Master's Thesis Presentation

On the Parameterized Complexity of SEMITOTAL DOMINATING SET On Graph Classes

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Motivation
Theory
Intractability
Kernel
Rule 1
Rule 2
Rule 3
References

Creative Introduction

Our Plan for Today

1. Motivation
2. Theory
   - Intractability
3. Kernel
   - Rule 1
   - Rule 2
   - Rule 3

Motivation

DOMINATING SET

Input	Graph $G = (V, E)$, $k \in \mathbb{N}$
Question	Is there a set $D \subseteq V$ of size at most k such that $N[D] = V$?

- The domination number is the minimum cardinality of a ds of G, denotes as $\gamma(G)$
- **Observation:** In connected G every $v \in D$ has another $z \in D$ with $d(v, z) \leq 3$.

Motivation

TOTAL DOMINATING SET

Input	Graph $G = (V, E)$, $k \in \mathbb{N}$
Question	Is there a set $D \subseteq V$ of size at most k such that for all $d_1 \in X$ exists $d_2 \in X \setminus \{d_1\}$ s.t. $d(d_1, d_2) \leq 1$?

- The total domination number is the minimum cardinality of a tds of G, denoted as $\gamma_t(G)$.

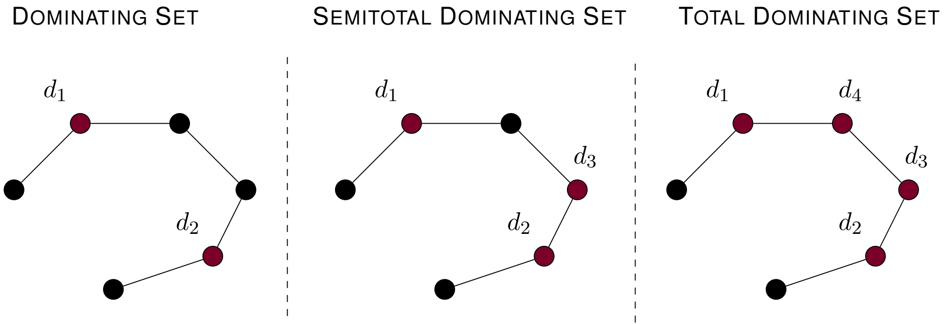
Motivation

SEMITOTAL DOMINATING SET

Input	Graph $G = (V, E)$, $k \in \mathbb{N}$
Question	Is there a subset $D \subseteq V$ with $\lvert D \rvert \leq k$ such that $N[D] = V$ and for all $d_1 \in X$ there exists another $d_2 \in X$ such that $d(d_1, d_2) \leq 2$?

- The semitotal domination number is the minimum cardinality of a sds of G, denoted as $\gamma_{2t}(G)$.
- **Observation**: $\gamma(G) \leq \gamma_{\mathbf{2t}}(\mathbf{G}) \leq \gamma t(G)$

Example: $\gamma(G) < \gamma_{\mathbf{2t}}(\mathbf{G}) < \gamma_t(G)$



Parameterized Complexity

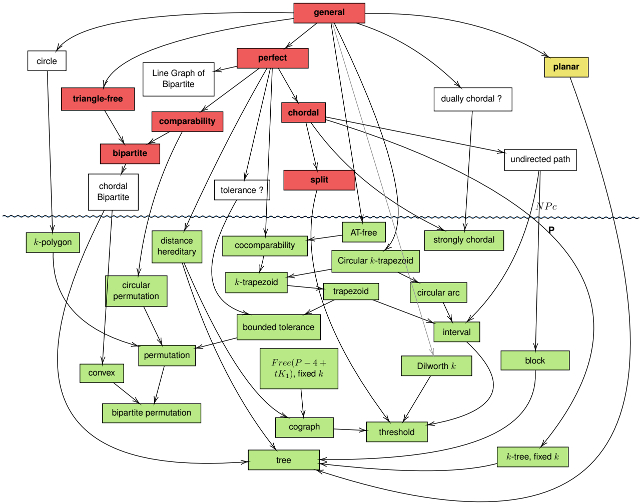
- NP-hard? We expect problem to be **at least** exponential
- **Idea:** Limit combinatorial explosion to some aspect of the problem
- **Goal:** Find an algorithm running in time $\mathcal{O}(f(k) \cdot n^c)$ for **some** parameter k
- In this work: by solution size
- **Techniques:** Kernelization, Bounded Search Trees, ...

If possible, the problem is **fixed-parameter tractable**.

