

# Exercise 16

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a)

$$P(S \cap W) = P(W \cap S) \quad I$$

$$P(S \cap W) = P(S)P(W|S) \quad II$$

$$P(W \cap S) = P(W)P(S|W) \quad III$$

I states that  $II = III$

$$\Rightarrow P(W)P(S|W) = P(S)P(W|S)$$

$$\Leftrightarrow P(S|W) = \frac{P(W|S) \cdot P(S)}{P(W)}$$

b)

$$P(S) = \frac{\text{Times soccer was played}}{\text{was played} + \text{was not played}} = \frac{9}{14}$$

$$P(W|S) = \prod_i P(x_i|S) \quad P(x_i|S) = P(S \cap x_i) = P(S)$$

$$P(\text{wind} = \text{high} | S = \text{yes}) = \frac{P(S = \text{yes}, \text{wind} = \text{high})}{P(S)} = \frac{3}{14} \cdot \frac{14}{9} = \frac{1}{3}$$

$$P(\text{humidity} = \text{high} | S = \text{yes}) = \frac{3}{9} = \frac{1}{3}$$

$$P(\text{temperature} = \text{cold} | S = \text{yes}) = \frac{1}{3}$$

$$P(\text{forecast} = \text{sunny} | S = \text{yes}) = \frac{2}{9}$$

$$\Rightarrow P(W|S) = \left(\frac{1}{3}\right)^3 \cdot \frac{2}{9} = \frac{2}{243}$$

(Table 1: row 2, column 1: 3 times high humidity when soccer was played (3 times total))

$$P(W = \text{table 2}) = P(\text{wind} = \text{high}) \cdot P(\text{humidity} = \text{low}) \cdot P(\text{temperature} = \text{cold}) \cdot P(\text{forecast} = \text{sunny})$$

$$= \frac{6}{14} \cdot \frac{7}{14} \cdot \frac{6}{14} \cdot \frac{3}{14}$$

$$= \frac{27}{1372}$$

$$\Rightarrow P(S = \text{yes} | W = \text{table 2}) = \frac{2}{243} \cdot \frac{9}{14} \cdot \frac{1372}{27} = \frac{156}{729}$$

$$\approx \underline{\underline{26,89\%}}$$

c)

The problem that occurs is, that there is no data for hot weather when soccer was played. That leads to  $P(\text{temperature} = \text{hot} | S = \text{yes}) = 0$   
 $\Rightarrow P(W = \text{table 3} | S = \text{yes}) = 0$  even though the probability should be  $> 0$

To solve that we can calculate  $P(S = \text{yes} | W) = 1 - P(S = \text{no} | W)$

$$P(S = \text{no} | W) = \frac{P(W|S = \text{no}) \cdot P(S = \text{no})}{P(W)}$$

$$P(S = \text{no}) = \frac{5}{14}$$

$$P(W|S = \text{no}) = \frac{2}{5} \cdot \frac{4}{5} \cdot \frac{1}{5} \cdot \frac{1}{5} = \frac{8}{625}$$

$$P(W) = \frac{8}{14} \cdot \frac{7}{14} \cdot \frac{1}{14} \cdot \frac{3}{14} = \frac{3}{686}$$

$$\Rightarrow P(S = \text{yes} | W = \text{table 3}) = 1 - \frac{8}{625} \cdot \frac{5}{14} \cdot \frac{686}{3}$$

## Exercise 17

a) Y: soccer

$$\begin{aligned} H(Y) &= -P(Y=\text{true}) \log_2(P(Y=\text{true})) - P(Y=\text{false}) \log_2(P(Y=\text{false})) \\ &= -\frac{9}{14} \log_2\left(\frac{9}{14}\right) - \frac{5}{14} \log_2\left(\frac{5}{14}\right) \\ &\approx \underline{\underline{0,94}} \end{aligned}$$

b) X: wind

$$\begin{aligned} H(Y|X) &= P(X=\text{true}) \cdot H(Y|X=\text{true}) \\ &\quad + P(X=\text{false}) \cdot H(Y|X=\text{false}) \end{aligned}$$

$$\begin{aligned} H(Y|X=\text{true}) &= -\frac{3}{6} \log_2\left(\frac{3}{6}\right) - \frac{3}{6} \log_2\left(\frac{3}{6}\right) \\ &= \underline{\underline{1}} \end{aligned}$$

$$\begin{aligned} H(Y|X=\text{false}) &= -\frac{6}{8} \log_2\left(\frac{6}{8}\right) - \frac{2}{8} \log_2\left(\frac{2}{8}\right) \\ &\approx \underline{\underline{0,811}} \end{aligned}$$

$$\begin{aligned} \Rightarrow H(Y|X) &= \frac{6}{14} \cdot H(Y|X=\text{true}) + \frac{8}{14} H(Y|X=\text{false}) \\ &\approx \underline{\underline{0,892}} \end{aligned}$$

$$\begin{aligned} \Rightarrow IG(X, Y) &= H(Y) - H(Y|X) \\ &= 0,94 - 0,892 = \underline{\underline{0,048}} \end{aligned}$$