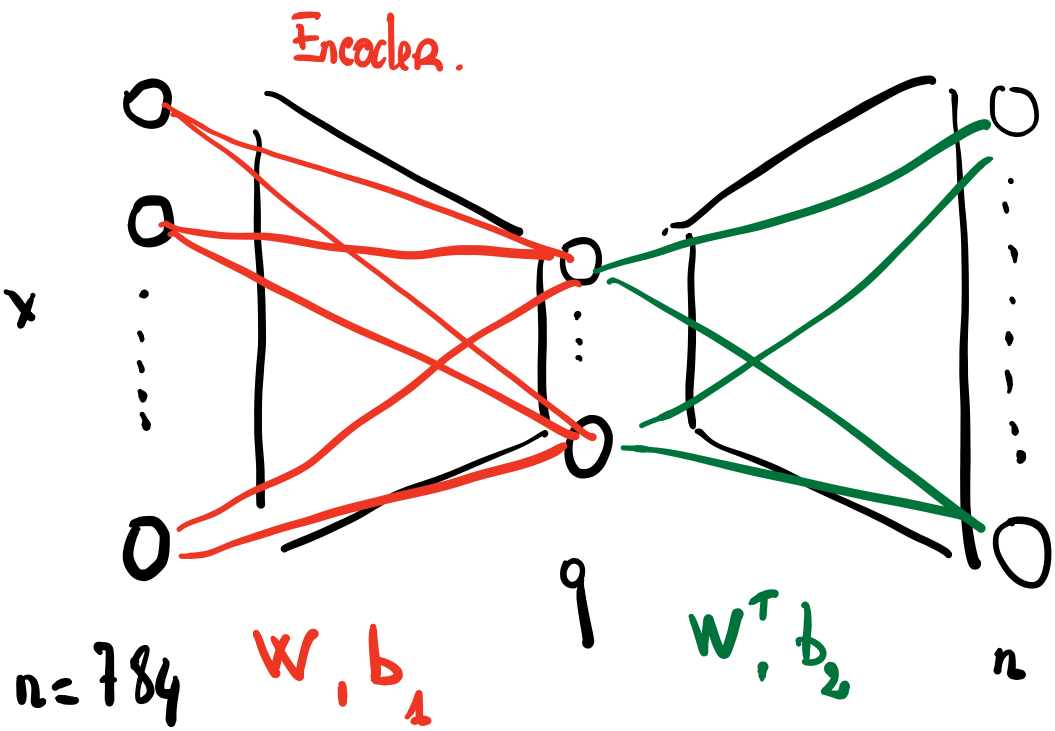
Autoencoders



Encoder: $f(x)=\sigma_1(Wx+b_1)$

$$g[f(x)] = g(h) = \sigma_2(W^T h + b_2)$$

$$\hat{x} = \boxed{g[f(x)] = \sigma_2\left[W^T\left(\sigma_1(Wx+b_1)\right)+b_2\right]}$$

$$L(x,\hat{x}) = \|x-\hat{x}\|^2$$

$$\boxed{\min_{W,b_1} L(x,\hat{x})} \rightarrow$$ SGD
Backpropagation

Principal Component Analysis.

$X \in \mathcal{M}_{np}(\mathbb{R})$: n data points
↳ each data $\in \mathbb{R}^p$

$$X = \begin{pmatrix} \text{———} \\ \text{———} \\ \vdots \\ \text{———} \end{pmatrix} \qquad x_i = \begin{pmatrix} \text{Age} \\ \text{Size} \\ \text{Size of foot} \\ \text{Size of legs} \end{pmatrix}$$

$\mathbb{R}^4 \to \mathbb{R}^2$

First: Normalize data : For each parameter (each column of x)

$$Y = X - \mathbb{1}^T g \qquad g \in \mathbb{R}^p = \begin{pmatrix} \overline{\text{Age}} \\ \overline{\text{Size}} \\ \vdots \end{pmatrix}$$

↳ Y has 0-mean columns

$$\boxed{Z = Y D_{1/\sigma}} \qquad D_{1/\sigma} = \begin{pmatrix} 1/\sigma_1 & & 0 \\ & \ddots & \\ 0 & & 1/\sigma_p \end{pmatrix}$$

$Z^T Z \in \mathcal{M}_p(\mathbb{R})$ symmetric

$\hookrightarrow \quad \underline{Z^T Z} = P^T \Lambda P \qquad \Lambda = \begin{pmatrix} \lambda_1 & & 0 \\ & \ddots & \\ 0 & & \lambda_p \end{pmatrix}$

