Parental Time Investment and Intergenerational Mobility*

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Abstract

This paper constructs a quantitative model of intergenerational mobility in which lifetime income mobility is shaped by various channels including parental time investments in children. The calibrated model delievers positive educational gradients in parental time investment, as observed in the data, and also successfully accounts for untargeted distributional aspects of income mobility, captured in the income quintile transition matrix. The model implies that removing the positive educational gradients in parental time investment during the whole childhood would reduce intergenerational income persistence nearly by 40 percent. Policy experiments suggest that subsidies to childhood investments that can diminish positive educational gradients in parental time investments would increase intergenerational mobility, and that there are better ways of subsidizing investments to achieve greater mobility in terms of aggregate output and welfare.

Keywords: Intergenerational elasticity; quintile transition matrix; parental investments; college education

JEL codes: E24, I24, J22

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1 Introduction

A large body of literature has found that intergenerational income mobility is low in the United States (Solon 1999; Chetty, Hendren, Kline and Saez 2014a). A growing empirical literature examining sources of such low mobility, as reviewed in Black and Devereux (2011), suggests that a key determinant of intergenerational mobility in the US is family background. However, it still remains to be understood which specific family factors are quantitatively relevant for low mobility and through what mechanisms such factors shape intergenerational persistence of lifetime income. The answers to these questions are essential for designing policies to increase intergenerational mobility. This paper develops a quantitative model of intergenerational mobility in which lifetime income mobility is shaped by multiple channels and investigates the quantitative effects of various mechanisms and policy changes on intergenerational mobility. In particular, the focus of this paper is on the role of parental time investment that has been hardly explored in the literature.

The model economy builds on a standard heterogeneous-agent incomplete-markets framework (Huggett 1993; and Aiyagari 1994) while following the tradition of Becker and Tomes (1986) in that altruistic parents care about their descendants' utility. Households are heterogeneous in multiple dimensions such as human capital, assets, education and age. Young parents, who face additional state variables for their child such as their human capital and learning ability, choose how much time and money to invest in their children in addition to standard consumption-savings and labor supply decisions. Children's human capital evolves according to the multiple-period production technology featuring dynamic complementarity and self-productivity, as highlighted by Cunha and Heckman (2007).[1] When children become young adults, they make their own college decision that affects their future life-cycle wage profiles. Parents can affect this decision indirectly through their parental investments and inter-vivos transfers to their children, as college decisions are affected by pre-college human capital and assets. Adult human capital is subject to idiosyncratic shocks, which cannot be fully insured since households have access to the non-state-contingent asset. Households face not only borrowing limits in each period but also across generations because parents are not allowed to borrow against their descendants' income.

[1] Dynamic complementarity denotes a higher productivity of investments with a higher current stock of human capital, and self-productivity refers to positive effects of human capital in one period on human capital in the next period.

The model economy is calibrated to US data by matching relevant target statistics. In particular, my calibration strategy requires the model economy to deliver positive educational gradients in parental time investment that are empirically consistent with those observed in the American Time Use Survey (ATUS) data (Guryan, Hurst and Kearney 2008; and Ramey and Ramey 2010). An important contribution of my paper relative to recent work in the literature (e.g., Lee and Seshadri 2019) is to evaluate a candidate model as a quantitative theory of intergenerational mobility by confronting it with the empirical income quintile transition matrix, and to thereby establish its success in explaining those *disaggregated* moments.[2] Specifically, I find that my model successfully replicates the quintile income transition matrix in US data, although the calibration targets only a single intergenerational mobility statistic (i.e., the correlation between the percentile rank of parents' income and that of children's income). In particular, the upward mobility rate – the probability of children from the parents in the bottom income quintile moving up to the first income quintile – in the model (7.0%) is strikingly close to 7.5% in US data (Chetty et al. 2014a).

Using the model economy, I conduct counterfactual exercises to investigate the role of various mechanisms including the parental time investment channel in shaping the intergenerational persistence of lifetime income. The model implies that removing heterogeneity in parental time investments reduces intergenerational mobility quite significantly. Having conducted this exercise in different stages of childhood, I find that removing educational gradients in parental time investments in all childhood periods decreases the intergenerational elasticity or rank correlation of lifetime income nearly by 40% and raises the upward mobility rate by 4 percentage points. It should be noted that these are equilibrium effects that allow parents to endogenously respond to use the other existing channels to transmit their economic status intergenerationally. I also examine the role of the other channels in shaping the intergenerational persistence of lifetime income. Interestingly, I find that shutting down inter-vivos transfers induces parents to rely more heavily on childhood human capital production to transmit their economic status, leading to a greater persistence of lifetime income across generations.

In light of the above findings, I also use my model to characterize the desirable properties of policy interventions to increase intergenerational mobility. I consider each policy's implications not

[2] This exercise is not commonly done in the literature. An early example of the model-generated quartile transition matrix in Fernandez and Rogerson (1998) shows that this is not a trivial task.

only for intergenerational mobility estimates, but also for aggregate output and welfare in order to evaluate whether those policies that do affect the intergenerational persistence of inequality are otherwise desirable for the overall economy. The first policy experiment I conduct aims to facilitate access to college by subsidizing college costs. I find that these policies, which do raise a fraction of college graduates, do not guarantee greater intergenerational mobility due to positive selection into college. More precisely, those who decide to go to college even before the policy change tend to have higher pre-college human capital as well as higher returns to college than marginal students do. Hence, facilitating college access does not substantially alter the relative standing of the marginal students, thereby having little effects on intergenerational mobility of lifetime income.

Given the quantitatively significant role of the parental time investment channel, it is natural to consider policies that affect parental time investments directly. However, parental time spent with children are typically home-based, and are not observable to government. My second set of policy experiments therefore considers subsidizing monetary investments in children that are complementary inputs to human capital production. I find that it is generally successful in increasing intergenerational mobility when government increases the size of public education investments directly. However, as private education spending is crowded out, output and welfare gains are not sizeable. On the other hand, I find that subsidizing private education spending could raise intergenerational mobility, accompanied by sizeable gains in output and welfare. I highlight that an important condition for this success is to subsidize private education in the period when the size of public investments is relatively small (e.g., early childhood). Specifically, subsidies to private education may not be able to induce poor parents to invest more time in the presence of sizeable public investments, and they could even reduce intergenerational mobility.

This paper builds on a growing literature that investigates intergenerational economic persistence in quantitative dynamic equilibrium models with heterogeneous households where the distribution of income evolves over time endogenously. Following a seminal study by Restuccia and Urrutia (2004) that presented a model that abstracts from potentially important features such as capital accumulation, valued leisure, idiosyncratic labor market shocks and multi-stage parental investments, recent papers increasingly consider models with richer environments (e.g., Holter 2015; Rauh 2017; Daruich 2019; Lee and Seshadri 2019 among others). The first distinguishing feature of my paper is its explicit focus on the parental time investment channel, which has so far received almost no attention in this literature. Lee and Seshadri (2019) is an exception that also models parental time investments.[3] However, the main counterfactual analysis and policy experiments, both of which focus on the parental time investment channel, differs from Lee and Seshadri (2019). Second, my paper is distinguished in this literature as it evaluates the calibrated model through not only the targeted empirical correlation of income across generations – as is standard in the literature – but also through the non-targeted US income quintile transition matrix.

In the literature using structural models that abstracts from early childhood development, initial conditions of adult human capital around early 20's are found to be crucial to account for lifetime income inequality (e.g., Keane and Wolpin 1997; Huggett, Ventura, and Yaron 2011). This result naturally implies that it is essential to look at what happens before early 20's in order to study how mobile lifetime income is over generations. Therefore, my model endogenizes various channels before adulthood to examine how lifetime income persistence is shaped by different forces before adulthood.

Finally, this paper is also related to the literature that uses equilibrium models of human capital investment across generations to study policies designed to raise human capital of children from disadvantaged families (e.g., Fernandez and Rogerson 1998; and Caucutt and Lochner 2020). So far, this literature has mostly focused on parents' inadequate financial investments in children's human capital. In contrast, my paper highlights the role of parental time investments in improving human capital of children from disadvantaged families.

The paper is organized as follows. Section 2 describes the model environment. Section 3 explains how the parameters of the baseline model economy are calibrated and discusses the relationship between parameters and target statistics in the model. Section 4 evaluates the baseline model economy as a quantitative theory of intergenerational mobility through non-targeted statistics implied by the model. Section 5 presents the counterfactual exercises to investigate the quantitative role of various mechanisms on intergenerational mobility, and Section 6 explores a series of policy experiments that illustrates the desirable properties of policies to increase intergenerational mobility. Section 7 concludes the paper.

[3] Erosa and Koreshkova (2007) and Zhu and Vural (2013) also present a model with endogenous parental time in a single childhood period. See also Morchio (2018) and Daruich (2019).

Table 1: Timeline of life-cycle events

	Parent's age										
	20-24	25-29	30-34	35-39	40-44	45-49	50-54	55-59	60-64	65-69	70-74
Model age	1	2	3	4	5	6	7	8	9	10	11
Key decisions	← – – – – – – – – – – – – – – – Consumption-savings – – – – – – – – – – – – – – →										
	← – – – – – – – – – – – – – Labor supply – – – – – – – – – – – – →									Retired – →	
	College		← – Parental – →			Inter-					
				Investments		vivos					
Child's model age			← – – Childhood – – →				1	2	3	4	5

2 Model

The model builds on a standard incomplete-markets general equilibrium framework where the economy consists of heterogeneous households, the representative firm and government.

2.1 Households

The economy is populated by overlapping generations of a continuum of households. A household is composed of an adult who lives with a child until the child grows up. One model period corresponds to five years, and an adult lives for eleven model periods (age 20-74) as an economic decision maker. In Table 1, I summarize the timeline of life-cycle events for a sample parent for illustration. The adult agent supplies labor beginning at period $j = 1$ (age 20) until retirement at the beginning of $j = 10$ (age 65). The agent then lives for two periods after retirement and dies at the end of period $j = 11$. In all periods, the agent makes a consumption-savings choice. The next generation is born when the agent enters the period $j = 3$. Then, the parent invests time and money in their children in periods $j = 3, 4, 5$. Before the child becomes independent ($j = 5$), the parent decides on inter-vivos transfers. The newly formed household faces the same lifetime structure as described above.

All households have identical preferences over consumption c and hours worked n, represented by a standard separable utility function

$$\frac{c^{1-\sigma}}{1-\sigma} - b\frac{n^{1+\chi}}{1+\chi} \tag{1}$$

with the disutility constant $b > 0$.

In each period while working, earnings y are subject to progressive taxation following the parametric form of Benabou (2002) and Heathcote, Storeletten, and Violante (2014). Specifically, after-tax earnings for those who earns y is given by

$$\lambda_j \left(y/\bar{y}\right)^{-\tau_j} y \tag{2}$$

where τ_j shapes the degree of progressivity, λ_j captures the scale of taxation and $\bar{y}$ denotes the average earnings. Note that τ_j and λ_j are indexed by age to allow the labor taxation to depend on the family structure, as in US data (e.g., Guner, Kaygusuz and Ventura 2014; Holter, Krueger and Stepanchuk 2019).

In all periods, capital income is taxed at the rate of τ_k unless the net worth is non-positive. Households receive transfers T and face borrowing constraints (Aiyagari 1994). Following Lee and Seshadri (2019), when r is the real interest rate, the borrowing limit $\underline{a}$ is given by

$$\underline{a} = -\frac{T}{1+r} \tag{3}$$

so that households are always able to pay back their debt in the next period.

This paper considers stationary environments in which market-clearing prices and aggregate quantities are constant over time. Therefore, the time index for the variables is omitted and a variable with a prime denotes its value in the next period. I now present the household's decision problems starting from period 1.

Period 1: College education A child becomes an independent economic decision maker in the model age $j = 1$ (20 years old) with three state variables in addition to j: a human capital stock of h, a level of asset holdings a, and the childhood learning ability ϕ. As discussed below, the first two state variables, h and a, are endogenously determined by their parents. Although the childhood learning ability is not directly relevant to those who already became an adult, it is still a state variable because it affects the learning ability of their child who is to be born in period $j = 3$. An important decision to be made in period $j = 1$ is whether to attain college education or not. Given the discrete nature of this choice, it is convenient to define the value of not completing college and the value of completing college separately.

