Language Technologies Institute

Carnegie Mellon University

# Advanced Multimodal Machine Learning

## Lecture 2.1: Basic Concepts

Louis-Philippe Morency

* Original version co-developed with Tadas Baltrusaitis

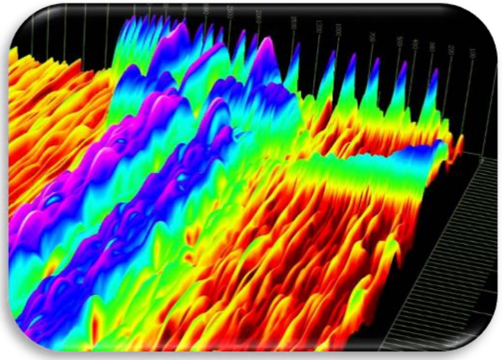
## Lecture Objectives

- Unimodal basic representations
  - Visual, language and acoustic modalities
- Data-driven machine learning
  - Training, validation and testing
  - Example: K-nearest neighbor
- Linear Classification
  - Score function
  - Two loss functions (cross-entropy and hinge loss)
- Course project “speed-dating”

# Multimodal Machine Learning

**Verbal**


**Vocal**



**Visual**

## Core Technical Challenges:

Representation    Translation    Alignment

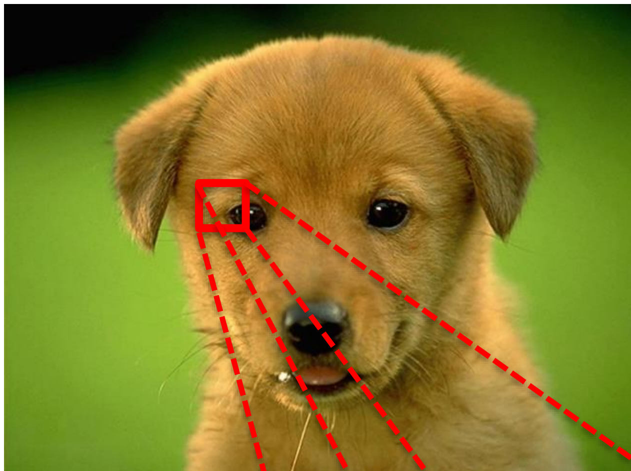
Fusion    Co-Learning

These challenges are non-exclusive.

# Unimodal Basic Representations

# Unimodal Classification – Visual Modality

Color image



Each pixel is represented in $\mathcal{R}^d$, $d$ is the number of colors ($d$=3 for RGB)

| | | | | | | | | |
|---|---|---|---|---|---|---|---|---|
| 88 | 82 | 84 | 88 | 85 | 83 | 80 | 93 | 102 |
| 88 | 80 | 78 | 80 | 80 | 78 | 73 | 94 | 100 |
| 85 | 79 | 80 | 78 | 77 | 74 | 65 | 91 | 99 |
| 38 | 35 | 40 | 35 | 39 | 74 | 77 | 70 | 65 |
| 20 | 25 | 23 | 28 | 37 | 69 | 64 | 60 | 57 |
| 22 | 26 | 22 | 28 | 40 | 65 | 64 | 59 | 34 |
| 24 | 28 | 24 | 30 | 37 | 60 | 58 | 56 | 66 |
| 21 | 22 | 23 | 27 | 38 | 60 | 67 | 65 | 67 |
| 23 | 22 | 22 | 25 | 38 | 59 | 64 | 67 | 66 |

Input observation $x_i$

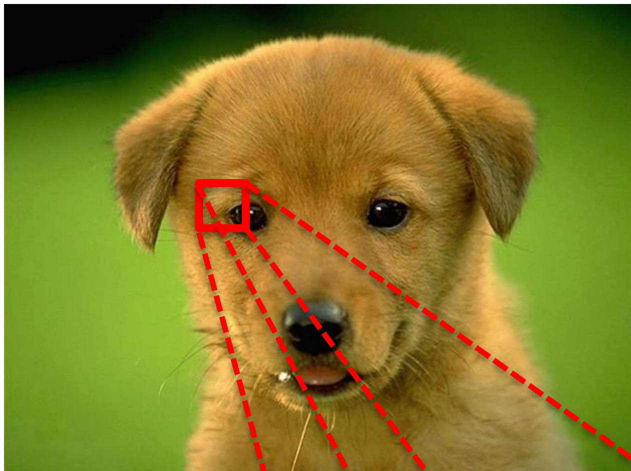
| |
|---|
| 88 |
| 88 |
| 85 |
| 38 |
| 20 |
| 22 |
| 24 |
| 21 |
| 23 |
| 82 |
| 80 |
| 79 |
| 35 |
| 25 |
| 26 |
| 28 |
| 22 |
| 22 |
| 84 |
| 78 |
| 80 |
| ⋮ |

**Binary classification problem**

Dog ?

label $y_i \in \mathcal{Y} = \{0,1\}$

# Unimodal Classification – Visual Modality



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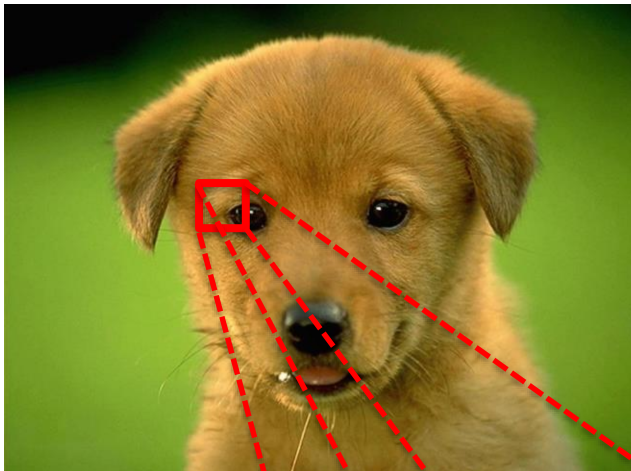
| 80 |
| ⋮ |

**Multi-class classification problem**

Duck
-or-
Cat
-or-
Dog
-or-
Pig
-or-
Bird ?

label $y_i \in \mathcal{Y} = \{0,1,2,3,\dots\}$

# Unimodal Classification – Visual Modality



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| 28 |
| 22 |
| 22 |
| 84 |
| 78 |
| 80 |
| ⋮ |

## Multi-label (or multi-task) classification problem

Duck?

Cat ?

Dog ?

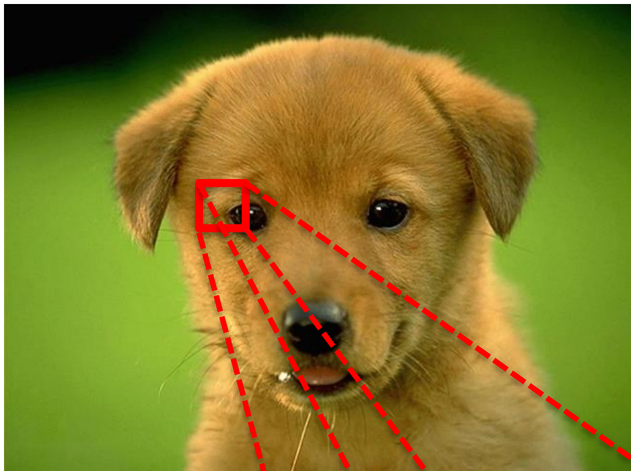
Pig ?

Bird ?

