Sheet03

May 9, 2022

1 Exercise 5

1.1 b)

[4]:
```
import numpy as np
from project_a1.random import LCG
import matplotlib.pyplot as plt

# Task b)

def check_array (value, array):
    for i in range(len(array)):
        if array[i] == value:
            return False
    return True

def periodtest(a1):
    seed1 = 0
    i = 0
    gen = LCG(seed = seed1, a = a1, c = 3, m = 1024)
    x = np.zeros(1024)
    gen.advance()
    while i <= 1024 and (gen.state != seed1) and check_array(gen.state, x):
        x[i] = gen.state
        gen.advance()
        i =  i + 1
    return i

print("a = 10: periodlength = ", periodtest(10))
print("a =  1: periodlength = ", periodtest(1))
print("a = 69: periodlength = ",periodtest(69))
print("a =  5: periodlength = ",periodtest(5))
print("a =  3: periodlength = ",periodtest(3))
print("a =  4: periodlength = ",periodtest(4))
```

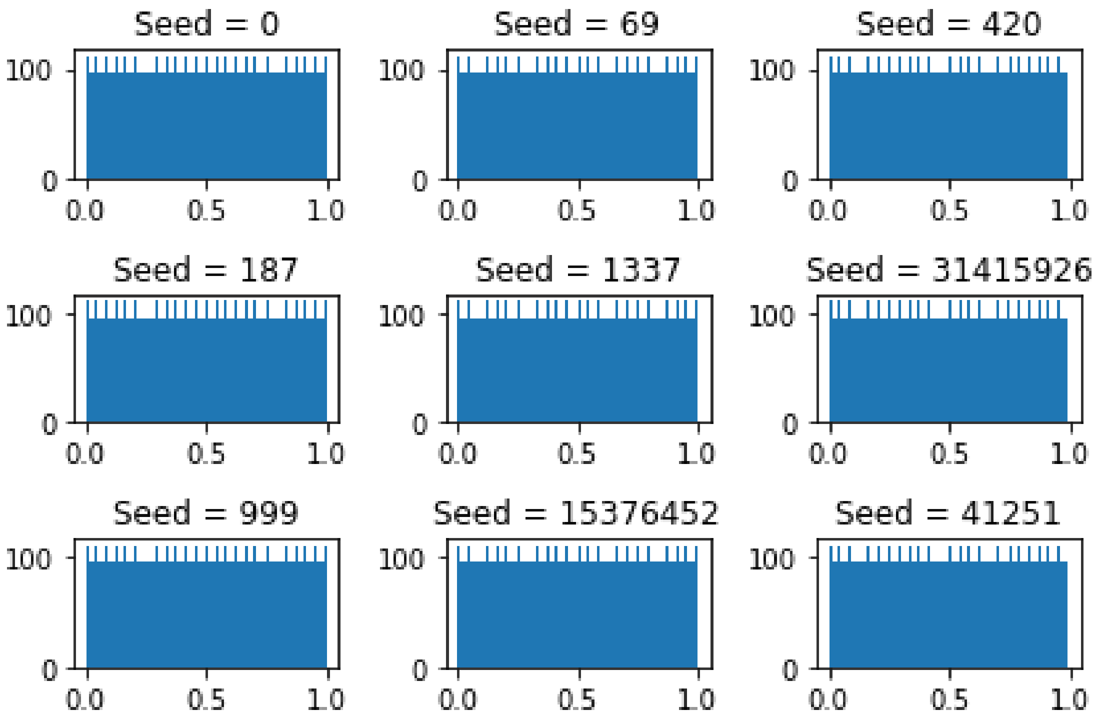
```
a = 10: periodlength =  10
a =  1: periodlength =  1023
a = 69: periodlength =  1023
```

```
a =  5: periodlength =  1023
a =  3: periodlength =  511
a =  4: periodlength =  5
```

Diffrent parameters a result in diffrent period lenghts. The Maximum period length is equal to the parameter $m = 1024$ and is reached for a's that fulfill the following conditions: - $a-1$ and m are divisible by 4 - each prime factor of m divides $a-1$ The prime factor of $m = 1024$ is 2 –> $a-1$ must be divisible by 2 and 4. E.g. $a = 69$ an $a = 5$ are such ideal numbers.