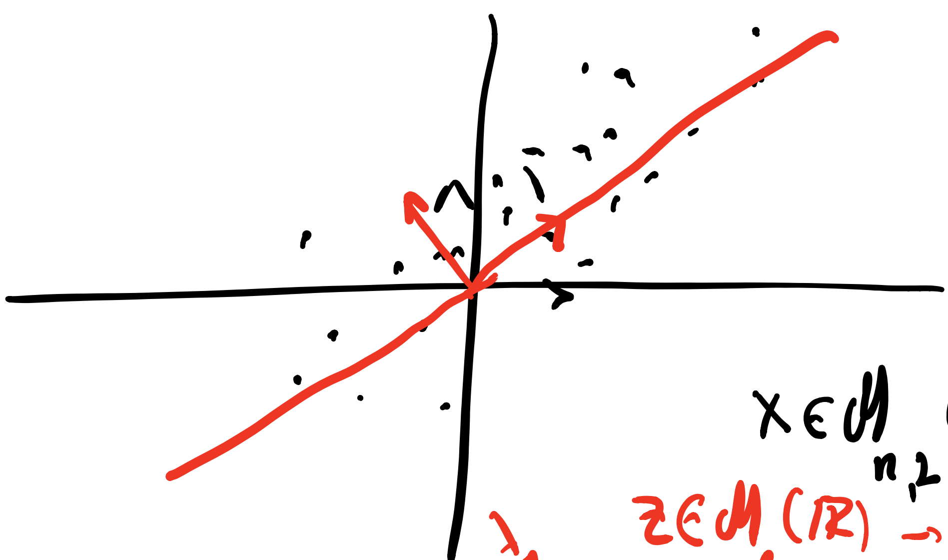
eigenvalues of $Z^T Z$.

P: eigenvectors

$P = \begin{pmatrix} p_1 & \cdots & p_p \end{pmatrix}$

- $\underline{p_1} \cdots \underline{p_p}$: principal components of $Z^T Z$
- $\lambda_1 \cdots \lambda_p$

$\dfrac{\lambda_1}{\lambda_1 + \cdots + \lambda_p}$: % of inertia explained by p_1



$X \in \mathcal{M}_{n,2}(\mathbb{R})$

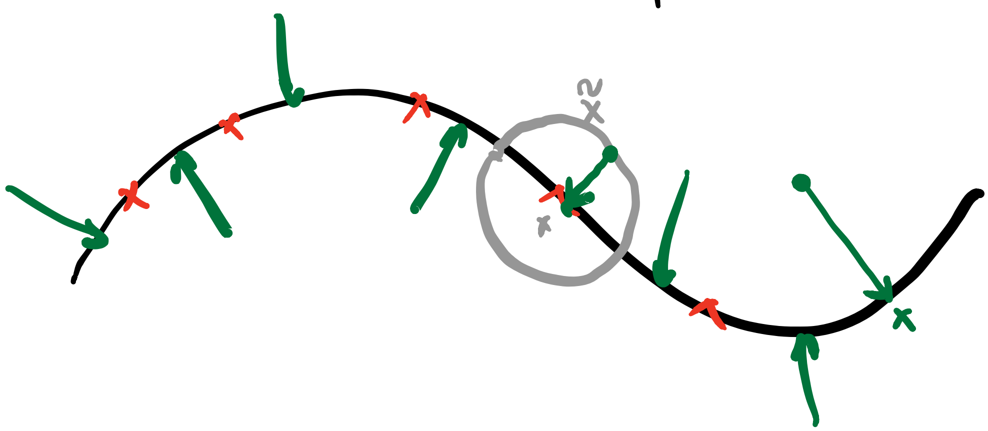
$Z \in \mathcal{M}_2(\mathbb{R}) \rightarrow p_1, p_2$

