1. a) $I=\int \frac{\ln(x^2+6x+10)}{(x+1)^2}\,dx = \begin{pmatrix} u=\ln(x^2+6x+10) & dv=\frac{dx}{(x+1)^2} \\ du=\frac{2x+6}{x^2+6x+10}\,dx & v=-\frac{1}{x+1} \end{pmatrix}$

$$\int \frac{dx}{(x+1)^2} = \begin{pmatrix} t=x+1 \\ dt=dx \end{pmatrix} = \int t^{-2}\,dt = -\frac{1}{t} = -\frac{1}{x+1}$$

$$I = -\frac{\ln(x^2+6x+10)}{x+1} + 2\int \frac{x+3}{(x^2+6x+10)(x+1)}\,dx$$

$x^2+6x+10$ је нерастављиво $(D=6^2-4\cdot 10<0)$

$$\frac{x+3}{(x^2+6x+10)(x+1)} = \frac{Ax+B}{x^2+6x+10} + \frac{C}{x+1} \quad /\cdot(x^2+6x+10)(x+1)$$

$$x+3 = (Ax+B)(x+1) + C(x^2+6x+10)$$

$$x+3 = (A+C)x^2 + (A+B+6C)x + B+10C$$

$A+C=0 \Rightarrow A=-C$

$A+B+6C=1 \Rightarrow -C+3-10C+6C=1$

$B+10C=3 \Rightarrow B=3-10C$

$3-5C=1$

$$\boxed{C=\frac{2}{5} \Rightarrow A=-\frac{2}{5},\quad B=-1}$$

$$\Rightarrow \int \frac{x+3}{(x^2+6x+10)(x+1)} = \int \frac{-\frac{2}{5}x-1}{x^2+6x+10}\,dx + \int \frac{\frac{2}{5}}{x+1}\,dx =$$

$$= -\frac{1}{5}\int \frac{2x+5}{x^2+6x+10}\,dx + \frac{2}{5}\int \frac{dx}{x+1} = -\frac{1}{5}\int \frac{2x+6-1}{x^2+6x+10}\,dx +$$

$$\frac{2}{5}\int \frac{dx}{x+1} = -\frac{1}{5}\int \frac{2x+6}{x^2+6x+10}\,dx + \frac{1}{5}\int \frac{dx}{x^2+6x+10} + \frac{2}{5}\int \frac{dx}{x+1}$$

$$\int \frac{2x+6}{x^2+6x+10}\,dx = \begin{pmatrix} t=x^2+6x+10 \\ dt=(2x+6)\,dx \end{pmatrix} = \int \frac{dt}{t} = \ln|t| = \ln|x^2+6x+10| =$$

$$= \ln(x^2+6x+10)$$

$$\int \frac{dx}{x^2+6x+10} = \int \frac{dx}{(x+3)^2+1} = \begin{pmatrix} t=x+3 \\ dt=dx \end{pmatrix} = \int \frac{dt}{t^2+1} = \operatorname{arctg} t =$$

$$= \operatorname{arctg}(x+3)$$

$$\int \frac{dx}{x+1} = \begin{pmatrix} t = x+1 \\ dt = dx \end{pmatrix} = \int \frac{dt}{t} = \ln|t| = \ln|x+1|$$

$$\Rightarrow I = -\frac{\ln(x^2+6x+10)}{x+1} + 2\left(-\frac{1}{5}\cdot\ln(x^2+6x+10) + \frac{1}{5}\operatorname{arctg}(x+3) + \frac{2}{5}\ln|x+3|\right) =$$

$$= -\frac{\ln(x^2+6x+10)}{x+1} - \frac{2\ln(x^2+6x+10)}{5} + \frac{2\operatorname{arctg}(x+3)}{5} + \frac{4\ln|x+1|}{5} + C$$

