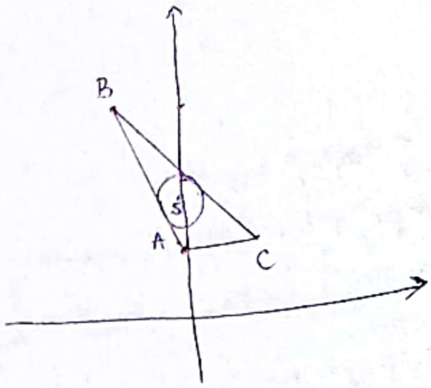
септембар 2 2022.

1. $A(0,1)$ $B(-1,4)$ $C(1,1)$



$$D_{ABC} = \begin{vmatrix} -1 & 3 \\ 1 & 0 \end{vmatrix} = -3 \Rightarrow \boxed{P_{\Delta} = \frac{1}{2}|D_{ABC}| = \frac{3}{2}}$$

$\boxed{D_{ABC} < 0 \Rightarrow \Delta ABC \text{ je „}-\text{" оријентације}}$

$S = ?$

$$s_A: \frac{\vec{AB}}{\|\vec{AB}\|} + \frac{\vec{AC}}{\|\vec{AC}\|} = \frac{-1,3}{2} + \frac{1,0}{1} = \left(-\frac{1}{2}+1, \frac{3}{2}\right) = \left(\frac{1}{2}, \frac{3}{2}\right)$$

$$s_C: \frac{\vec{CA}}{\|\vec{CA}\|} + \frac{\vec{CB}}{\|\vec{CB}\|} = \frac{-1,0}{1} + \frac{-2,3}{\sqrt{4+9}} = \left(-1-\frac{2}{\sqrt{13}}, \frac{3}{\sqrt{13}}\right)$$

$$s_A \cap s_C = \{S\}$$

$$s_A: \begin{aligned} x &= \frac{1}{2}t \\ y &= 1 + \frac{3}{2}t \end{aligned} \qquad s_C: \begin{aligned} x &= \left(-1-\frac{2}{\sqrt{13}}\right)s + 1 \\ y &= 1 + \frac{3}{\sqrt{13}}s \end{aligned}$$

$$\frac{1}{2}t = -s - \frac{2s}{\sqrt{13}} + 1 \quad / 2\sqrt{13}$$

$$1 + \frac{3}{2}t = 1 + \frac{3}{\sqrt{13}}s \quad / 2\sqrt{13}$$

$$\sqrt{13}t + 2\sqrt{13}s + 4s = 2\sqrt{13}$$

$$\cancel{2\sqrt{13}} + 3\sqrt{13}t = \cancel{2\sqrt{13}} + 6s$$

$$\sqrt{13}t = 2s \Rightarrow s = \frac{\sqrt{13}t}{2}$$

$$\sqrt{13}t + \cancel{2}\sqrt{13}\cdot\frac{\sqrt{13}t}{\cancel{2}} + \cancel{4}^{2}\,\frac{\sqrt{13}t}{\cancel{2}_{1}} = 2\sqrt{13}$$

$$3\sqrt{13}t + 13t = 2\sqrt{13}$$

$$t = \frac{2\sqrt{13}}{3\sqrt{13}+13}$$

$$s = \frac{\sqrt{13}}{\cancel{2}}\cdot\frac{\cancel{2}\sqrt{13}}{3\sqrt{13}+13} = \frac{13}{3\sqrt{13}+13}$$

$$S\left(\frac{\sqrt{13}}{3\sqrt{13}+13}, 1 + \frac{3\sqrt{13}}{3\sqrt{13}+13}\right) \Rightarrow \boxed{S\left(\frac{\sqrt{13}}{3\sqrt{13}+13}, \frac{6\sqrt{13}+13}{3\sqrt{13}+13}\right)}$$

c_1:

$$x = 1 + \left(\frac{\sqrt{13}}{3\sqrt{13}+13} - 1\right)t$$
$$y = 1 + \left(\frac{3\sqrt{13}}{3\sqrt{13}+13}\right)t$$