Puppy ?

label vector $\boldsymbol{y_i} \in \mathcal{Y}^m = \{0,1\}^m$

# Unimodal Classification – Visual Modality



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Input observation $\boldsymbol{x_i}$

| |
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| 25 |
| 26 |
| 28 |
| 22 |
| 22 |
| 84 |
| 78 |
| 80 |
| ⋮ |

**Multi-label (or multi-task) regression problem**

Age ?

Height ?

Weight ?

Distance ?

Happy ?

label vector $\boldsymbol{y_i} \in \mathcal{Y}^m = \mathbb{R}^m$

# Unimodal Classification – Language Modality

**Written language**

★★★★★ **Masterful!**

By Antony Witheyman - January 12, 2006

Ideal for anyone with an interest in disguises who likes to see the subject tackled in a humourous manner.

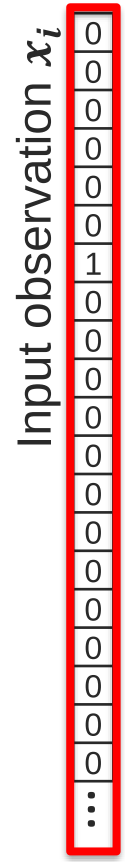
0 of 4 people found this review helpful

**Spoken language**

```
               MARTHA(CON'T)
Look around you. Look at all the
great things you've done and the
people you've helped.

               CLARK
But you've only put up the good
things they say about me.

               MARTHA
Clark, honey. If I were to use the
bad things they say I could cover
the barn, the house and the
outhouse.
```




**"one-hot" vector**

$|x_i|$ = number of words in dictionary

## Word-level classification

Part-of-speech ?
(noun, verb,…)

Sentiment ?
(positive or negative)

Named entity ?
(names of person,…)

# Unimodal Classification – Language Modality

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the barn, the house and the
outhouse.
```

Input observation $x_i$

$$x_i = \begin{bmatrix} 0 \\ 1 \\ 0 \\ 0 \\ 1 \\ 0 \\ 1 \\ 0 \\ 0 \\ 0 \\ 0 \\ 1 \\ 0 \\ 0 \\ 0 \\ 0 \\ 1 \\ 0 \\ 0 \\ 0 \\ \vdots \end{bmatrix}$$

**"bag-of-word" vector**

$|x_i|$ = number of words in dictionary

**Document-level classification**

Sentiment ?

(positive or negative)

# Unimodal Classification – Language Modality

**Written language**

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                 MARTHA
Clark, honey. If I were to use the
bad things they say I could cover
the barn, the house and the
outhouse.
```

Input observation $x_i$

| |
|---|
| 0 |
| 1 |
| 0 |
| 0 |
| 1 |
| 0 |
| 1 |
| 0 |
| 0 |
| 0 |
| 0 |
| 1 |
| 0 |
| 0 |
| 0 |
| 0 |
| 1 |
| 0 |
| 0 |
| 0 |
| ⋮ |

**"bag-of-word" vector**

$|x_i|$ = number of words in dictionary

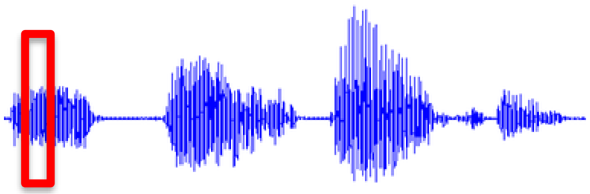
**Utterance-level classification**

Sentiment ?

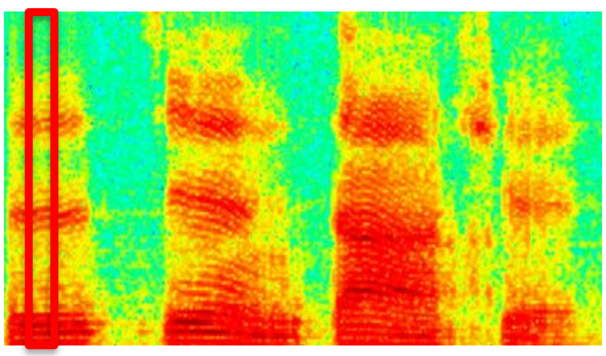
(positive or negative)

# Unimodal Classification – Acoustic Modality

## Digitalized acoustic signal



- Sampling rates: 8~96kHz
- Bit depth: 8, 16 or 24 bits
- Time window size: 20ms
  - Offset: 10ms



**Spectogram**

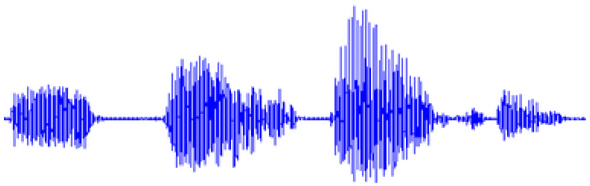
Input observation $x_i$

| |
|---|
| 0.21 |
| 0.14 |
| 0.56 |
| 0.45 |
| 0.9 |
| 0.98 |
| 0.75 |
| 0.34 |
| 0.24 |
| 0.11 |
| 0.02 |

Spoken word ?

# Unimodal Classification – Acoustic Modality

## Digitalized acoustic signal



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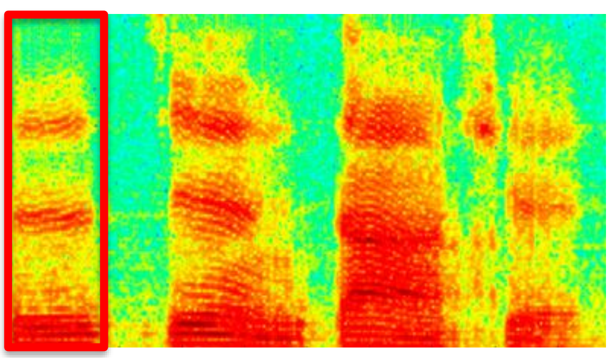


**Spectogram**

Input observation $x_i$

| |
|---|
| 0.21 |
| 0.14 |
| 0.56 |
| 0.45 |
| 0.9 |
| 0.98 |
| 0.75 |
| 0.34 |
| 0.24 |
| 0.11 |
| 0.02 |
| 0.24 |
| 0.26 |
| 0.58 |
| 0.9 |
| 0.99 |
| 0.79 |
| 0.45 |
| 0.34 |
| 0.24 |
| ⋮ |

Emotion ?

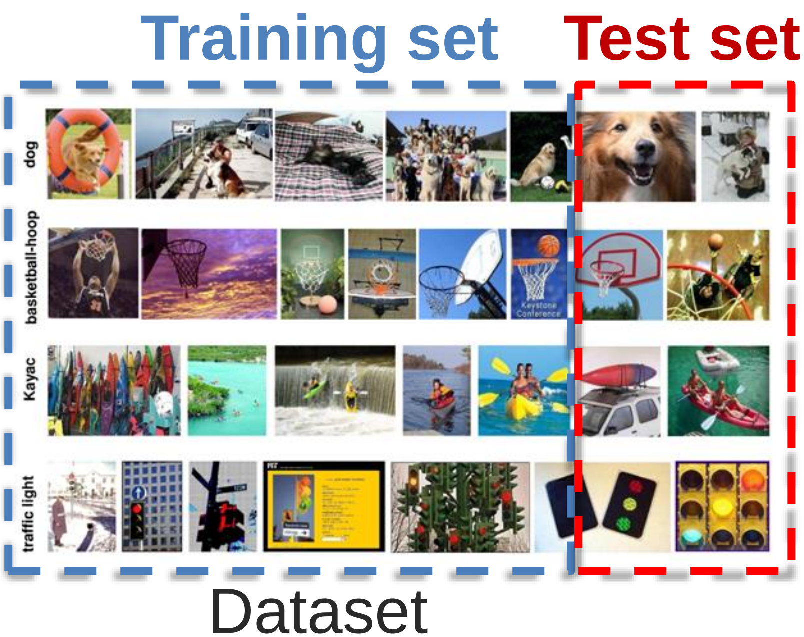
Spoken word ?