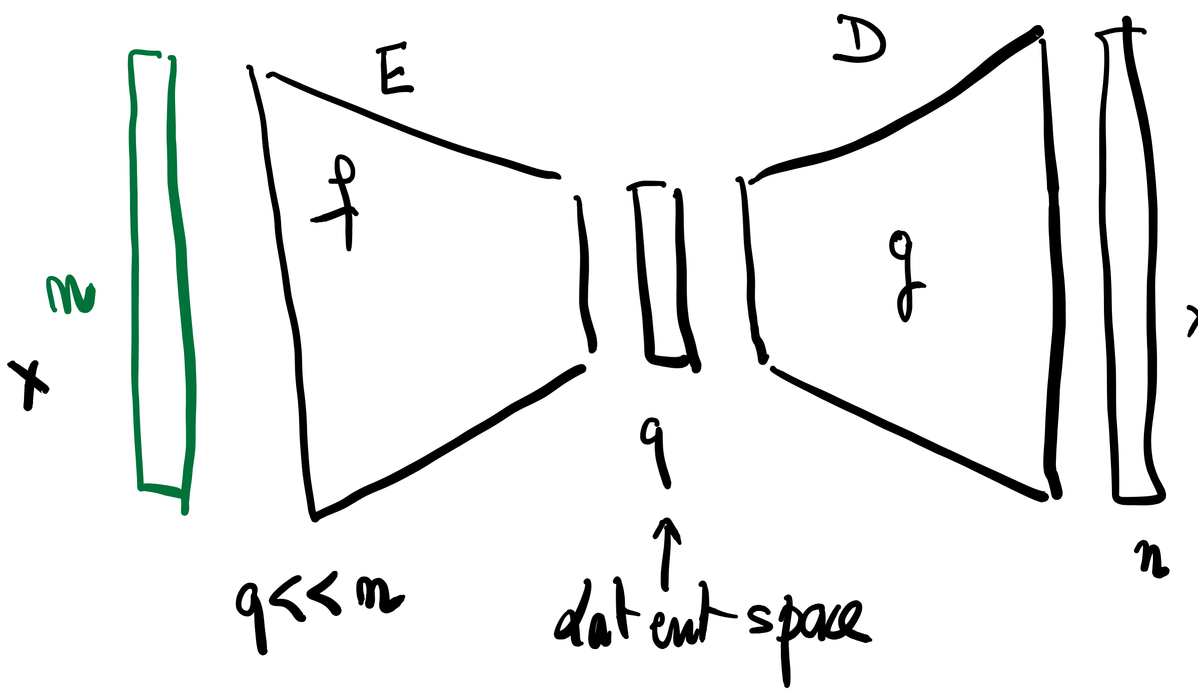
$\dfrac{\lambda_1}{\lambda_1 + \lambda_2}$.

PCA: dimension reduction technique

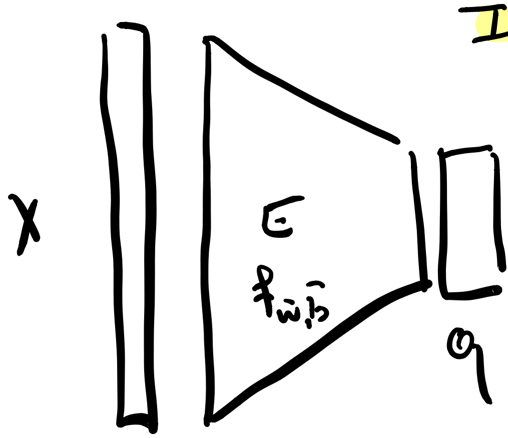
: objective : explain as much as possible the variation of the original data.

Vector field in latent space





Inside latent space



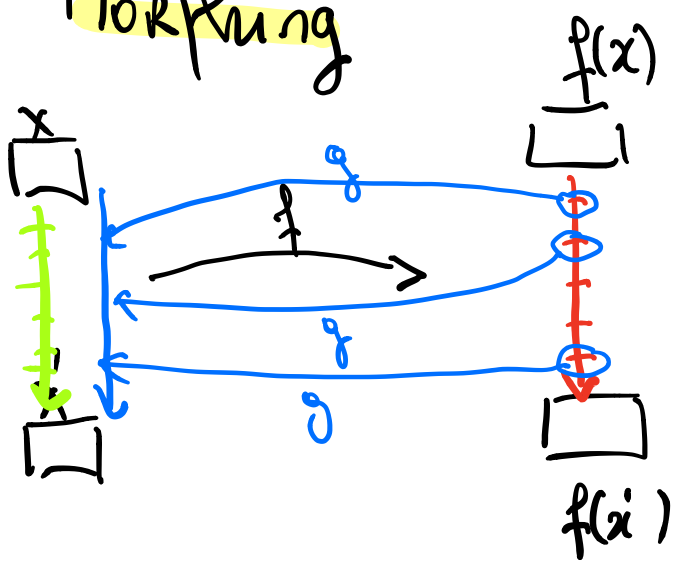
$x \in \mathbb{R}^n \to \underline{h} \in \mathbb{R}^q$

$q << n$

$\boxed{\text{Data compression}}$

↳ Inside latent space.

Morphing



linear interpolation

$\mathbb{R}^n$