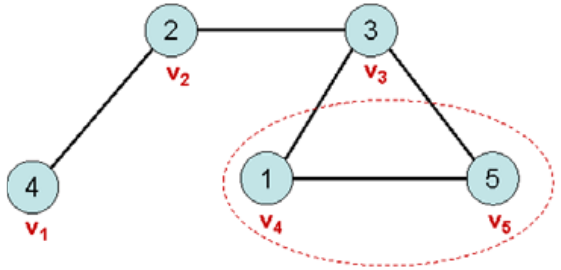
MINIMUM BANDWIDTH

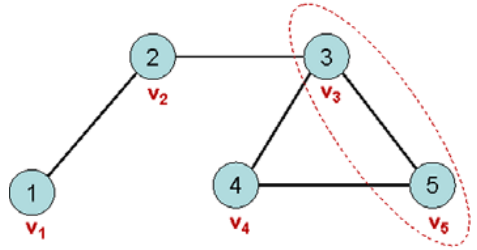
Opis problema

- G = (V,E); |V | = n
- f : V ⟶ {1, 2, . . . , n}
- Bf (G) = max{|f(u) – f(v)| : (u, v) ∈ E}
- Naći f* takvo da je Bf∗ (G) = min{Bf (G)}



B(f) =max$_v$ B(f,v) = max {2, 2, 2, 4, 4} = 4

$$A(f) = \begin{pmatrix} 1 & 0 & 1 & 0 & 1 \\ 0 & 1 & 1 & 1 & 0 \\ 1 & 1 & 1 & 0 & 1 \\ 0 & 1 & 0 & 1 & 0 \\ 1 & 0 & 1 & 0 & 1 \end{pmatrix}$$



B(f') =max$_v$ B(f',v) = max {1, 1, 2, 1, 2} = 2

$$A(f') = \begin{pmatrix} 1 & 1 & 0 & 0 & 0 \\ 1 & 1 & 1 & 0 & 0 \\ 0 & 1 & 1 & 1 & 1 \\ 0 & 0 & 1 & 1 & 1 \\ 0 & 0 & 1 & 1 & 1 \end{pmatrix}$$

Simulirano kaljenje

- DRSA (2015)
- DRSA sa početnim pohlepnim rešenjem

Algorithm 1: DRSA algoritam

```
Data: t_max, greedy_init
Result: Najbolje pronađeno rešenje w
T_0 ← 1000;
α ← 0.99;
T_f ← 10^−7;
L ← 40;
L_f ← 10 * n * m;
γ ← compute-increment-factor(T_0,T_f,L,L_f) ;
start_time ← time();
if greedy_init = true then
    x ← greedy-algorithm();
else
    x ← generate-random-solution();
end
w ← x;
while T > T_f do
    improvement ← false;
    for i ← 1 to L do
        y ← g(x);
        if f(y) < f(x) then
            x ← y;
        else if random(0, 1) < e^−(f(y)−f(x))/T then
            x ← y;
        if f(x) < f(w) then
            w ← x;
            improvement ← true;
    end
    if improvement = false then
        T ← T * α;
        L ← L * γ;
    if time()−start_time > t_max then
        break
end
return w
```

Interna reprezentacija rešenja

- Dualna reprezentacija
- $\pi = (\pi_1, \pi_2, \ldots, \pi_n)$
- π_i predstavlja oznaku dodeljenu čvoru i
- $\rho = (\rho_1, \rho_2, \ldots, \rho_n)$
- čvoru ρ_i dodeljena oznaka i

Funkcija susedstva

- REX operator (0.6)
- NEX operator (0.2)
- ROT operator (0.2)

Funkcija evaluacije

- d
- di predstavlja broj apsolutnih razlika jednakih i
- v
- vi broj mogućih vrednosti koje di može da ima
- f = β + δ

Algorithm 2: Preslikavanje vektora d u δ vrednost

Data: d, v, β
Result: δ
$\delta \leftarrow 0$;
for $i \leftarrow 0$ **to** β **do**
  $\delta \leftarrow (\delta + d_i)/v_i$;
end

Pohlepni algoritam

- BFS obilazak koji počinje od čvora sa najmanjim stepenom
- Obeležavanje čvorova redom kojim se na njih naišlo

Algoritam grube sile

- Proverava redom sve permutacije skupa {1, 2, . . . , n} i nalazi optimalno obeležavanje
- O(n!)