б)

$$f(x) = \begin{cases} \cos x \ln\frac{1+x}{1-x}, & x \in [-\frac{1}{2}, \frac{1}{2}) \\ 5, & x = \frac{1}{2} \\ \sqrt{1-\cos^2 x}, & x \in (\frac{1}{2}, 2\pi] \end{cases}$$

$$\int_{-\frac{1}{2}}^{2\pi} f(x)\,dx = \int_{-\frac{1}{2}}^{\frac{1}{2}} f(x)\,dx + \underbrace{\int_{\frac{1}{2}}^{\frac{1}{2}} f(x)\,dx}_{0} + \int_{\frac{1}{2}}^{2\pi} \sqrt{1-\cos^2 x}\,dx =$$

$\sin x \geq 0$ за $x \in (\frac{1}{2}, \pi]$

$\sin x \leq 0$ за $x \in (\pi, 2\pi]$

$$= \underbrace{\int_{-\frac{1}{2}}^{\frac{1}{2}} \overbrace{\underbrace{\cos x}_{\text{парна}} \cdot \underbrace{\ln\frac{1+x}{1-x}}_{\text{непарна}}}^{\text{непарна}}\,dx}_{0} + \int_{\frac{1}{2}}^{2\pi} \sqrt{\sin^2 x}\,dx = \int_{\frac{1}{2}}^{2\pi} |\sin x|\,dx =$$

$$= \int_{\frac{1}{2}}^{\pi} \sin x\,dx + \int_{\pi}^{2\pi} -\sin x\,dx = -\cos x\Big|_{\frac{1}{2}}^{\pi} + \cos x\Big|_{\pi}^{2\pi} =$$

$$= -(\underset{-1}{\cos\pi} - \cos\tfrac{1}{2}) + (\underset{1}{\cos 2\pi} - \underset{-1}{\cos\pi}) = 1 + \cos\tfrac{1}{2} + 2 = 3 + \cos\tfrac{1}{2}$$

$$g(x) = \ln\frac{1+x}{1-x} = \ln(1+x) - \ln(1-x)$$

$$g(-x) = \ln\frac{1-x}{1+x} = \ln(1-x) - \ln(1+x) = -g(x) \Rightarrow g \text{ је непарна}$$

в)

$$I = \int_0^{\frac{\pi}{2}} \frac{x\sin 2x}{\sin^4 x + \cos^4 x}\,dx = \left(\begin{matrix} t = \frac{\pi}{2} - x \\ dt = -dx \end{matrix} \quad \begin{array}{c|c|c} x & 0 & \pi/2 \\ \hline t & \pi/2 & 0 \end{array} \right) =$$

$$= \int_{\pi/2} \frac{\left(\frac{\pi}{2}-t\right)\sin\left(\pi-2t\right)}{\sin^4\left(\frac{\pi}{2}-t\right)+\cos^4\left(\frac{\pi}{2}-t\right)}\cdot(-dt) = \int_0 \frac{\left(\frac{\pi}{2}-t\right)\sin 2t}{\cos^4 t+\sin^4 t}\,dt =$$

$$= \frac{\pi}{2}\int_0^{\pi/2} \frac{\sin 2t}{\cos^4 t+\sin^4 t}\,dt - \underbrace{\int_0^{\pi/2} \frac{t\sin 2t}{\cos^4 t+\sin^4 t}\,dt}_{I}$$

$$\Rightarrow \quad 2I = \frac{\pi}{2}\int_0^{\pi/2} \frac{\sin 2t}{\cos^4 t+\sin^4 t}\,dt$$

$$\Rightarrow \quad I = \frac{\pi}{\cancel{4}_2}\int_0^{\pi/2} \frac{\cancel{2}\sin t\cos t}{\cos^4 t+\sin^4 t}\,dt = \left(\begin{array}{l} u=\sin t \\ du=\cos t\,dt \end{array} \quad \begin{array}{c|c|c} t & 0 & \pi/2 \\ \hline u & 0 & 1 \end{array}\right) =$$

$$= \frac{\pi}{2}\cdot\int_0^1 \frac{u\,du}{(1-u^2)^2+u^4} = \frac{\pi}{2}\int_0^1 \frac{u\,du}{2u^4-2u^2+1} = \left(\begin{array}{l} s=u^2 \\ ds=2u\,du \end{array} \quad \begin{array}{c|c|c} u & 0 & 1 \\ \hline s & 0 & 1 \end{array}\right)$$

$$= \frac{\pi}{2}\cdot\int_0^1 \frac{\frac{1}{2}\,ds}{2s^2-2s+1} = \frac{\pi}{4}\int_0^1 \frac{ds}{2\left(s^2-s+\frac{1}{2}\right)} = \frac{\pi}{8}\int_0^1 \frac{ds}{\left(s-\frac{1}{2}\right)^2+\frac{1}{4}} =$$

