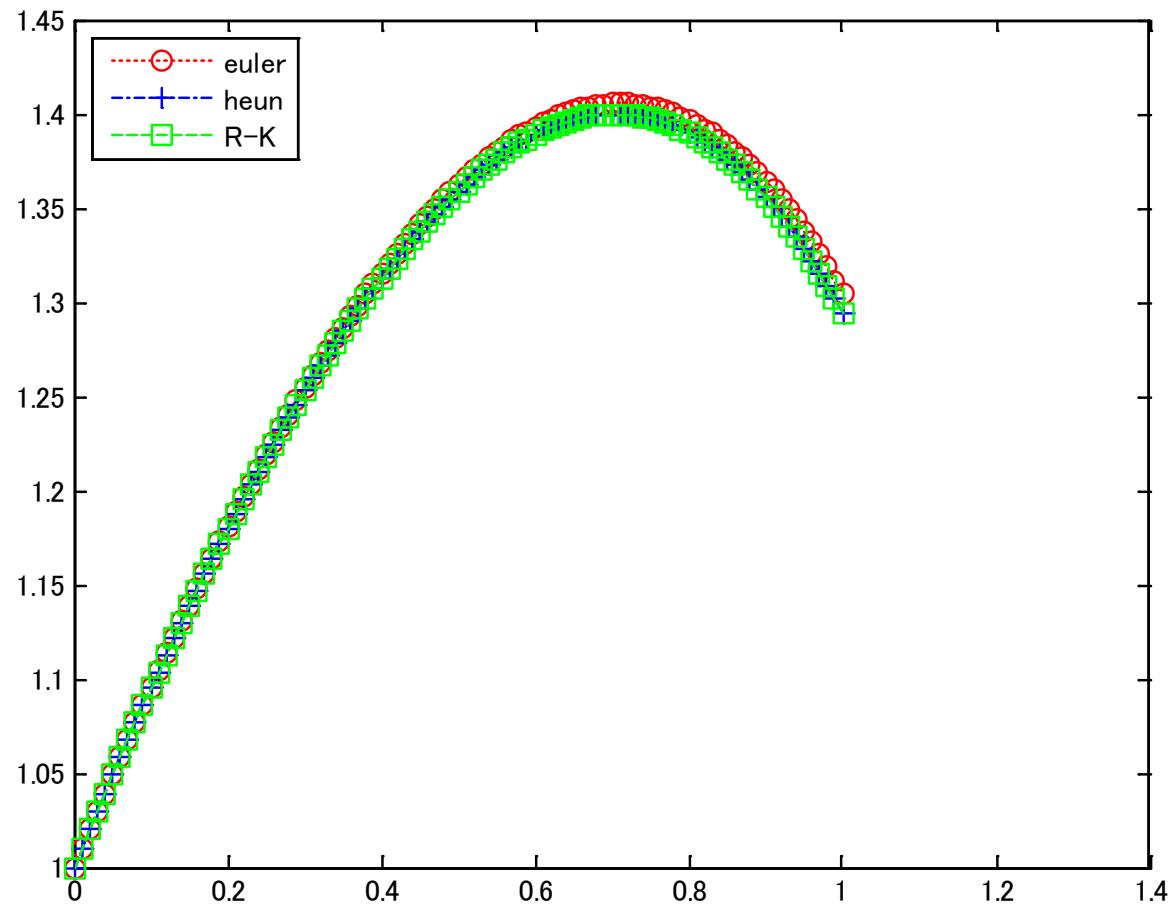
Wir rechnen die Näherungswerte von $x = x(t)$ mit Euler-Verfahren, Heun-Verfahren und Runge – Kutta-Verfahren für $h = 0.1, 0.01, 0.001$

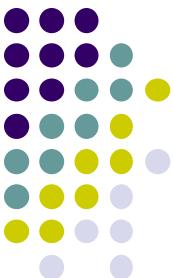
(Dieses Problem lässt sich nicht geschlossen lösen.)