Voice quality ?

# Data-Driven Machine Learning

# Data-Driven Machine Learning

1. **Dataset:** Collection of labeled samples $\mathrm{D}: \{x_i, y_i\}$
2. **Training:** Learn classifier on training set
3. **Testing:** Evaluate classifier on hold-out test set



# Simple Classifier ?


Traffic light

-or-

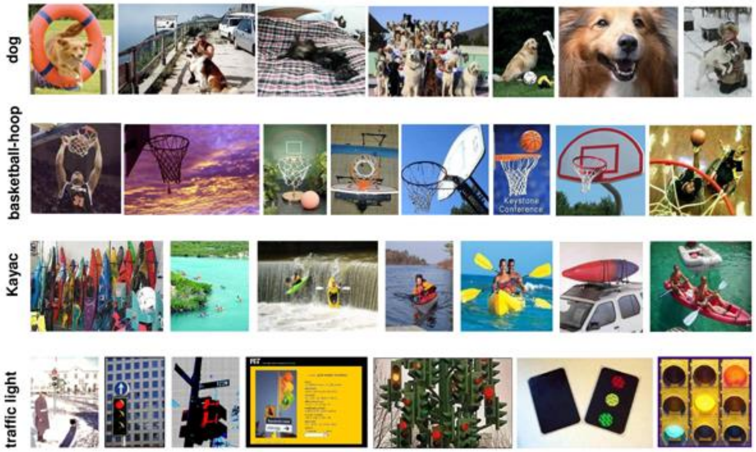
Dog

-or-

Basket

-or-

Kayak ?

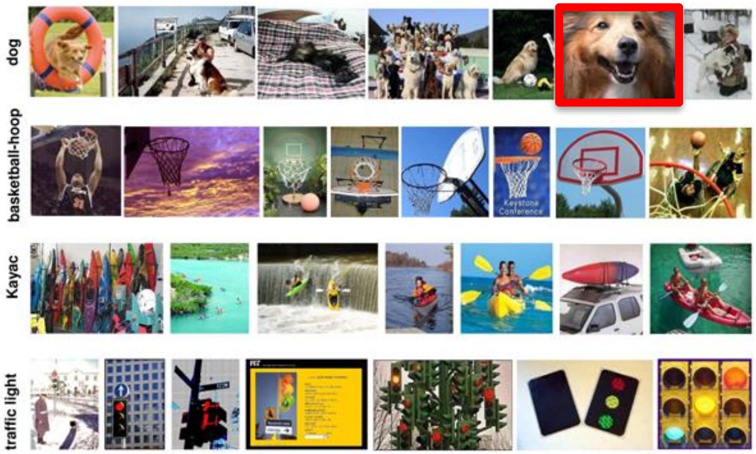



Dataset

# Simple Classifier: Nearest Neighbor

?

Traffic light
-or-
Dog
-or-
Basket
-or-
Kayak ?




Training

# Nearest Neighbor Classifier

- Non-parametric approaches—key ideas:
  - *“Let the data speak for themselves”*
  - *“Predict new cases based on similar cases”*
  - *“Use multiple local models instead of a single global model”*

- What is the complexity of the NN classifier w.r.t training set of N images and test set of M images?
  - at training time?

    O(1)
  - At test time?

    O(N)

# Simple Classifier: Nearest Neighbor

## Distance metrics

L1 (Manhattan) distance:

$$d_1(x_1, x_2) = \sum_j \left| x_1^j - x_2^j \right|$$

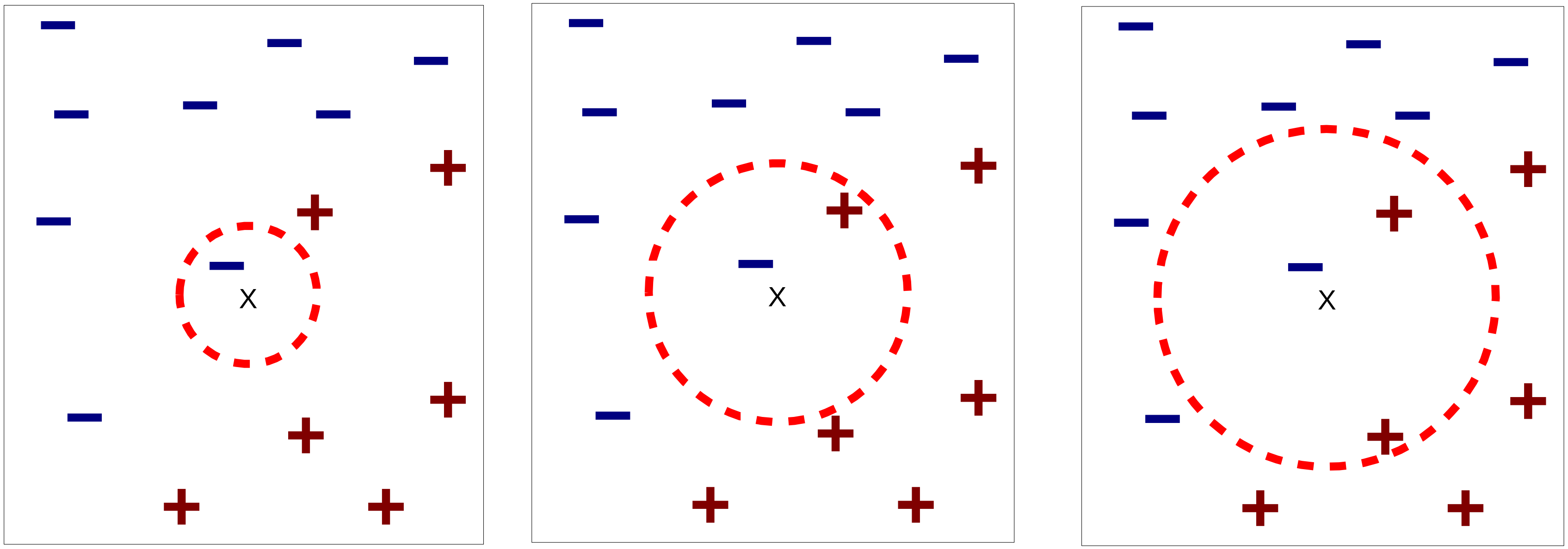
L2 (Eucledian) distance:

$$d_2(x_1, x_2) = \sqrt{\sum_j \left( x_1^j - x_2^j \right)^2}$$

**Which distance metric to use?**

First hyper-parameter!

