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Interferometry

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1. Motivation

In this experiment we aim at determining the refractive index of air and glass by using the priciples of interferometry. A Sagnac interferometer is used to achieve effects of interference.

2. Theory

When two wavefronts meet, interference phenomena can occur under certain conditions. Interference means that the waves add up according to the superposition principle. This can result in intensity maxima and minima. In the following, the physics behind interference and the connection to other physical properties like the refractive index is examined based on the example of the Sagnac interferometer.

2.1. Coherence

Two waves can interfere with other when they are coherent, meaning a constant phase relation is given. It is differentiated between temporal and spacial coherence. For the temporal coherence, the phase relation stays the same for an infinet time. Spacial coherence describes the constant phase relation regarding the spacial direction of propagation.
In reality, the will be hardly any waves that are perfectly coherent. Nevertheless, a coherence length can be identified as the distance between waves under which the waves are sufficiently coherent. The degree of coherence γ_{12} is given by

$$\gamma_{12}(\tau) = \frac{\langle E_1(t+\tau)E_2^*(t)r\rangle}{\sqrt{\langle|E_1|^2\rangle\langle|E_2|^2\rangle}}.CHECKFORMULA$$

It becomes clear that the lower $|\gamma_{12}|$ the less the light is coheren with $0 \leq |\gamma_{12}| \leq 1$.

2.2. Polarization

Another important property of light is its polarization. The polarization of a light beam describes the direction in which the electric or magnetic field oscillates. Normal sunlight is unpolarized and thus has no distinguished oscillation direction of the electric or magnetic field. There are several ways to polarise light beams, for example polarization filters that only let light pass that is linearly polarized under a certain angle.
Another way to polarize light is the usage of polarizing beam splitter cubes (PBSC). When entering a PBSC, the input light beam is split into a p-polarized and a s-polarized part as depicted in Figure 1.

2.3. Contrast

The previously described interference of waves results in intensity maxima and minima. This allows for a definition of the constrast K via

$$K = \frac{I_{\max} - I_{\min}}{I_{\max} + I_{\min}}. \tag{1}$$

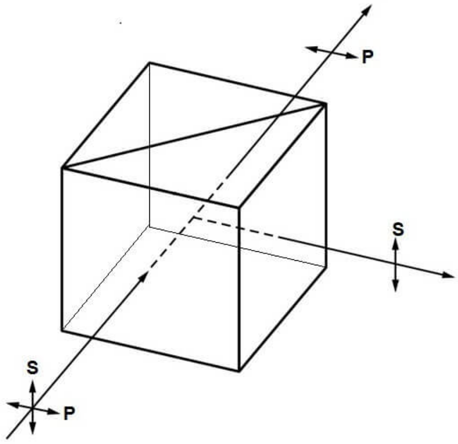

Figure 1: Visualization of the beam path in a PBSC [1].

The intensities $I_{\max}$ and $I_{\min}$ can be measured. For the derivation of a theoretical function, the intensity of the resulting wave needs to be examined. The ansatz with the superposition priciple yields

$$\begin{aligned} I &\propto \langle |E_1 \cos(\omega t) + E_2 \cos(\omega t + \delta)|^2 \rangle \\ &= \langle E_1^2 \cos^2(\omega t) + 2E_1 E_2 \cos(\omega t) \cos(\omega t + \delta) + E_2^2 \cos^2(\omega t + \delta) \rangle \\ &= \frac{E_1^2}{2} + E_1 E_2 \cos(\delta) + \frac{E_2^2}{2} \end{aligned}$$

Here, the path difference is labled as δ. For a path difference $\delta = 2\pi n$, constructive interference is achieved, whereas $\delta = (2n+1)\pi$ delivers destructive interference. Thus, the maximum and mimimum intensity calculates as

$$I_{\max/\min} \propto E_1^2 + E_2^2 \pm E_1 E_2.$$

In case of an interferometer with an PBSC, the two light beams are polarized perpendicular to each other and the amplitudes E_1 and E_2 depend on the input polarization angle ϕ. This is the polarization angle at which the light enters the PBSC. Hence, the amplitudes result as

$$\begin{aligned} E_1 &= E_0 \cos(\phi) = \sqrt{E_1 + E_2} \cos(\phi) \\ E_2 &= E_0 \sin(\phi) = \sqrt{E_1 + E_2} \sin(\phi). \end{aligned}$$