$h = 0.1$

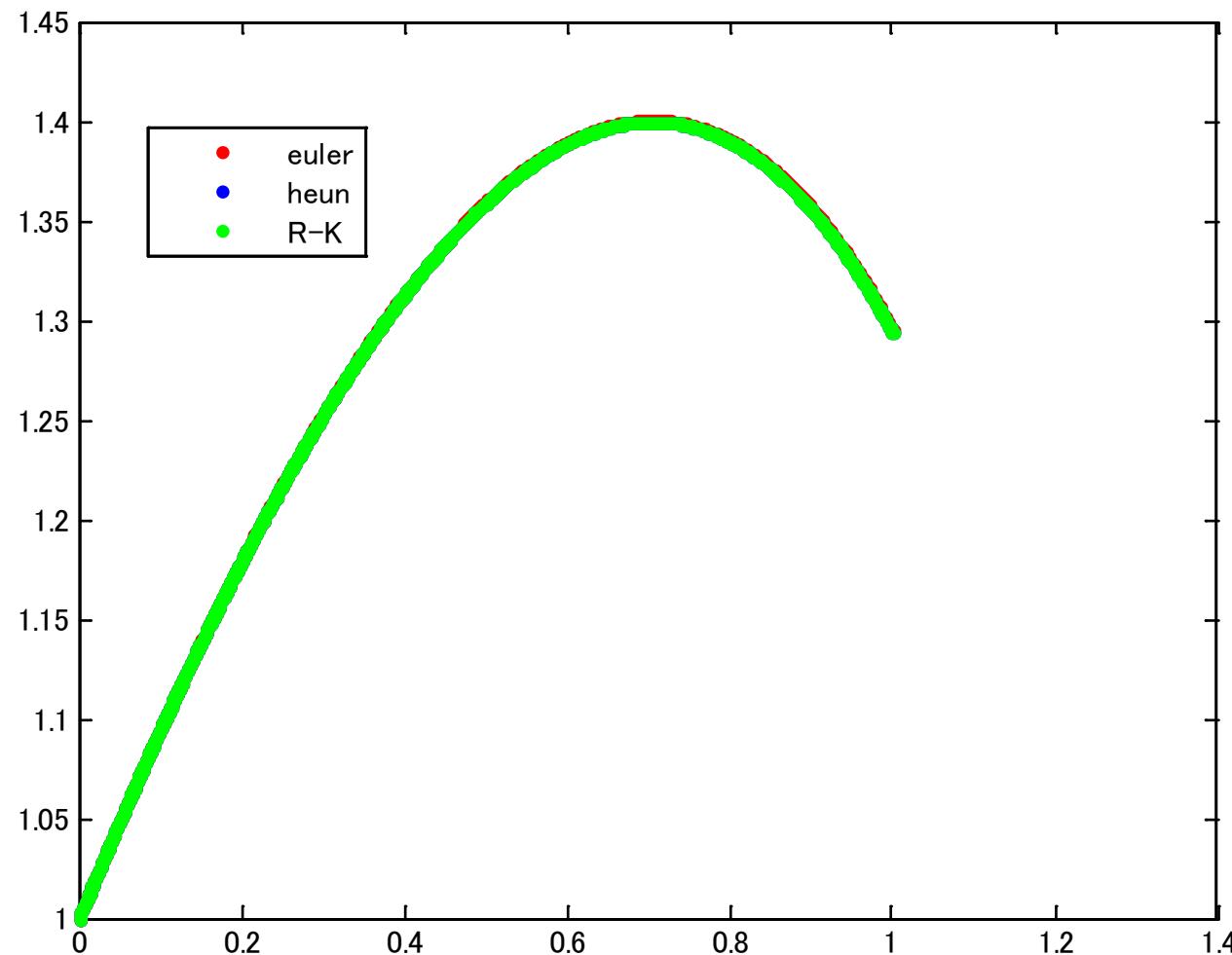


$h = 0.01$





$h = 0.001$





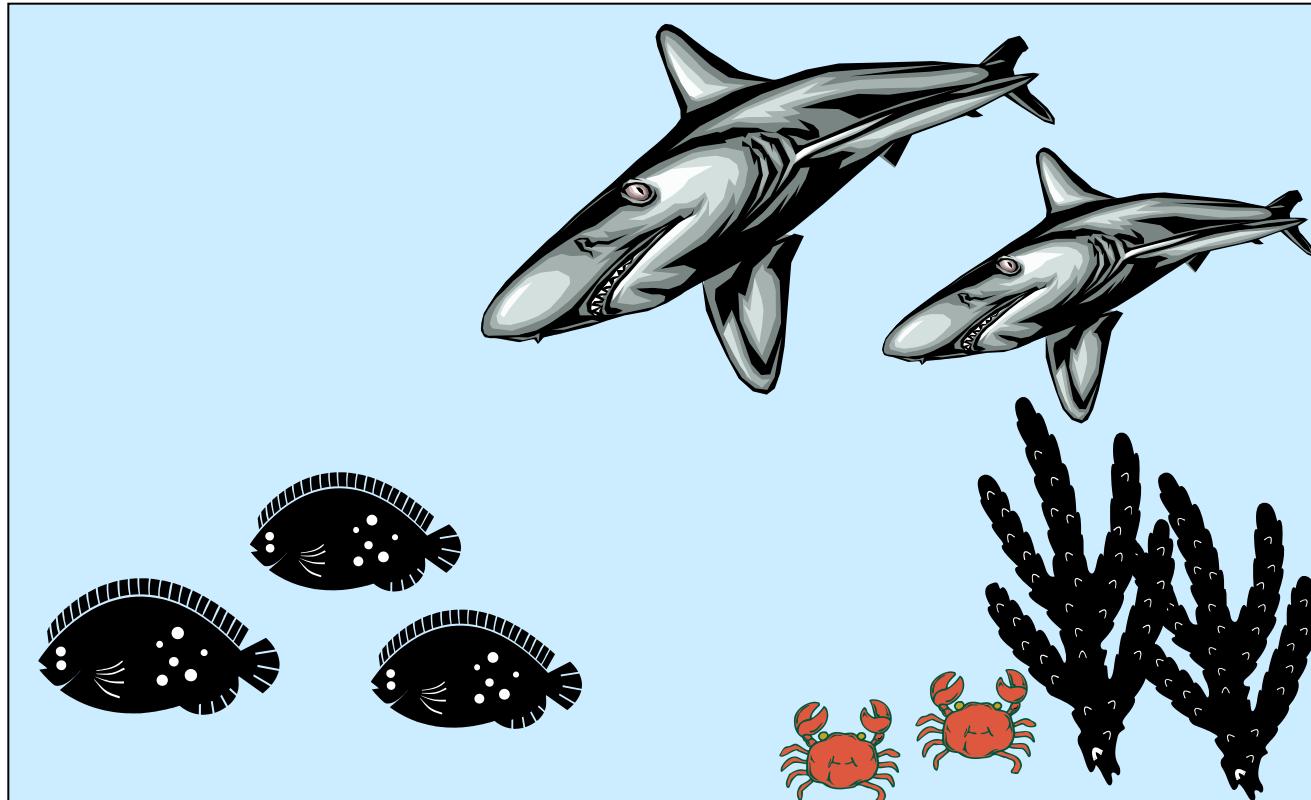
Anwendungsbeispiele

1) Lotka-Volterra-Model

Beute → Seezunge

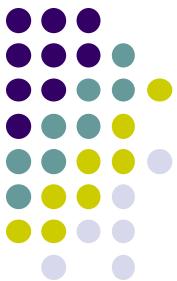
Räuber → Hai

Wie die Anzahl der Seezungen und der Haie sich verändert?



Adriatisches Meer

Americanischer Mathematiker Lotka und Italienischer Mathematiker Volterra entwickelten ein mathematisches Modell für die Interaktion zwischen Seezungen und Hai.



$f(t)$: Anzahl der Seezunge (zeitabhängig)

$g(t)$: Anzahl des Hai (zeitabhängig)

Voraussetzung

- a: Reproduktionsrate der Seezunge ohne Störung und bei großem Nahrungsangebot
- c: Sterberate des Haies, wenn keine Seezunge vorhanden ist
- b: Fressrate des Haies pro Seezunge
- d: Reproduktionsrate des Haies pro Seezunge

$$f'(t) = af(t) \quad (a > 0)$$

$$g'(t) = -cg(t) \quad (c > 0)$$

$$-bg(t) \quad (b > 0)$$

$$df(t) \quad (d > 0)$$

$$f'(t) = af(t) - bf(t)g(t)$$

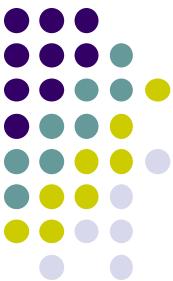
$$g'(t) = -cg(t) + df(t)g(t)$$

Lotka – Volterra Modell



Numerische Ergebnisse (a = 0.01, b = d = 0.0001, c=0.05)

