Equivalence of dose-finding methods

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Suppose that we have the following working model,

$$\tau_i = \psi_m(d_i, a) = \alpha_{mi}^{\exp(a)},$$

then we aim to show that the expected value of $\psi_m(d_i, a)$ is equivalent to the expected value of τ_i. Hence we have both,

$$\mathrm{E}(\tau_i) = \int_0^1 \tau_i f_m(\tau_i|\Omega_j) d\tau_i \text{ and,}$$

$$\mathrm{E}(\psi_m(d_i, a)) = \int_{-\infty}^{\infty} \psi_m(d_i, a) f_m(a|\Omega_j) da.$$

$$\begin{aligned}
\Rightarrow \frac{d\tau_i}{da} &= \alpha_{mi}^{\exp(a)} \log(\alpha_{mi}) \exp(a), \\
&= \alpha_{mi}^{\exp(a)} \log(\alpha_{mi}^{\exp(a)}), \\
&= \psi_m(d_i, a) \log(\psi_m(d_i, a)), \\
\Rightarrow d\tau_i &= \psi_m(d_i, a) \log(\psi_m(d_i, a)) da.
\end{aligned}$$

Moreover, since we know that

$$f_m(\tau_i|\Omega_j) = \left| \frac{1}{\tau_i \log(\tau_i)} \right| f_m(\psi_m^{-1}(d_i, \tau_i)|\Omega_j),$$

then by substitution we obtain the following,

$$\begin{aligned}
\mathrm{E}(\tau_i) &= \int_0^1 \tau_i f_m(\tau_i|\Omega_j) d\tau_i, \\
&= \int_{-\infty}^{\infty} \psi_m(d_i, a)^2 \log(\psi_m(d_i, a)) \left| \frac{1}{\psi_m(d_i, a) \log(\psi_m(d_i, a))} \right| f_m(a|\Omega_j) da, \\
&= \int_{-\infty}^{\infty} \psi_m(d_i, a) f_m(a|\Omega_j) da, \\
&= \mathrm{E}(\psi_m(d_i, a))
\end{aligned}$$

From this result, it is trivial to show that,

$$\begin{aligned}
\int_0^1 \tau_i g(\tau_i|\Omega_j) d\tau_i &= \int_0^1 \tau_i \left(\sum_m p(m|\Omega_j) f_m(\tau_i|\Omega_j) \right) d\tau_i, \\
&= \sum_m p(m|\Omega_j) \int_0^1 \tau_i f_m(\tau_i|\Omega_j) d\tau_i, \\
&= \sum_m p(m|\Omega_j) \int_{-\infty}^{\infty} \psi_m(d_i, a) f_m(a|\Omega_j) da.
\end{aligned}$$