First, the household's value of not going to college ($\kappa = 1$) is given by

$$N(h,a,\phi) = \max_{\substack{c \geq 0;\ a' \geq \underline{a} \\ n \in [0,1]}} \left\{ \frac{c^{1-\sigma}}{1-\sigma} - b\frac{n^{1+\chi}}{1+\chi} + \beta \mathbb{E}_{z'} V_2(h', a', \kappa, \phi) \right\} \tag{4}$$

$$\begin{aligned} \text{subject to} \quad & c + a' \leq \lambda_j \left(w_\kappa hn/\bar{y}\right)^{-\tau_j} w_\kappa hn + P + T \\ & P = (1+r)\,a - \tau_k r \max\{a, 0\} \\ & h' = \exp(z')\gamma_{1,\kappa} h \\ & \kappa = 1 \end{aligned}$$

where w_κ is the rental price of human capital for skill type κ per unit hours of work, r is the interest rate and P is the initial asset saved and transferred by parents (inter-vivos transfers). Human capital evolves at the gross growth rate of $\gamma_{j,\kappa}$, which depends on age j and education κ to capture the empirical age-profile of wage for different skilled workers, and is subject to the idiosyncratic shock (or market luck) z. As in Huggett et al. (2011), I assume that z follows an i.i.d. normal distribution with mean zero and the standard deviation of σ_z. Note that although z is drawn from an i.i.d. distribution, its effect persists over the rest of the life because z is not a shock to earnings but rather a shock to human capital. The idiosyncratic shocks z cannot be fully insured because a is not a state-contingent asset. As h' is uncertain due to z', households take expectation on the next period value V_2.

To define the value of going to college, it is useful to discuss how the college education affects households in the model. On the one hand, college degree affects the agent's life-cycle wages in two ways. First, college education allows them to enter the skilled labor market (i.e., $\kappa = 2$), receiving w_2 over the life cycle. Second, college changes the life cycle profile of wages through $\{\gamma_{j,\kappa}\}_{j=1}^{8}$. On the other hand, college is costly and requires a stochastic fixed cost of $\psi(\xi, a)$, as in the recent literature (e.g., see Caucutt and Lochner 2020). Specifically, the college cost is defined as

$$\psi(\xi, a) = \max\left\{\exp(\xi) - \iota \exp(-a), 0\right\} \tag{5}$$

where ξ is an exogenous source of stochastic fixed costs, following an i.i.d. normal distribution with the mean of μ_ξ and the standard deviation of σ_ξ. With the degree parameter being positive $\iota > 0$, the second component is designed to capture needs-based scholarships because $\exp(-a)$ is positive and decreases with a. The max operator makes sure that the college cost stays non-negative.

Then, the value of going to college after the realization of ξ is given by

$$C(h, a, \phi, \xi) = \max_{\substack{c \geq 0;\ a' \geq \underline{a} \\ n \in [0,1]}} \left\{ \frac{c^{1-\sigma}}{1-\sigma} - b\frac{n^{1+\chi}}{1+\chi} + \beta \mathbb{E}_{z'} V_2(h', a', \kappa, \phi) \right\}$$

$$\begin{aligned} \text{subject to} \quad & c + a' + \psi(\xi, a) \leq \lambda_j \left(w_\kappa hn/\bar{y}\right)^{-\tau_j} w_\kappa hn + P + T \\ & P = (1+r)\,a - \tau_k r \max\{a, 0\} \\ & h' = \exp(z')\gamma_{1,\kappa} h \\ & \kappa = 2 \end{aligned} \tag{6}$$

where additional elements reflect the benefits and costs of college education, as described above.

Households make a discrete choice regarding college education after observing a draw of ξ. The expected value at the beginning of $j = 1$ is then defined as

$$V_1(h, a, \phi) = \mathbb{E}_\xi \max\left\{N(h, a, \phi), C(h, a, \phi, \xi)\right\}. \tag{7}$$

Period 2: Young adult without children In this period, households face a standard life cycle problem. That is, households make consumption-savings and labor supply decisions.

$$V_2(h, a, \kappa, \phi) = \max_{\substack{c \geq 0;\ a' \geq \underline{a} \\ n \in [0,1]}} \left\{ \frac{c^{1-\sigma}}{1-\sigma} - b\frac{n^{1+\chi}}{1+\chi} + \beta \mathbb{E}_{z', \phi' | \phi} V_3(h', a', \kappa, \phi') \right\}$$

$$\text{subject to} \quad c + a' \leq \lambda_j \left(w_\kappa hn/\bar{y}\right)^{-\tau_j} w_\kappa hn + (1+r)a - \tau_k r \max\{a, 0\} + T$$
$$h' = \exp(z')\gamma_{2,\kappa} h.$$

The only non-standard element is about taking expectation over the ability of the child to be born next period (i.e., ϕ') because each household is going to be endowed with a child whose ability is drawn stochastically at the beginning of period $j = 3$. I assume that it is correlated across generations, following an AR(1) process in logs

$$\log \phi' = \rho_\phi \log \phi + \epsilon_\phi \tag{8}$$

where $\epsilon_\phi \sim N(0, \sigma_\phi^2)$. The exogenous source of a positive correlation of human capital across generations – which is standard in the literature (e.g., Restuccia and Urrutia 2004; Herrington 2015; Holter 2015; Rauh 2017; Lee and Seshadri 2019) – may capture not only genetic transmission but also any residual intergenerational persistence not explained by modelled elements.

Periods 3-5: Parental investments At the beginning of $j = 3$, a child is born with the learning ability of ϕ. The child's human capital at the end of childhood is affected by parental inputs and government inputs in periods $j = 3, 4, 5$, and learning ability. The human capital production technology captures how these affect the whole process. My modelling approach builds on the childhood skill formation literature (Cunha and Heckman 2007) in encompassing the features that skill formation is a multi-stage process and that investments in different periods are complementary.

I first describe how parental inputs and government inputs are aggregated in each period. Investment inputs are of the forms of time and money. Let I_j denote the total investment inputs in period j, aggregated following the constant elasticity of substitution technology

$$I_j = \left(\theta_j^x \left(x_j/\bar{x}\right)^{\zeta_j} + \left(1 - \theta_j^x\right) \left(\left(e_j + g_j\right)/\bar{m}\right)^{\zeta_j}\right)^{\frac{1}{\zeta_j}}, \tag{9}$$

where x_j is parental time, e_j is private education spending, g_j denotes public education investment, $\theta_j^x \in (0, 1)$ captures the relative share of time investments in period j.[4] Note that $\zeta_j \leq 1$ shapes the

[4]For notational convenience, the technology is indexed by the parent's age j, given that there is a one-to-one

elasticity of substitution between time and money in period j, $\frac{1}{1-\zeta_j}$, and is allowed to be general, as compared to unit elasticity in Lee and Seshadri (2019). Since time and money have different units, each input is entered after being normalized by their corresponding unconditional means, which is useful for calibrating ζ_j.[5] As is standard in the literature, private and public monetary investments are assumed to be perfect substitutes (e.g., Restuccia and Urrutia 2004; Holter 2015).

Given the aggregated inputs in period j, the developed human capital at the end of period 5, $h_{c,6}$, is determined by the technology f

$$h_{c,6} = \phi f(I_3, I_4, I_5). \tag{10}$$

where $\frac{\partial^2 f}{\partial I_i \partial I_j} > 0$, implying dynamic complementarity (Cunha and Heckman 2007; Caucutt and Lochner 2020). As in Lee and Seshadri (2019), the technology features unit elasticity of substitution across periods (Del Boca, Flinn and Wiswall 2014) and constant returns to scale. The following recursive formulation is convenient to capture this technology over the whole periods

$$\begin{aligned} h_{c,j+1} &= \phi I_j^{\theta_j^I} h_{c,j}^{1-\theta_j^I} \quad \text{if } j = 5 \\ &= I_j^{\theta_j^I} h_{c,j}^{1-\theta_j^I} \quad \text{if } j = 3, 4 \end{aligned} \tag{11}$$

where $\theta_j^I \in (0, 1)$.[6]

I now describe the decision problem of parents, which incorporates the human capital investment choices described above. I assume that the child shares the household consumption c, according to the household equivalence scale q, and does not make time allocation decisions relevant to the

relationship between children's age and the parent's age in the model.

[5] Specifically, $\bar{x} = \frac{1}{3}\sum_{j=3}^{5} \bar{x}_j$ and $\bar{m} = \frac{1}{3}\sum_{j=3}^{5} (\bar{e}_j + g_j)$ where $\bar{x}_j$ and $\bar{e}_j$ are the average of x and e in period j, respectively. As shown by Cantore and Levine (2012), normalization is necessary for the analysis of changing the elasticity of substitution parameter unless it is fixed at one (Cobb-Douglas).

[6] One can easily recover f in (10) by

$$\begin{aligned} h_{c,6} &= \phi I_5^{\theta_5^I} h_{c,5}^{1-\theta_5^I} \\ &= \phi I_5^{\theta_5^I} \left(I_4^{\theta_4^I} h_{c,4}^{1-\theta_4^I} \right)^{1-\theta_5^I} \\ &= \phi I_5^{\theta_5^I} \left(I_4^{\theta_4^I} \left(I_3^{\theta_3^I} h_{c,3}^{1-\theta_3^I} \right)^{1-\theta_4^I} \right)^{1-\theta_5^I}. \end{aligned}$$

household's economic status during childhood. The following functional equation summarizes a parent's decision problem for $j = 3$:

$$V_3(h, a, \kappa, \phi) = \max_{\substack{c,e\geq 0;\ a'\geq \underline{a} \\ x,n\in[0,1]}} \left\{ \frac{(c/q)^{1-\sigma}}{1-\sigma} - b\frac{n^{1+\chi}}{1+\chi} - \varphi x + \beta \mathbb{E}_{z'} V_4(h', a', \kappa, h_c', \phi) \right\}$$

$$\begin{aligned} \text{subject to} \quad & c + a' + e \leq \lambda_j \left(w_\kappa hn/\bar{y}\right)^{-\tau_j} w_\kappa hn + (1+r)\,a - \tau_k r \max\{a, 0\} + T \\ & x + n \leq 1 \\ & h' = \exp(z')\gamma_{3,\kappa} h \\ & h_c' = \left(\theta_3^x \left(x/\bar{x}\right)^{\zeta_3} + (1-\theta_3^x)\left(\left(e+g\right)/\bar{m}\right)^{\zeta_3}\right)^{\frac{\theta_3^I}{\zeta_3}} h_c^{1-\theta_3^I}. \end{aligned} \tag{12}$$

where $\varphi > 0$ captures the disutility of time investments and (12) is obtained by combining (9) and (11).[7] Note that parents have an incentive to invest their time x and money e in their children because these investments would lead to greater human capital at the end of childhood according to the production technology (10). On the other hand, these investments are costly: parental time reduces utility and private education spending reduces income available for consumption and savings.

For $j = 4, 5$, the decision problem is similarly defined as

$$V_j(h, a, \kappa, h_c, \phi) = \max_{\substack{c,e\geq 0;\ a'\geq \underline{a} \\ n,x\in[0,1]}} \left\{ \frac{(c/q)^{1-\sigma}}{1-\sigma} - b\frac{n^{1+\chi}}{1+\chi} - \varphi x + \beta \mathbb{E}_{z'} V_{j+1}(h', a', \kappa, h_c', \phi) \right\}$$

[7] Given the exogenous transmission of learning ability, the initial human capital when a child is just born is assumed to be homogeneous: $h_c = 1$ (see e.g., Herrington, 2015; and Lee and Seshadri, 2019).

$$
\begin{aligned}
\text{subject to} \quad & c + a' + e \leq \lambda_j \left(w_\kappa hn/\bar{y}\right)^{-\tau_j} w_\kappa hn + (1+r)\, a - \tau_k r \max\{a, 0\} + T \\
& n + x \leq 1 \\
& h' = \exp(z')\gamma_{j,\kappa} h \\
& h_c' = \left(\theta_j^x \left(x/\bar{x}\right)^{\zeta_j} + \left(1 - \theta_j^x\right) \left(\left(e+g\right)/\bar{m}\right)^{\zeta_j}\right)^{\frac{\theta_j^I}{\zeta_j}} h_c^{1-\theta_j^I} \quad \text{if } j = 4 \qquad (13) \\
& \quad = \phi \left(\theta_j^x \left(x/\bar{x}\right)^{\zeta_j} + \left(1 - \theta_j^x\right) \left(\left(e+g\right)/\bar{m}\right)^{\zeta_j}\right)^{\frac{\theta_j^I}{\zeta_j}} h_c^{1-\theta_j^I} \quad \text{if } j = 5. \qquad (14)
\end{aligned}
$$

where the state vector additionally includes the child's human capital at the beginning of the period, h_c. Recall that the state variable κ can take either 1 or 2, depending on the college decision made in the period $j = 1$.