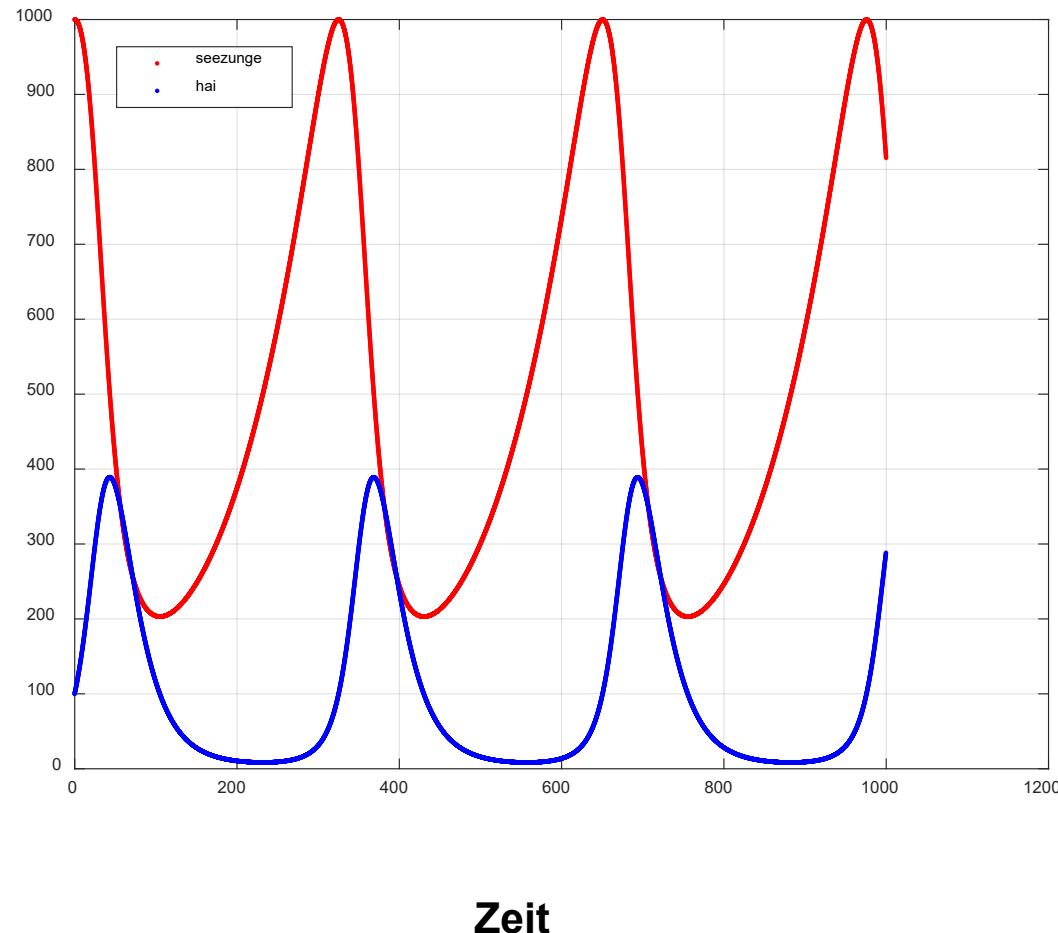
| | | | |
|-------------------|--------------------|-------------------|--------------------|
| 0.000000000000000 | 1000.0000000000000 | 0.000000000000000 | 100.0000000000000 |
| 0.100000000000000 | 999.99749583335165 | 0.100000000000000 | 100.50125124738071 |
| 0.200000000000000 | 999.98996666717426 | 0.200000000000000 | 101.00500995791107 |
| 0.300000000000000 | 999.97738750383076 | 0.300000000000000 | 101.51128353587602 |
| 0.400000000000000 | 999.95973334948997 | 0.400000000000000 | 102.02007931990096 |
| 0.500000000000000 | 999.93697921606554 | 0.500000000000000 | 102.53140458134952 |
| 0.600000000000000 | 999.90910012318136 | 0.600000000000000 | 103.04526652270535 |
| 0.700000000000000 | 999.87607110016302 | 0.700000000000000 | 103.56167227593808 |
| 0.800000000000000 | 999.83786718805482 | 0.800000000000000 | 104.08062890085351 |
| 0.900000000000000 | 999.79446344166297 | 0.900000000000000 | 104.60214338342793 |
| 1.000000000000000 | 999.74583493162538 | 1.000000000000000 | 105.12622263412673 |
| 1.100000000000000 | 999.69195674650746 | 1.100000000000000 | 105.65287348620735 |
| 1.200000000000000 | 999.63280399492396 | 1.200000000000000 | 106.18210269400652 |
| 1.300000000000000 | 999.56835180768746 | 1.300000000000000 | 106.71391693121188 |
| 1.400000000000000 | 999.49857533998306 | 1.400000000000000 | 107.24832278911822 |
| 1.500000000000000 | 999.42344977356947 | 1.500000000000000 | 107.78532677486804 |
| 1.600000000000000 | 999.34295031900683 | 1.600000000000000 | 108.32493530967690 |
| 1.700000000000000 | 999.25705221791065 | 1.700000000000000 | 108.86715472704331 |
| 1.800000000000000 | 999.16573074523239 | 1.800000000000000 | 109.41199127094350 |
| 1.900000000000000 | 999.06896121156649 | 1.900000000000000 | 109.95945109401094 |
| 2.000000000000000 | 998.96671896548401 | 2.000000000000000 | 110.50954025570091 |
| ⋮ | ⋮ | ⋮ | ⋮ |

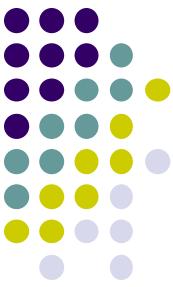


Veranschaulichung

Lotka – Volterra Modell (Graphik 1)