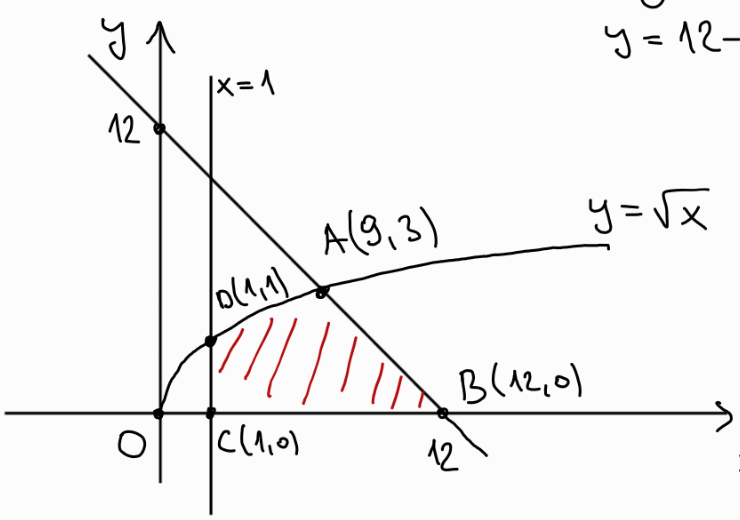
$$= \left(\begin{array}{l} y=s-\frac{1}{2} \\ dy=ds \end{array} \quad \begin{array}{c|c|c} s & 0 & 1 \\ \hline y & -\frac{1}{2} & \frac{1}{2} \end{array}\right) = \frac{\pi}{8}\cdot\int_{-\frac{1}{2}}^{\frac{1}{2}} \underbrace{\frac{dy}{y^2+\frac{1}{4}}}_{\text{парна}} = \frac{\pi}{4}\cdot\int_0^{\frac{1}{2}} \frac{dy}{y^2+\left(\frac{1}{2}\right)^2} =$$

$$= \frac{\pi}{4}\cdot 2\,\mathrm{arctg}(2y)\Big|_0^{\frac{1}{2}} = \frac{\pi}{2}\Big(\underbrace{\mathrm{arctg}\,1}_{=\frac{\pi}{4}} - \underbrace{\mathrm{arctg}\,0}_{=0}\Big) = \frac{\pi^2}{8}$$

2. a) $y=\sqrt{x}$, $\underbrace{y=0}_{x\text{-оса}}$, $x+y-12=0$, $x=1$

$$\Updownarrow$$

$y=12-x \rightarrow$ сече x-осу у $(12,0)$, а y-осу у $(0,12)$



координате тачке D:

$x=1$

$y=\sqrt{x}=1$

Координате пресечне тачке A:

$$y=\sqrt{x} \Rightarrow x=y^2$$
$$x+y-12=0$$
$$y^2+y-12=0$$
$$(y+4)(y-3)=0$$
$$y=-4 \quad \vee \quad y=3$$

немогуће јер $y\geq 0$

$$x=9$$
$$A(9,3)$$

<u>Обим:</u> $CB=12-1=11$, $BA=\sqrt{(12-9)^2+(0-3)^2}=3\sqrt{2}$, $CD=1-0=1$

Дужину дела параболе $y=\sqrt{x}$ између тачака D и A рачунамо по формули

$$l=\int_1^9\sqrt{1+(y')^2}\,dx=\int_1^9\sqrt{1+\left(\frac{1}{2\sqrt{x}}\right)^2}\,dx=\int_1^9\sqrt{1+\frac{1}{4x}}\,dx=\int_1^9\sqrt{\frac{4x+1}{4x}}\,dx=$$

