sheet02E4

May 1, 2022

1 Sheet 02

1.1 Exercise 4 Maxwell velocity distribution

The normalisation constant can be determined by integrating over $f(v)$. This integral must be equal to one, so that the resulting function is a normalised probability density. Here and in the following it is sufficient to calculate the integrals form 0 to ∞, because the function describes the absolute value of the velocity. $\rightarrow v \geq 0$

$\int_0^\infty N\mathrm{e}^{-\frac{mv^2}{2k_bT}} 4\, v^2 \mathrm{d}x \stackrel{!}{=} 1$

After solving the integral the equation can be solved for N.

$\rightarrow N = \frac{1}{\left(2\pi\frac{k_bT}{m}\right)^{\frac{3}{2}}}$

1.1.1 a)

The most probable velocity is, where the function $f(v)$ reaches its maximum. So the most probable velocity is calculated by searching where the derivative of the distribution is equal to zero.

$f'(v_m) = 4\mathrm{e}^{-\frac{mv_m^2}{2kT}} \frac{N\pi v_m(2kT - mv_m^2)}{kT} = 0$

$\rightarrow$The most prbable velocity $v_m = \sqrt{\frac{2k_bT}{m}}$

With this N can be expressed by using v_m.

$\rightarrow N = \frac{1}{(\sqrt{\pi}v_m^2)^3}$

1.1.2 b)

The mean of a distribution is defined as $\langle v\rangle = \int_{-\infty}^\infty vf(v)\mathrm{d}v$.

$\langle v\rangle = \int_0^\infty N\mathrm{e}^{-\frac{mv^2}{2k_bT}} 4\, v^3 \mathrm{d}x \rightarrow \langle v\rangle = \frac{8Nk_b^2\pi T^2}{m^2} \rightarrow \langle v\rangle = \frac{2}{\sqrt{\pi}v_m^2}$

1.1.3 e)

The variance is defined as $\mathrm{Var}(x) = \int_0^\infty (x - \mathrm{E}(x))^2 f(x)\mathrm{d}x$.

$\mathrm{Var}(v) = \int_0^\infty (v - \langle v\rangle)^2 (N\mathrm{e}^{-\frac{mv^2}{2kT}}) 4\pi v^2)\mathrm{d}v$

With $\mathrm{Var}(v) = \sigma_v$ we can calculate the standard deviation.

$$\sigma_v = \sqrt{\frac{2\pi N\cdot\left(\sqrt{\pi Tkm}\left(\sqrt{2}\langle v\rangle^2 Tkm+3\sqrt{2}\,T^2k^2\right)-8\langle v\rangle T^2k^2m\right)}{m^3}}$$