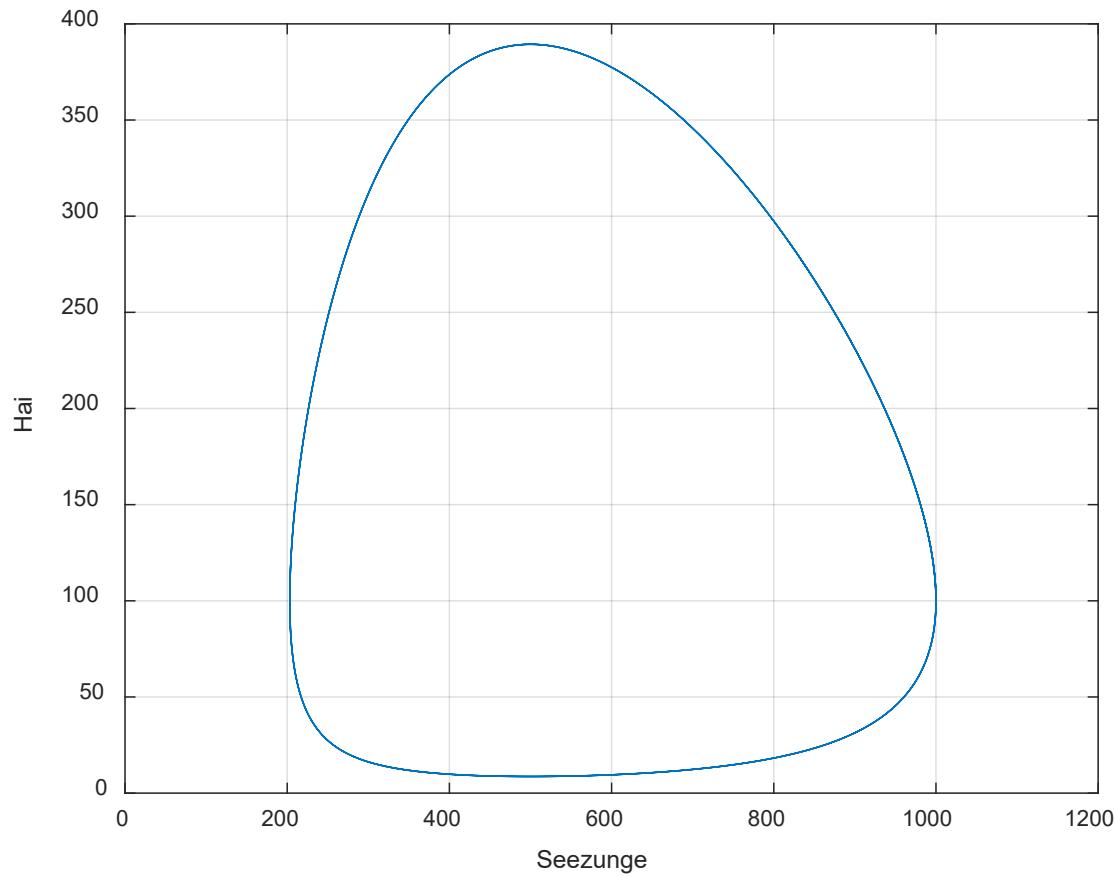
Anfangswert: $x(0)=1000$, $y(0)=100$

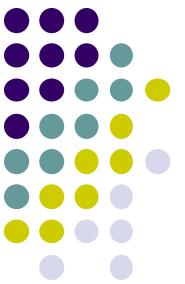




Lotka – Volterra Modell (Graphik 2)

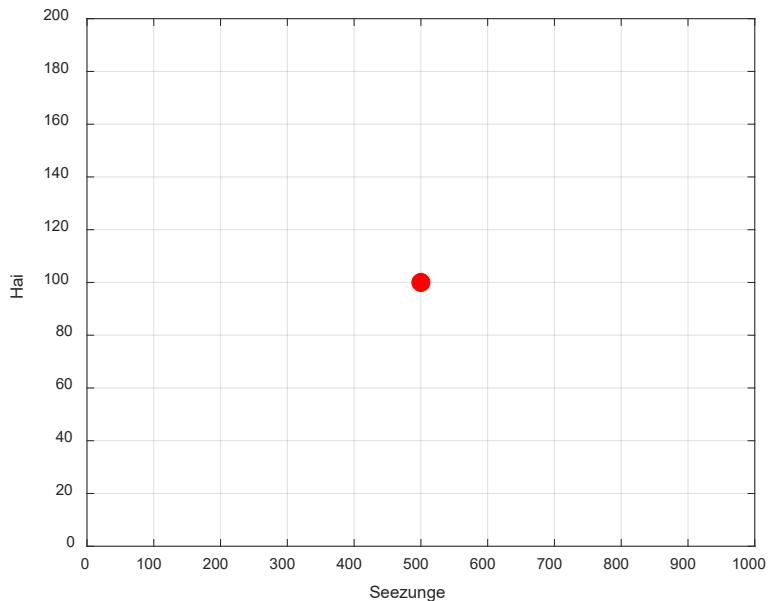
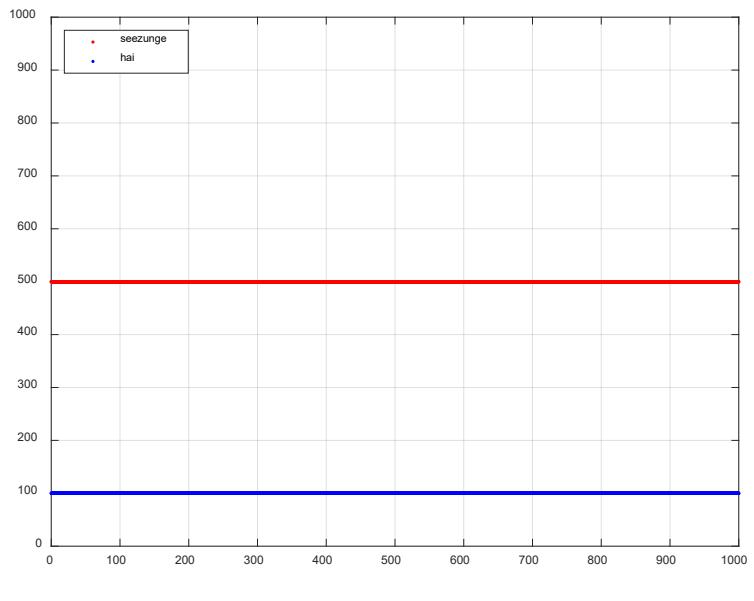
Anfangswert: $x(0)=1000$, $y(0)=100$