$$=\frac{1}{2}\int_1^9\sqrt{\frac{4x+1}{x}}\,dx=\left(\begin{array}{llll} t=\sqrt{\dfrac{4x+1}{x}} & t^2x=4x+1 & dx=\dfrac{-2t\,dt}{(t^2-4)^2} & \begin{array}{c|c|c} x & 1 & 9 \\ \hline t & \sqrt{5} & \frac{\sqrt{37}}{3} \end{array} \\ t^2=\dfrac{4x+1}{x} & x=\dfrac{1}{t^2-4} & & \end{array}\right)=$$

$$=\frac{1}{2}\int_{\sqrt{5}}^{\sqrt{37}/3} t\cdot\frac{-2t}{(t^2-4)^2}\,dt=\int_{\sqrt{37}/3}^{\sqrt{5}}\frac{t^2}{(t^2-4)^2}\,dt=\int_{\sqrt{37}/3}^{\sqrt{5}}\frac{t^2}{(t-2)^2(t+2)^2}\,dt$$

$$\frac{t^2}{(t-2)^2(t+2)^2}=\frac{A}{t-2}+\frac{B}{(t-2)^2}+\frac{C}{t+2}+\frac{D}{(t+2)^2}\quad \Big/\cdot (t-2)^2(t+2)^2$$

$$t^2=A(t-2)(t+2)^2+B(t+2)^2+C(t-2)^2(t+2)+D(t-2)^2$$

изједначимо коефицијенте уз $1, t, t^2, t^3$ или убацујемо конкретне вредности за t

$t=2$: $\quad 4=B\cdot 16 \Rightarrow \boxed{B=\frac{1}{4}}$

$t=-2$: $\quad 4=D\cdot 16 \Rightarrow \boxed{D=\frac{1}{4}}$

$t=1$: $\quad 1=-9A+9B+3C+D \Rightarrow -9A+3C=1-9B-D=-\frac{3}{2}$

$$\Rightarrow \; -3A + C = -\frac{1}{2} \Rightarrow C = 3A - \frac{1}{2}$$

$$t=-1: \quad 1 = -3A + B + 9C + 9D \Rightarrow -3A + 9C = 1 - B - 9D = -\frac{3}{2}$$

$$\Rightarrow -A + 3C = -\frac{1}{2}$$

$$-A + 3\left(3A - \frac{1}{2}\right) = -\frac{1}{2}$$

$$8A = 1$$

$$\boxed{\begin{aligned} &A = \frac{1}{8} \\ &C = \frac{3}{8} - \frac{1}{2} = -\frac{1}{8} \end{aligned}}$$

$$\Rightarrow \; l = \frac{1}{8}\int_{\sqrt{37}/3}^{\sqrt{5}} \frac{dt}{t-2} + \frac{1}{4}\int_{\sqrt{37}/3}^{\sqrt{5}} \frac{dt}{(t-2)^2} - \frac{1}{8}\int_{\sqrt{37}/3}^{\sqrt{5}} \frac{dt}{t+2} + \frac{1}{4}\int_{\sqrt{37}/3}^{\sqrt{5}} \frac{dt}{(t+2)^2}$$

$$\int \frac{dt}{t-2} = \left(\begin{array}{l} s = t-2 \\ ds = dt \end{array} \right) = \int \frac{ds}{s} = \ln|s| = \ln|t-2|$$

и слично $\displaystyle\int \frac{dt}{t+2} = \ln|t+2|$

$$\int \frac{dt}{(t-2)^2} = \left(\begin{array}{l} s = t-2 \\ ds = dt \end{array} \right) = \int \frac{ds}{s^2} = -\frac{1}{s} = -\frac{1}{t-2}$$