Period 6: Inter-vivos transfers At the end of $j = 6$, the child becomes independent. The decision problem in this period thus includes a decision for inter-vivos transfers a_c, which is transferred at the beginning of $j = 7$ to the next generation that forms a new household entering $j = 1$. This financial transfer could help their child's college decision financially. The decision problem is summarized by

$$
V_6(h, a, \kappa, h_c, \phi) = \max_{\substack{c \geq 0;\ a' \geq \underline{a} \\ n \in [0,1] \\ a_c' \geq [0, \bar{a}_c]}} \left\{ \frac{(c/q)^{1-\sigma}}{1-\sigma} - b \frac{n^{1+\chi}}{1+\chi} + \beta \mathbb{E}_{z'} \left[V_7(h', a', \kappa) + \eta V_1(h_c', a_c', \phi) \right] \right\} \qquad (15)
$$

$$
\begin{aligned}
\text{subject to} \quad & c + a' + a_c' \leq \lambda_j \left(w_\kappa hn/\bar{y}\right)^{-\tau_j} w_\kappa hn + (1+r)\, a - \tau_k r \max\{a, 0\} + T \\
& h' = \exp(z')\gamma_{6,\kappa} h \\
& h_c' = \gamma_c h_c
\end{aligned}
$$

where the continuation value now includes the initial value function of the child, defined above in (7), weighted by the degree of altruism $\eta > 0$. As is clear in the continuation value term, the intergenerational link is modeled following the dynastic utility approach in the sense that parents care about their child's utility, which in turn depends on the next generation's utility, and so on.

This recursive structure linked by altruism combines successive generations as a single dynasty as in Becker and Tomes (1986). Finally, a'_c cannot be negative, meaning that households are not allowed to borrow from their child's future income.

Periods 7-11: Without children Once the child becomes an adult, the state variables do not have to include h_c and ϕ. The decision problems in the remaining periods are standard. Households make consumption-savings and labor supply decisions in periods $j = 7, 8, 9$ (age 50-64) until they retire in $j = 10$ (age 65). The household's problem in $j = 7, 8, 9$ is summarized by

$$V_j(h, a, \kappa) = \max_{\substack{c \geq 0;\ a' \geq \underline{a} \\ n \in [0,1]}} \left\{ \frac{c^{1-\sigma}}{1-\sigma} - b\frac{n^{1+\chi}}{1+\chi} + \beta \mathbb{E}_{z'} V_{j+1}(h', a', \kappa) \right\} \quad \text{if } j = 7, 8, 9 \tag{16}$$

$$\begin{aligned} \text{subject to} \quad & c + a' \leq \lambda_j \left(w_\kappa hn/\bar{y}\right)^{-\tau_j} w_\kappa hn + (1+r)\,a - \tau_k r \max\{a, 0\} + T \\ & h' = \exp(z')\gamma_{j,\kappa} h. \end{aligned}$$

When households retire ($j = 10, 11$), they receive social security pension payments Ω.[8] The value in the retirement stages is given by

$$V_j(h, a, \kappa) = \max_{c \geq 0;\ a' \geq \underline{a}} \left\{ \frac{c^{1-\sigma}}{1-\sigma} + \beta V_{j+1}(h, a', \kappa) \right\} \tag{17}$$

$$\text{subject to} \quad c + a' \leq (1+r)\,a - \tau_k r \max\{a, 0\} + T + \Omega$$

and $V_{j=12}(\cdot) = 0$.

[8] This assumption on the flat pension benefit is quite common (e.g., Abbott et al. 2019). I have considered a version of the model with a more realistic pension that increases with human capital in a concave manner. Given the nature and focus of this paper, this change has very little effects on the quantitative results in this paper.

2.2 Firm's problem and government

A representative firm produces output with constant returns to scale technology. The production function is assumed to be Cobb-Douglas

$$Y = K^{\alpha} H^{1-\alpha} \tag{18}$$

where K is aggregate capital stock, H denotes the aggregate labor input and $\alpha \in (0,1)$. The aggregate labor input H is then defined as

$$H = [\nu H_1^{\rho} + (1-\nu) H_2^{\rho}]^{\frac{1}{\rho}} \tag{19}$$

where $\rho < 1$ determines the elasticity of substitution $(1/(1-\rho))$ between skilled workers H_2 and unskilled workers H_1.

The representative firm in competitive markets solves the following profit maximization problem:

$$\max \{Y - w_1 H_1 - w_2 H_2 - (r+\delta) K\}$$

where δ is the capital depreciation rate. The first order conditions are

$$[K] : \alpha K^{\alpha-1} H^{1-\alpha} = r + \delta \tag{20}$$

$$[H_1] : (1-\alpha) K^{\alpha} H^{-\alpha} \frac{1}{\rho} [\nu H_1^{\rho} + (1-\nu) H_2^{\rho}]^{\frac{1}{\rho}-1} \nu \rho H_1^{\rho-1} = w_1 \tag{21}$$

$$[H_2] : (1-\alpha) K^{\alpha} H^{-\alpha} \frac{1}{\rho} [\nu H_1^{\rho} + (1-\nu) H_2^{\rho}]^{\frac{1}{\rho}-1} (1-\nu) \rho H_2^{\rho-1} = w_2 \tag{22}$$

Government tax revenues from labor income and capital income are spent on four categories: (i) social security pension payments Ω to retirees; (ii) lump-sum transfers T to all households, (iii) public education for children $\{g_j\}_{j=3}^{5}$; and (iv) government spending G that is not directly valued by households. Government balances its budget each period.

2.3 Equilibrium

Let $x_j \in X_j$ denote the age-specific state space defined according to the household's recursive problems in the previous subsection. A stationary recursive competitive equilibrium is a collection of factor prices w_1, w_2, r, the household's decision rules, value functions $V_j(x_j)$, government policies and age-specific measures π_j over x_j such that

1. Given the government policies and factor prices, household decision rules solve the household's life cycle optimization problems defined in the previous subsection, and $V_j(x_j)$ are the associated value functions;

2. Factor prices w_1, w_2, r are competitively determined according to (20), (21) and (22);

3. Markets clear:

$$K = \sum_{j=1}^{11} \mu_j \int a_j d\pi_j$$
$$H_s = \sum_{j=1}^{11} \mu_j \int h_j n_j(x_j) d\pi_j(\cdot|\kappa = s), \quad s = 1, 2;$$

4. Government budget balances: the sum of transfers payments, social security pension payments, public education spending and the residual government spending $G(\geq 0)$ is equal to the sum of labor income tax revenues and capital income tax revenues;

5. The vector of age-specific measures of households $\pi = (\pi_1, \pi_2, ..., \pi_{11})$ is the fixed point of $\pi(X) = P(X, \pi)$ where $P(X, \cdot)$ is a transition function determined by the household decision rules and the exogenous probability distributions, and X is the generic subset of the Borel σ-algebra B, defined over the state space $\mathbf{X} = \prod_{j=1}^{11} X_j$.

3 Calibration

I calibrate parameter values of the baseline model economy to match relevant US statistics. As is standard, there are two sets of parameters. The first set of parameters is chosen externally without using model-generated data while the second set of parameters is determined internally. I now describe them in detail.

3.1 Parameters calibrated externally

The two curvature parameters in the utility function, σ and χ, govern the household's willingness to substitute intertemporally. I set the value of σ equal to 1.5 so that the intertemporal elasticity of substitution for consumption is 2/3 and the value of χ equal to 4/3, which implies the Frisch elasticity of 0.75 (Chetty, Guren, Manoli and Weber 2013). As discussed in the previous section, when a parent lives with a child, consumption in the utility function is replaced by c/q. The value of q is set to 1.59 based on the OECD equivalence scale. Next, I set $\bar{a}_c$ to be 25% of average income, the size of which roughly corresponds to the exemption limit of gift tax in the US.[9]

The gross growth rates of human capital during adulthood $\{\gamma_{j,\kappa}\}_{j=1}^{8}$ for each education level κ govern the life cycle profile of wages for high-skilled and low-skilled workers. Table $A2$ in Appendix reports these 16 values computed based on the estimates in the PSID samples of Rupert and Zanella (2015). The key features captured by these estimates are that (i) the growth rates are much higher in the early adult periods and then diminish as they become older; and that (ii) college-educated households face significantly higher growth rates than those without a college degree. The parameter γ_c that maps childhood human capital to adulthood human capital is set to 20.98 so that the output in the baseline model is normalized to be 1.

I now move on to the parameters related to government. As noted earlier, labor income taxation is progressive, and the degree of progressivity differs by the household structure. Table $A3$ in Appendix reports how these values are chosen for each j. A key feature to note is that progressivity is higher for households with a child. The tax rate for capital income τ_k is set to 0.36. Both labor and capital taxation parameters are based on the estimates in Holter et al. (2019). To obtain the size of public education investments, I follow the approach by Restuccia and Urrutia (2004) and Holter (2015) in that education spending by state and federal government are treated as public investments whereas education spending by local government is treated as private investments. This is motivated by the fact that early education in the US is largely locally-financed. Using the

[9] I have considered the models with alternative values for $\bar{a}_c$ around this value. The main quantitative results herein are not very sensitive to the value of $\bar{a}_c$ after the model is recalibrated to match the target statistics.

information in 2016 from the Education at Glance published by the OECD, I obtain the public investments in periods 3-5 relative to mean income to be 0.060, 0.098 and 0.111, respectively.[10] It is important to note that public education spending increases with child's education stage. Next, following Lee and Seshadri (2019), the size of government transfers T is set to 2% of output to capture welfare programs. Finally, the value of Ω is set to imply that the social security replacement rate is 40%.

As for the production sector, the capital share in the aggregate US data leads to the choice of $\alpha_K = 0.36$. The five-year capital depreciation rate δ is computed under 2.5% of the quarterly depreciation rate. These parameter values are within the range commonly used in the quantitative macroeconomics literature (e.g., Krusell and Smith 1998). Finally, I set $\rho = 1/3$ so that the elasticity of substitution between skilled and unskilled is 1.5 (Ciccone and Peri 2005).[11]

3.2 Parameters calibrated internally

The rest of the parameters are calibrated internally. Table 2 summarizes a set of parameters that are jointly calibrated by simulating the model economy. These parameter values are determined as minimizers of the distance between the relevant statistics from the data and those from the model-generated data. Despite a relatively large number of parameters and targets, there are clear relationships between them, and the model matches the target statistics quite well. I now explain the role of these parameters in the model, and illustrate how each parameter is related to its target statistic, as summarized in Table 2. All statistics regarding time-use are obtained from the 2003-2017 waves of the American Time Use Survey (ATUS), combined with the Current Population Survey (CPS). More details on the data can be found in Appendix.

Preference First, β is households' discount factor. The relevant target for this parameter is set to be the annual interest rate of 4%, as is standard in the literature. The equilibrium capital-output ratio is 2.92 at the annual frequency, which is in line with US data. The next parameter b determines the disutility constant for hours worked. The relevant target for b is set to be the average weekly

[10] The details are available in Appendix. These values are in line with the estimates in Lee and Seshadri (2019).

[11] As this elasticity is important for policy exercises that strongly influence college decisions, in Appendix, I also present the policy exercise results based on $\rho = 2/3$, or the elasticity of 3, which is close to the value used in Abbott et al. (2019).