Lotka – Volterra Modell

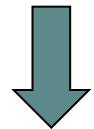
Anfangswert: $x(0)=\textcolor{red}{500}$, $y(0)=100$





$$(f(t), g(t)) = (0, 0), \left(\frac{c}{d}, \frac{a}{b} \right)$$

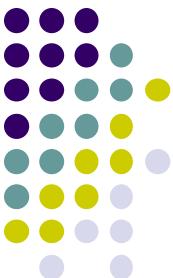
$$\Leftrightarrow af(t) - bf(t)g(t) = -cg(t) + df(t)g(t) = 0$$



$$f'(t) = af(t) - bf(t)g(t) = 0$$

$$g'(t) = -cg(t) + df(t)g(t) = 0$$

$$a = 0.01, b = d = 0.0001, c = 0.05 \Rightarrow \left(\frac{c}{d}, \frac{a}{b} \right) = (500, 100)$$



2) Modellierung eines Ökosystems

Bezeichnet man die Menge Gras, die Zahl der Hasen und die Zahl der Füchse mit g , h bzw. f , so kann man folgendes Modell aufstellen.

$$g'(t) = 1 - h(t)g(t),$$

$$h'(t) = h(t)(g(t) - f(t)),$$

$$f'(t) = f(t)h(t) - c_1 f(t) - c_2 \sqrt{f(t)}, \quad c_1 > 0, c_2 > 0, c_1 + c_2 = \text{const.}$$

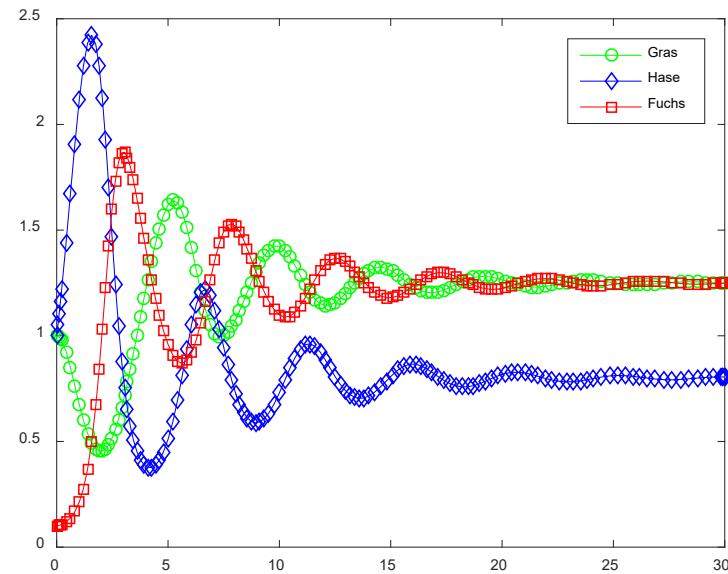
Mögliche Anfangsbedingungen sind $g(0) = h(0) = 1$, $f(0) = 0.1$.

Beispiele:

Parameter: $c_1 = 0.8$, $c_2 = 0$

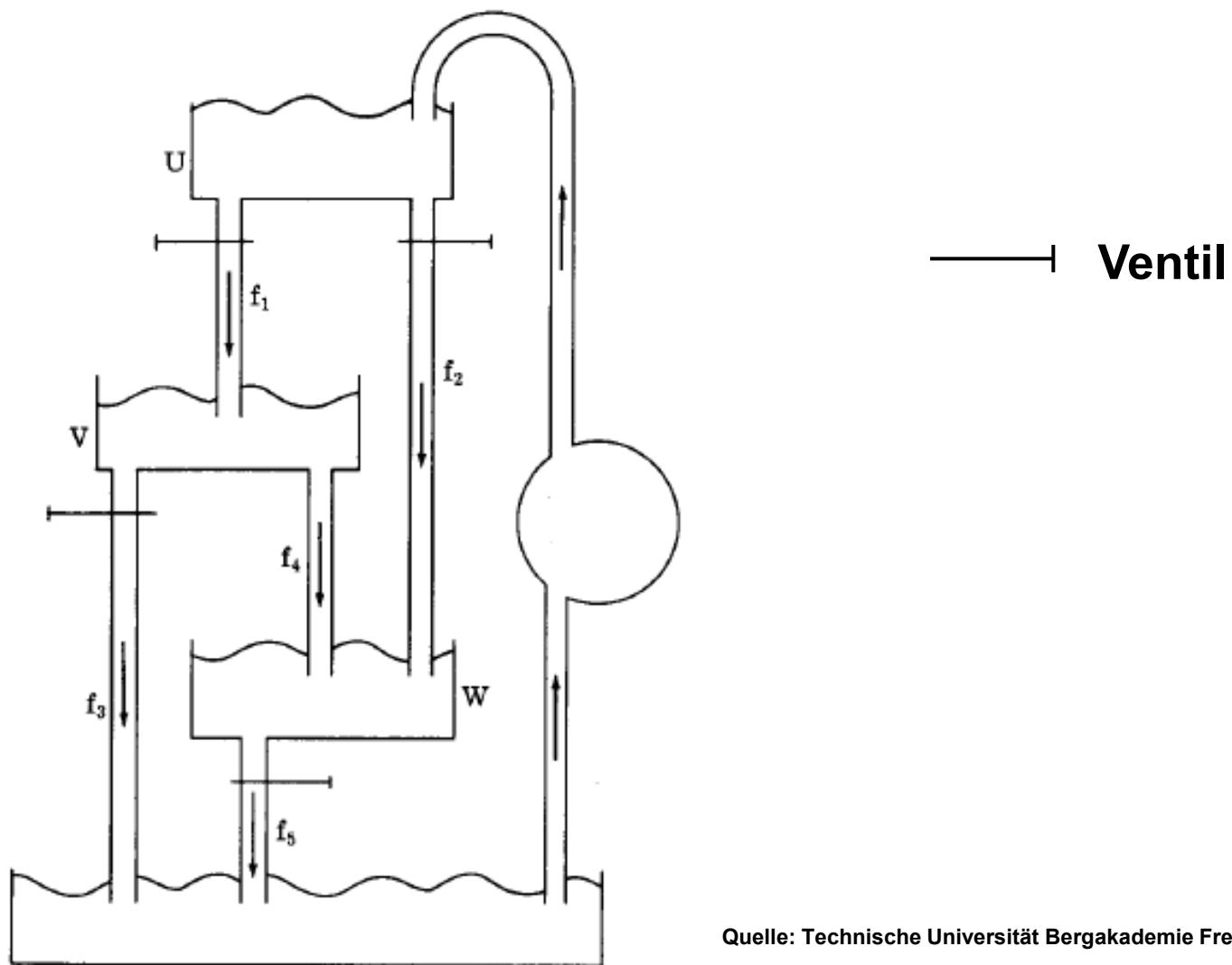
oder

Parameter: $c_1 = 0.3$, $c_2 = 0.5$

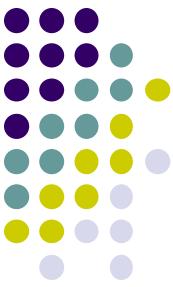




3) Wasserkreislauf



Quelle: Technische Universität Bergakademie Freiberg



Physikalische Grundlagen

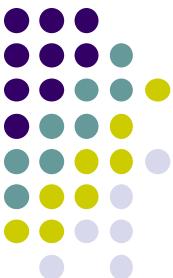
Evangelista Torricelli (1608-1647):

Abflussgeschwindigkeit $\sqrt{2gh}$, $g = 9.81$ (Gravitationsbeschleunigung),
 h = Höhe des Wasserspiegels.

Abflussrate als Funktion des im Behälter befindlichen Wasservolumens v
(falls es sich um einen Zylinder mit Grundfläche A handelt)

$$f(v) = a\sqrt{2gv/A} = c\sqrt{v} \quad \text{mit} \quad c := a\sqrt{2g/A}.$$

Der Parameter c kann über a variiert werden, wenn der Abfluss einen Hahn besitzt. Wir sprechen von einem **Steuerungsparameter**.



Mathematisches Modell

$U(t), V(t), W(t), R(t)$: Wassermengen zur Zeit t in den Behältern.

f_1, \dots, f_5 : Abflussfunktionen mit den Steuerungsparametern
 c_1, \dots, c_5 .