# Definition of K-Nearest Neighbor



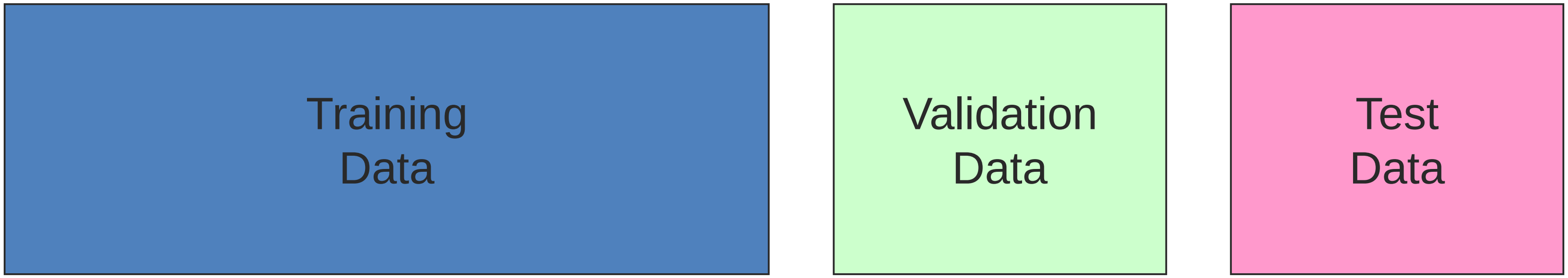

(a) 1-nearest neighbor (b) 2-nearest neighbor (c) 3-nearest neighbor

**What value should we set K?**

Second hyper-parameter!

# Data-Driven Approach

1. **Dataset:** Collection of labeled samples $\mathrm{D}: \{x_i, y_i\}$
2. **Training:** Learn classifier on training set
3. **Validation:** Select optimal hyper-parameters
4. **Testing:** Evaluate classifier on hold-out test set



# Evaluation methods (for validation and testing)

- Holdout set: The available data set *D* is divided into two disjoint subsets,
  - the *training set* $D_{train}$ (for learning a model)
  - the *test set* $D_{test}$ (for testing the model)
- Important: training set should not be used in testing and the test set should not be used in learning.
  - Unseen test set provides a unbiased estimate of accuracy.
- The test set is also called the holdout set. (the examples in the original data set *D* are all labeled with classes.)
- This method is mainly used when the data set *D* is large.
- Holdout methods can also be used for validation

# Evaluation methods (for validation and testing)

- n-fold cross-validation: The available data is partitioned into *n* equal-size disjoint subsets.
- Use each subset as the test set and combine the rest *n*-1 subsets as the training set to learn a classifier.
- The procedure is run *n* times, which give *n* accuracies.
- The final estimated accuracy of learning is the average of the *n* accuracies.
- 10-fold and 5-fold cross-validations are commonly used.
- This method is used when the available data is not large.

## Evaluation methods (for validation and testing)

- Leave-one-out cross-validation: This method is used when the data set is very small.
- Each fold of the cross validation has only a single test example and all the rest of the data is used in training.
- If the original data has *m* examples, this is *m*-fold cross-validation
- It is a special case of cross-validation

# Linear Classification: Scores and Loss

# Linear Classification (e.g., neural network)

Image

(Size: 32*32*3)

?

1. Define a (linear) score function
2. Define the loss function (possibly nonlinear)
3. Optimization

# 1) Score Function

Image

(Size: 32*32*3)

Duck ?
Cat ?
Dog ?
Pig ?
Bird ?

**What should be the prediction score for each label class?**

For linear classifier:

Input observation *($i^{th}$ element of the dataset)* [3072x1]

$$f(x_i; W, b) = W x_i + b$$

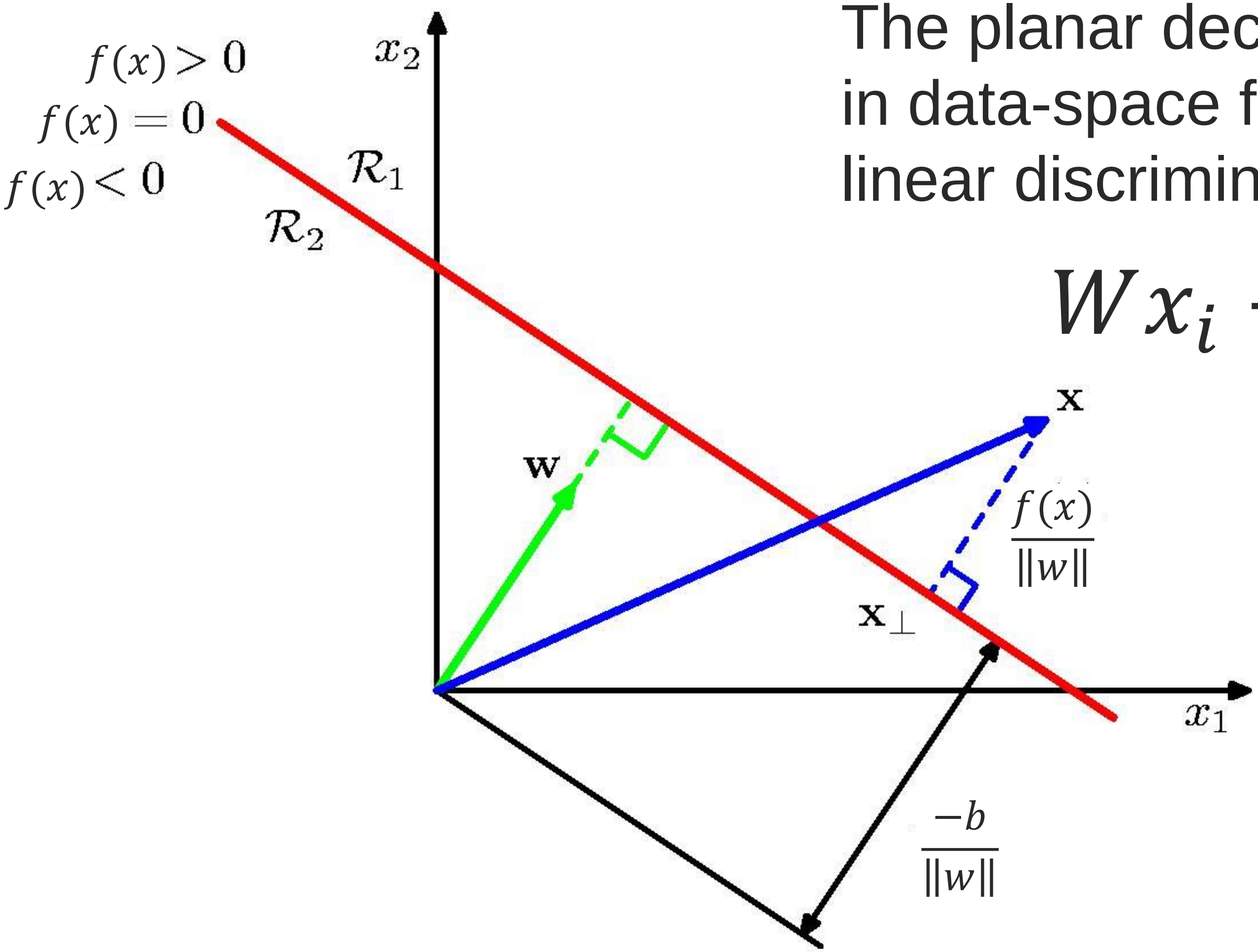
Class score [10x1]

Weights [10x3072]

Bias vector [10x1]

Parameters [10x3073]

# Interpreting a Linear Classifier




The planar decision surface in data-space for the simple linear discriminant function:

$$W x_i + b > 0$$

# Some Notation Tricks – Multi-Label Classification

$$W = [W_1 \quad W_2 \quad \ldots \quad W_N]$$

$$f(x_i; W, b) = Wx_i + b \longrightarrow f(x_i; W) = Wx_i$$

| Weights | x | Input | + | Bias | | Weights | x | Input |
|---|---|---|---|---|---|---|---|---|
| [10x3072] | | [3072x1] | | [10x1] | | [10x3073] | | [3073x1] |