Complexity Comparison

Graph Class	Dominating Set		Semitotal Dominating Set		Total Dominating Set	
	classical	Parameterized	classical	Parameterized	classical	Parameterized
bipartite	NPc [4]	W_2 [39]	NPc [25]	W_2 (this)	NPc [32]	W_2 (cite!)
line graph of bipartite	NPc [28]	?	NPc [19]	? (?)	NPc [35]	?
circle	NPc [26]	W_1 [7]	NPc [27]	? (?)	NPc [35]	W_1 [7]
chordal	NPc [6]	W_2 [39]	NPc [25]	W_2 (this)	NPc [37]	W_1 [11] by *split*
s-chordal , $s > 3$	NPc [33]	W_2 [33]	? (?)	? (?)	NPc [33]	W_1 [33]
split	NPc [4]	W_2 [39]	NPc [25]	W_2 **this**	NPc [37]	W_1 [11]
3-claw-free	NPc [14]	FPT [14]	Prob. Unk	Prob. Unk	NPc [35]	Unknown
t-claw-free, $t > 3$	NPc [14]	W_2 [14]	Prob. Unknown	Unknown	NPc [35]	Prob. Unknown
chordal bipartite	NPc [36]	? (?)	NPc [25]	?	P [15]	
planar	NPc (Sources!)	FPT [2]	NPc	**FPT (this)**	NPc	FPT [20]
undirected path	NPc [6]	FPT [18]	NPc [24]	?	NPc [31]	?
dually chordal	P [8]		? (attempted [19])		P [30]	
strongly chordal	P [17]		P [40]		NPc [17]	
AT-free	P [29]		P [27]		P [29]	
tolerance	P [22]		?		?	
block	P [17]		P [24]		P [10]	
interval	P [12]		P [38]		P [5]	
bounded clique-width	P [13]		P [13]		P [13]	
bounded mim-width	P [3, 9]		P [19]		P [3, 9]	

Status SEMITOTAL DOMINATING SET



Fixed-Parameter Intractability

- Class NP corresponds to whole hierarchy $W[i]$ in parameterized setting.
- Problems at least $W[1]$-hard considered **fixed-parameter intractable**
- DOMINATING SET is $W[2]$-complete
- **Tool for Proving Hardness**: FPT Reductions, preserving the parameter

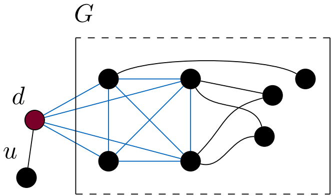
Warmup: Intractability Results

ω_2 hard on split, chordal and bipartite graphs

- **Split Graph:** $G = \texttt{Clique} + \texttt{IndependentSet}$

Split Graphs

SEMITOTAL DOMINATING SET on *split* and *chordal* graphs is ω_2-hard

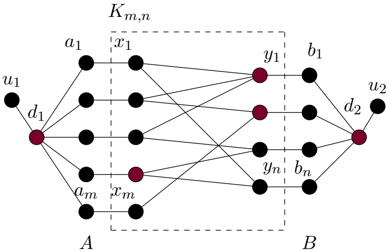


Proof by fpt-reduction from PLANAR DOMINATING SET on split graphs:

1. **Construct** G* by adding v with pendant z to clique. G* split
2. If ds D in G, $D* = D \cup \{v\}$ is sds $D*$.
3. If sds $D*$ in $G*$, $D \setminus \{v\}$ is D in G
4. Parameter k only changed by constant

Bipartite Graphs

SEMITOTAL DOMINATING SET on *bipartite* graphs is ω_2-hard



Proof by fpt-reduction from PLANAR DOMINATING SET on bipart. graphs:

1. **Construct** Add new neighbor to each vertex and add d_1, d_2, u_1, u_2
2. If ds D in G, then $D* = D \cup \{d_1, d_2\}$ is sds in $G*$
3. Assume sds $D*$ in $G*$. If $a_i \in D*$ (b_i), flip. $D = D * \setminus \{d_1, d_2\}$ is ds in G

A Linear Kernel for PLANAR SEMITOTAL DOMINATING SET

Another Explicit kernel for a Dominating Problem

Kernelization

- **Idea:** Preprocess an instance using *Reduction Rules* until hard *kernel* is found.

Kernelization

- **Idea:** Preprocess an instance using *Reduction Rules* until hard *kernel* is found.

$$\overbrace{(I,\ k) \in Q}^{n}$$

$$\Big\downarrow \ \mathcal{O}\left(n^c\right)$$

$$\underbrace{(I',\ k') \in Q}_{g(k)}$$

Related Works

Problem	Size	Source
PLANAR DOMINATING SET	$67k$	[16]
PLANAR TOTAL DOMINATING SET	$410k$	[20]
PLANAR SEMITOTAL DOMINATING SET	$xxxxk$	**This work**
PLANAR EDGE DOMINATING SET	$14k$	[23]
PLANAR EFFICIENT DOMINATING SET	$84k$	[23]
PLANAR RED-BLUE DOMINATING SET	$43k$	[21]
PLANAR CONNECTED DOMINATING SET	$130k$	[34]
PLANAR DIRECTED DOMINATING SET	Linear	[1]

Main Theorem

The Main Theorem

SEMITOTAL DOMINATING SET parameterized by solution size admits a linear kernel on planar graphs. There exists a polynomial-time algorithm that, given a planar graph (G, k), either correctly reports that (G, k) is a NO-instance or returns an equivalent instance (G', k) such that $|V(G')| \leq xxx \cdot k$.

