Blatt 6

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Exercise 12

a) $\vec{\mu}_0 = \begin{pmatrix} (1+2+1.5+2+2+3)/6 \\ (1+1+2+2+3+3)/6 \end{pmatrix} = \begin{pmatrix} 23/12 \\ 2 \end{pmatrix}$

$\vec{\mu}_1 = \begin{pmatrix} (1.5+2.5+3.5+2.5+3.5+4.5)/6 \\ (1+1+1+2+2+2)/6 \end{pmatrix} = \begin{pmatrix} 3 \\ 3/2 \end{pmatrix}$

$$S_0 = \sum_{i=1}^{6} (x_i - \vec{\mu}_0) = \begin{pmatrix} -11/12 \\ -1 \end{pmatrix}\begin{pmatrix} -11/12 & -1 \end{pmatrix} + \begin{pmatrix} 1/12 \\ -1 \end{pmatrix}\begin{pmatrix} 1/12 & -1 \end{pmatrix} + \begin{pmatrix} -5/12 \\ 0 \end{pmatrix}\begin{pmatrix} -5/12 & 0 \end{pmatrix} + \begin{pmatrix} 1/12 \\ 0 \end{pmatrix}\begin{pmatrix} 1/12 & 0 \end{pmatrix} + \begin{pmatrix} 1/12 \\ 1 \end{pmatrix}\begin{pmatrix} 1/12 & 1 \end{pmatrix} + \begin{pmatrix} 13/12 \\ 1 \end{pmatrix}\begin{pmatrix} 13/12 & 1 \end{pmatrix}$$

$$= \begin{pmatrix} (11/12)^2 & 11/12 \\ 11/12 & 1 \end{pmatrix} + \begin{pmatrix} 1/12^2 & -1/12 \\ -\frac{1}{12} & 1 \end{pmatrix} + \begin{pmatrix} (5/12)^2 & 0 \\ 0 & 0 \end{pmatrix} + \begin{pmatrix} (1/12)^2 & 0 \\ 0 & 0 \end{pmatrix} + \begin{pmatrix} (1/12)^2 & 1/12 \\ 1/12 & 1 \end{pmatrix} + \begin{pmatrix} (13/12)^2 & 13/12 \\ 13/12 & 1 \end{pmatrix} = \begin{pmatrix} \frac{318}{144} & \frac{24}{12} \\ \frac{24}{12} & 4 \end{pmatrix} = \begin{pmatrix} \frac{53}{24} & 2 \\ 2 & 4 \end{pmatrix}$$

$\begin{pmatrix} 3 \\ 3/2 \end{pmatrix}$

$$S_1 = \begin{pmatrix} -\frac{3}{2} \\ -\frac{1}{2} \end{pmatrix}\begin{pmatrix} -\frac{3}{2} & -1/2 \end{pmatrix} + \begin{pmatrix} -\frac{1}{2} \\ -\frac{1}{2} \end{pmatrix}\begin{pmatrix} -\frac{1}{2} & -\frac{1}{2} \end{pmatrix} + \begin{pmatrix} 1/2 \\ -1/2 \end{pmatrix}\begin{pmatrix} 1/2 & -1/2 \end{pmatrix} + \begin{pmatrix} -\frac{1}{2} \\ 1/2 \end{pmatrix}\begin{pmatrix} -\frac{1}{2} & \frac{1}{2} \end{pmatrix} + \begin{pmatrix} 1/2 \\ 1/2 \end{pmatrix}\begin{pmatrix} 1/2 & 1/2 \end{pmatrix} + \begin{pmatrix} 3/2 \\ 1/2 \end{pmatrix}\begin{pmatrix} 3/2 & 1/2 \end{pmatrix}$$

$$= \begin{pmatrix} 9/4 & 3/4 \\ 3/4 & 1/4 \end{pmatrix} + \begin{pmatrix} 1/4 & 1/4 \\ 1/4 & 1/4 \end{pmatrix} + \begin{pmatrix} 1/4 & -\frac{1}{4} \\ -1/4 & 1/4 \end{pmatrix} + \begin{pmatrix} 1/4 & -1/4 \\ -1/4 & 1/4 \end{pmatrix} + \begin{pmatrix} 1/4 & 1/4 \\ 1/4 & 1/4 \end{pmatrix} + \begin{pmatrix} 9/4 & 3/4 \\ 3/4 & 1/4 \end{pmatrix} = \begin{pmatrix} 11/2 & 3/2 \\ 3/2 & 3/2 \end{pmatrix}$$

$$+\begin{pmatrix} & \\ 1/4 & 1/4 \end{pmatrix} + \begin{pmatrix} 1/4 & 1/4 \\ 3/4 & 1/4 \end{pmatrix} = \begin{pmatrix} 11/2 & 3/2 \\ 3/2 & 3/2 \end{pmatrix}$$

$$S_{0,1} = \begin{pmatrix} \frac{185}{24} & 7/2 \\ 7/2 & 11/2 \end{pmatrix}$$

d) Calculation of the inverse matrix $S_{0,1}^{-1}$, lamba should be an solution to the eigenvalueproblem $S_{01}^{-1} S_B \vec{\lambda} = D\vec{\lambda}$

$$\left(\begin{array}{cc|cc} \frac{185}{24} & 7/2 & 1 & 0 \\ 7/2 & 11/2 & 0 & 1 \end{array}\right) \Leftrightarrow \left(\begin{array}{cc|cc} 1 & 0 & \frac{264}{1447} & \frac{-168}{1447} \\ 0 & 1 & \frac{-168}{1447} & \frac{370}{1447} \end{array}\right)$$

$$\vec{e}_{\vec{\lambda}} = S_{0,1}^{-1} (\vec{\mu}_0 - \vec{\mu}_1) = \frac{1}{1447} \begin{pmatrix} 264 & -168 \\ -168 & 370 \end{pmatrix} \begin{pmatrix} \frac{13}{12} - 3 \\ 2 - \frac{3}{2} \end{pmatrix}$$

$$= \frac{1}{1447} \begin{pmatrix} 264 & -168 \\ -168 & 370 \end{pmatrix} \begin{pmatrix} -\frac{23}{12} \\ 1/2 \end{pmatrix} = \frac{1}{1447} \begin{pmatrix} -370 \\ 367 \end{pmatrix}$$

e) Precision: $P_{\lambda_{cut}} = \frac{4}{4+0} = 1$

Recall: $R_{\lambda_{cut}} = \frac{4}{4+2} = \frac{4}{6} = \frac{2}{3}$

Accuracy: $= \frac{4+6}{4+6+2} = \frac{10}{12} = \frac{5}{6}$

the λ_{cut} is defined in the Jupyter Notebook