With the help of this relation, the equation (2.3) simplifies to

$$I_{\max/\min} \propto I_{\text{Input}}(1 \pm 2 \sin(\phi) \cos(\phi)). \tag{2}$$

The theoretical function for the contrast of a interferometer with a PBSC is only depended on the polarization angle of the light relative to the horizontal plane of the PBSC. It is given as

$$K = \left|\frac{(1 + 2\sin(\phi)\cos(\phi)) - (1 - 2\sin(\phi)\cos(\phi))}{(1 + 2\sin(\phi)\cos(\phi)) + (1 - 2\sin(\phi)\cos(\phi))}\right| \tag{3}$$

$$= |2\sin(\phi)\cos(\phi)|. \tag{4}$$

2.4. Connection with Interference and Refractive Index

When a light beam enters a medium, the propagation speed changes. This leads to a path difference relative to a light beam that does not propagate the medium. The discussed interference effects lead to intensity maxima and minima and the number of maxima is given as

$$M = \frac{\delta}{2\pi}.$$

2.4.1. Refractive Index of Glass

2.5. Refractive Index of Air

3. Experimental setup and measurement process

4. Auswertung

4.1. Dependence of the contrast on the polarisation angle

At first the dependence of the contrast on the polarisation angle phi is analysed. The measurements listed in Table 1 show the minimum and maximum intensity of the lasers interference for different polarisation angles ϕ. In order to compute the contrast, **??**

Table 1: Measurements for the polarisation angle dependence of the contrast K.

ϕ / °	I_{min1} / V	I_{max1} / V	I_{min2} / V	I_{max2} / V	I_{min3} / V	I_{max3} / V
0	1.60	1.75	1.57	1.77	1.63	1.78
15	0.97	1.57	0.94	1.49	0.95	1.54
30	0.51	1.28	0.52	1.27	0.53	1.30
45	0.39	1.31	0.40	1.34	0.40	1.36
60	0.53	1.74	0.54	1.78	0.53	1.76
75	0.96	2.23	0.97	2.13	0.98	2.21
90	1.87	2.05	1.97	2.41	2.01	2.24
105	1.81	3.10	1.78	3.22	1.84	3.53
120	1.31	4.30	1.35	4.23	1.41	4.47
135	1.15	5.06	1.23	4.78	1.21	5.05
150	1.39	4.44	1.44	4.32	1.47	4.54
165	1.65	3.16	1.69	3.19	1.73	3.33
180	1.66	1.87	1.64	1.82	1.80	2.01

EQREF HERE is used to calculate the contrast K for each measuremnt series. After that the average values and the standard deviations of the three measurements are calculated. The corresponding datapoints are shown in Figure 2. The theory law of the