1.2 d) - f)

[6]:
```
# Tasks d) - f)

# Numpy generator
rng = np.random.default_rng(420)
x, y, z = rng.uniform(size = (3, 10000))

# Our LCG
gen = LCG(seed = 0, a = 1601, c = 3456, m = 10000)          # Exercise
 ↪Parameters
x2, y2, z2 = gen.uniform(size = (3, 10000))

good_gen = LCG(seed = 0, a = 625, c = 6571, m = 31104)      # Good Parameters
x3, y3, z3 = good_gen.uniform(size = (3, 10000))

# Dependency on start value

values = [0, 69, 420, 187, 1337, 31415926, 999, 15376452, 41251]
fig0 = plt.figure()

for i in range(len(values)):
    ax = fig0.add_subplot(3, 3, i+1)
    tempgen = LCG(seed = values[i], a = 1601, c = 3456, m = 10000)
    ax.hist(tempgen.uniform(size = 10000), bins = 100)
    ax.set_title(f"Seed = {values[i]}")

fig0.tight_layout()
plt.show()
```



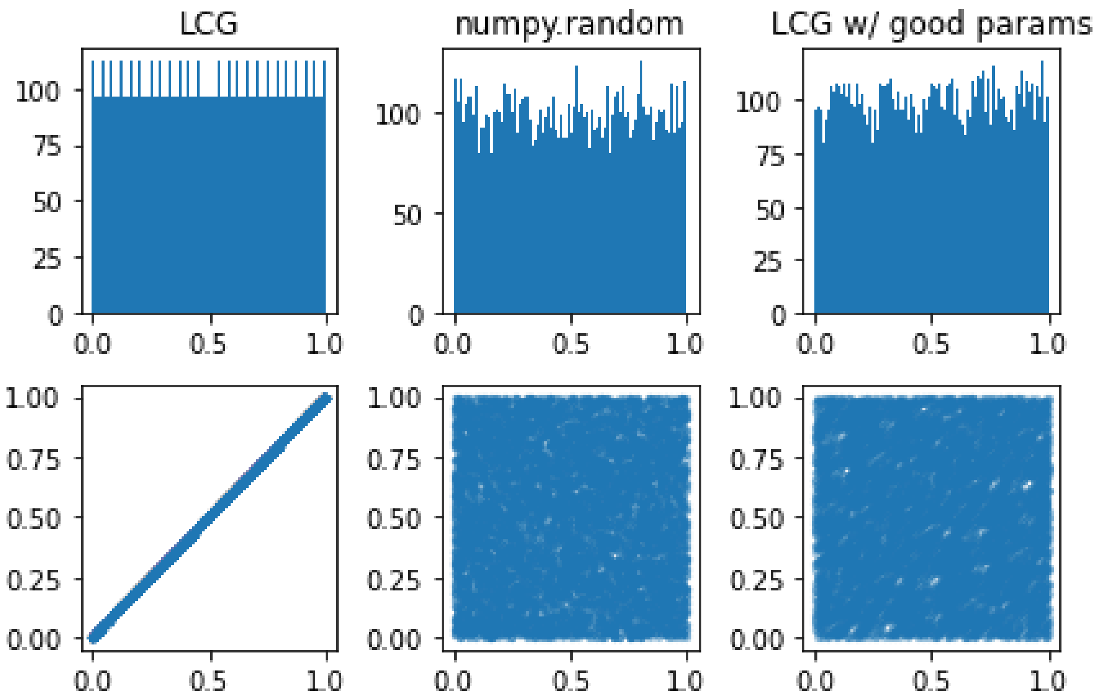
As it can be seen in the graphic diffrent start values (seeds) do not lead to a diffrent behaviour of the generator.