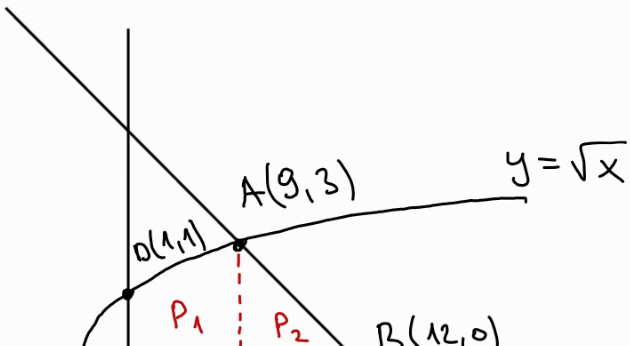
и слично $\displaystyle\int \frac{dt}{(t+2)^2} = -\frac{1}{t+2}$

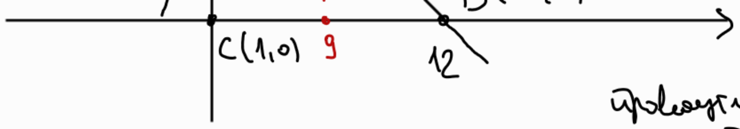
$$\Rightarrow \; l = \frac{1}{8}\ln|t-2|\Big|_{\sqrt{37}/3}^{\sqrt{5}} - \frac{1}{4}\cdot\frac{1}{t-2}\Big|_{\sqrt{37}/3}^{\sqrt{5}} - \frac{1}{8}\ln|t+2|\Big|_{\sqrt{37}/3}^{\sqrt{5}}$$

$$-\frac{1}{4}\cdot\frac{1}{t+2}\Big|_{\sqrt{37}/3}^{\sqrt{5}} = \ldots$$

$$O = AB + BC + CD + l = 12 + 3\sqrt{2} + l$$

<u>Површина</u>:



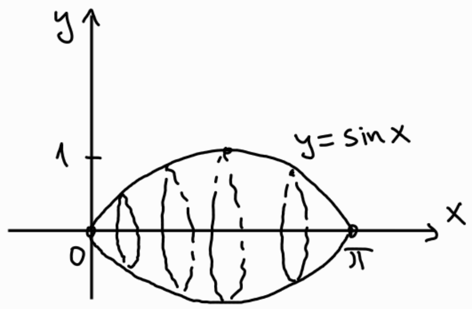


правоугли троугао са катетама 3 и 3

$$P = P_1 + P_2 = \int_1^9 \sqrt{x}\,dx + \frac{3\cdot 3}{2} = \frac{x^{\frac32}}{\frac32}\Bigg|_1^9 + \frac{9}{2} =$$

$$= \frac{2}{3}\cdot\left(9^{\frac32} - 1^{\frac32}\right) + \frac{9}{2} = \frac{2}{3}\cdot 26 + \frac{9}{2} = \frac{131}{6}$$

б)



$$y' = \cos x$$

$$P = 2\pi\int_0^{\pi} y\sqrt{1+(y')^2}\,dx = 2\pi\int_0^{\pi}\sin x\sqrt{1+\cos^2 x}\,dx = \left(\begin{array}{l} t=\cos x \\ dt = -\sin x\,dx \end{array}\quad \begin{array}{c|c|c} x & 0 & \pi \\ \hline t & 1 & -1 \end{array}\right)$$

$$= 2\pi\int_1^{-1}\sqrt{1+t^2}\,(-dt) = 2\pi\int_{-1}^{1}\underbrace{\sqrt{1+t^2}}_{\text{парна}}\,dt = 4\pi\int_0^1\sqrt{1+t^2}\,dt =$$

$$= \left(\begin{array}{l} t = \operatorname{sh} u \\ dt = \operatorname{ch} u\,du \end{array}\quad u = \operatorname{arcsh} t = \ln\left(t+\sqrt{t^2+1}\right)\quad \begin{array}{c|c|c} t & 0 & 1 \\ \hline u & 0 & \ln(1+\sqrt2) \end{array}\right) =$$

$$= 4\pi\int_0^{\ln(1+\sqrt2)}\underbrace{\sqrt{1+\operatorname{sh}^2 u}}_{\operatorname{ch} u}\cdot\operatorname{ch} u\,du = 4\pi\int_0^{\ln(1+\sqrt2)}\operatorname{ch}^2 u\,du = 4\pi\int_0^{\ln(1+\sqrt2)}\left(\frac{e^u+e}{2}\right)^2 du =$$