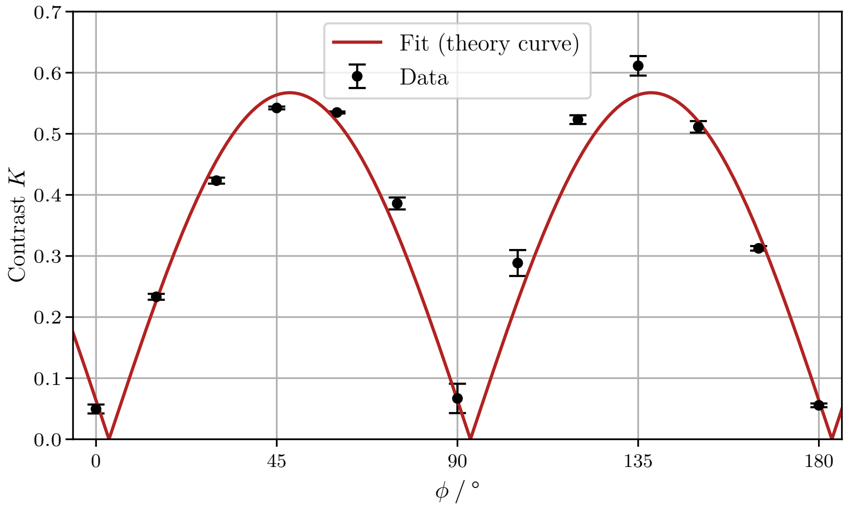

Figure 2: Averaged measurements of the contrast K against the polarisation angle ϕ and fit using *scipy* [2].

angle dependence is given by **?? EQREF HERE**. Here, a function of the form

$$K = 2K_0 \cdot |\sin(\phi - \delta)\cos(\phi - \delta)|$$

is fitted to the data points. The offset δ is used to compensate for deviations in the experimental setup. The fitparameters follow as

$$K_0 = 0.57 \pm 0.01 \qquad \delta = (3.23 \pm 0.58)°$$

using the *python* extension *scipy* [2] and only include statistical uncertainties. The resulting fit function is also displayed in Figure 2. For the following measurements the polarisation angle is set to 45°.

4.2. Refraction index of glass

For the determination of the refraction index of glass the double glass holder is placed in the two beams. The two glass panes are already tilted by an angle $\Theta_0 = \pm 10°$. Using **?? EQREF HERE** the number if maxima passing the center of the interference spectrum is given by

$$M = \frac{\Delta\phi_+ + \Delta\phi_-}{2\pi}$$

where $\Delta\phi_\pm$ is the phase shift induced by the glass panes tilted by $\pm 10°$. This expression can be simplified to

$$M = \frac{2T}{\lambda} \cdot \frac{n-1}{n} \cdot \Theta_0 \theta \tag{5}$$

where $\lambda = 632.99\,\mathrm{nm}$ is the wavenlenght of the laser and $T = 1\,\mathrm{mm}$ is the thickness of the glass panes. The number of measured maxima is again averaged over the ten measurement series. The values are shown in Table 2. The experimental value of the

Table 2: Measurements of the maxima M passing the center of the interference spectrum and corresponding tilt angle θ.

θ	$\overline{M}$
2	6.00 ± 0.45
4	12.30 ± 0.46
6	19.20 ± 0.75
8	25.30 ± 0.64

refraction index of glass follows from a linear fit to the datapoints. The datapoints and the resulting fit are shown in Figure 3. The resulting value is $n_{\mathrm{glass}} = 1.484 \pm 0.008$.

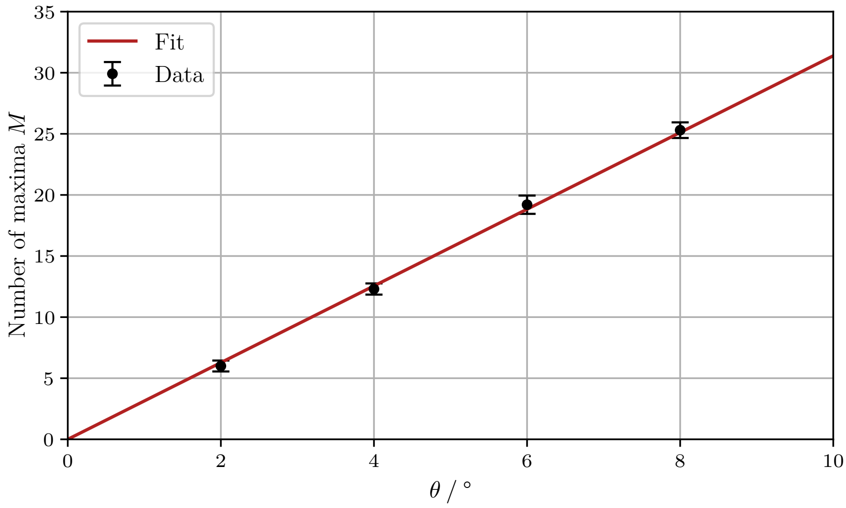

Figure 3: Averaged measurements of the number of maxima $\overline{M}$ against the tilt angle θ and fit using *scipy* [2].