The bias vector will be the last column of the weight matrix

Add a "1" at the end of the input observation vector

# Some Notation Tricks

General formulation of linear classifier: $f(x_i; W, b)$

"dog" linear classifier:

$f\left(x_i; W_{dog}, b_{dog}\right)$ or

$f(x_i; W, b)_{dog}$ or $f_{dog}$

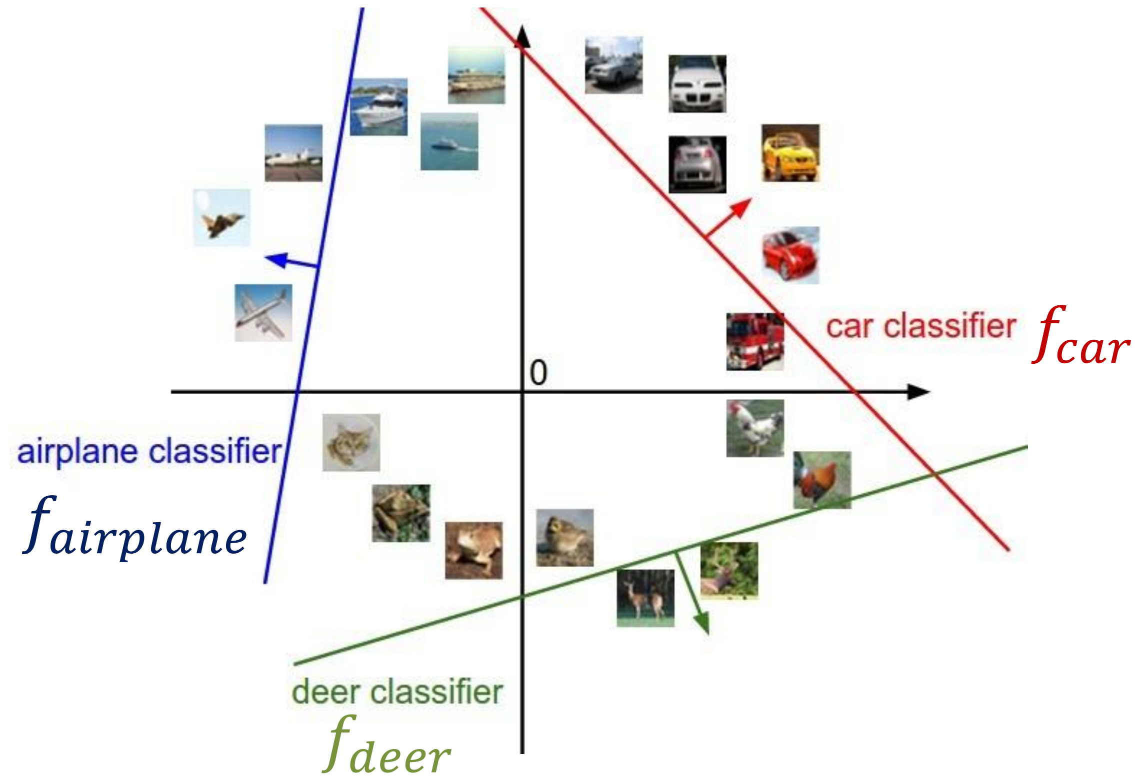
Linear classifier for label *j*:

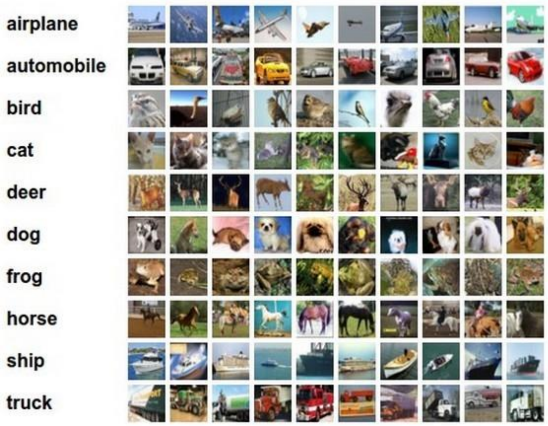
$f\left(x_i; W_j, b_j\right)$ or

$f(x_i; W, b)_j$ or $f_j$

# Interpreting Multiple Linear Classifiers

$$f(x_i; W_j, b_j) = W_j x_i + b_j$$




CIFAR-10 object recognition dataset

# Linear Classification: 2) Loss Function

(or cost function or objective)

**Scores** $f(x_i; W)$ — **Label** $y_i = 2\ (dog)$ → **Loss** $L_i = ?$

Image $x_i$

(Size: 32*32*3)

| | |
|---|---|
| 0 (duck) ? | -12.3 |
| 1 (cat) ? | 45.6 |
| 2 (dog) ? | 98.7 |
| 3 (pig) ? | 12.2 |
| 4 (bird) ? | -45.3 |

How to assign only one number representing how "unhappy" we are about these scores?

**Multi-class problem**

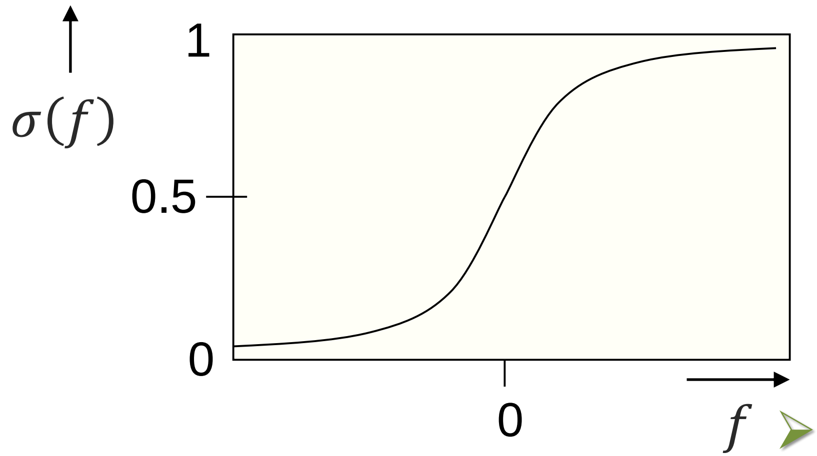
**The loss function quantifies the amount by which the prediction scores deviate from the actual values.**

A first challenge: how to normalize the scores?

# First Loss Function: Cross-Entropy Loss

(or logistic loss)

Logistic function: $$\sigma(f) = \frac{1}{1+e^{-f}}$$




➢ **Score function**

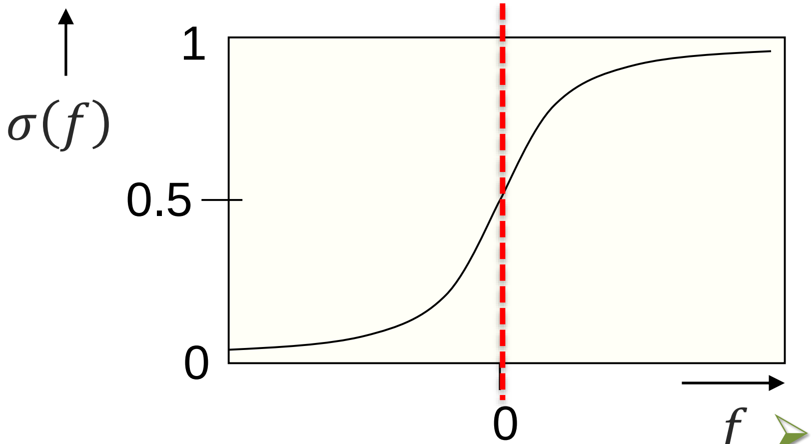
# First Loss Function: Cross-Entropy Loss

