CS331-HW13-Lukang-Sun

December 2, 2021

p1. (see Figure 1.) $f_1(x,y) := ax^2 + by^2 + \sin(x), f_2(x,y) := cx^2 + dy^2 + \cos(y), f(x,y) = \frac{1}{2}f_1(x,y) + \frac{1}{2}f_2(x,y), \tau = 1$, the initial point $w_1^0 = w_2^0 = (1,1)$. In the four experiments, I choose $(a,b,c,d) = (5,3,2,7,),(50,3,2,70)$, $(50,300,2,70),(50,300,200,7)$ respectively. The error is $\mathrm{E}\left[||\nabla f(x)||^2\right]$. In these experiments, no matter what the value of (a,b,c,d) are, they all converge very fast(actually all take 2 steps till the error almost converges to 0).

p2.

Proof.

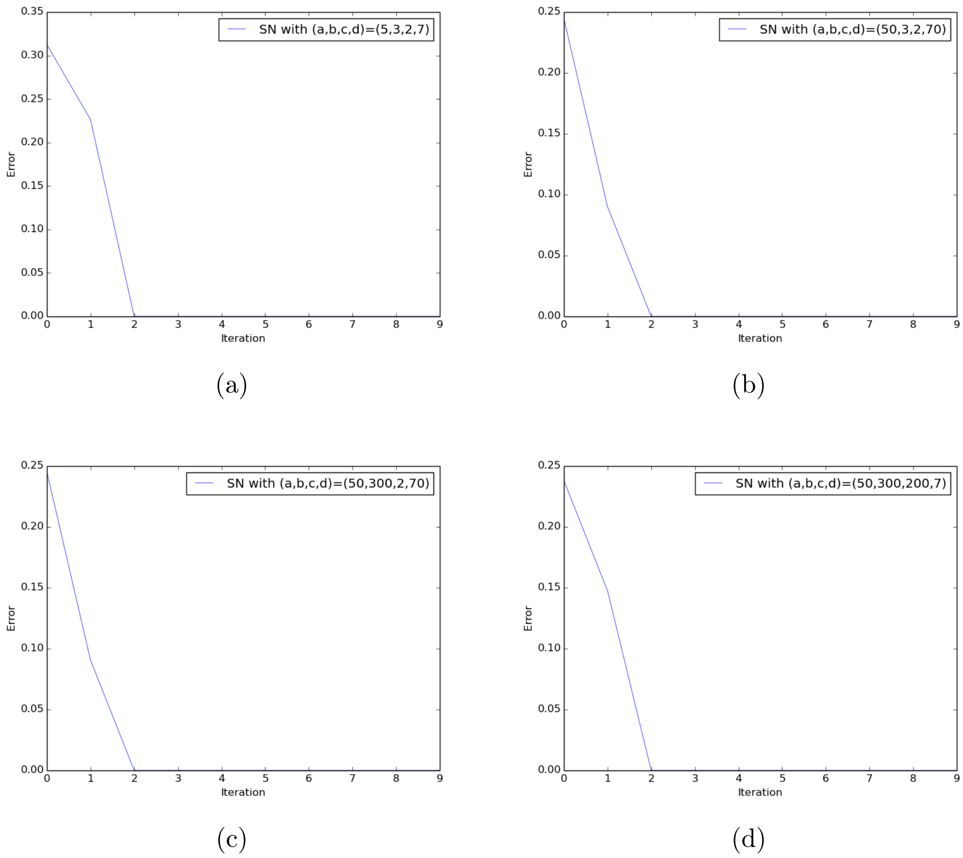
$$\nabla f(x) = Ax + b, \quad \text{for any } x \in \mathbb{R}^n,$$