$$\vec{CS} = \left(\frac{\sqrt{13} - 3\sqrt{13} - 13}{3\sqrt{13}+13}, \frac{3\sqrt{13}}{3\sqrt{13}+13}\right)$$

$$\vec{CS} = \left(\frac{-2\sqrt{13} - 13}{3\sqrt{13}+13}, \frac{3\sqrt{13}}{3\sqrt{13}+13}\right)$$

$$\vec{n}_{CS} = \left(\frac{3\sqrt{13}}{3\sqrt{13}+13}, \frac{2\sqrt{13}+13}{3\sqrt{13}+13}\right)$$

cs: $ax + by + c = 0$

$c = ?$

$$c = -a - b \Rightarrow \frac{-3\sqrt{13} - 2\sqrt{13} - 13}{3\sqrt{13}+13} = \frac{-5\sqrt{13} - 13}{3\sqrt{13}+13}$$

A: $$\cancel{\frac{3\sqrt{13}}{3\sqrt{13}+13}}\cdot 0 + \overbrace{\frac{2\sqrt{13}+13}{3\sqrt{13}+13} - \frac{5\sqrt{13}+13}{3\sqrt{13}+13}}^{-3\sqrt{13}} = 0 < 0$$

B: $$\frac{-3\cancel{\sqrt{13}}}{3\sqrt{13}+13} + \frac{8\cancel{\sqrt{13}} + 13\cdot 8}{3\sqrt{13}+13} + \frac{-5\cancel{\sqrt{13}} - 13}{3\sqrt{13}+13} = 0 > 0$$

$\boxed{\Rightarrow \text{тачке A и B су са супротне стране праве } c_1}$

(2) $P(360, 420)$ $Q(520, 520)$

$R_{P,\varphi} = ?$ $\varphi = \frac{\pi}{2}$

$$R_{P,\varphi} = T_{\vec{OP}} \circ R_{\varphi} \circ T_{\vec{PO}}$$

$$R_{P,\varphi} = \begin{pmatrix} 1 & 0 & 360 \\ 0 & 1 & 420 \\ 0 & 0 & 1 \end{pmatrix} \cdot \begin{pmatrix} 0 & -1 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 1 \end{pmatrix} \cdot \begin{pmatrix} 1 & 0 & -360 \\ 0 & 1 & -420 \\ 0 & 0 & 1 \end{pmatrix} =$$

$$= \begin{pmatrix} 0 & -1 & 360 \\ 1 & 0 & 420 \\ 0 & 0 & 1 \end{pmatrix} \cdot \begin{pmatrix} 1 & 0 & -360 \\ 0 & 1 & -420 \\ 0 & 0 & 1 \end{pmatrix} = \begin{pmatrix} 0 & -1 & 780 \\ 1 & 0 & 60 \\ 0 & 0 & 1 \end{pmatrix}$$

$$\begin{pmatrix} x' \\ y' \end{pmatrix} = \begin{pmatrix} 0 & -1 \\ 1 & 0 \end{pmatrix} \begin{pmatrix} x \\ y \end{pmatrix} + \begin{pmatrix} 780 \\ 60 \end{pmatrix}$$

$$\left.\begin{aligned} x' &= -y + 780 \\ y' &= x + 60 \end{aligned}\right\} \Rightarrow \boxed{P'(360, 420)} = P$$

$$\boxed{Q'(260, 580)}$$

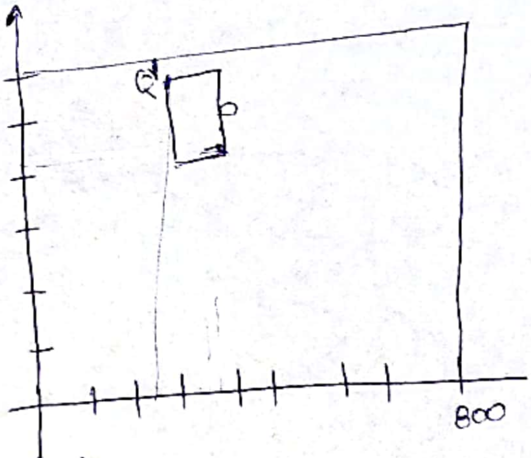
$$C = \frac{(360,420) + (260,580)}{2} = (310, 500)$$