(or logistic loss)

Logistic function: $$\sigma(f) = \frac{1}{1 + e^{-f}}$$

Logistic regression: (two classes) $$p(y_i = \text{"}dog\text{"} | x_i; w) = \sigma\left(w^T x_i\right)$$

**= true** for two-class problem




➢ **Score function**

# First Loss Function: Cross-Entropy Loss

(or logistic loss)

Logistic function:

$$\sigma(f) = \frac{1}{1 + e^{-f}}$$

Logistic regression:
(two classes)

$$p(y_i = "dog" | x_i; w) \quad = \sigma\left(w^T x_i\right)$$

**= true**
for two-class problem

Softmax function:
(multiple classes)

$$p(y_i | x_i; W) = \frac{e^{f_{y_i}}}{\sum_j e^{f_j}}$$

# First Loss Function: Cross-Entropy Loss

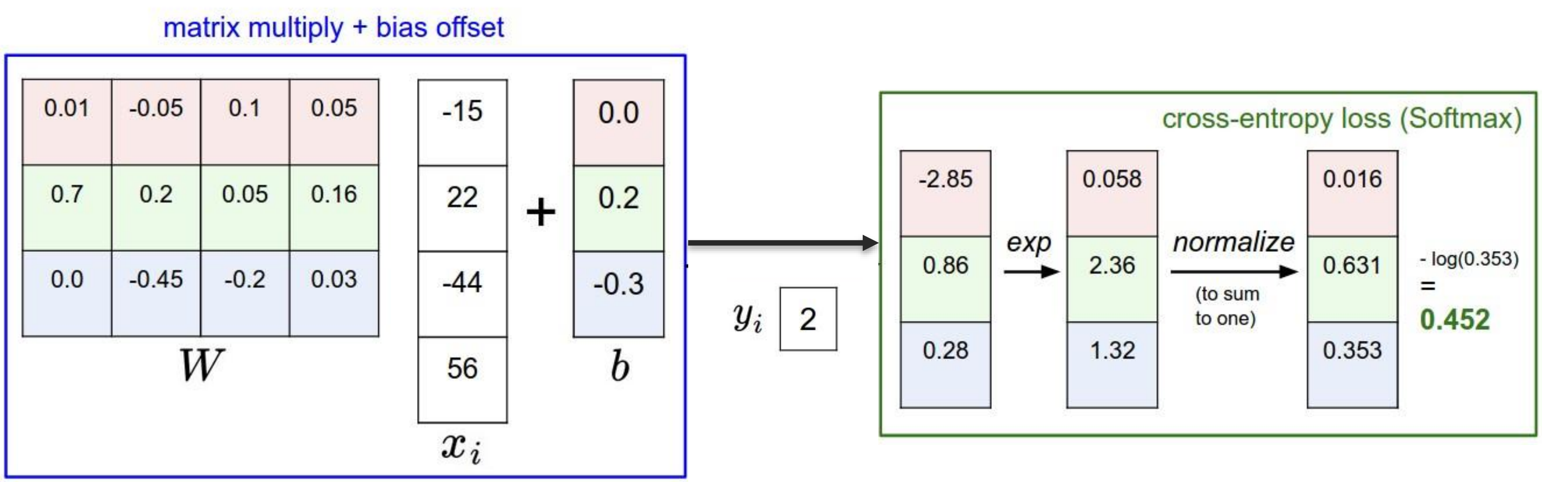
(or logistic loss)

Cross-entropy loss:

$$L_i = \boxed{-\log\left(\boxed{\frac{e^{f_{y_i}}}{\sum_j e^{f_j}}}\right)}$$

Softmax function

Minimizing the negative log likelihood.



# Second Loss Function: Hinge Loss

(or max-margin loss or Multi-class SVM loss)

$$\boxed{L_i} = \boxed{\sum_{j \neq y_i}} \max(0, \boxed{f(x_i, W)_j - f(x_i, W)_{y_i}} + \Delta)$$

loss due to example i

sum over all incorrect labels

difference between the correct class score and incorrect class score

## Second Loss Function: Hinge Loss

(or max-margin loss or Multi-class SVM loss)

$$L_i = \sum_{j \neq y_i} \max(0, f(x_i, W)_j - f(x_i, W)_{y_i} + \Delta)$$

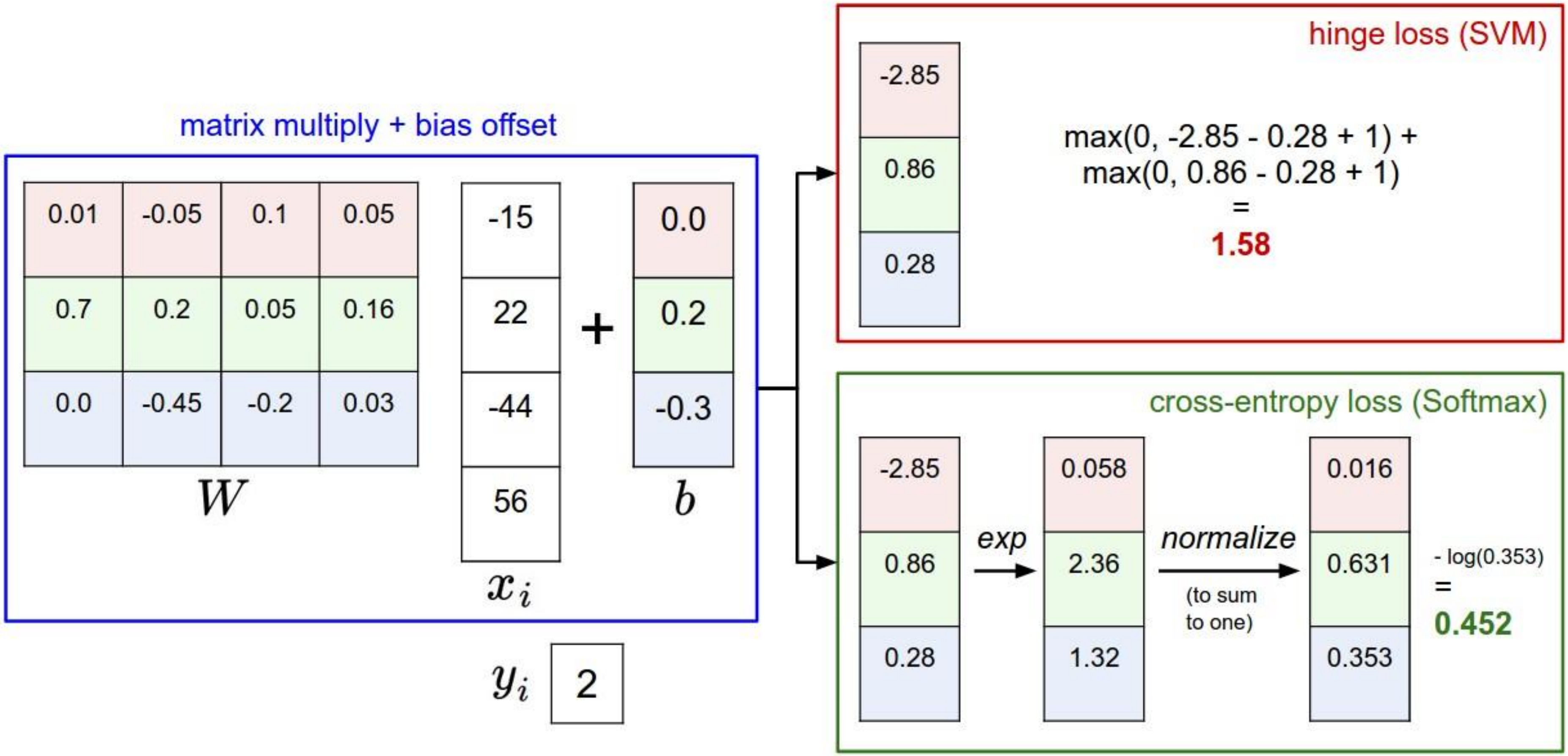
e.g. 10

Example: $f(x_i, W) = [13, -7, 11]$

$y_i = 0$

$$L_i = \max(0, -7 - 13 + 10) + \max(0, 11 - 13 + 10)$$

# Two Loss Functions




How to find the optimal W?