Table 2: Internally calibrated parameters and target statistics

Parameter		Target statistics	Data	Model
Preference				
β	.971	Equilibrium real interest rate (annual)	0.04	0.04
b	23.4	Average hours of work (age 30-64)	.287	.287
η	.315	Average inter-vivos transfers/GDP per-capita	.056	.057
Childhood human capital production				
θ_3^x	.919	Average parental time investments in period 3	.061	.061
θ_4^x	.219	Average parental time investments in period 4	.036	.036
θ_5^x	.043	Average parental time investments in period 5	.020	.020
θ_3^I	.401	Average parental monetary investments in period 3	.098	.096
θ_4^I	.262	Average parental monetary investments in period 4	.113	.110
θ_5^I	.185	Average parental monetary investments in period 5	.128	.126
ζ_3	-3.96	Educational gradients in parental time in period 3 (%)	20.9	20.5
ζ_4	-1.33	Educational gradients in parental time in period 4 (%)	14.8	15.0
ζ_5	-1.65	Educational gradients in parental time in period 5 (%)	20.2	20.3
College				
ν	.534	Fraction with a college degree (%)	34.2	34.0
ι	.106	Degree of positive selection	.50	.479
μ_ξ	.389	Average college expenses/GDP per-capita	.140	.142
δ_ξ	.595	Observed college wage gap (%)	75.0	81.2
Remaining parameters				
ρ_ϕ	.097	Intergenerational correlation of percentile-rank income	.341	.346
σ_ϕ	.597	Gini wage	.370	.373
σ_z	.144	Slope of variance of log wage from age 25-29 to age 55-59	.180	.184

hours of work for those whose age is between 30 and 65. This leads to $30.16/105 = 0.287$, provided that the weekly feasible time endowment is $105 (= 15 \times 7)$ hours, excluding time for sleeping and basic personal care. Similarly, the disutility parameter φ affects the average parental time investments in all periods. Given the calibration strategy using θ_j^x to control average parental time investments in period j as described below, φ is calibrated together with b such that the marginal disutility of parental time is given by the marginal disutility of work evaluated at the mean hours worked ($\varphi = b\bar{n}^{\chi}$). Finally, η is calibrated to match the average inter-vivos transfers. Recall that the role of the inter-vivos transfers in the model is to provide young households with financial resources that help complete college education. The relevant target is thus the total parental transfers made for children during the college years. More precisely, I sum up the money from parents and college transfers from age 18 to age 26, reported in Table 4 of Johnson (2013), while accounting for the fraction of recipients. This leads to the ratio of average parental financial transfers to the five-year GDP per-capita, which is 0.056.

Childhood human capital production There are three parameters $-$ θ_j^x, θ_j^I and ζ_j $-$ in the per-period production (12), (13) and (14). To calibrate these parameters, I use the clear linkages between each parameter and its corresponding target moment in the model economy. First, θ_j^x determines the relative importance of time investments (as compared to monetary investments), and it clearly increases the average parental time investment in period j. Therefore, for each j, the target moment for θ_j^x is set to be the mean parental time investments in period j. To compute statistics regarding parental time investment, I focus on parental time spent directly with children that can promote development of children's human capital (see Appendix for details). A notable feature of these moments is that it is highest in early years (0.061 in the model or 6.4 hours per week) and decreases with children's age.

Next, given the values of θ_j^x and g_j, a higher θ_j^I strongly increases parental monetary investments in period j. Therefore, I set the mean private education spending in period j as a target moment for θ_j^I. As discussed above, average private education spending in the data is constructed as the sum of not only private spending but also local government spending as in Restuccia and Urrutia (2004) and Holter (2015) because public schools are largely funded through local taxes in the US.

This leads to the target statistics of $0.098, 0.113$ and 0.128 for $j = 3, 4$ and 5, respectively (see Appendix for details). Unlike the parental time inputs, note that money inputs increase with children's education stage.

Finally, ζ_j governs the elasticity of substitution between time and money in period j. In US data, more educated parents spend more time with children (Guryan et al. 2008; Ramey and Ramey 2010). I use this elasticity of substitution to replicate this salient fact.[12] The empirical moments are obtained from the ATUS data. Educational gradients, estimated while controlling for some observable characteristics of parents, are around 20%, meaning that college-educated parents spend 20 percent more time than parents without a college degree.[13] In order to match the stage-specific educational gradients, the baseline specification that allows ζ_j to differ by j. Note that the calibration leads to the elasticity of substitution to be lower in $j = 3$ (0.20) than later periods (0.43 and 0.38 in $j = 4$ and 5, respectively). In other words, parental time and financial investments are more complementary to each other when children are very young.[14]

College The parameter ν in the aggregate production function (19) is calibrated to match the fraction with a college degree (34.2%), as in Lee and Seshadri (2019). In the US, people with higher pre-college human capital are more likely to have a college degree. Specifically, the probability of becoming a four-year college graduate for the top pre-college human capital quintile is about 50 percentage-point higher than that for the bottom quintile, according to Heckman, Stixrud and Urzua (2006). Recall that the value of ι in the cost of college (5) governs the relative strength of need-based scholarships in determining college costs. As ι increases, more asset poor households would be able to go to college (holding other things constant), thereby reducing the degree of positive selection. Therefore, I choose this as a target statistic to discipline the degree of positive selection into college in the model.

The target statistic for μ_ξ in the model is set to be the equilibrium ratio of average (tuition

[12] Zhu and Vural (2013) show how the complementarity between time and money in human capital production affects the wage gradient of parental time in an analytically tractable model with two-period-lived overlapping generations and a single parental investment period.

[13] Precisely, the education gradient refers to the percentage difference in mean parental time investments between education groups. See Appendix for details.

[14] In Appendix, I consider an alternative calibration strategy where $\zeta_j = \zeta$ for all j. When the model is calibrated in this way by matching the overall education gradient (without targeting age-specific gradients), the model implies that educational gradients in parental time investment would increase with children's age.

and non-tuition) expenses after financial aid to per capita GDP. Specifically, I first compute the average ratio of annual college tuition and required fees (excluding room and board) for four-year institutions to the per capita real GDP for the recent periods 1990-2011, which is 0.22 according to the Digest of Education Statistics (2011, Table 349) and the Bureau of Economic Analysis. In order to approximate actual costs faced by students, I also include the non-tuition expenses such as books, other supplies, commuting costs, and room and board expenses that would not have to be paid by a person who chooses not to go to college, as in Abbott et al. (2019). These non-tuition expenses amount to approximately 30% of the average tuition and fees. In 2000-2001, the average grants (federal, state/local, and institutional) received by full-time students in four-year colleges weighted by numbers enrolled are approximately 50% of the average tuition and fees. Based on the above information and assuming that college completion takes four years, the equilibrium ratio of average financial college costs to the five-year GDP is 0.14. Finally, as the variability of college cost draws σ_ξ increases, the observed wage premium tends to decline. The observed college premium, or the ratio between the average wage of those with a college degree and the average wage of those without a college degree ranges from 70 to 80% in the ATUS samples depending on the age bands. Thus, I choose 75% as a target, which is also in the range of recent estimates in Heathcote, Perri and Violante (2010).

Remaining parameters A higher ρ_ϕ leads to a higher degree of economic associations across generations. I set its relevant target as the rank correlation of family income of 0.341 (Chetty et al. 2014a), which has been relatively stable in the US (Chetty, Hendren, Kline, Saez, and Turner 2014b). Due to the data limitation, Chetty et al. (2014a) estimate intergenerational persistence using the proxy income variable instead of lifetime income. The rank correlation from the model, which is used as a target statistic, is also obtained based on the proxy incomes equivalently defined as in Chetty et al. (2014a) (see Section 5 for the precise definition of proxy income).

Recall that the idiosyncratic shocks to adult human capital z, following a normal distribution, have mean zero with the standard deviation of σ_z. Since both σ_ϕ and σ_z are exogenous sources of the cross-sectional dispersion of wages in the model, I choose the Gini coefficient of wage (0.37) as a target statistic. Note that, although the degree of wage inequality monotonically increases with either σ_ϕ or σ_z, their economic mechanism is very different. This is because σ_ϕ affects the variability of the initial condition in human capital while σ_z affects households over the working life. Specifically, holding the overall dispersion of wage constant, in the case when σ_z is relatively larger, households would experience more volatile idiosyncratic shocks to human capital, the effect of which accumulates over the life cycle. As a result, the life-cycle profile of wage inequality would become steeper. Therefore, I choose the difference between the variance of log wage at age 55-59 and that of log wage at age 25-29 as an additional target to pin down the relative contribution of each shock process to the overall wage inequality.[15] These statistics on wage inequality in US data for recent periods, obtained from Heathcote et al. (2010), are reported in Table 2.

4 Assessing the model as a quantitative theory of intergenerational mobility

Prior to the quantitative exercises in the next sections such as counterfactual and policy experiments, this section evaluates the baseline model economy as a quantitative theory of intergenerational mobility. I consider three measures of intergenerational mobility: (i) the IGE; (ii) the rank correlation; and (iii) the quintile transition matrix. The intergenerational mobility estimates reported below are based on family income in order to be consistent with US data counterparts from Chetty et al. (2014a). Specifically, in Chetty et al. (2014a), family income is the five-year per parent average of the pre-tax income defined as either the sum of Adjusted Gross Income, tax-exempt interest income and the non-taxable portion of Social Security and Disability benefits (if a tax return is filed) or the sum of wage earnings, unemployment benefits, and gross social security and disability benefits (otherwise). In the model, family income is the five-year per parent sum of labor earnings, interest income, and social security benefits. It is worth noting that family income is more preferred to measure intergenerational mobility of the economic status when samples include not only males but also females (Chadwick and Solon 2002), which is the case in Chetty et al. (2014a) as well as in my gender-neutral model.

[15] With the help of this target, the model replicates the lifecycle inequality of wages and earnings over the age quite well, as shown in Appendix (Figure $A1$).

IGE and rank correlation The first measurement is the IGE, a conventional way to measure the degree of intergenerational persistence in the literature. The IGE is the slope coefficient obtained by running the following log-log regression equation:

$$\log \mathcal{Y}_{child} = \rho_0 + \rho_1 \log \mathcal{Y}_{parent} + \varepsilon \tag{23}$$

where $\mathcal{Y}$ is supposed to be permanent income. The IGE provides a straightforward interpretation: a 1% increase in parental permanent income is associated with a ρ_1% increase in their children's permanent income. Thus, a high ρ_1 implies low intergenerational mobility. The second way to measure intergenerational mobility is to use a rank-rank specification instead of a log-log specification (Chetty et al. 2014a; 2014b). In other words, I estimate the slope parameter after replacing log income with the percentile rank of income within one's own generation in (23). The slope coefficient in a rank-rank specification (or the rank correlation) has a similar interpretation: a one percentage-point increase in parent's percentile rank is associated with a ρ_1 percentage-point increase in their children's percentile rank.[16] Unlike the IGE, the rank correlation is less sensitive to the treatment of zero income observations and is relatively robust to the point of measurement in the income distribution (Chetty et al. 2014a; 2014b).

In the literature estimating intergenerational mobility, the biggest challenge is the data requirement: we need a data set that contains career-long income histories (or permanent income) for at least two successive generations. Due to the data limitation, in practice, permanent income is replaced with proxy income measured at a point in the life cycle. For purposes of comparison, I present model statistics based on proxy income defined similarly to Chetty et al. (2014a). Specifically, in Chetty et al. (2014a), child's income is measured when children are around 30 years old, averaged over two years. The parent's income is averaged over five years when parents are roughly around 45 years old. Accordingly, in the model, the age at which the parent's income is measured is set to be 45-49 ($j = 6$), and the age at which the child's income is measured is 30-34 ($j = 3$). In addition, I also compute the intergenerational persistence measures using present-value lifetime

[16] Note that the rank-rank slope estimate is simply equal to the correlation coefficient in percentile rank (or Spearman correlation) since the independent and dependent variables, both of which are normalized by transforming the income level to the percentile ranks, have the same variance.

Table 3: Intergenerational persistence estimates

	U.S. data	Model	
	Chetty et al. (2014a)	Proxy income	Lifetime income (discounted)
IGE: log-log slope	0.344	0.318	0.376
Rank corr: rank-rank slope	0.341	0.346	0.367

Notes: The log-log slope estimate is obtained from a univariate regression equation where the dependent variable is the child's log income and the independent variable is the parent's log income. The rank-rank slope estimate is obtained from an equivalent regression equation replacing log transformation with the percentile rank.

income discounted according to the equilibrium real interest rate (Haider and Solon 2006).

Table 3 reports these first two measures (i.e., slope estimates) from the model and the data. The first column shows estimates from US data in Chetty et al. (2014a). Recall that the rank-rank slope using proxy income has been used as a calibration target. The estimate of the log-log slope (IGE) using lifetime income is 0.376, which is close to the estimates around 0.4 in Solon (1999). Moreover, note that this estimate using lifetime income is considerably larger than the estimate of 0.309 using proxy income. This is in line with findings in the empirical studies noting that the short-term income (even multi-year averages) may not represent the permanent income, thereby leading to the attenuation bias in estimating the persistence of income across generations. The bias is smaller in the estimate of the rank-rank slope using proxy income instead of lifetime income (0.346 versus 0.367).