$$= \pi\int_0^{\ln(1+\sqrt2)}\left(e^{2u}+2+e^{-2u}\right)du = \pi\cdot\left(\frac12 e^{2u} + 2u - \frac12 e^{-2u}\right)\Bigg|_0^{\ln(1+\sqrt2)} =$$

$$= \pi\cdot\left(\frac12 e^{2\ln(1+\sqrt2)} - \frac12\cdot 1 + 2\ln(1+\sqrt2) - \left(\frac12 e^{-2\ln(1+\sqrt2)} - \frac12\cdot 1\right)\right) =$$

$$= \pi\cdot\left(\frac12\cdot(1+\sqrt2)^2 - \cancel{\tfrac12} + 2\ln(1+\sqrt2) - \frac12\cdot\overset{1\cdot(\sqrt2-1)^2}{\frac{1}{(1+\sqrt2)^2}} + \cancel{\tfrac12}\right) =$$

$$= \pi\cdot\left(\frac12\left(1+2\sqrt2+2\right) + 2\ln(1+\sqrt2) - \frac12\cdot\frac{(\sqrt2-1)^2}{1^2}\right) =$$

$$= \pi \cdot \left(\frac{3}{2} + \sqrt{2} + 2\ln(1+\sqrt{2}) - \frac{1}{2}\left(2 - 2\sqrt{2} + 1\right)\right) = \pi \cdot \left(\cancel{\frac{3}{2}} + \sqrt{2} + 2\ln(1+\sqrt{2}) - \cancel{\frac{3}{2}} + \sqrt{2} \right)$$

$$= 2\pi\left(\sqrt{2} + \ln(1+\sqrt{2})\right)$$

3. a) $\displaystyle\sum_{n=1}^{\infty} \underbrace{\left(e^{\frac{1}{n^2}} - \cos\frac{2}{n}\right)\left(n - \ln(1+n)\right)}_{a_n}$

$e^{\frac{1}{n^2}} > e^0 = 1 \geq \cos\frac{2}{n}$

$\ln(1+n) \leq n$

$\Rightarrow a_n \geq 0 \Rightarrow$ апсолутна конв. је исто што и обична

$e^{\frac{1}{n^2}} \sim 1 + \frac{1}{n^2}, \; n \to \infty$

$\cos\frac{2}{n} \sim 1 - \frac{\left(\frac{2}{n}\right)^2}{2} = 1 - \frac{2}{n^2}, \; n \to \infty$

$$n - \ln(1+n) = n\Big(1 - \underbrace{\frac{\ln(1+n)}{n}}_{\xrightarrow{n\to\infty} 0}\Big) \sim n \cdot 1 = n, \; n \to \infty$$

јер $\ln n \ll n, \; n \to \infty$

$$\Rightarrow a_n \sim \left(1 + \frac{1}{n^2} - \left(1 - \frac{2}{n^2}\right)\right) \cdot n = \frac{3}{n^2} \cdot n = \underset{\neq 0}{\text{const}} \cdot \frac{1}{n^1}$$

$\displaystyle\sum_{n=1}^{\infty} \frac{1}{n^1}$ (Д) $(1 \leq 1) \overset{\text{II п.к.}}{\Longrightarrow} \displaystyle\sum_{n=1}^{\infty} a_n$ (Д), па не конв. ни условно

б) $$\sum_{n=1}^{\infty} \frac{n \cos n\pi}{\sqrt{n^4 + 2024}} = \sum_{n=1}^{\infty} \underbrace{(-1)^n \overbrace{\frac{n}{\sqrt{n^4+2024}}}^{b_n}}_{a_n}$$

АК: $$|a_n| = b_n = \frac{n}{\sqrt{n^4+2024}} = \frac{n}{\sqrt{n^4\Big(1 + \underbrace{\frac{2024}{n^4}}_{\to 0}\Big)}} \sim \frac{n}{\sqrt{n^4}} = \frac{n}{n^2} = \frac{1}{n^1}, \; n \to \infty$$

$\overset{\text{II п.к.}}{\Longrightarrow} \displaystyle\sum_{n=1}^{\infty} a_n$ не конв. апсолутно

К: $b_n \sim \frac{1}{n}, \; n \to \infty \Rightarrow \lim\limits_{n\to\infty} b_n = 0$

b_n монотон?