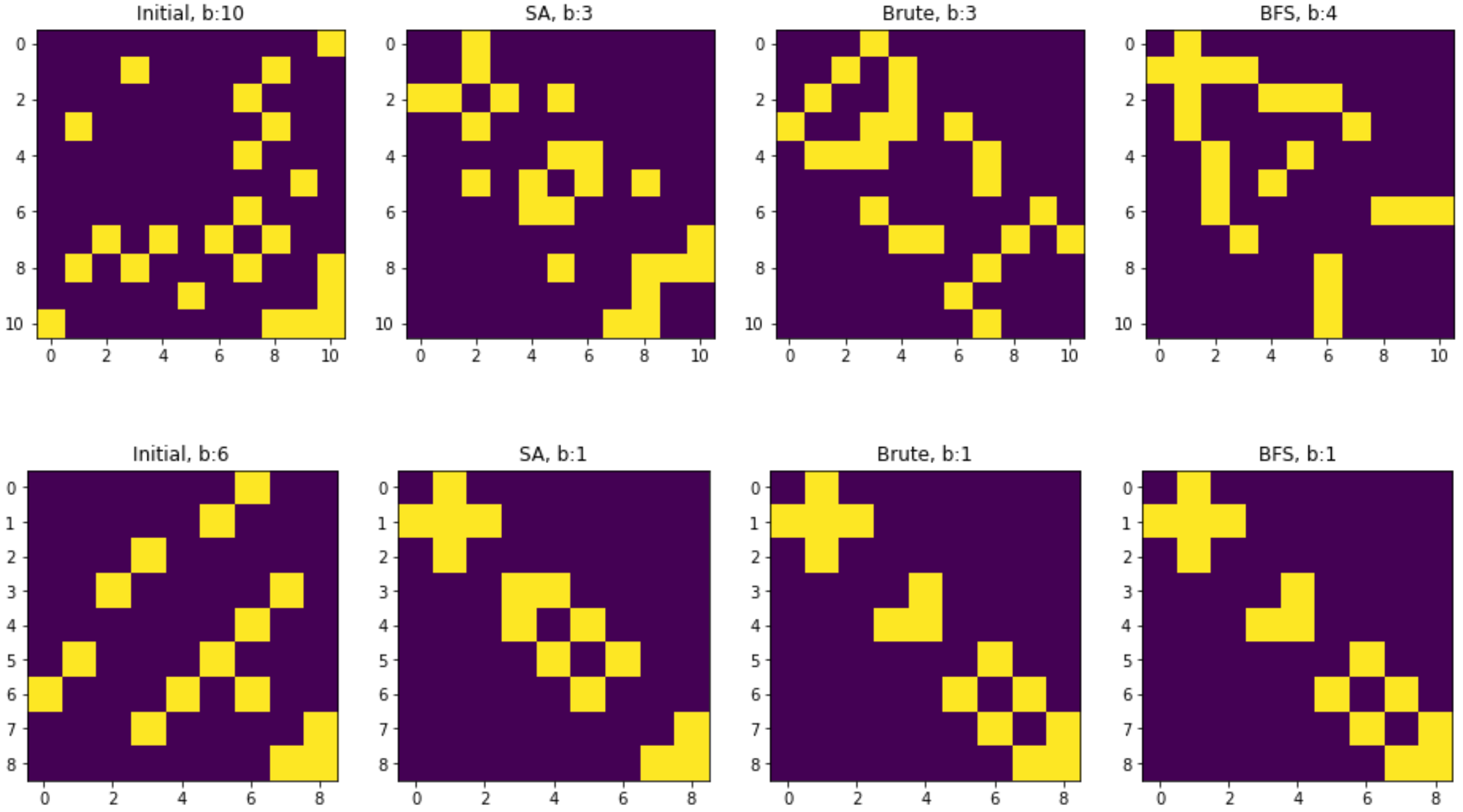
Eksperimentalni rezultati

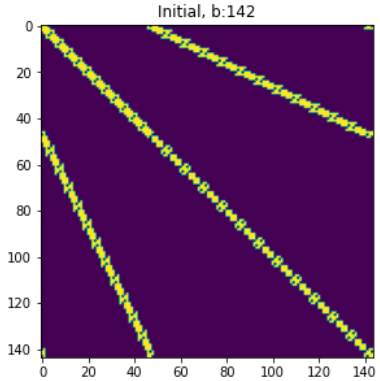
ime	n	Gruba sila		DRSA		DRSA_BFS		BFS	
		β	t (s)	β	t (s)	β	t (s)	β	t (s)
graph_9_2	9	2	0.0199893	2	0.0111269	2	0.0157975	3	2.9521e-05
graph_8_1	8	3	0.00167633	3	1.33463e-05	3	2.75613e-05	4	2.1336e-05
graph_13_1	13	3	447.559	3	0.0215134	3	2.2192e-05	3	4.6908e-05
graph_12_1	12	2	31.4245	2	0.0248881	2	0.0240763	3	3.5489e-05
graph_11_1	11	3	2.26534	3	0.0026155	3	0.00252803	4	3.3563e-05
graph_10_1	10	2	0.172391	2	0.0040499	2	0.00437372	3	2.7731e-05
graph_9_1	9	1	0.0150303	1	0.00629719	1	1.54993e-05	1	2.6432e-05

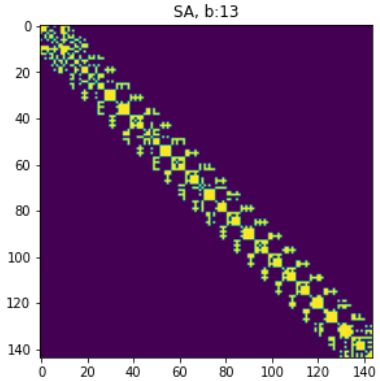
Tabela 1: Rezultati na slučajno generisanim matricama

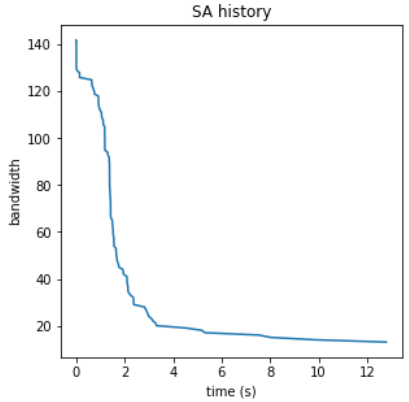
ime	n	Originalni DRSA	DRSA		DRSA_BFS		BFS	
		β	β	t (s)	β	t (s)	β	t (s)
will199	199	64	66	14.3757	66.3333	12.7677	195	0.0106505