Quintile transition matrix I now use the quintile transition matrix as a way of evaluating how successful a candidate model is as a quantitative theory of intergenerational mobility. The income quintile transition matrix is a 5 by 5 matrix where the (a, b) element gives the conditional probability that a child's lifetime income is in the b-th quintile of his generation's distribution, provided that his parent's income is in the a-th quintile of her own generation's distribution. This matrix provides a richer description of how economic status is transmitted across generations than do the first two measures of correlations. Given that calibration targets do not include any elements in the income quintile transition matrix and that the same correlation of income across generations can be obtained from different disaggregated moments in the quintile transition matrix, comparison

Table 4: Income quintile transition matrices: data vs. model

Unit: %	U.S. data					Model									
	Chetty et al. (2014a)					Proxy income					Lifetime income				
Parent	Child quintile					Child quintile					Child quintile				
quint.	1st	2nd	3rd	4th	5th	1st	2nd	3rd	4th	5th	1st	2nd	3rd	4th	5th
1st	33.7	28.0	18.4	12.3	7.5	35.0	25.5	18.1	14.4	7.0	36.2	26.3	16.2	14.6	6.8
2nd	24.2	24.2	21.7	17.6	12.3	25.7	22.0	21.8	17.7	12.7	25.8	22.6	21.7	18.0	11.9
3rd	17.8	19.8	22.1	22.0	18.3	18.3	19.3	22.1	21.3	18.9	18.9	18.5	23.7	20.3	18.6
4th	13.4	16.0	20.9	24.4	25.4	13.7	18.1	20.5	22.0	25.7	12.6	17.6	21.2	22.3	26.3
5th	10.9	11.9	17.0	23.6	36.5	7.3	15.1	17.5	24.6	35.7	6.5	15.0	17.3	24.8	36.5

of the model output to the empirical quintile transition matrix would be a straightforward way of evaluating a model as a quantitative theory of intergenerational mobility.[17]

Table 4 compares the transition matrix obtained from US data (Chetty et al. 2014a) to the transition matrices using the model-generated data. Three features are worth noting in the transition matrix from US data. First, it shows that the observed positive correlations of income across generations are not simply due to the intergenerational poverty trap but are also due to the rich families that sustain their economic status intergenerationally. Specifically, the probability of children remaining in the bottom quintile when their parents' income lies in the bottom quintile is 33.7%, and the probability of children staying in the top quintile when their parents' income belong to the top quintile is even higher: 36.5%. Second, there is quite a bit of mobility in the middle of the income distribution. For instance, children born into the third quintile parents are almost equally likely to be located in any income quintiles ($18-22\%$). Third, both upward mobility, measured by a probability of moving up from the bottom quintile to the top quintile, and downward mobility, measured by a probability of moving down from the top quintile to the bottom quintile, are quite low (7.5% and 10.9%, respectively).

The middle panel of Table 4 shows that the model is able to account for the above salient features in the US income quintile transition matrix strikingly well despite the fact the calibration only targets the overall correlation of income across generations. In particular, the model generates a high probability of staying in the bottom quintile (35.0%) and the even higher probability of

[17] Note that this is in the same spirit as the model validation exercises in the quantitative macroeconomics literature on income and wealth inequality. For instance, the same high Gini coefficient can be due to a various combination of sizeable poor households and super rich households.

staying in the top quintile (35.7%). The model also predicts a substantial degree of mobility in the middle of the income distribution: children born into the third quintile parents are almost equally likely to end up with any quintiles ($18-22\%$). Finally, the upward mobility rate is 7.0% in the model, which is very close to the data counterpart (7.5%).

The right panel of Table 4 reports the quintile transition matrix when lifetime income is used. As shown in Table 3, intergenerational mobility is slightly lower when lifetime income is used. This is evident from higher probabilities of remaining in the bottom (36.2%) and in the top (36.5%) income quintiles. The upward mobility rate is also slightly lower at 6.8% in terms of lifetime income. In order to quantify the effects of various channels and policy changes on intergenerational mobility more accurately,.the following sections will focus on the intergenerational mobility measures using lifetime income instead of proxy income, the latter of which is subject to attenuation biases (Haider and Solon 2006).

5 Sources of intergenerational lifetime income mobility

In this section, I assess the quantitative importance of various channels in shaping the intergenerational mobility of lifetime income and inspect mechanisms through which each channel affects mobility. I first focus on the role of the parental time investment – the key channel of interest in this paper. Then, I examine other channels that shape intergenerational mobility in the model. It is important to note that the quantitative significance of various channels studied herein reveal the total equilibrium effects of shutting down one channel in the presence of other channels that could be either reinforcing or dampening. In addition, note that the counterfactual exercises in this section are not meant to be realistic; instead, the goal is to clearly demonstrate the role of each channel through marked restrictions in the model.

5.1 Parental time investment channel

In the first set of counterfactual exercises, I explore the role of the parental time investment channel – the focus of this paper. In the baseline model, households choose to invest different amounts of time and money endogenously. To quantify the importance of heterogeneity in parental time investments across households, I shut down educational gradients in parental time investments by imposing that all parents invest the same amount of time at its average from the baseline specification. Note that this exercise is feasible given the nature of the time endowment being equal across households.[18]

Table 5 reports the results. Because parental investments are made in multiple periods, I consider 7 different combinations of removing educational gradients in parental time investments. I first impose $x_j = \bar{x}_j$ for each period j individually. The results show that intergenerational mobility measures change quite significantly. Both IGE and the rank correlation fall by 0.05 (or 13-14%) in periods $j = 3, 5$ and by 0.03-0.04 (or 10%) in period $j = 4$. The upward mobility rate goes up by 0.9-1.2 percentage points (or 14-18%). It should be noted that these are equilibrium effects where parents can endogenously respond using the other existing channels that can strengthen intergenerational association. In fact, Table 5 shows that educational gradients in parental time in the other periods when parents are not constrained by the restriction $x_j = \bar{x}_j$ become higher and that parental financial transfers increase. Despite these counter-efforts, intergenerational mobility becomes higher, thereby suggesting the quantitative importance of heterogeneity in parental time investments in shaping intergenerational mobility.

Table 5 also reports when I impose $x_j = \bar{x}_j$ for two periods simultaneously. The results show that the overall effects of shutting down heterogeneity in parental time in two periods are slightly larger than the sum of the individual effects when it is done separately. This is natural given dynamic complementarity in the production technology (10). The last row of Table 5 shows that removing heterogeneity in parental time investments in all the childhood periods would reduce both IGE and the rank correlation by 0.142 (or 38-39%) and increase the upward mobility rate by 4 percentage points, highlighting the strong quantitative role of the parental time investment channel.

To better understand the mechanism through which the parental time investment channel affects intergenerational income mobility, it is useful to look at the equilibrium relationship between parental time and monetary investments. The upper three figures of Figure 1 show their relationship in the baseline model economy for each period $j = 3, 4, 5$. The bottom three figures show their counterparts in the counterfactual exercise where $x_3 = \bar{x}_3$ is imposed.

[18] In contrast, it is generally not feasible to consider a counterfactual exercise that imposes the same amount of private money investments due to the evident income inequality both in the model and in the data.

Table 5: Quantitative effects of heterogeneity in parental time investments on intergenerational mobility

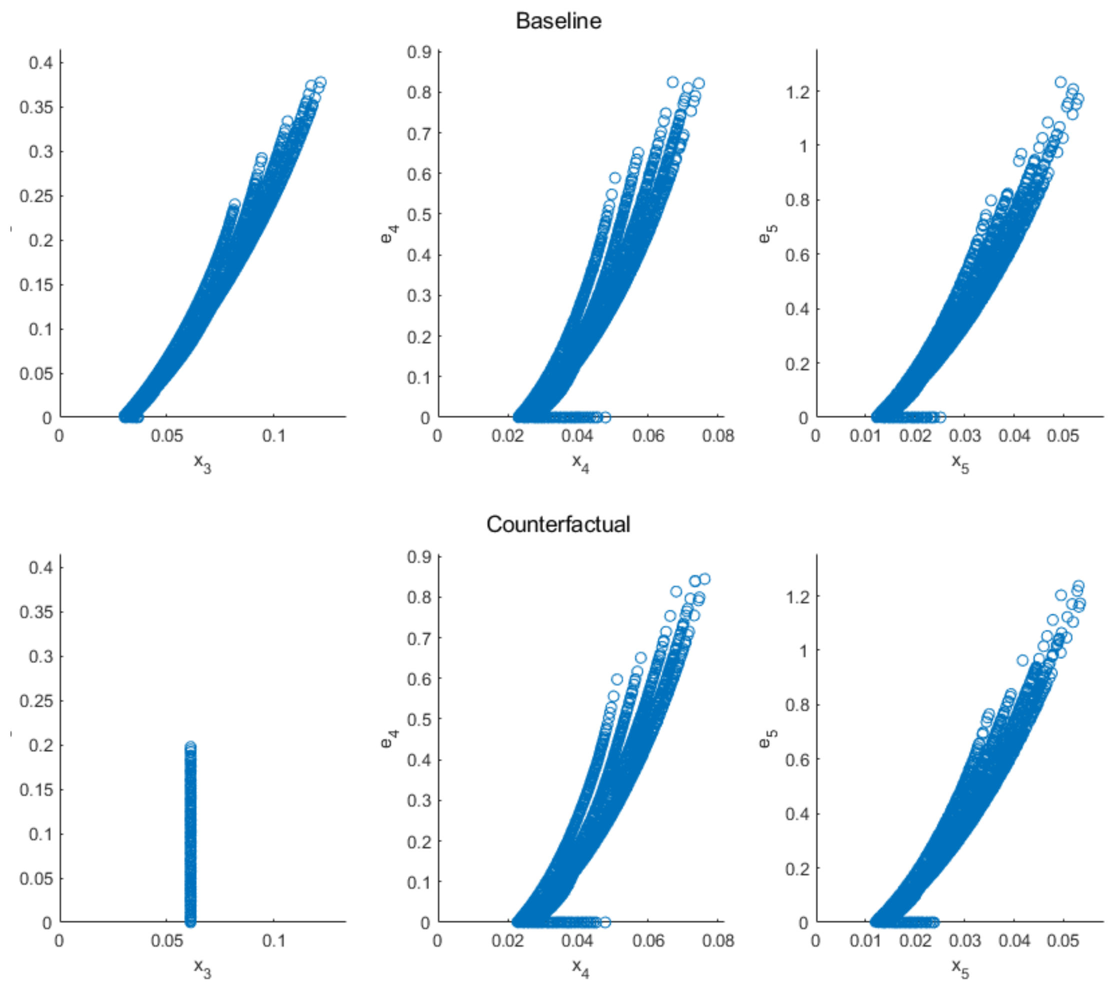
	IGE	Rank corr	Upward mobility (%)	Educ. gradient in x_j (%) $j=3$	$j=4$	$j=5$	Mean P/Y
Baseline	.376	.367	6.8	20.5	15.0	20.3	.057
Counterfactuals							
(1) Single period							
- $x_3 = \bar{x}_3$	.327	.317	7.9	0.0	15.3	20.5	.059
- $x_4 = \bar{x}_4$	.340	.330	7.7	21.1	0.0	20.6	.060
- $x_5 = \bar{x}_5$	.327	.317	8.0	20.9	15.5	0.0	.061
(2) Two periods							
- $x_3 = \bar{x}_3$ & $x_4 = \bar{x}_4$	.289	.280	9.0	0.0	0.0	21.3	.062
- $x_4 = \bar{x}_4$ & $x_5 = \bar{x}_5$	.288	.279	9.1	21.6	0.0	0.0	.064
- $x_3 = \bar{x}_3$ & $x_5 = \bar{x}_5$	.275	.265	9.3	0.0	16.0	0.0	.063
(3) All periods							
- $x_j = \bar{x}_j$ for $j = 3, 4, 5$	.234	.225	10.7	0.0	0.0	0.0	.067

In the upper figures, one can easily notice a clear positive association between time (x-axis) and money (y-axis) due to strong complementarities between these two inputs in each period. Note that this also means that rich parents are investing much more money than poor parents because they are investing more (complementary) time inputs. On the other hand, when parents are constrained to invest the fixed mean time $\bar{x}_3$ in the bottom-left figure, we can see that the variation of monetary investments decrease significantly, especially driven by disappearance of high investments. This explains why intergenerational mobility increases, even with higher educational gradients in parental time in other periods $j = 4, 5$ and greater amounts of parental transfers, as shown in Table 5.