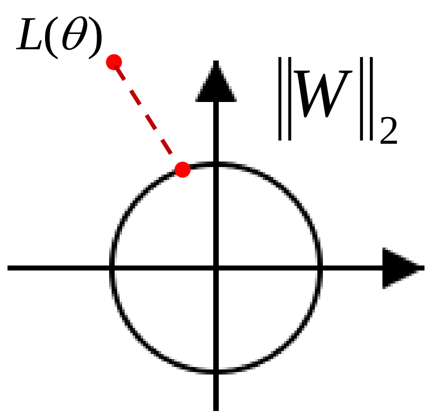
# Regularization

Regularization factor

$$L_i = -\log\left(\frac{e^{f_{y_i}(x_i;W)}}{\sum_j e^{f_j(x_i;W)}}\right) + \lambda R(W)$$

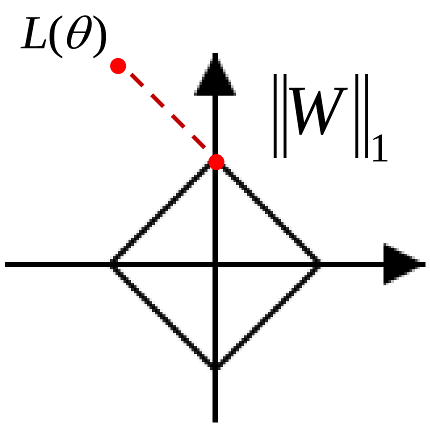
**L-2 Norm (Gaussian prior):**

$$R(W) = \|W\|_2$$




**L-1 Norm (Laplacian prior):**

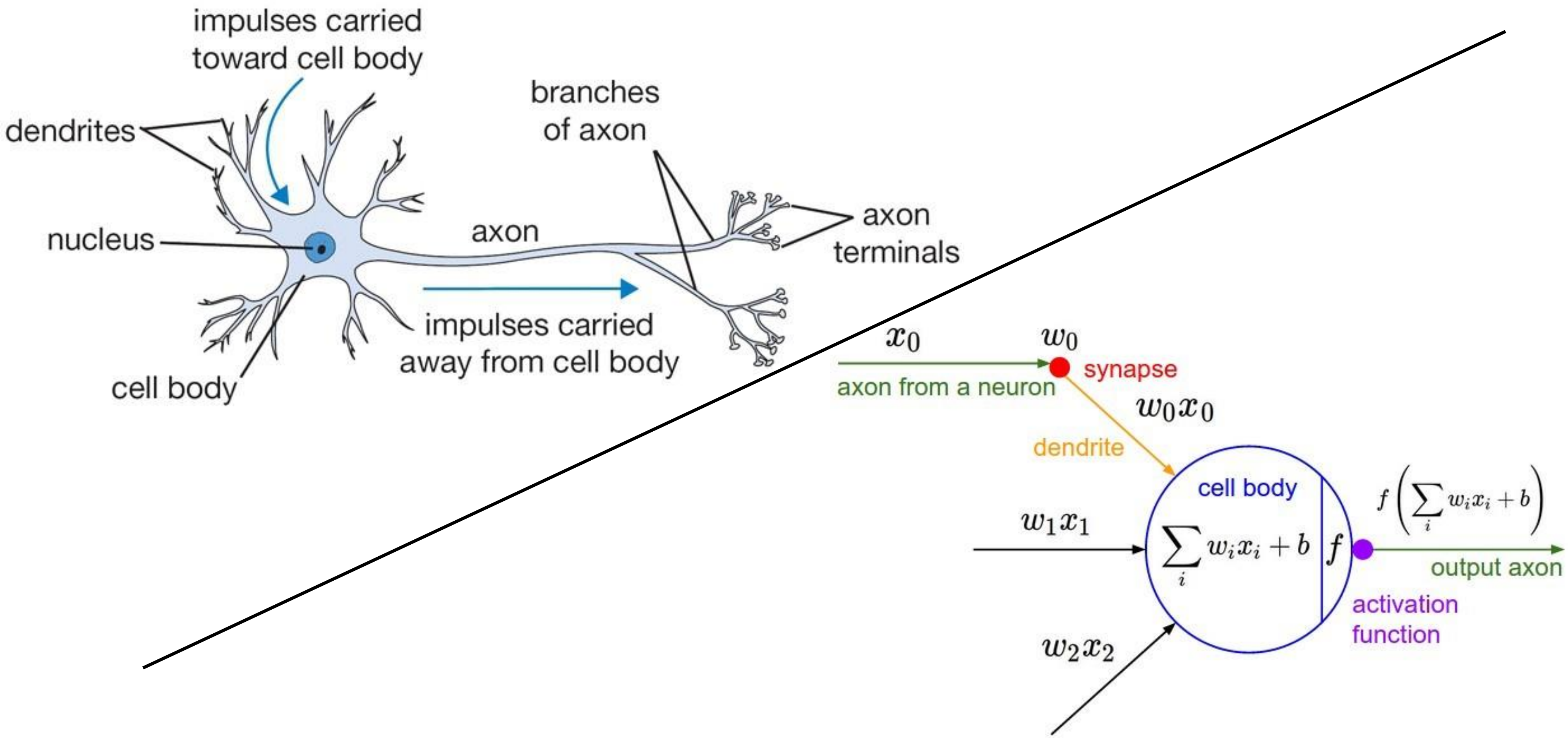
$$R(W) = \|W\|_1$$



# Basic Concepts: Neural Networks

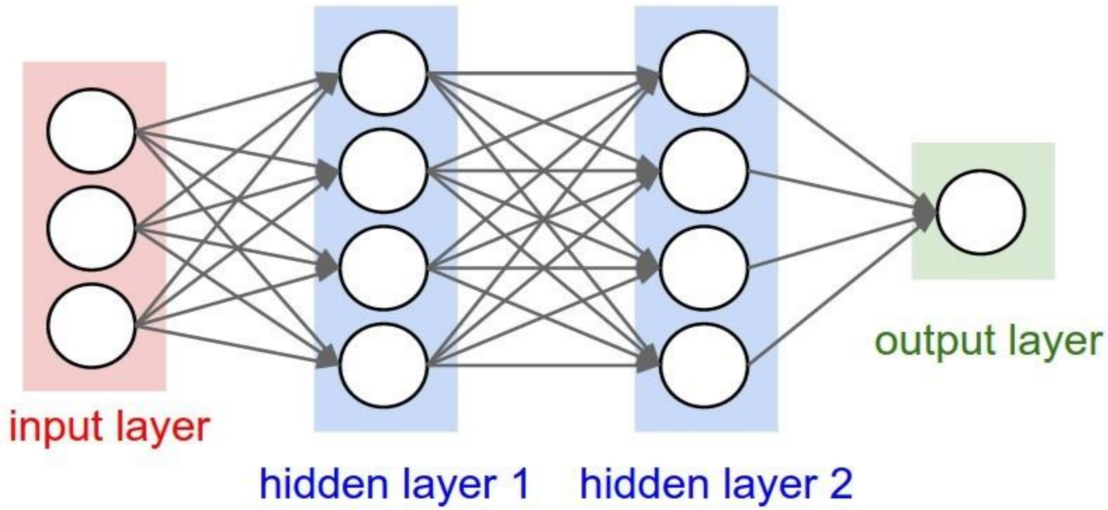
# Neural Networks – inspiration

- Made up of artificial neurons



# Neural Networks – score function

- Made up of artificial neurons
  - Linear function (dot product) followed by a nonlinear activation function
- Example a Multi Layer Perceptron