pores_1	30	7	7.66667	0.18529	7.66667	0.184772	14	0.00010385
can_161	161	18	21.6667	11.0212	32	3.17277	33	0.00386305
sherman1	1000	46	99.3333	295.163	68	0.14067	68	0.139427
can_292	292	38	42.3333	119.004	44.6667	76.7704	77	0.0132052
can_144	144	13	13	14.6074	13	12.0245	20	0.00332474
lund_a	147	23	25	17.2851	23	0.00435125	23	0.00342371
arc130	130	63	63	188.655	64	98.1438	128	0.00249965
fs_760_1	760	37	79.3333	294.541	77.6667	295.994	206	0.0765773
gent113	113	27	27.3333	5.27794	28.3333	12.2223	110	0.00226914
ash85	85	9	10.3333	2.2452	10	2.55389	15	0.00125526
nos3	960	44	181	297.752	81	0.125799	81	0.117394
ash292	292	19	22.3333	76.4161	22	148.146	33	0.0136006
bcsstk05	153	20	20.6667	23.3754	21.3333	19.2524	26	0.00421739
ibm32	32	11	11	0.19647	11	0.177752	28	0.000122744

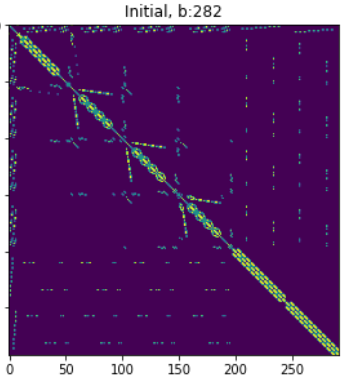
Tabela 2: Rezultati na odabranim matricama iz skupa Harwell-Boeing