5.2 Other channels shaping intergenerational mobility

I now move on to other channels that shape the intergenerational mobility of lifetime income in the model. In the model economy, parents can transfer money to their child when she becomes independent. An important role of this money is to provide financial help for their child's college decision. In Table 6, the first counterfactual exercise reported is to shut down the inter-vivos transfer

Figure 1: Time and money investments in simulated data



Notes: The upper panels are obtained from the baseline model whereas the bottom panels are obtained from a counterfactual exercise where $x_3 = \bar{x}_3$ for all households. Each dot represents the choices of a simulated sample for monetary investments (y-axis) and time investments (x-axis).

channel by setting $a_c = 0$. It is striking to note that intergenerational persistence estimates, both IGE and rank correlation, become slightly higher (or mobility goes down) in this counterfactual exercise. In fact, Table 6 shows that richer parents, who are not allowed to transfer money to their children, choose to invest more time in young children instead. This substitution towards parental time investment leads to greater educational gradients in parental time, which in turn raises intergenerational income persistence and reduce upward mobility.

Public education investments $\{g_j\}_{j=3}^{5}$ in the model are provided to everyone equally. Therefore, their presence is expected to dampen intergenerational association in the model. To explore the equilibrium effects of the public education investment channel, the next three rows of Table 6 report the results from setting $g_j = 0$ for $j = 3, 4, 5$. As expected, we can see that intergenerational mobility measures indicate lower mobility in the absence of public investments. In particular, the effects are stronger for the periods $j = 4, 5$ where the size of public investments are greater in the baseline economy. For instance, IGE would increase by 0.02 (or 5-6%) when public investments are eliminated in either $j = 4$ or $j = 5$.

The next row shows the result when the exogenous source of intergenerational persistence is shut down by setting $\rho_\phi = 0$. Note that the calibrated persistence of ϕ may capture genetic transmission that would tend to increase ϕ but also any other factors that are not modeled herein that could in principle also reduce the calibrated ϕ. Given that the calibrated ρ_ϕ was positive, we can see that shutting down ability transmission reduces both IGE and the rank correlation quite considerably by 20%. In particular, the upward mobility rate goes up by 2.2 percentage points (or 34%). These results show that the external transmission of ability is a quantitatively important source of lifetime income persistence in the model.

Lastly, I also examine the role of idiosyncratic shocks over the life cycle by setting $\sigma_z = 0$. Note that the most immediate consequence of this restriction is to limit *intragenerational* mobility because idiosyncratic shocks essentially play a role of moving up and down the ranking of adults' wage over the life cycle. When this is shut down, initial conditions at the beginning of adulthood becomes much more important in determining lifetime income, because the initial gap would be simply amplified through steeper wage growth rates among the college-educated. This implies that parental influence on child's lifetime income becomes greater, and as a result, intergenerational

Table 6: Quantitative effects of other mechanisms on intergenerational mobility

	IGE	Rank corr	Upward mobility (%)	Educ. gradient in x_j (%) $j=3$	$j=4$	$j=5$	Mean T_p/Y
Baseline	.376	.367	6.8	20.5	15.0	20.3	.057
Counterfactuals							
- No inter-vivos transfers	.394	.385	6.6	21.5	15.9	21.1	.000
- $g_3 = 0$	.381	.371	6.6	20.4	14.5	19.7	.058
- $g_4 = 0$	.396	.383	6.0	20.1	17.5	19.7	.058
- $g_5 = 0$	.397	.384	5.9	20.2	14.6	23.5	.057
- No persistence in ϕ	.300	.291	9.0	20.1	14.8	19.7	.059
- No idiosyncratic shocks	.395	.377	4.8	20.7	15.7	20.8	.054

mobility decreases quite significantly without adulthood idiosyncratic shocks to human capital.

6 Policy experiments

In this section, the baseline model economy is used to consider various policies that can be considered as tools to influence intergenerational mobility. I consider universal (or flat) policies that can avoid stigmatization especially when it comes to family policies (Heckman 2008). The main objective of this section is to examine and illustrate desirable properties of effective policies to increase intergenerational mobility. In doing so, I also examine the implications of such policies for aggregate output and welfare.[19] That way, we could better evaluate whether policy changes that raise intergenerational mobility are otherwise desirable for the economy. All the policies are designed to be financed through G to satisfy the government budget constraint without changing taxes. For illustrative convenience, all monetary values are expressed in (approximately) 2011 US dollar under the assumption that the annual GDP per capita in the baseline model is \$50,000, a value close to nominal GDP per capita in 2011.

[19] Welfare changes are measured by a consumption equivalent premium, as is standard in the literature. Specifically, I measure the percentage change in consumption for all agents in the baseline model that makes them indifferent to living in the alternative economy using the utilitarian social welfare function.

6.1 Subsidizing college education

College is often believed to be a means of upward mobility. I first consider subsidizing college costs as a way of providing easier access to college. Specifically, the college cost (5) in the budget constraint (6) is replaced by

$$(1 - s_c)\psi(\xi, a) \tag{24}$$

where $s_c \in [0, 1]$ is a subsidy rate. I consider the two levels of the rate: 10% and 20%. For each level of s_c, I also report the results obtained while holding prices (w_1, w_2 and r) fixed at the baseline level, denoted as FP (fixed prices).

Some interesting results emerge in Table 7. Clearly, the college fraction increases sharply when s_c increases in the case of FP. For instance, with respect to 20% subsidy, college graduates increase by 8.8 percentage points. However, intergenerational mobility measures are barely affected despite substantial increases in college graduates. Note that the FP results are not equilibrium outcomes because prices are held fixed and are not market-clearing. When prices are allowed to adjust in general equilibrium, we can see that the increases of the college fraction are dramatically dampened.[20] Again, we can see that all intergenerational mobility measures change only marginally.

To better understand this insignificance of college subsidies on intergenerational mobility, it is useful to understand how the college choice is made. In the model, college decisions depend not only on financial conditions but also on pre-college human capital. The discrete decision rule for college education features threshold-based behavior. More precisely, holding other things constant, the college decision rule is to get a college degree if his or her human capital is above some threshold level. The reason is that the return to college, which is accumulated over the life cycle through higher growth rates, increases with their pre-college human capital level. This property of the college decision rule leads to positive selection in equilibrium, meaning that those who have higher pre-college human capital is more likely to be college-educated. Note that this relationship is not perfect because college costs are stochastic and depends negatively on assets.

To visualize the quantitative importance of pre-college human capital that exists in the model,

[20] This general equilibrium effect hinges on the elasticity of substitution between skilled and unskilled workers, shaped by the parameter ρ. In Appendix, I consider an alternative calibration where I double ρ. The same policy exercises show that the college fraction increases more in general equilibrium, but is still much weaker than the case with fixed prices.

Table 7: Effects of providing easier access to college

	Baseline	$s_c = 0.1$		$s_c = 0.2$	
		FP	GE	FP	GE
IGE	.376	.376	.375	.376	.375
Rank correlation	.367	.367	.366	.367	.366
Upward mobility (%)	6.8	6.7	6.8	6.7	6.8
College fraction (%)	34.0	38.1	34.3	42.8	34.5
Observed college premium (%)	81.2	81.6	79.3	83.1	77.7
Educ. gradient in x_j (%)					
- $j = 3$	20.5	20.8	20.1	21.2	19.8
- $j = 4$	15.0	15.0	14.6	15.1	14.4
- $j = 5$	20.3	20.3	19.8	20.5	19.3
Aggregate output *(% chg)*	-	-	0.0	-	-0.1
Aggregate capital *(% chg)*	-	-	-0.1	-	-0.1
Consumption equiv.*(%)*	-	+0.2	+0.1	+0.5	+0.3

Notes: FP (fixed prices) refers to the case where prices are held constant at the baseline level. Output changes are not reported in these cases. Average labor productivity is defined as aggregate output per total hours worked. Welfare gains are the consumption equivalent premium measured by a percentage change in consumption required for all agents to be indifferent to living in an alternative economy.

Figure 2: Probabilities of being college graduates at age 30 relative to unconditional mean

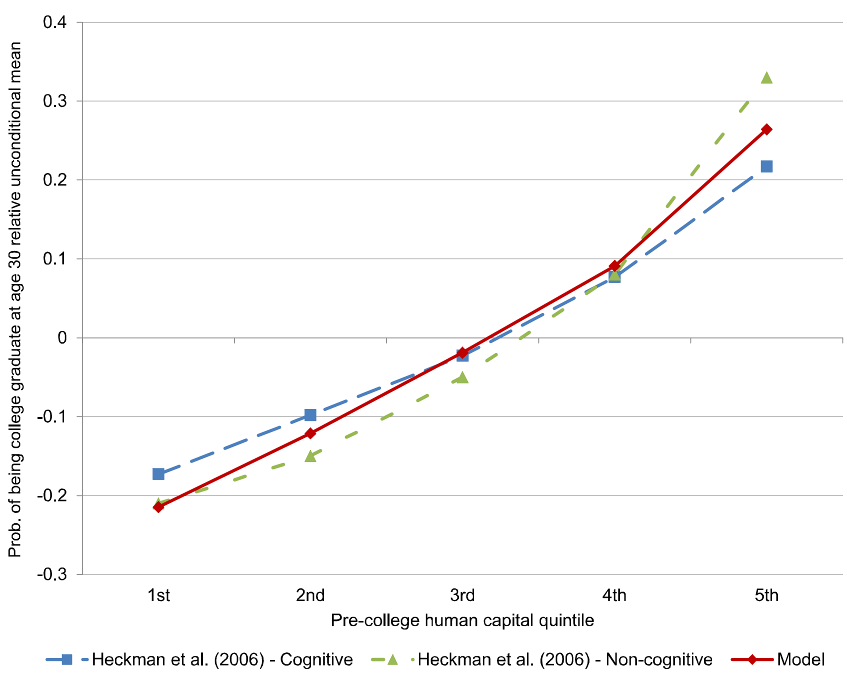


Figure 2 plots the probability of being a college graduate at age 30 by the quintiles of pre-college human capital. The data counterparts are from Heckman et al. (2006) by either cognitive factors or non-cognitive factors.[21] It clearly shows that high pre-college human capital raises the probability of becoming a college graduate, indicating positive selection into college both in the model and in the data. Note that the model produces a slope, which captures the strength of selection, in line with the data, as the calibration strategy roughly targets this overall slope.

By now it should be clearer why college subsidies turned out to be ineffective in affecting intergenerational mobility. Given the property of the college decision rule featuring positive selection as shown above, the marginal households affected by the subsidy tend to have lower pre-college human capital than those who would go to college already. On average, those marginal college graduates do accumulate more of human capital but only up to the level less than those who already choose to go to college, leading to little rank reversals. Therefore, such college subsidies that could potentially induce more people to go to college are difficult to affect intergenerational mobility.

6.2 Subsidizing childhood education

The counterfactual analysis in Section 5 has shown that the parental time investment channel is quantitatively important in shaping intergenerational mobility, calling for policy interventions that affect parental time investment behavior. However, in practice, it is very difficult for government to directly influence parental time investments because time spent with children is mostly home-based and is not observable to government. Therefore, in this subsection, I consider two kinds of policies on monetary investments in children instead, noting that they can indirectly influence parental time investment behavior.

The first is a subsidy $s_{e,j}$ proportional to private education spending e. In other words, the left-hand side of the resource constraint in period j is replaced by

$$c + a' + (1 - s_{e,j})e. \tag{25}$$

I consider providing this subsidy for each period $j = 3, 4, 5$ separately. The subsidy rate is chosen as

[21] The samples considered in Heckman et al. (2006) have a lower unconditional mean probability. To focus on the slope rather than the level, Figure 2 plots probabilities relative to the unconditional mean probability.

Table 8: Effects of increasing quantity of parental time investment

	Baseline	$s_{e,j} = 0.2$			$\Delta g_j = 0.02/Y$		
		$j = 3$	$j = 4$	$j = 5$	$j = 3$	$j = 4$	$j = 5$
IGE	.376	.367	.378	.380	.366	.363	.365
Rank correlation	.367	.358	.368	.370	.359	.355	.357
Upward mobility (%)	6.8	7.0	6.5	6.5	7.2	7.4	7.2
Mean (% chg)							
- $\bar{e}_3$ (relative to Y)	.096	+22.1	-2.3	-2.2	-20.3	-0.6	-0.4
- $\bar{e}_4$	.110	-1.6	+35.4	-1.5	-0.4	-15.6	-0.2
- $\bar{e}_5$	.126	-1.4	-1.3	+37.4	-0.6	-0.5	-13.4
- $\bar{x}_3$ (hrs/wk)	6.4	+11.4	+0.6	+0.5	+0.5	-0.1	-0.1
- $\bar{x}_4$	3.8	+0.6	+9.0	+0.5	-0.3	+2.3	-0.1
- $\bar{x}_5$	2.1	+0.9	+0.8	+11.3	-0.4	-0.2	+2.2
Educ. gradient in x_j (%)							
- $j = 3$	20.5	18.2	20.0	20.0	19.3	20.6	20.6
- $j = 4$	15.0	14.5	15.6	14.6	15.0	11.9	15.0
- $j = 5$	20.3	19.6	19.7	21.2	20.1	20.2	17.2
Aggregate output *(% chg)*	-	+3.1	+2.6	+2.3	+0.2	+0.5	+0.4
Aggregate capital *(% chg)*	-	+2.8	+2.3	+2.0	+0.3	+0.4	+0.2
Consumption equiv. *(%)*	-	+3.3	+2.5	+2.1	+0.7	+0.9	+0.7

20% for each case. As this policy is expected to encourage parental monetary investments, it could, in principle, boost parental time investments that are complementary inputs in the skill formation technology.