# Basic NN building block

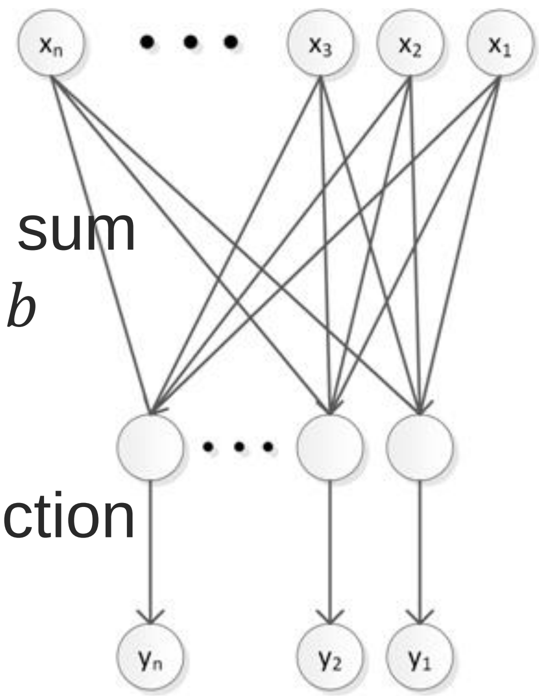
- Weighted sum followed by an activation function

Input

Weighted sum
$Wx + b$

Activation function

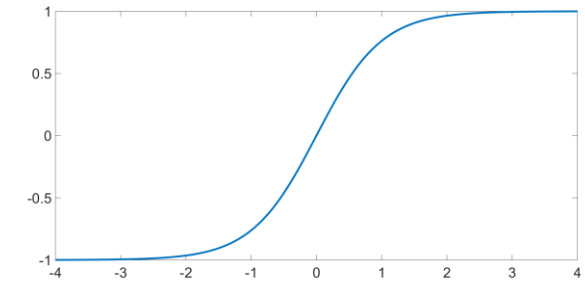
Output




$$y = f(Wx + b)$$

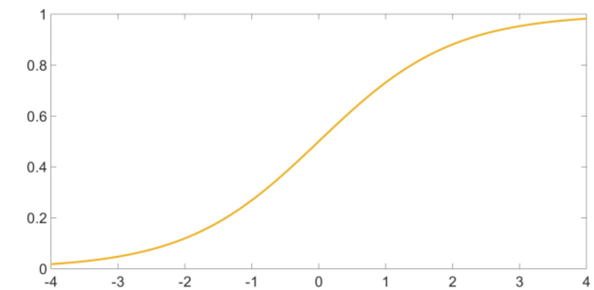
# Neural Networks – activation function

- $f(x) = \tanh(x)$



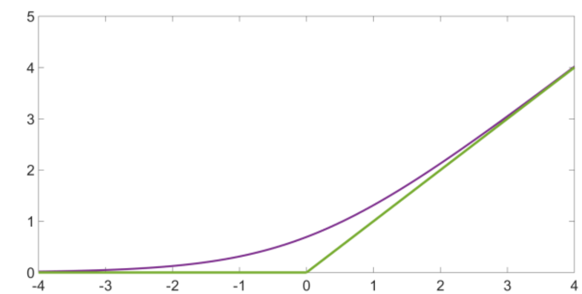
tanh(x)

- Sigmoid - $f(x) = (1 + e^{-x})^{-1}$



sigmoid

- Linear – $f(x) = ax + b$
- ReLU $f(x) = \max(0, x) \sim \log(1 + \exp(x))$
  - Rectifier Linear Units
  - Faster training - no gradient vanishing
  - Induces sparsity



ReLU (soft and hard)

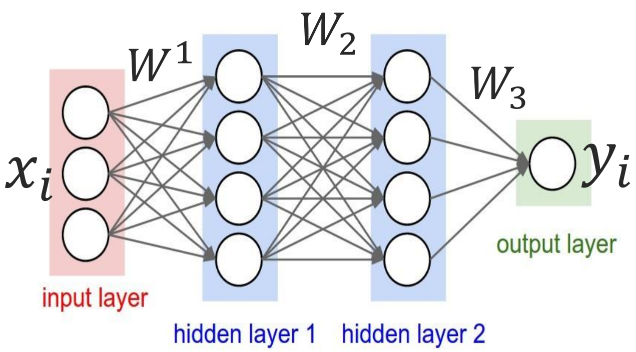
# Multi-Layer Feedforward Network

Activation functions (individual layers)

$$f_{1;W_1}(x) = \sigma(W_1 x + b_1)$$

$$f_{2;W_2}(x) = \sigma(W_2 x + b_2)$$

$$f_{3;W_3}(x) = \sigma(W_3 x + b_3)$$




Score function

$$y_i = f(x_i) = f_{3;W_3}(f_{2;W_2}(f_{1;W_1}(x_i)))$$

Loss function (e.g., Euclidean loss)

$$L_i = (f(x_i) - y_i)^2 = (f_{3;W_3}(f_{2;W_2}(f_{1;W_1}(x_i))))^2$$

# Neural Networks inference and learning

- Inference (Testing)
  - Use the score function ($y = f(\boldsymbol{x}; W)$)
  - Have a trained model (parameters $W$)
- Learning model parameters (Training)
  - Loss function ($L$)
  - Gradient – will talk about next week
  - Optimization – will talk about next week and during week 8

# Loss function (1)

- Loss function is often made up of three parts

$$L = L_{data} + \lambda_1 L_{regularization} + \lambda_2 L_{constraints}$$

- Data term
  - How well our model is explaining/predicting training data (e.g. cross-entropy loss, Euclidean loss)

$$\sum_i L_i = -\sum_i \log\left(\frac{e^{f_{y_i}(x_i;W)}}{\sum_j e^{f_j(x_i;W)}}\right)$$

$$\sum_i L_i = \sum_i \left(y_i - f(x_i, W)\right)^2$$

# Loss function (2)

- Loss function is often made up of three parts

$$L = L_{data} + \lambda_1 L_{regularization} + \lambda_2 L_{constraints}$$

- Regularization/Smoothness term
  - Prevent the model from becoming too complex
  - e.g. $||W||_2$ for parameters smoothness
  - e.g. $||W||_1$ for parameter sparsity
- $\lambda_1$ is a hyper-parameter
- Optional, but almost never omitted

# Loss function (3)

- Loss function is often made up of three parts

$$L = L_{data} + \lambda_1 L_{regularization} + \lambda_2 L_{constraints}$$

- Additional constraints
  - Optional and not always used
  - Help with certain models (e.g. coordinated multimodal representation)
  - e.g. Triplet loss, hinge ranking loss, reconstruction loss
  - Will talk more during multimodal representation lecture