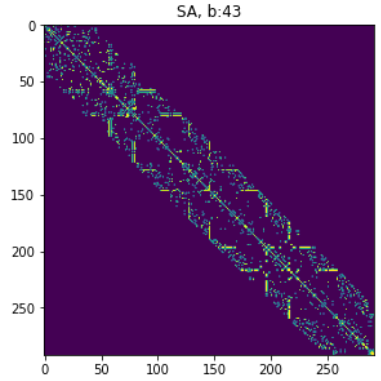

Initial, b:10
SA, b:3
Brute, b:3
BFS, b:4
Initial, b:6
SA, b:1
Brute, b:1
BFS, b:1

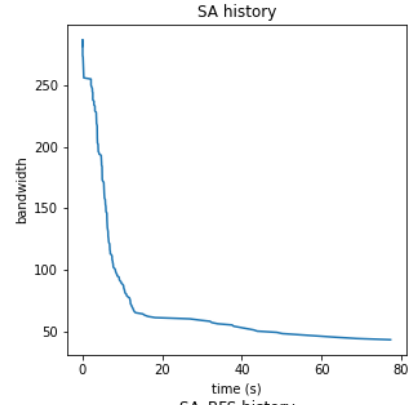

Initial, b:142

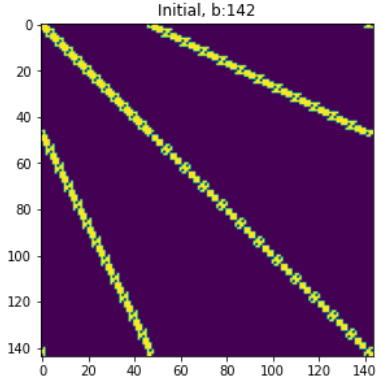

SA, b:13

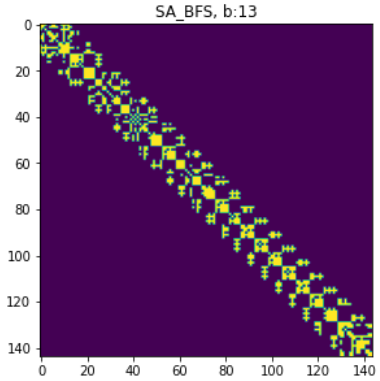

SA history
bandwidth
time (s)

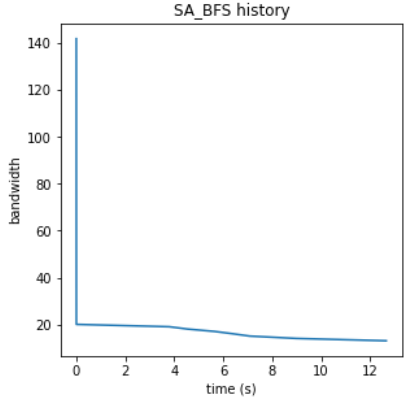

Initial, b:282

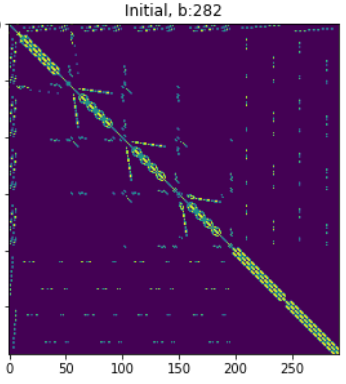

SA, b:43

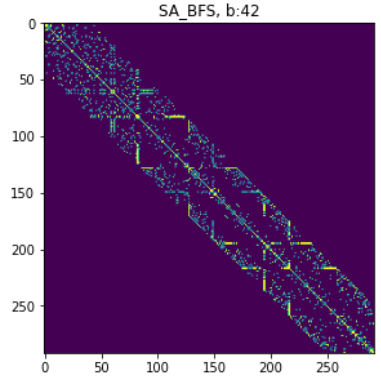

SA history
bandwidth
time (s)