The second policy tool I consider is to increase public investments g_j directly. The idea is similar to the previous one: by increasing (public) monetary investments, parents are indirectly incentivized to invest more time that is a complementary input to human capital production. However, the key difference to note is that an increase in g_j would affect not only time investment choices but also private education spending. The magnitude of changes is set to 2% of mean income in each period.

Table 8 summarizes the results for each policy exercise conducted in different target periods. I first focus on the effects of subsidies to private education spending. As expected, we can see that this subsidy $s_{e,j}$ increases not only monetary investments (22-37%) but also time investments substantially (around 10%) in the period j targeted by each policy. However, it is interesting to note that intergenerational mobility increases in the case of subsidy in period 3 whereas it goes down in the other cases ($s_{e,4} = 0.2$ and $s_{e,5} = 0.2$). Why does the higher average time investment have such opposite effects on intergenerational mobility? To better understand this, Figure 3 plots the percentage change in parental time investments by income quintile for the private education subsidies in three target periods. Note that, although the effect of each subsidy (i.e., $s_{e,j}$) on the *unconditional* mean of parental time investment in its corresponding period j is very similar (around 10%), their impact across the distribution differs sharply. More specifically, increases in parental time investments in lower income quintiles are much more pronounced in the case of $s_{e,3}$ while parental time investments in the bottom quintile respond very weakly in the cases of $s_{e,4}$ and $s_{e,5}$. As a result, educational gradients in parental time investments are reduced only in the case of $s_{e,3}$ (from 20.5% to 18.2%), yet they become higher in the cases of $s_{e,4}$ (from 15.0% to 15.6%) and $s_{e,5}$ (from 20.3% to 21.2%), explaining why intergenerational mobility changes in the opposite direction.

A natural question is then why parental time investments among the bottom quintile respond weakly with respect to subsidies to private education spending in later periods $j = 4$ and 5. The key reason for this is that public investments are relatively larger in later periods ($g_4 = 0.098$ and $g_5 = 0.111$) compared to the early period ($g_3 = 0.060$). Given the relatively large public investments, Figure 4 that plots the distribution of private education expenditures across households in each period j clearly shows that greater fractions of households are essentially constrained near zero private spending, crowded out by public investments.[22] For these households who are more likely to be poor, the subsidy on private education is less effective, which in turn weakens their parental time investment responses.[23]

Table 9 also reports the results when government directly increases public education spending. As expected, there are crowding-out effects. That is, private education spending strongly declines in the period when public investments rise. Consequently, we can also see that parental time investments rise only weakly (by less than 2%) in the targeted period, despite a sizeable increase

[22] The shares of households who spend near zero private education investment are prominent as a result of the assumption that public and private education investments are perfect substitutes – a standard, yet relatively strong assumption (e.g., Restuccia and Urrutia, 2004; Holter, 2015). It should be viewed as illustrative to better understand the theoretical mechanism.

[23] In Appendix, Table $A7$ reports the results when $s_{e,5}$ is set to 0.2 while g_5 is also set to be zero. In this case, educational gradients in parental time in $j = 5$ also diminishes and intergenerational mobility increases. Thus, it clearly corroborates the importance of the size of public investments when it comes to heterogeneous policy effects of private education subsidies.

Figure 3: Effects of subsidies to e in period j on parental time across income quintile

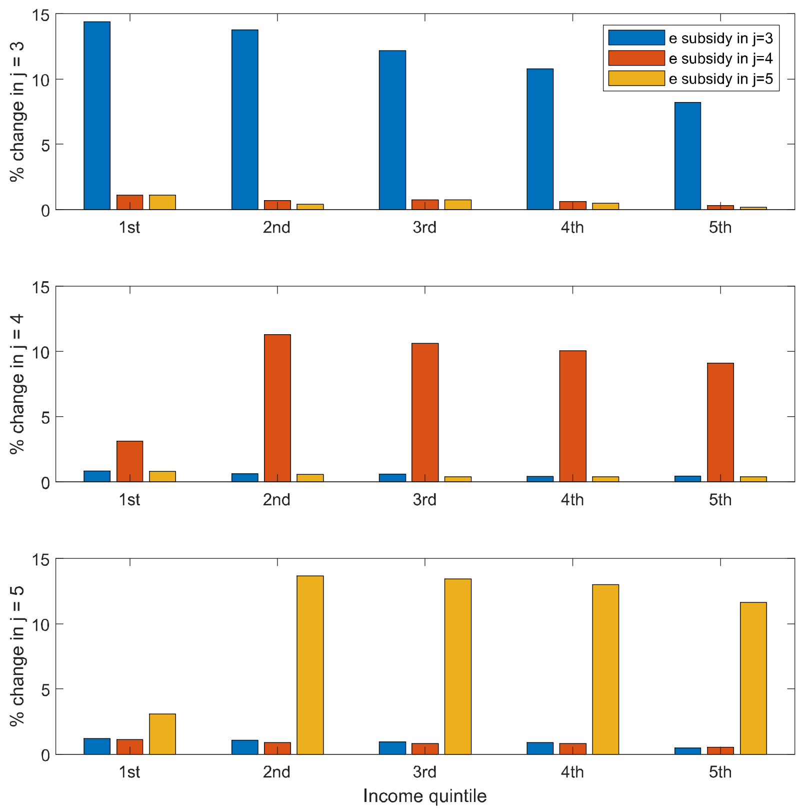
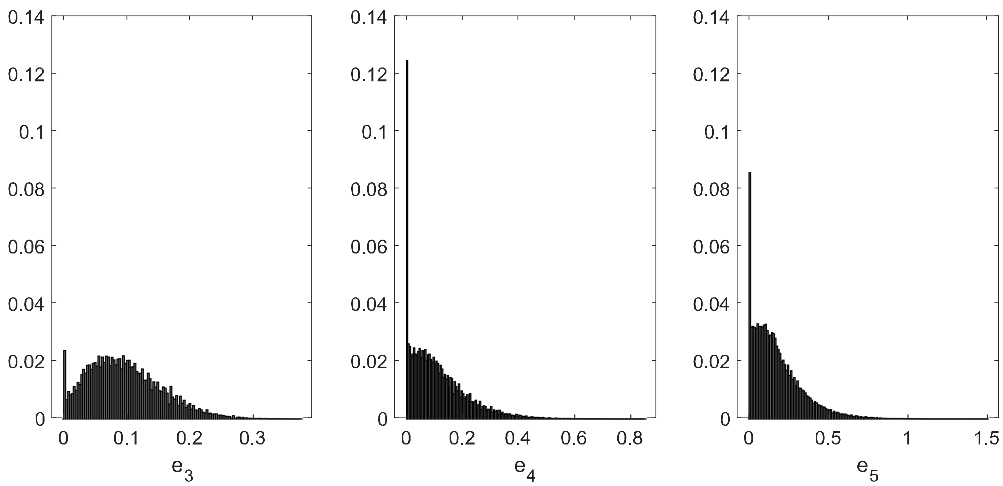


Figure 4: Distributions of simulated private education spending in the baseline economy



in public education spending. One thing to note is that regardless of the choice of the period for increasing public investments, education gradients in parental time investments tend to decline (especially in the targeted period), leading to greater intergenerational mobility. This is precisely related to the distribution of private investments, as shown in Figure 4. Because poor households are generally near zero investments, crowding-out effects of public investments disproportionately affect relatively richer households more, thereby reducing the gap in parental investments.

The overall lesson from the policy exercises above is that it is possible to increase intergenerational mobility through subsidizing parental investments during childhood. In particular, the results highlight some pros and cons of (i) proportional subsidies to private investments and (ii) direct expansions of government spending in education. The former tends to give rise to larger output and welfare gains, yet it may rather decrease intergenerational mobility when public investments are already sizeable (e.g., later childhood periods). The latter tends to induce greater mobility regardless of timing of policy target periods but crowding-out effects generally lead to relatively small output and welfare gains.

7 Conclusion

This paper has presented a quantitative model that encompasses various standard elements in the quantitative macroeconomics literature such as general equilibrium and incomplete markets as well as endogenous human capital development such as multiple-period childhood skill formation and college. I have found that the model successfully accounts for positive educational gradients in parental time investments as well as untargeted distributional aspects of intergenerational persistence of income, as observed in US data. I have investigated the quantitative role of various mechanisms and found that nearly 40% of the intergenerational persistence of lifetime income is reduced when educational gradients in parental time investments are eliminated, despite the alternative endogenous channels that parents could rely on to strengthen the intergenerational association. The policy experiments I examined in this paper illustrate that effective policies that are intended to increase intergenerational mobility should focus on narrowing down the gap in parental time investments. While doing so, the model also implies that there are better ways to achieve greater mobility in terms of sides effects on aggregate output and welfare.

The purpose of the policy exercises in this paper was to provide some important characteristics in designing actual policies to increase intergenerational mobility. An interesting avenue for future work is to design a more effective and implementable policy scheme that keeps the universal nature. For instance, a more ideal policy should specifically induce a greater amount of high-quality time investment towards able children born into low human capital families, as what the social planner who maximizes social welfare would do. In addition, it is important to note that this paper abstracts from spillover effects. Consider an example of play-centers. If parents can (i) learn parenting skills while watching how other parents spend time with their children in such centers or (ii) share valuable information directly regarding parenting while spending time in such centers, they could potentially increase their parenting quality at home as well. These spillovers effects could potentially strengthen the effects of the aforementioned policies on intergenerational mobility.

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A Appendix

A.1 Time-use data

Statistics regarding time-use are computed using the 2003-2017 waves of the ATUS, combined with the CPS. For all statistics reported, the ATUS statistical weights are used. To compute average hours worked and the fraction holds a college degree, I consider both men and women and include those whose age is greater than or equal to 30 and less than 65. A person is college-educated if the highest level of completed school or highest degree received is Bachelor's degree or above.

To construct the key variable of parental time investments, I focus on interactive activities that require the existence of both a parent and a child in a common space. Such categories include reading to/with children, playing with children, doing arts and crafts with children, playing sports

Table A1: Education gradients in parental time investments

	$j = 3$	$j = 4$	$j = 5$
College-educated	1.342	.561	.416
	(.133)	(.109)	(.091)
Sex	-2.62	-1.51	-1.20
	(.123)	(.101)	(.083)
Age	-.041	.016	.023
	(.009)	(.007)	(.006)
Married	-.911	-.318	-.102
	(.085)	(.064)	(.053)
R^2	.023	.014	.017
Average x	6.43	3.78	2.06

Notes: Numbers in parentheses are standard errors. The dependent variable is parental time x (weekly hours).

with children, talking with/listening to children, looking after children as a primary activity, caring for and helping children, doing homework, doing home schooling, and other related educational activities. For the time investment variable, I further restrict the sample to households who have any child and whose age is between 21 and 55 (inclusive), as in Guryan et al. (2008). The statistics by the model period is based on the age of youngest child: $j = 3, 4, 5$ correspond to age 0-4; age 5-9; age 10-14, respectively. Educational gradients in parental time investments are obtained by regressing parental time on a college indicator variable while controlling for sex, age, and marital status, as reported in Table $A1$. In fact, the coefficients on college are quite stable when control variables are added, in line with Guryan et al. (2008).

The time-diary survey also reports secondary activities and part of them may also include childcare. However, since the childcare time recorded as secondary activities is expected to be less active and the same hours may not be effective as an input to skill formation (Del Boca et al. 2014), I do not consider the time of childcare recorded as secondary activities, and only focus on childcare activities reported as a main activity.

A.2 More on parameter values calibrated externally

Table $A2$ reports the gross growth rates of human capital by age and education, computed based on the estimates from the PSID samples in Rupert and Zanella (2015). Table $A3$ reports the estimates

Table A2: Gross growth rates of human capital by age and education

$j =$	1	2	3	4	5	6	7	8
$\gamma_{j,1}$	1.231	1.052	1.017	1.004	0.998	0.995	0.994	0.994
$\gamma_{j,2}$	1.317	1.152	1.101	1.063	1.032	1.004	0.975	0.942

Notes: The above values are computed based on the estimates from the PSID samples in Rupert and Zanella (2015).

