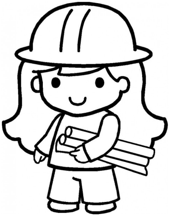

Architecture design

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Architecture

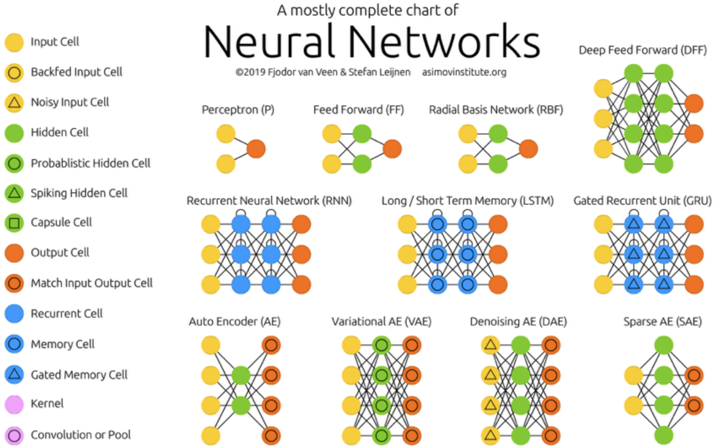
Architecture

- Number of layers (depth)
- Number of neurons per layer
- Type of neurons
- Type of connections between neurons / layers

Classical Network architectures

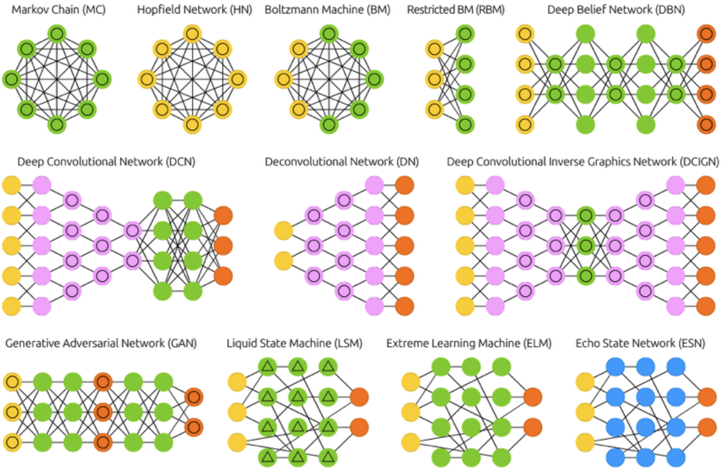
- Most networks are organized into groups of layers, arranged in a chain structure
- Each layer is a function of the previous one

A (MOSTLY) COMPLETE ZOO[1]



[1] source: The Asimov Institute

A (mostly) complete Zoo



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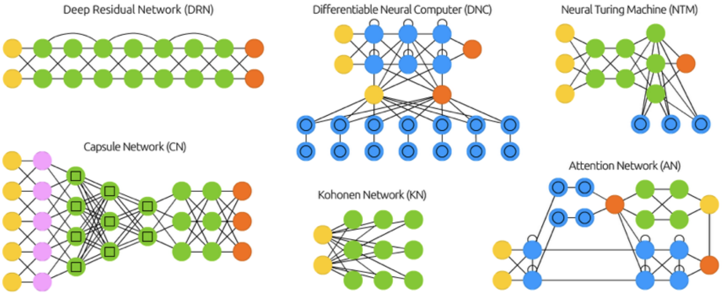
A (MOSTLY) COMPLETE ZOO



Universal approximation

Theorem (Cybenko 1989; Hornik et al, 1991)

σ: bounded, non-constant continuous function, - I_d: d-dimensional hypercube - $C(I_d)$ space of continuous functions on I_d.
$(\forall f \in C(I_d))(\forall \epsilon > 0)(\exists q > 0, v_i, \mathbf{w_i}, b_i, i \in [\![1 \ldots q]\!])$ such that

$$F(\mathbf{x}) = \sum_{i=1}^{q} v_i \sigma(\mathbf{w}^{\mathbf{T}}\mathbf{x} + b)$$

satisfies $\sup_{\mathbf{x} \in I_d} \mid f(\mathbf{x}) - F(\mathbf{x}) \mid < \epsilon$

And so..

A feed-forward network with a single hidden layer containing a finite number of neurons can approximate continuous functions on compact subsets of $\mathbb{R}^d$, under mild assumptions on the activation function

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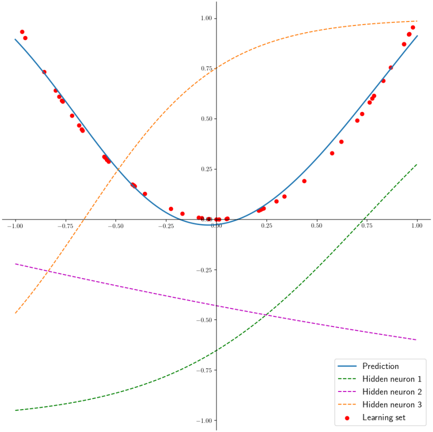
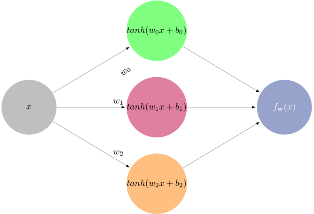
Universal approximation



$f(x) = x^2, |Z| = 50$

A simple example

- $|Z|$ points uniformly sampled (red) over the definition set
- 1 hidden layer MLP, 3 neurons.
- $tanh$ activation function, and linear output neurons
- network output : blue curve
- hidden neurons outputs: dashed curves



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Universal approximation

Good news but...

- Does not inform about good/bad architectures, the number of neurons q nor how they relate to the optimization procedure
- Bounds on size of the single-layer network exist for a broad class of functions....
- But worst case is exponential / q

Bad news :-(: No Free Lunch theorem

There is no universal procedure for examining a training set of samples and choosing a function that will generalize to points not in training set

Effect of depth

Up to now

- A feedforward network with a single layer is sufficient to represent "any" function
- But the layer may be infeasibly large and may fail to generalize well
- Using deeper models can reduce number of units required and reduce generalization error

Theorem (Montúfar et al, 2014)

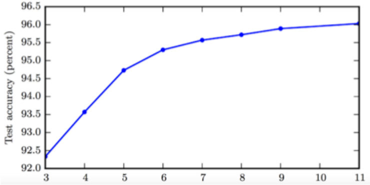
A MLP with ReLU as activation functions, p inputs, L hidden layers with $q \geq p$ neurons can compute functions having $\Omega\left(\left(\frac{q}{p}\right)^{(L-1)p} q^p\right)$ linear regions.

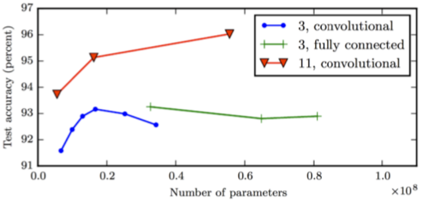
Properties

- The number of linear regions of deep models grows exponentially in L and polynomially in q.
- Even for small values of L and q, deep rectifier models are able to produce substantially more linear regions than shallow models.

Effect of depth

- Test accuracy consistently increases with depth
- Increasing parameters without increasing depth is not as effective





Deep architectures express a useful prior over the space of functions the model learns

Specialized architectures

CNN, RNN,...Discussed in future lectures

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