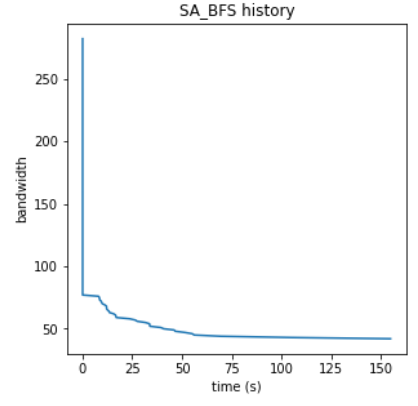

Initial, b:142

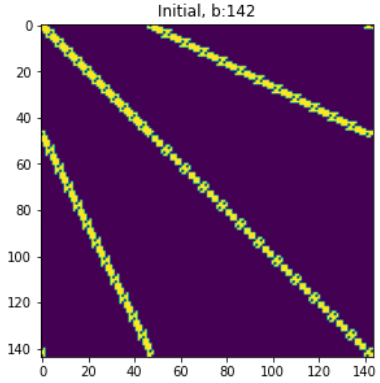

SA_BFS, b:13

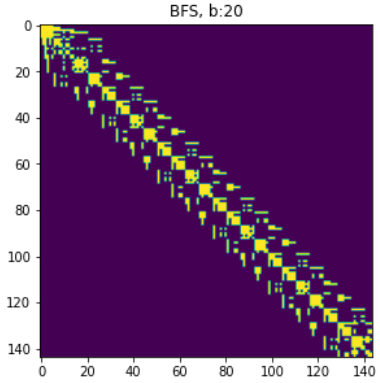

SA_BFS history
bandwidth
time (s)

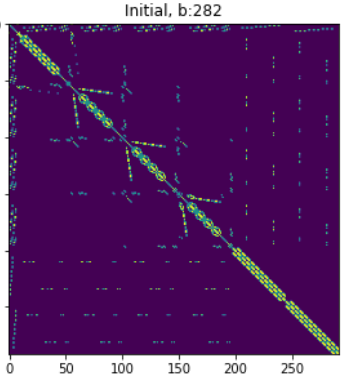

Initial, b:282

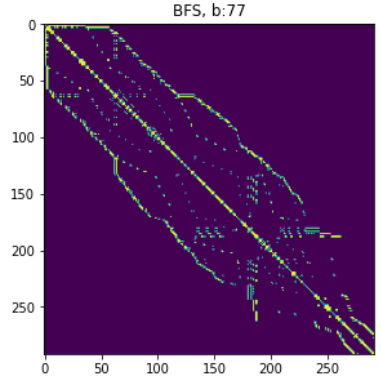

SA_BFS, b:42

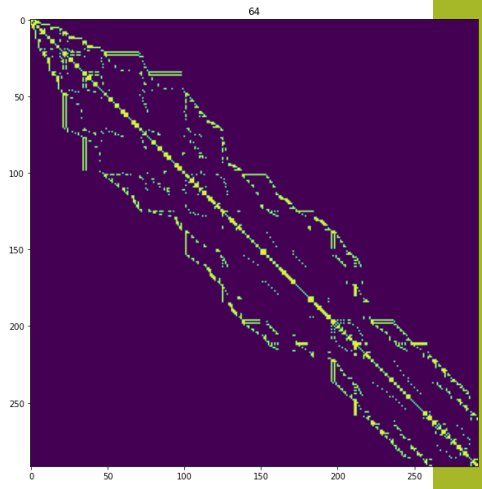

SA_BFS history
bandwidth
time (s)