Table A3: Parameter values for progressive taxation and public education investments

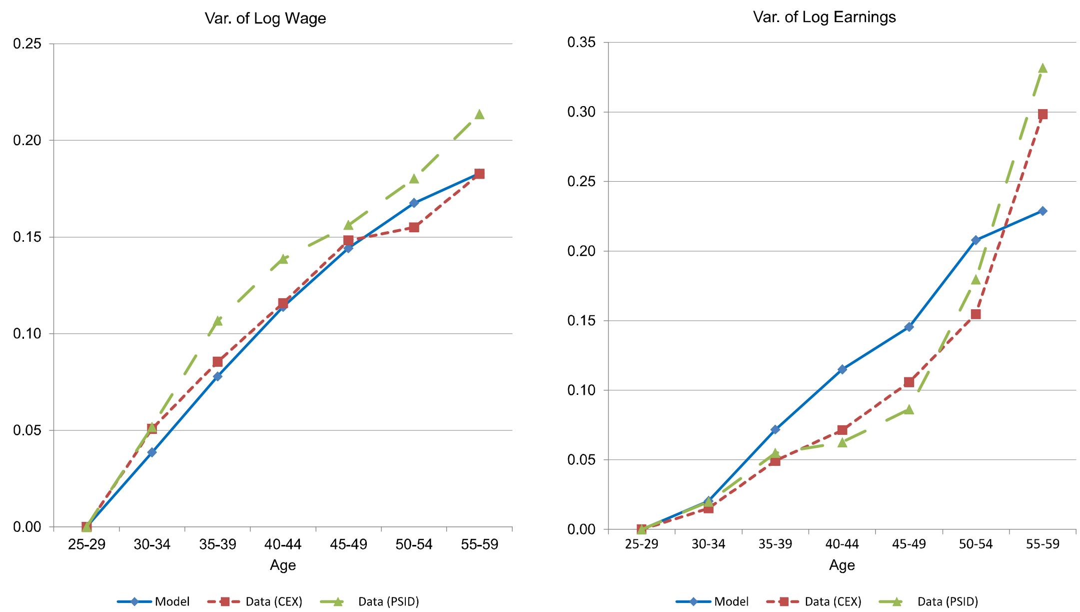
	τ_j	λ_j		g_j
$j = 1, 2$	.1106	.8177	$j = 3$	0.060
$j = 3, ..., 6$	.1585	.9408	$j = 4$	0.098
$j = 7, 8, 9$	.1080	.8740	$j = 5$	0.111

Notes: τ_j and λ_j are based on the estimates in Holter et al. (2019). Public education investments g_j are based on 2019 Education at a Glance (OECD).

of two parameters shaping the progressive taxation by age, obtained from Holter et al. (2019). Note that for $j = 1, 2$, estimates for single households are used, and for the later periods, estimates for married households are used (either with a child for $j = 3, ..., 6$ or without children for $j = 7, 8, 9$).

To compute the public education and private education investments (money), I use the 2016 information published in 2019 Education at a Glance by the OECD. In terms of mapping from the model period to education stages, I consider pre-primary as $j = 3$, primary as $j = 4$, and secondary as $j = 5$ in the model. As explained in the main text, I follow the approach of Restuccia and Urrutia (2004) and Holter (2015) by treating state and federal government spending as public investments whereas local government spending is part of private investments. By using the local share of public spending as 0.49, I obtain the adjusted shares of private and public investments for each period. Then, private and public investments are obtained by multiplying the total education expenditure per child ($j = 3$) or per student ($j = 4, 5$) at each education stage. Note that both mean private investments and mean public investments are approximately in line with the estimates in Lee and Seshadri (2019) based on the micro-level data with a relatively small number of samples.

Figure A1: Lifecycle inequality in the model and the data



A.3 Life cycle inequality

Figure $A1$ shows the life cycle inequality for wages and earnings in the model and the data. The data source is Heathcote et al. (2010). As in Heathcote et al. (2010), the unit of the y-axis is the variance of log relative to the initial age. The figures show that the model replicates the quantitative patterns of life cycle inequality that the dispersion of both wages and earnings increases with age.

A.4 Sensitivity analysis

First, I consider a calibration strategy where I match the overall educational gradient rather than period-specific educational gradients in parental time investments. The overall fit of the model is good except for educational gradients in parental time investment, which increase with age monotonically, as can be seen in Table $A4$. The counterfactual exercises regarding the role of parental time investment heterogeneity are roughly similar to the benchmark model in the main text while it is also clear to see that shutting down heterogeneity in parental time investments in period 3 has slightly weaker effects on intergenerational mobility in this case. This should not be

Table A4: Alternative calibration 1: Quantitative effects on intergenerational mobility

	IGE	Rank corr	Upward mobility (%)	Educ. gradient in x_j (%) $j=3$	$j=4$	$j=5$	Mean P/Y
Baseline	.378	.369	6.7	17.2	21.5	24.7	.056
Counterfactuals							
- $x_3 = \bar{x}_3$	.338	.328	7.6	0.0	21.7	24.9	.058
- $x_4 = \bar{x}_4$	.332	.322	7.8	17.7	0.0	25.1	.059
- $x_5 = \bar{x}_5$	.319	.308	8.1	17.4	21.8	0.0	.059
- $x_3 = \bar{x}_3$ & $x_4 = \bar{x}_4$	.290	.280	8.8	0.0	0.0	25.6	.062
- $x_4 = \bar{x}_4$ & $x_5 = \bar{x}_5$	.270	.260	9.4	18.1	0.0	0.0	.063
- $x_3 = \bar{x}_3$ & $x_5 = \bar{x}_5$	.275	.265	9.3	0.0	22.6	0.0	.062
- $x_j = \bar{x}_j$ for $j = 3, 4, 5$	.224	.216	10.7	0.0	0.0	0.0	.066
- No inter-vivos transfers	.395	.385	6.4	18.1	22.3	25.4	.000
- $g_3 = 0$	.383	.372	6.5	17.0	20.7	23.8	.057
- $g_4 = 0$	.395	.382	6.0	16.5	23.0	23.7	.057
- $g_5 = 0$	.398	.385	5.9	16.6	20.8	27.3	.055
- No persistence in ϕ	.315	.306	8.6	17.0	21.1	24.1	.058
- No idiosyncratic shocks	.395	.377	5.0	17.2	22.0	25.1	.053

Notes: The above results are based on the alternative calibration that imposes $\zeta_j = \zeta$ for $j = 3, 4, 5$.

surprising since the baseline model in this alternative calibration generates a lower educational gradient in parental time in the first place (17.2%). The overall effect of heterogeneity in parental time investments (the final row) gives a similar quantitative effect that both IGE and rank correlation of lifetime income would decrease nearly by 40%.

I also consider an alternative calibration in that the persistence of ability is imposed to be higher at $\rho_\phi = 0.15$. Then, I recalibrate the model with the same set of target statistics excluding only the intergenerational correlation of income (which is the main target of the parameter ρ_ϕ in the main text). Table $A5$ summarizes the quantitative role of various mechanisms in this alternative calibration. Note that the baseline model in this alternative calibration features lower intergenerational mobility as I do not allow ρ_ϕ to be calibrated to match the observed rank correlation. Nevertheless, it is interesting to note that the quantitative role of various channels is very similar

Table A5: Alternative calibration 2: Quantitative effects on intergenerational mobility

	IGE	Rank corr	Upward mobility (%)	Educ. gradient in x_j (%) $j=3$	$j=4$	$j=5$	Mean P/Y
Baseline	.418	.409	5.6	20.6	14.8	20.2	.056
Counterfactuals							
- $x_3 = \bar{x}_3$	.371	.361	6.7	0.0	15.3	20.7	.059
- $x_4 = \bar{x}_4$	.384	.374	6.5	21.1	0.0	20.6	.060
- $x_5 = \bar{x}_5$	.371	.360	6.8	20.8	15.3	0.0	.060
- $x_3 = \bar{x}_3$ & $x_4 = \bar{x}_4$	.334	.324	7.7	0.0	0.0	21.4	.062
- $x_4 = \bar{x}_4$ & $x_5 = \bar{x}_5$	.333	.323	7.9	21.6	0.0	0.0	.063
- $x_3 = \bar{x}_3$ & $x_5 = \bar{x}_5$	.320	.309	8.1	0.0	15.9	0.0	.062
- $x_j = \bar{x}_j$ for $j = 3, 4, 5$	.279	.270	9.3	0.0	0.0	0.0	.066
- No inter-vivos transfers	.436	.426	5.5	21.5	15.7	21.0	.000
- $g_3 = 0$	.423	.413	5.5	20.5	14.5	19.9	.058
- $g_4 = 0$	.436	.423	5.0	20.2	17.2	19.7	.057
- $g_5 = 0$	.438	.425	5.0	20.3	14.6	23.4	.056
- No persistence in ϕ	.305	.295	8.8	20.2	15.0	19.9	.060
- No idiosyncratic shocks	.440	.424	4.0	20.9	15.7	21.1	.053

Notes: The above results are based on the alternative calibration that imposes $\rho_\phi = 0.15$.

to the baseline calibration in the main text. As the change in mobility measures are similar in magnitude, percentage changes in correlations relative to the baseline are smaller but percentage changes in upward mobility becomes larger. This is because the baseline model in this alternative calibration features higher IGE and rank correlation yet lower upward mobility in the first place.

Table *A6* shows the policy exercises on subsidizing college with a different elasticity of substitution between skilled and unskilled workers since policy effects may be sensitive to this elasticity. Specifically, I set the value of ρ to $2/3$ so that the elasticity becomes 3. This value is quite close to 3.3 in Abbott et al. (2019). The results show that the general equilibrium results are much stronger with the higher elasticity although the magnitudes are still much less than the fixed price cases. More importantly, it is worth noting that the effects on intergenerational mobility are nearly unaffected by this elasticity.

Table A6: Alternative calibration 3: Effects of providing easier access to college

	(1)	(2)	(3)	(4)	(5)
	Baseline	$s = 0.1$		$s = 0.2$	
		FP	GE	FP	GE
IGE	.376	.376	.376	.376	.375
Rank correlation	.367	.367	.366	.367	.366
Upward mobility (%)	6.8	6.7	6.8	6.7	6.8
College fraction (%)	34.1	38.1	34.6	42.9	35.0
Observed college premium (%)	80.4	81.6	78.8	83.3	78.1
Educ. gradient in x_j (%)					
- $j = 3$	20.4	20.8	20.1	21.2	19.9
- $j = 4$	14.9	15.0	14.6	15.1	14.4
- $j = 5$	20.0	20.3	19.6	20.5	19.4
Aggregate output *(% chg)*		-	-0.0	-	-0.1
Aggregate capital *(% chg)*		-	-0.1	-	-0.1
Consumption equiv.*(%)*		+0.2	+0.1	+0.5	+0.3

Notes: The above results are based on the alternative calibration where the elasticity of substitution between skilled and unskilled workers are set to 3.

Table $A7$ reports the exercise that subsidizes private education spending in period 5 ($s_{e,5} = 0.2$). Here, the important difference (compared to the counterpart in the main text) is that the baseline economy features zero public investment ($g_5 = 0$). Unlike the result in the main text, we can see that $s_{e,5}$ reduces educational gradients in parental time investments and increases mobility in this case. This illustrates the importance of the size of public investments when determining the effects of subsidies to private education spending on intergenerational mobility, as highlighted in the main text.

Table A7: Effects of private investment subsidy in the absence of public investment

	$g_5 = 0$	$g_5 = 0$ $s_{e,5} = 0.2$
IGE	.397	.391
Rank correlation	.384	.379
Upward mobility (%)	5.9	6.0
Mean (% chg)		
- $\bar{e}_3$ (relative to Y)	.097	.095
- $\bar{e}_4$	.111	.109
- $\bar{e}_5$	.235	.284
- $\bar{x}_3$ (hrs/wk)	6.5	6.6
- $\bar{x}_4$	3.8	3.9
- $\bar{x}_5$	2.1	2.3
Educ. gradient in x_j (%)		
- $j = 3$	20.2	19.7
- $j = 4$	14.6	14.3
- $j = 5$	23.5	22.5
Aggregate output *(% chg)*	-	+2.3
Aggregate capital *(% chg)*	-	+2.3
Consumption equiv. *(%)*	-	+2.8