$p = p(t)$: „Pumpenfunktion“

Änderungsraten der Wasservolumina: Zuflüsse weniger Abflüsse, d.h.

$$U'(t) = p(t) - f_1(U(t)) - f_2(U(t))$$

$$V'(t) = f_1(U(t)) - f_3(V(t)) - f_4(V(t))$$

$$W'(t) = f_2(U(t)) + f_4(V(t)) - f_5(W(t))$$

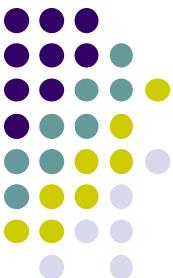
$$R'(t) = f_3(V(t)) + f_5(W(t)) - p(t).$$

$$f_i(v) = c_i \sqrt{v}$$

Addiert man alle Gleichungen, so ergibt sich

$$U'(t) + V'(t) + W'(t) + R'(t) = 0$$

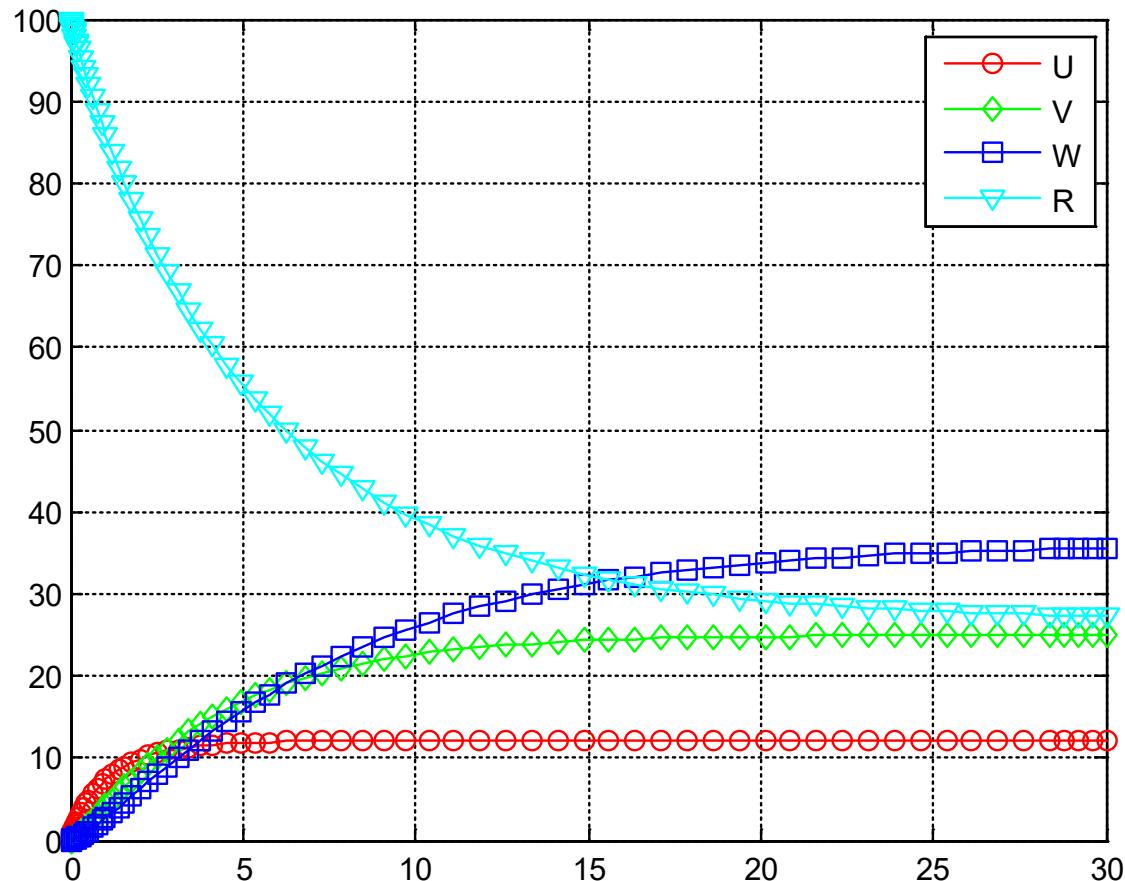
→ Die Gesamtwassermenge verändert sich nicht.



Beispiel:

$$U(0) = V(0) = W(0) = 0, R(0) = 100$$

$$c_1 = \sqrt{12}, c_2 = c_4 = \sqrt{2}, c_3 = 1, c_5 = 2, p \equiv 17$$





Aufgabe 9.1 Räuber-Beute Gleichungen

$$\begin{cases} \frac{dx}{dt} = ax - bxy \\ \frac{dy}{dt} = -cy + dxy \end{cases} \quad 0 \leq t \leq 1000$$

$$(a= 0.01, b = d = 0.0001, c=0.05)$$

Rechnen Sie die Näherungswerte von $x = x(t)$, $y = y(t)$ mit Runge – Kutta-Verfahren unter folgenden Bedingungen.

$$h = 0.1$$

Anfangswerte:

- 1) $x(0)=y(0)=100$
- 2) $x(0)=500, y(0)=100$
- 3) $x(0)=700, y(0)=100$
- 4) $x(0)=1000, y(0)=100$