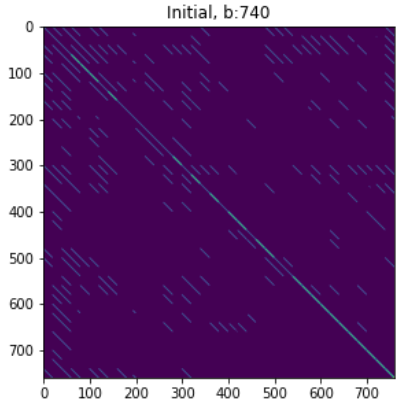

Initial, b:142

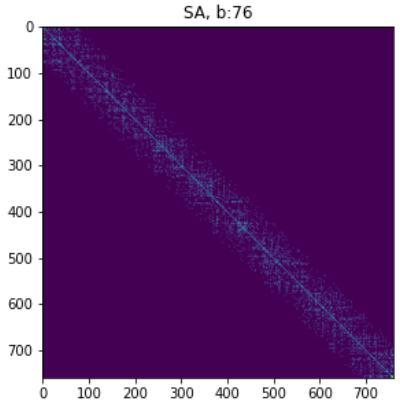

BFS, b:20

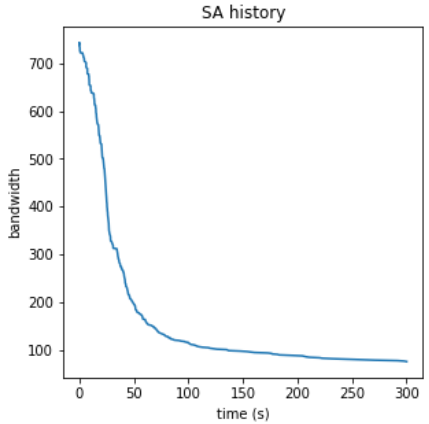

Initial, b:282

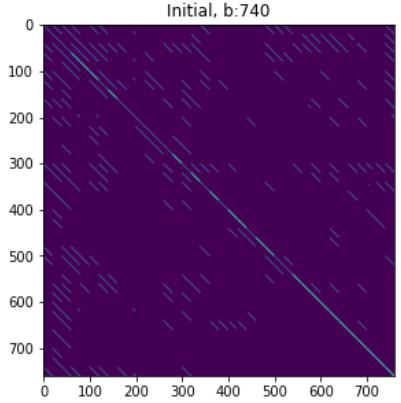

BFS, b:77

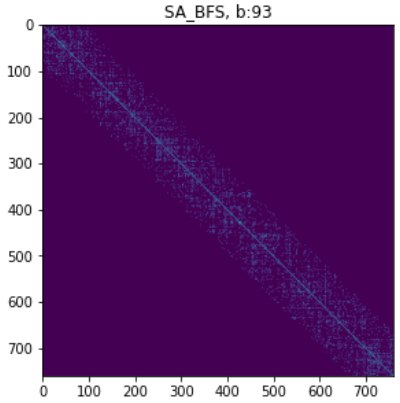

64

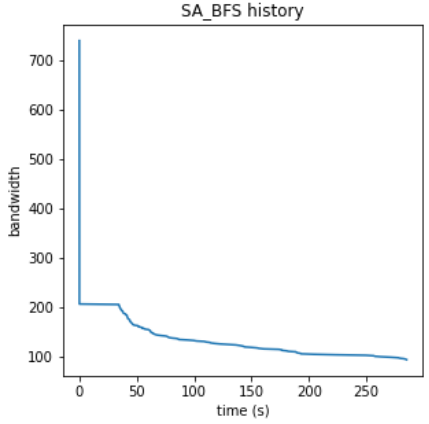

Initial, b:740

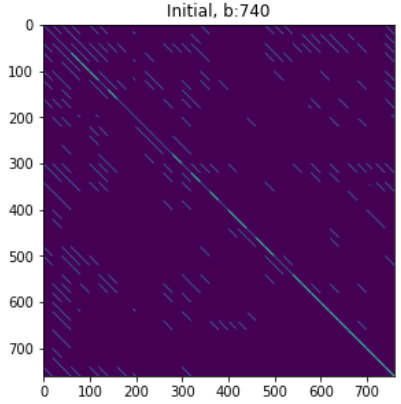

SA, b:76

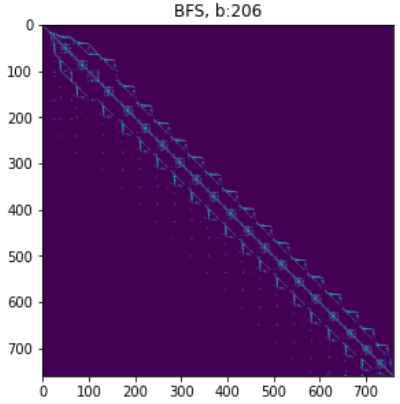

SA history
bandwidth
time (s)