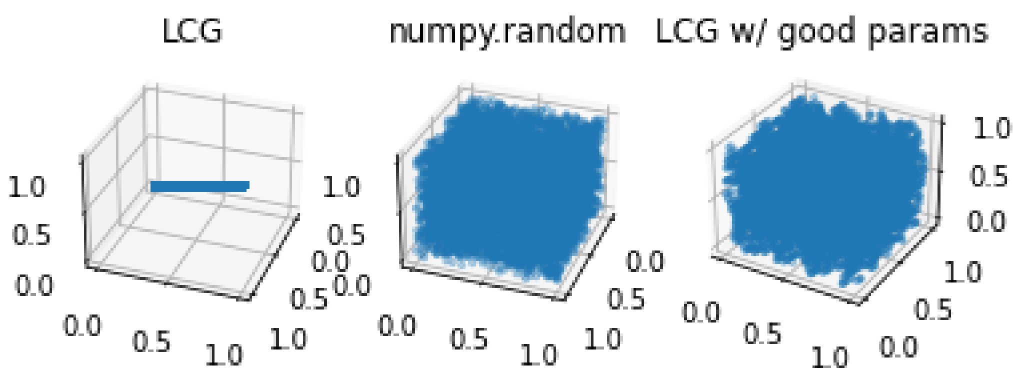
In the following the given LCG is compared to *numpy*'s generator and the same LCG with better parameters.

[7]:
```
# Comparison of our LCG vs numpy.random

fig = plt.figure()

ax1 = fig.add_subplot(2, 3, 1)
ax1.hist(y2, bins = 100)            # Histogram of our "random" numbers
ax1.set_title("LCG")

ax2 = fig.add_subplot(2, 3, 4)
ax2.scatter(
    x2, y2,
    s=5,
    #smaller points
    alpha=0.3,
    # 70% transparency
)

ax3 = fig.add_subplot(2, 3, 2)
```

```
ax3.hist(y, bins = 100)             # histogram of numpy's random numbers
ax3.set_title("numpy.random")

ax4 = fig.add_subplot(2, 3, 5)
ax4.scatter(
    x, y,
    s=5,
    #smaller points
    alpha=0.3,
    #70% transparency
)

ax5 = fig.add_subplot(2, 3, 3)
ax5.hist(y3, bins = 100)             # histogram of numpy's random numbers
ax5.set_title("LCG w/ good params")

ax6 = fig.add_subplot(2, 3, 6)
ax6.scatter(
    x3, y3,
    s=5,
    #smaller points
    alpha=0.3,
    #70% transparency
)

fig.tight_layout()


# 3D Scatter-Plots

plt.show()

fig2 = plt.figure()

ax = fig2.add_subplot(1, 3, 1, projection='3d')

ax.scatter(
    x2, y2, z2,
    s=5,
    # smaller points
    alpha=0.3,
    # 70% transparency
)
ax.set_title("LCG")
# set the orientation of the 3d axis

ax.view_init(elev=30, azim=20)
```

```
ax2 = fig2.add_subplot(1, 3, 2, projection='3d')

ax2.scatter(
    x, y, z,
    s=5,
    # smaller points
    alpha=0.3,
    # 70% transparency
)
ax2.set_title("numpy.random")

# set the orientation of the 3d axis

ax2.view_init(elev=30, azim=20)

ax3 = fig2.add_subplot(1, 3, 3, projection='3d')

ax3.scatter(
    x3, y3, z3,
    s=5,
    # smaller points
    alpha=0.3,
    # 70% transparency
)
ax3.set_title("LCG w/ good params")
# set the orientation of the 3d axis

ax.view_init(elev=30, azim=20)

plt.show()
```





The plots on the leftern side show, that the given LCG ($a = 1601$, $c = 3456$ and $m = 10000$) does not match the requirements of a good random generator. The periodlength is too short for high amounts of random numbers.