$$C' = (400, 300)$$

$$C \to C'$$

$$\lambda = ?$$

$$\left.\begin{aligned} 800 : 160 &= 5 \\ 600 : 100 &= 6 \end{aligned}\right\} \Rightarrow \lambda = 5$$



$$H: \begin{pmatrix} 400 \\ 300 \end{pmatrix} = \begin{pmatrix} 5 & 0 \\ 0 & 5 \end{pmatrix} \begin{pmatrix} 310 \\ 500 \end{pmatrix} + \begin{pmatrix} a \\ b \end{pmatrix}$$

$$400 = 1550 + a \Rightarrow a = -1150$$

$$300 = 2500 + b \Rightarrow b = -2200$$

$$\boxed{H: \begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} 5 & 0 \\ 0 & 5 \end{pmatrix} \begin{pmatrix} x' \\ y' \end{pmatrix} \begin{pmatrix} -1150 \\ -2200 \end{pmatrix}}$$

(4.)

а) Скуп свих равни које садрже тачку $p = \alpha \cap \beta$ је дат једначином

$$\boxed{\gamma: \lambda_1(a_1x + b_1y + c_1z + d_1) + \lambda_2(a_2x + b_2y + c_2z + d_2) = 0}$$

б) $\lambda = ?$

$\alpha: 3x - y + z - 17 = 0$

$\beta: x + 2y - z - 8 = 0$

$\gamma: 2x - 3y + 2z + \lambda = 0$

$\gamma: \lambda_1(3x - y + z - 17) + \lambda_2(x + 2y - z - 8) = 0$

$\lambda_1 3x - \lambda_1 y + \lambda_1 z - 17\lambda_1 + \lambda_2 x + 2y\lambda_2 - z\lambda_2 - 8\lambda_2 = 0$

$x(3\lambda_1 + \lambda_2) + y(2\lambda_2 - \lambda_1) + z(\lambda_1 - \lambda_2) - 17\lambda_1 - 8\lambda_2 = 0$

$3\lambda_1 + \lambda_2 = 2$

$-\lambda_1 + 2\lambda_2 = -3 \quad /3$

$3\lambda_1 + \lambda_2 = 2$

$-3\lambda_1 + 6\lambda_2 = -9 \quad +$

$7\lambda_2 = -7 \Rightarrow \boxed{\lambda_2 = -1}$

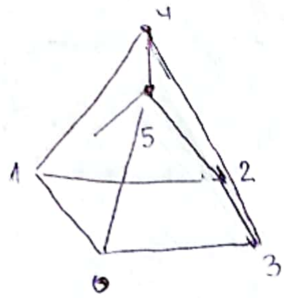
$\boxed{\lambda_1 = 1}$

$-17\lambda_1 - 8\lambda_2 = \lambda$

$-17 + 8 = \lambda$

$\boxed{\lambda = -9}$

5.



$\mathcal{I} = \{0, \ldots, 5\}$

$\mathcal{P} = \{P_0, P_1, P_2\}$

$P_0 = \langle 0, 1, 2, 3 \rangle$

$P_1 = \langle 1, 0, 5, 4 \rangle$

$P_2 = \langle 2, 3, 4, 5 \rangle \overset{P_1}{\overset{\circ}{=}} \langle 5, 4, 3, 2 \rangle = 1$ или ће 54 или ће 32
имати исту орј. 2x

$I = \{\overset{\leftrightarrows}{01}, 12, \overset{\rightleftarrows}{23}, 30, 05, \overset{\leftrightarrows}{54}, 34, 45, 41\}$

$\boxed{-1 \text{ није оријентабилна}}$