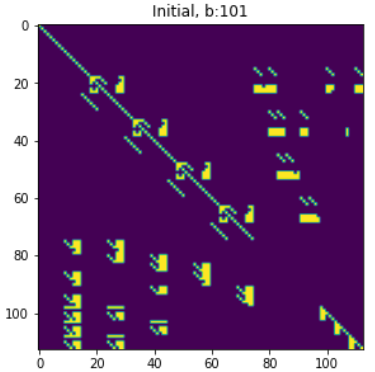

Initial, b:740

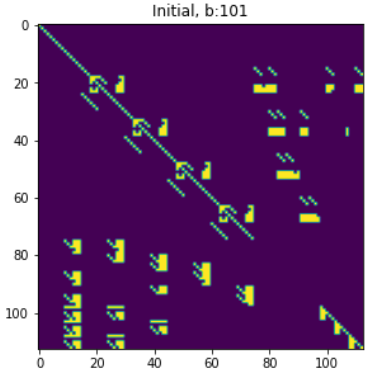

SA_BFS, b:93

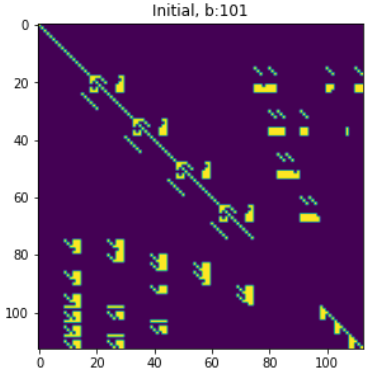

SA_BFS history
bandwidth
time (s)


Initial, b:740


BFS, b:206


Initial, b:101


Initial, b:101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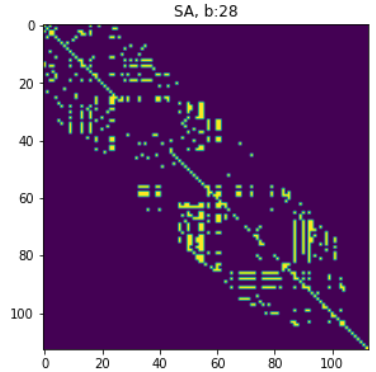
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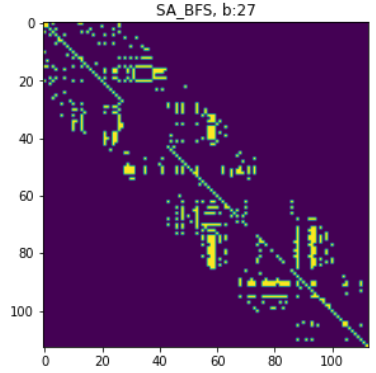
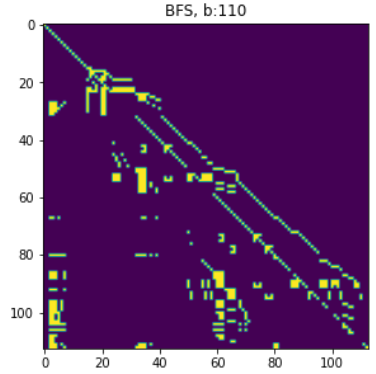
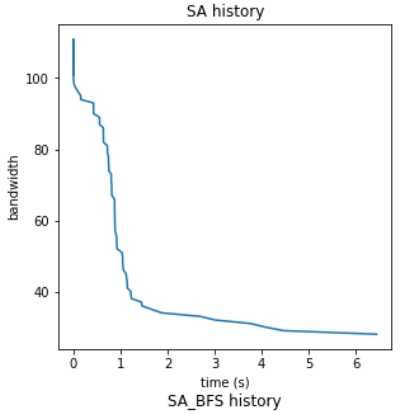
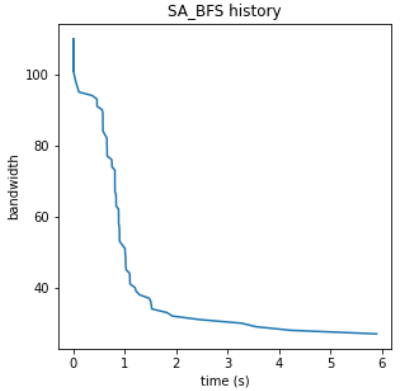

SA_BFS, b:27


BFS, b:110


SA history
bandwidth
time (s)


SA_BFS history
bandwidth
time (s)