4.3. Refraction index of air

To determine the refraction index of air, the measurements listed in Table 3 are used. Again the average over the five measurement series is calculated.

Table 3: Measurements of the maxima M passing the center of the interference spectrum and pressure p in the air chamber.

p / mbar	M_1	M_2	M_3	M_4	M_5
8	0	0	0	0	0
50	2	2	2	2	2
100	4	4	4	4	4
150	6	6	6	7	6
200	8	9	9	9	9
250	10	11	11	11	11
300	13	13	13	13	13
350	15	15	15	15	15
400	17	17	17	17	17
450	19	19	19	19	19
500	21	21	21	21	21
550	23	23	23	23	23
600	25	25	25	25	25
650	27	28	28	27	27
700	29	30	30	30	30
750	32	32	32	32	32
800	34	34	34	34	34
850	36	36	36	36	36
900	38	38	38	38	38
950	40	40	40	40	40
981	41	41	41	41	41

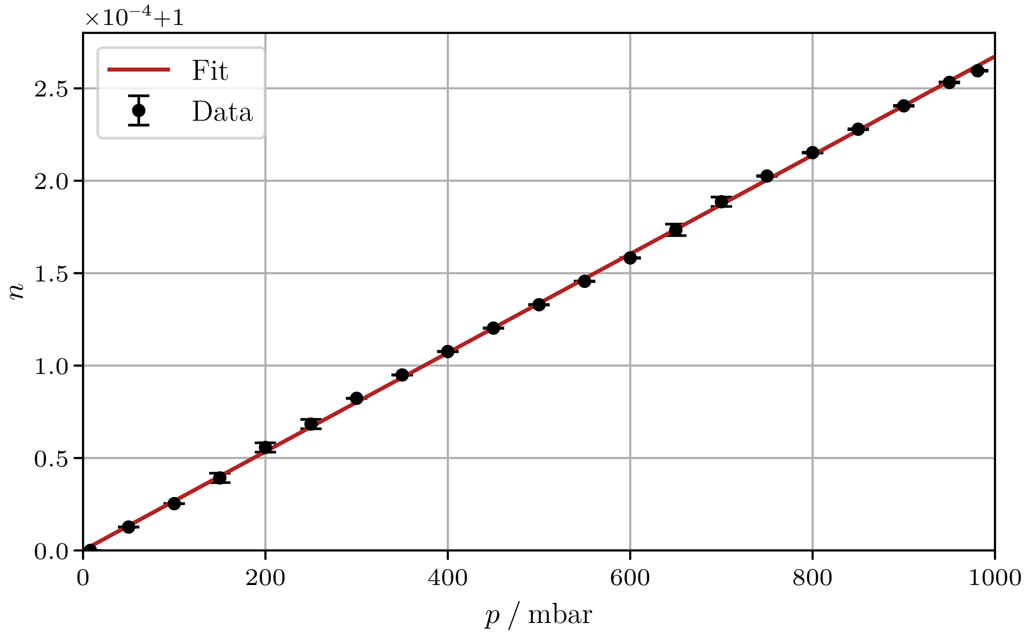

Figure 4: Plot.

5. Diskussion

References

[1] *Polarisierende Strahlteilerwürfel.* Artifex Engineering GmbH Co KG. URL: `https://artifex-engineering.com/de/optiken/strahlteiler/polarisierende-strahlteilerwuerfel/` (visited on 16/11/2023).

[2] Pauli Virtanen et al. 'SciPy 1.0: Fundamental Algorithms for Scientific Computing in Python'. In: *Nature Methods* 17 (2020), pp. 261–272. DOI: `10.1038/s41592-019-0686-2`.

A. Anhang

A.1. Originaldaten