уочимо $f : [1, +\infty) \to \mathbb{R}$, $f(x) = \dfrac{x}{\sqrt{x^4+2024}}$

$\Big| \cdot \sqrt{x^4+2024}$

$$f'(x) = \frac{1 \cdot \sqrt{x^4+2024} - x \cdot \frac{4x^3}{2\sqrt{x^4+2024}}}{\ldots}$$

$$f'(x) = \frac{\quad \ldots \quad 2\sqrt{x^4+2024}}{x^4+2024} =$$

<0 за $x > \sqrt[4]{2024}$

$$= \frac{x^4+2024-2x^4}{(x^4+2024)^{\frac{3}{2}}} = \frac{2024-x^4}{(x^4+2024)^{\frac{3}{2}}}$$

>0

$\Rightarrow f \searrow$ на $[\sqrt[4]{2024}, +\infty)$

$\Rightarrow b_n \searrow$ почев од некој n_0 $(n_0=7)$

Лајбниц $\Rightarrow \sum\limits_{n=1}^{\infty} a_n$ (K) $\Rightarrow \sum\limits_{n=1}^{\infty} a_n$ условно конв.

в) $\sum\limits_{n=1}^{\infty} \underbrace{\sqrt{2^n}\left(\frac{3n-2}{3n+1}\right)^{n^2+n}}_{a_n}$ $\qquad a_n>0 \Rightarrow$ апсолутна конв. ѓе исто што и обична

$$a_n = (2^n)^{\frac{1}{2}}\left(\frac{3n-2}{3n+1}\right)^{n^2+n} = 2^{\frac{n}{2}}\cdot\left(\frac{3n-2}{3n+1}\right)^{n^2+n}$$

$$\sqrt[n]{a_n} = \left(2^{\frac{n}{2}}\cdot\left(\frac{3n-2}{3n+1}\right)^{n^2+n}\right)^{\frac{1}{n}} = 2^{\frac{1}{2}}\cdot\left(\frac{3n-2}{3n+1}\right)^{n+1}$$

$$\lim_{n\to\infty}\left(\frac{3n-2}{3n+1}\right)^{n+1} = \lim_{n\to\infty}\left(1-\frac{3}{3n+1}\right)^{n+1} = \lim_{n\to\infty}\left(1+\frac{1}{-\frac{3n+1}{3}}\right)^{n+1} =$$

$$= \lim_{n\to\infty}\underbrace{\left(\left(1+\frac{1}{-\frac{3n+1}{3}}\right)^{-\frac{3n+1}{3}}\right)}_{\to e}^{-\frac{3}{3n+1}\cdot(n+1)} = \lim_{n\to\infty} e^{-\frac{3n+3}{3n+1}} = e^{-1} = \frac{1}{e}$$

$\Rightarrow \lim\limits_{n\to\infty}\sqrt[n]{a_n} = \frac{\sqrt{2}}{e} < 1 \overset{\text{Коши}}{\Rightarrow} \sum\limits_{n=1}^{\infty} a_n$ (K), па и (АК)

$\Rightarrow$ не конв. условно

4. $f_n(x) = x^n e^{-(n+4)x}$

прво т.п.т. $\qquad x \in [0,1]$ фиксирано

1° $x=0$

$$\lim_{n\to\infty} f_n(x) = \lim_{n\to\infty} f_n(0) = \lim_{n\to\infty} 0^n \cdot e^{-(n+4)\cdot 0} = 0$$

2° $x \in (0,1)$

$$\lim_{n\to\infty} f_n(x) = \lim_{n\to\infty} \underbrace{x^n}_{\to 0} \cdot e^{\overbrace{-(n+4)x}^{\to -\infty}} = 0 \cdot e^{-\infty} = 0$$