so

$$\nabla f(x) - \nabla f(y) = A(x-y).$$

□

p3. (see Figure 2.) $f_1(x,y) := ax^2 + by^2 + \sin(x), f_2(x,y) := cx^2 + dy^2 + \cos(y), f(x,y) = \frac{1}{2}f_1(x,y) + \frac{1}{2}f_2(x,y)$, the initial point $x^0 = (1,1)$. In the four experiments, I choose $(a,b,c,d) = (5,3,2,7,),(5,3,20,7)$ respectively. The error is $\sqrt{f(x) - f(x^*)}$ and $||B_k - H_*^{-1}||^2_{F(H_*)}$ respectively, where $x^* = (-0.0199960017324, 0)$. I want to test that: 1.Error $\sqrt{f(x) - f(x^*)}$ decreases exponentially, 2. Error $||B_k - H_*^{-1}||^2_{F(H_*)}$ decreases exponentially, 3. These errors' decreasing rate is independent of the condition number. I choose these problems because the theory predicts these results and I can verify these through experiments, actually the results verify the prediction.



(a) (b) (c) (d)

Figure 1: The four pictures show the error with respect to the iteration number.Note the the first error is evaluated at point at x^1 instead of $x^0 = w_1^0 = w_2^0 = (1,1)$ in my experiments.

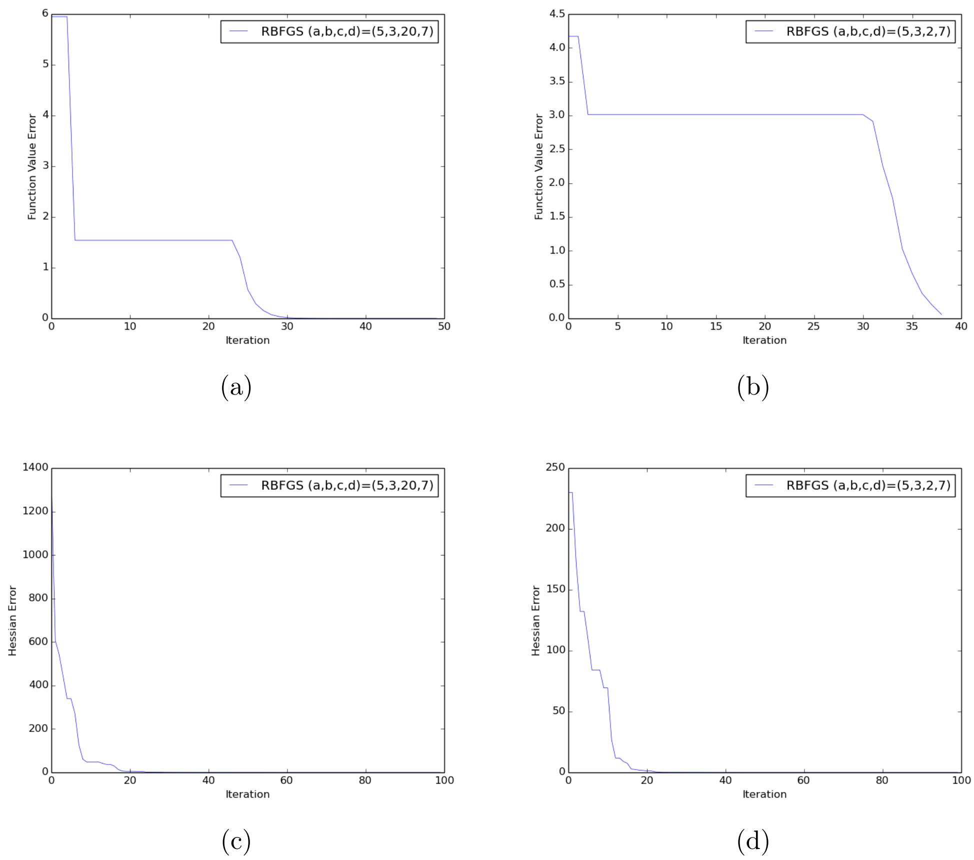


Figure 2: (a) and (b) show $\sqrt{f(x) - f(x^*)}$ converges exponentially fast with different (a, b, c, d), note there are some iteration steps that the error doesn't drop, this is because of the Monotonic option step. (c) and (d) show $||B_k - H_*^{-1}||^2_{F(H_*)}$ converges exponentially fast with different (a, b, c, d). $x^0 = (1, 1)$ in my experiments.