

# Project 1:

# Collaborative Filtering

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# Problem: Collaborative Filtering

A recommender system technique to predict users' ratings of objects (in our case, movies), with the help of the known ratings of other users and items. We predict the missing values in a large, sparse,  $\mathbb{R}^{m \times n}$  dimension matrix,  $R$ , which when completed will fully describe the ratings of all users for all objects. This will enable personalized recommendations of new items the users have not yet rated.



# Our methods

- Matrix factorization with gradient descent
  - L2 Normalization
  - Bias Terms
  - Momentum
  - Naive rounding + fitting output to input distribution
- Graph method

We also considered using hyperdimensional computing

Evaluated with RMSE and accuracy

# Matrix Factorization

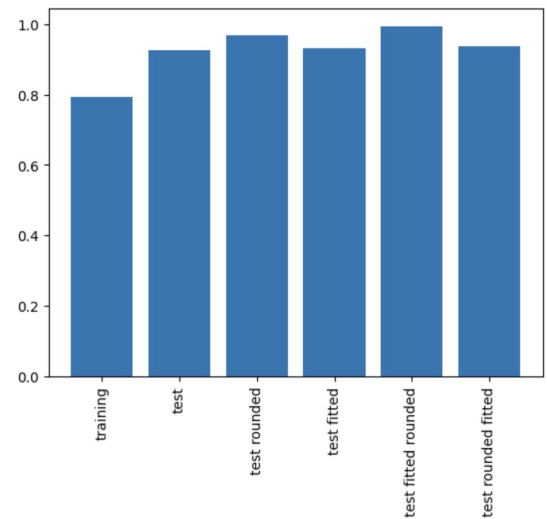
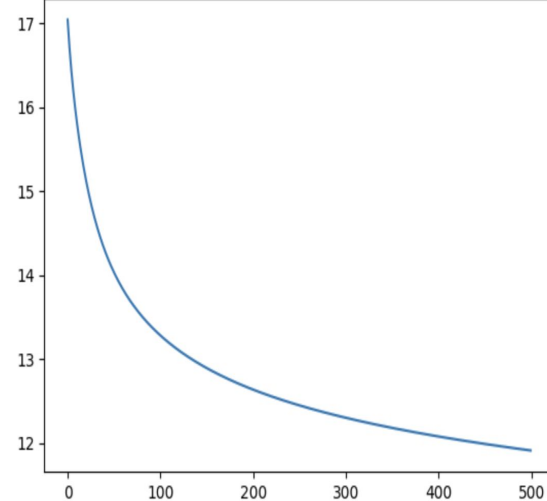
Decomposition into  $UV^T$  with L2 normalization and bias terms for U and V

Loss function:

$$J(\mathbf{U}, \mathbf{V}) = 1/2(\text{tr}(\mathbf{R}^T \mathbf{R}) - 2 \text{tr}(\mathbf{R}^T \mathbf{U} \mathbf{V}^T) + \text{tr}(\mathbf{V} \mathbf{U}^T \mathbf{U} \mathbf{V}^T) + (\text{tr}(\mathbf{U}^T \mathbf{U}) + \text{tr}(\mathbf{V}^T \mathbf{V})))$$

Update functions:

$$\mathbf{U} \leftarrow \mathbf{U} (1 - \alpha \cdot \lambda) + \mathbf{E} \mathbf{V} \quad \mathbf{V} \leftarrow \mathbf{V} (1 - \alpha \cdot \lambda) + \mathbf{E}^T \mathbf{U}$$



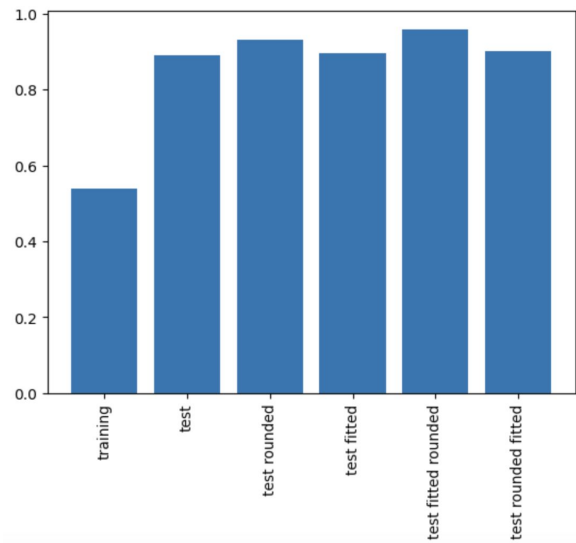
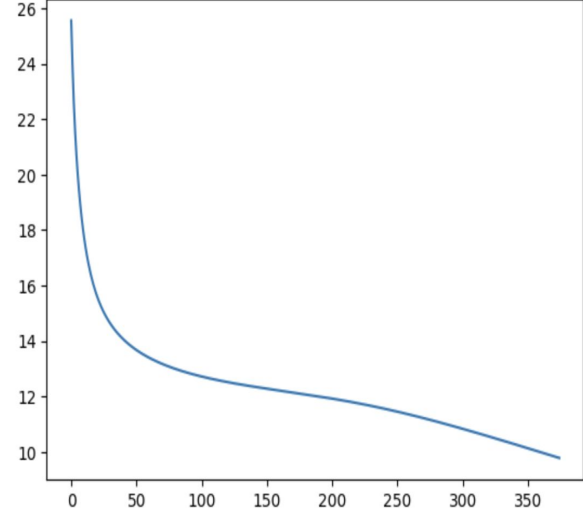
# Momentum

