

PERCEPTRONS AND MULTILAYER PERCEPTRONS

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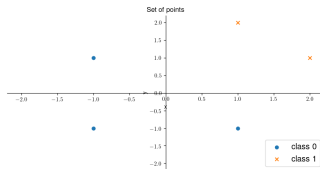
LEARNING THE PERCEPTRON

A toy example

- ▶ 5 points $x_i = (x_i^1, x_i^2)$
- ▶ 2 classes (0,1)
- ▶ Linear perceptron $w^T x + b > 0$?
- ▶ Error e = real class - predicted class
- ▶ Init: $(w_0, w_1, b) = (-0.1, 0.1, 0)$
- ▶ Parameters update:

$$1 \quad w_{k+1} = w_k + ex$$

$$2 \quad b_{k+1} = b_k + e$$



x^1	x^2	c
1	2	1
2	1	1
-1	1	0
-1	-1	0
1	-1	0

LEARNING THE PERCEPTRON

w_0	w_1	b	x^1	x^2	c	$z = \mathbf{w}^T x + b$	$H(z)$	e
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LEARNING THE PERCEPTRON

w_0	w_1	b	x^1	x^2	c	$z = \mathbf{w}^T \mathbf{x} + b$	$H(z)$	e
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-0.1	0.1	0	2	1	1	-0.1	0	1

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-0.1	0.1	0	1	2	1	0.1	1	0
-0.1	0.1	0	2	1	1	-0.1	0	1
1.9	1.1	1	-1	1	0	0.2	1	-1

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2.9	0.1	0	-1	-1	0	-3	0	0

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2.9	0.1	0	-1	-1	0	-3	0	0
2.9	0.1	0	1	-1	0	2.8	1	-1

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2.9	0.1	0	1	-1	0	2.8	1	-1
1.9	1.1	-1	1	2	1	3.1	1	0
1.9	1.1	-1	2	1	1	3.9	1	0

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-0.1	0.1	0	2	1	1	-0.1	0	1
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1.9	1.1	-1	-1	1	0	-0.8	0	0

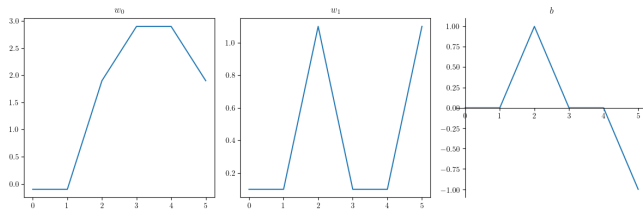
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LEARNING THE PERCEPTRON



And the decision boundary is $1.9x^1 + 1.1x^2 - 1 = 0$