3° $x = 1$

$$\lim_{n\to\infty} f_n(x) = \lim_{n\to\infty} f_n(1) = \lim_{n\to\infty} 1^n \cdot e^{-(n+4)} = e^{-\infty} = 0$$

$\Rightarrow$ $f_n \to f$, $n\to\infty$ na $[0,1]$, gdje je $f \equiv 0$

$f_n \rightrightarrows f$, $n \to \infty$ na $[0,1]$?

$$L = \lim_{n\to\infty} \sup_{x\in[0,1]} |f_n(x) - f(x)| \overset{?}{=} 0$$

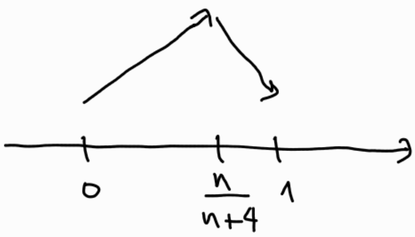
$$|f_n(x) - f(x)| = |x^n e^{-(n+4)x} - 0| = x^n e^{-(n+4)x}$$

$$f_n'(x) = n x^{n-1} e^{-(n+4)x} + x^n \cdot e^{-(n+4)x} \cdot (-(n+4)) =$$

$$= \underbrace{x^{n-1} e^{-(n+4)x}}_{\geq 0} \left(n - (n+4)x\right)$$

$\Rightarrow$ $f_n'(x) \geq 0 \iff n - (n+4)x \geq 0 \iff x \leq \frac{n}{n+4}$

$\Rightarrow$ $f_n \nearrow$ na $\left[0, \frac{n}{n+4}\right]$ i $f_n \searrow$ na $\left[\frac{n}{n+4}, 1\right]$



$$\Rightarrow \sup_{x\in[0,1]} |f_n(x) - f(x)| = f_n\left(\frac{n}{n+4}\right) = \left(\frac{n}{n+4}\right)^n \cdot e^{-(n+4)\cdot\frac{n}{n+4}} =$$

$$= \left(\frac{n}{n+4}\right)^n \cdot e^{-n}$$

$$L = \lim \left(\frac{n}{n+4}\right)^n \cdot e^{-n} = \lim \left(1 - \frac{4}{n+4}\right)^n \cdot e^{-n} =$$

$$= \lim_{n\to\infty} \underbrace{\left(\left(1+\frac{1}{-\frac{n+4}{4}}\right)^{-\frac{n+4}{4}}\right)}_{\to e}{}^{-\frac{4}{n+4}\cdot n} \cdot e^{-n} = \lim_{n\to\infty} \underbrace{e^{-\frac{4n}{n+4}}}_{\to e^{-4}} \cdot \underbrace{e^{-n}}_{\to 0} =$$

$$= e^{-4}\cdot 0 = 0 \quad \Rightarrow \quad f_n \rightrightarrows f \equiv 0, \ n\to\infty \text{ на } [0,1]$$

б) $\sum\limits_{n=1}^{\infty} f_n(x)$ РК на $[0,1]$?

У делу а) смо доказали да је $\sup\limits_{x\in[0,1]} |f_n(x)| = \left(\frac{n}{n+4}\right)^n \cdot e^{-n}$ за све $n\in\mathbb{N}$

$$\Rightarrow (\forall n\in\mathbb{N})(\forall x\in[0,1]) \quad |f_n(x)| \leq \left(\frac{n}{n+4}\right)^n \cdot e^{-n} =: c_n$$

$$\sum_{n=1}^{\infty} c_n = \sum_{n=1}^{\infty} \left(\frac{n}{n+4}\right)^n \cdot e^{-n} \quad \text{(K) по Кошију, јер је}$$

$$\lim_{n\to\infty} \sqrt[n]{c_n} = \lim_{n\to\infty} \underbrace{\frac{n}{n+4}}_{\to 1} \cdot e^{-1} = \frac{1}{e} < 1$$

Вајерштрас $\Rightarrow$ $\sum\limits_{n=1}^{\infty} f_n(x)$ РК на $[0,1]$.