Basic momentum to smooth out gradient and improve speed

Loss function remains the same

Update functions:

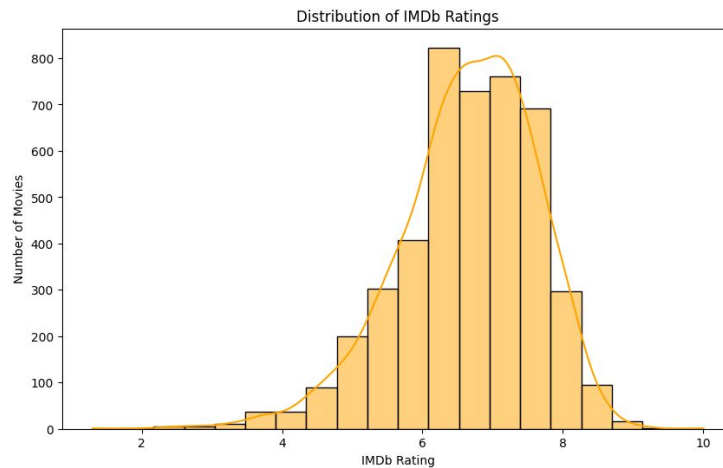
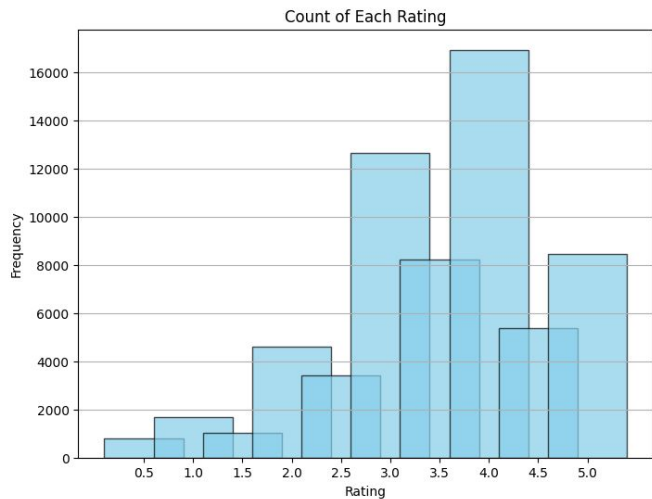
$$\begin{aligned} \mathbf{vU} &\leftarrow m * \mathbf{vU} + \alpha(\mathbf{E}^T \mathbf{U} - \lambda \mathbf{U}) & \mathbf{vV} &\leftarrow m * \mathbf{vV} + \alpha(\mathbf{E}^T \mathbf{U} - \lambda \mathbf{V}) \\ \mathbf{U} &\leftarrow \mathbf{U} + \mathbf{vU} & \mathbf{V} &\leftarrow \mathbf{V} + \mathbf{vV} \end{aligned}$$



# Fitting + Rounding

Shifted and scaled output distribution by mean and standard deviation of input distribution

Rounded to the nearest half integer

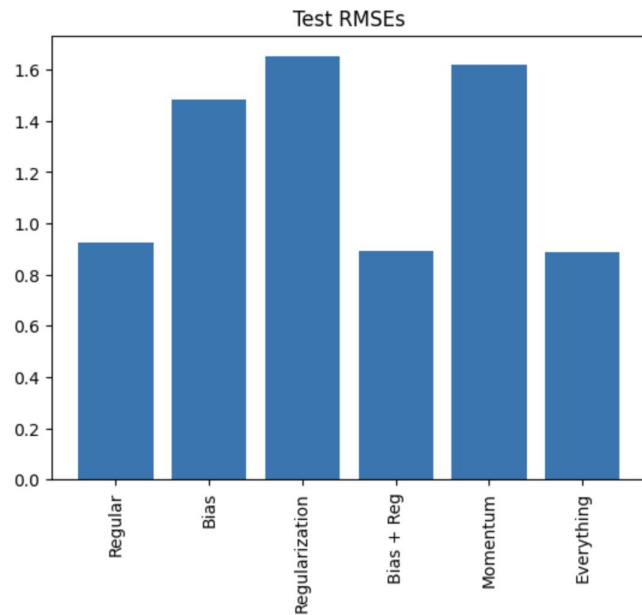


# Experimental Results

Our best performing set of hyperparameters for matrix factorization came out to be:

- $k=160$
- $\alpha=0.0008$
- $\text{momentum}=0.1$
- $L2\ \mu = L2\ \lambda=0.01$
- $\text{epochs}=375$

Which gave us an RMSE of 0.888



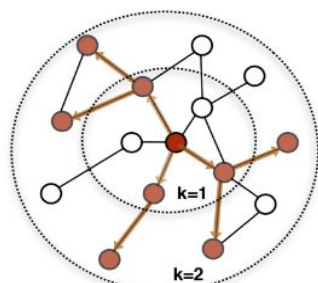
# Graph Method

Heterogeneous graph with nodes are **users** and **movies**, and edges as **interactions** (ratings) between users and movies

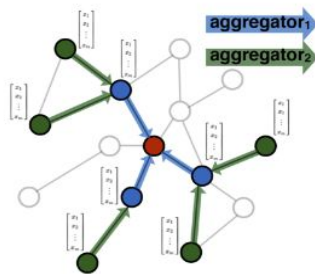
Initialize nodes as feature vectors

- For users, **one-hot encoding of size 610** (for 610 users in total)
- For movies, **embeddings with movie metadata** from IMDB (recovered using OMDB API)

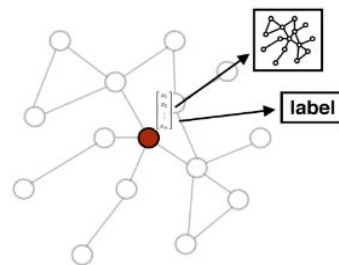
To actually predict the ratings, we perform an edge prediction task and predict the edge label, the rating a user would give to a movie



1. Sample neighborhood



2. Aggregate feature information from neighbors



3. Predict graph context and label using aggregated information



# Training the model

## Encoding

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**Algorithm 1:** GraphSAGE embedding generation (i.e., forward propagation) algorithm

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**Input :** Graph  $\mathcal{G}(\mathcal{V}, \mathcal{E})$ ; input features  $\{\mathbf{x}_v, \forall v \in \mathcal{V}\}$ ; depth  $K$ ; weight matrices  $\mathbf{W}^k, \forall k \in \{1, \dots, K\}$ ; non-linearity  $\sigma$ ; differentiable aggregator functions  $\text{AGGREGATE}_k, \forall k \in \{1, \dots, K\}$ ; neighborhood function  $\mathcal{N} : v \rightarrow 2^{\mathcal{V}}$

**Output :** Vector representations  $\mathbf{z}_v$  for all  $v \in \mathcal{V}$

```
1  $\mathbf{h}_v^0 \leftarrow \mathbf{x}_v, \forall v \in \mathcal{V};$ 
2 for  $k = 1 \dots K$  do
3   for  $v \in \mathcal{V}$  do
4      $\mathbf{h}_{\mathcal{N}(v)}^k \leftarrow \text{AGGREGATE}_k(\{\mathbf{h}_u^{k-1}, \forall u \in \mathcal{N}(v)\});$ 
5      $\mathbf{h}_v^k \leftarrow \sigma \left( \mathbf{W}^k \cdot \text{CONCAT}(\mathbf{h}_v^{k-1}, \mathbf{h}_{\mathcal{N}(v)}^k) \right)$ 
6   end
7    $\mathbf{h}_v^k \leftarrow \mathbf{h}_v^k / \|\mathbf{h}_v^k\|_2, \forall v \in \mathcal{V}$ 
8 end
9  $\mathbf{z}_v \leftarrow \mathbf{h}_v^K, \forall v \in \mathcal{V}$ 
```

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## Decoding

$\mathbf{Z} = \text{CONCAT}(\mathbf{z}_{\text{user}}, \mathbf{z}_{\text{movie}})$

$\mathbf{R}' = \text{LeakyReLU}(\mathbf{Wd} \times \mathbf{Z})$

$\mathbf{z}_{\text{user}}$  and  $\mathbf{z}_{\text{movie}}$  are the embeddings of the user and movie nodes

$\mathbf{Wd}$  is the weight matrix of the linear layer of the decoder

$\mathbf{R}$  is the predicted rating of the edge that joins the user and movie

To train, we minimize MSE throughout continual forward/back passes

# Experimental Results