2 Exercise 6

2.1 a)

$$f(x) = \begin{cases} N\mathrm{e}^{-x/\tau} & 0 \leq x < \infty \\ 0 & else \end{cases}$$

Determination of normilization constant N: $\int_0^\infty N\mathrm{e}^{-x/\tau}\mathrm{d}x \stackrel{!}{=} 1$

$\Leftrightarrow N[-\tau \cdot \mathrm{e}^{-x/\tau}]_0^\infty = N \cdot \tau = 1$

$\Leftrightarrow N = \frac{1}{\tau}$

Transformation of uniform Distribution u to x with propability density $f(x)$: $u = F(x) = [-\mathrm{e}^{-y/\tau}]_0^x = -\mathrm{e}^{-x/\tau} + 1$

$\Leftrightarrow 1 - u = \mathrm{e}^{-x/\tau} \qquad |\ln(...)$

$1 - u \in [0, 1]$, because $u \in [0, 1]$

$\Leftrightarrow \ln(1 - u) = -x/\tau$

$\Leftrightarrow x = -\ln(1 - u) \cdot \tau$

2.2 b)

$$f(x) = \begin{cases} Nx^{-n} & x_{\text{min}} \le x \le x_{\text{max}} \\ 0 & \textit{else} \end{cases}$$

Determination of normilization constant N: $\int_{x_{\text{min}}}^{x_{\text{max}}} Nx^{-n}\mathrm{d}x \stackrel{!}{=} 1$

$\Leftrightarrow N[\frac{1}{1-n}x^{1-n}]_{x_{\text{min}}}^{x_{\text{max}}} = 1$

$\Leftrightarrow N\frac{1}{1-n}\left(x_{\text{max}}^{1-n} - x_{\text{min}}^{1-n}\right) = 1$

$\Leftrightarrow N = \frac{1-n}{x_{\text{max}}^{1-n} - x_{\text{min}}^{1-n}}$

Transformation of uniform Distribution u to x with propability density $f(x)$: $u = F(x) = N[\frac{y^{1-n}}{1-n}]_{x_{\text{min}}}^x$

$\Leftrightarrow \frac{u(1-n)}{N} = x^{1-n} - x_{\text{min}}^{1-n}$

$\Leftrightarrow x^{1-n} = \frac{u(1-n)}{N} + x_{\text{min}}^{1-n}$

$\Leftrightarrow x = \sqrt[1-n]{\frac{u(1-n)}{N} + x_{\text{min}}^{1-n}} \qquad |N = \frac{1-n}{x_{\text{max}}^{1-n} - x_{\text{min}}^{1-n}}$

$\Leftrightarrow x = \sqrt[1-n]{u(x_{\text{max}}^{1-n} - x_{\text{min}}^{1-n}) + x_{\text{min}}^{1-n}}$

$\Leftrightarrow x = x_{\text{min}} \sqrt[1-n]{u\left((\frac{x_{\text{max}}}{x_{\text{min}}})^{1-n} - 1\right) + 1}$

2.3 c)

$f(x) = \frac{1}{\ } \frac{1}{1+x^2}$

Transformation of uniform Distribution u to x with propability density $f(x)$: $u = F(x) = \frac{1}{\ } \int_{-\infty}^x \frac{1}{1+y^2}\mathrm{d}y$

$\Leftrightarrow \ \cdot u = \arctan(x) + \frac{\ }{2}$

$\Leftrightarrow x = \tan\left(\ (u - \frac{1}{2})\right)$

2.3.1 Interpretation of results in ‘distributions.pdf’ and ‘lcg.pdf’

Since the histogram matches the distributions curve, the methods we implemented seem to work fine :)