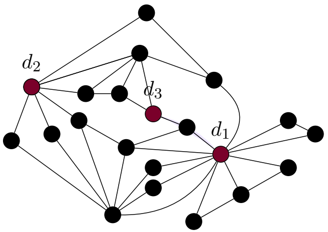
The xxxStone: Regions

Introduced by Alber et al. [2], decomposition technique for planar graph.

Region (Simplified)

Given plane G and $v, w \in V$, a region is a closed subset, such that

- there are two non-crossing (but possibly overlapping) boundary paths
- Every vertex in R belongs to $N(v, w)$



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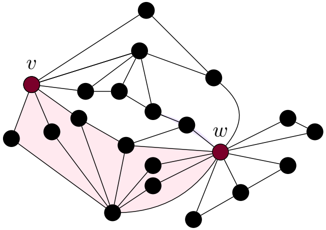

Introducing D-region decomposition

D*-region decomposition* [2]

Given $G = (V, W)$ and $D \subseteq V$, a D*-region decomposition* is a set $\mathfrak{R}$ with poles in D such that:

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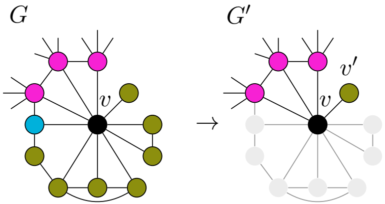
Splitting up $N(v)$

Rule 1, Appetizer: Shrinking $N_3(v)$

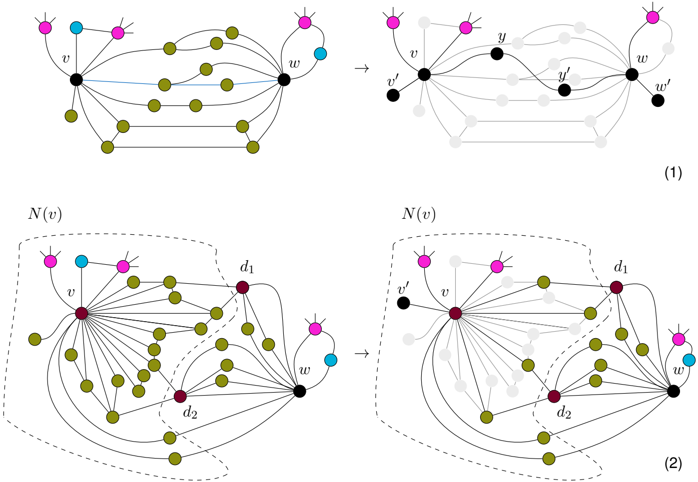
Let $G = (V, E)$ be a graph and let $v \in V$. If $|N_3(v)| \geq 1$:

- remove $N_{2,3}(v)$ from G,
- add a vertex v' and an edge $\{v, v'\}$.



- **Idea:** Removing isolated vertices
- Correctness: Omitted

Rule 2



Rule 3: Shrinking the size of simple regions

Let $G = (V, E)$ be a plane graph, $v, w \in V$ and R be a simple region between v and w. If $|V(R) \setminus \{v, w\}| \geq 5$ apply the following:

Case 1: If $G[R \setminus \partial R] \cong P_3$, then:

- remove $V(R \setminus \partial R)$
- add vertex y with edges $\{v, y\}$ and $\{y, w\}$

Case 2: If $G[R \setminus \partial R] \ncong P_3$, then

- remove $V(R \setminus \partial R)$
- add vertices y, y' and four edges $\{v, y\}$, $\{v, y'\}$, $\{y, w\}$ and $\{y', w\}$

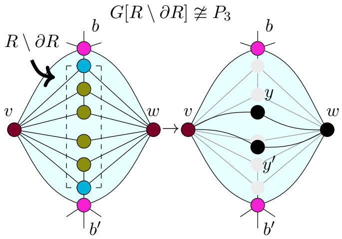
Rule 3: Shrinking the size of simple regions

Case 1: If $G[R \setminus \partial R] \cong P_3$, then:

- **remove $V(R \setminus \partial R)$**
- **add vertex y with edges $\{v, y\}$ and $\{y, w\}$**

Case 2: If $G[R \setminus \partial R] \not\cong P_3$, then

- remove $V(R \setminus \partial R)$
- add vertices y, y' and four edges $\{v, y\}$, $\{v, y'\}$, $\{y, w\}$ and $\{y', w\}$



Rule 3: Shrinking the size of simple regions

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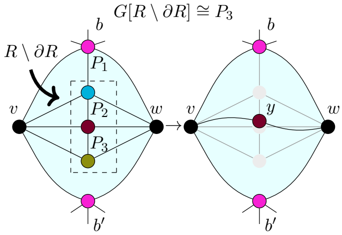
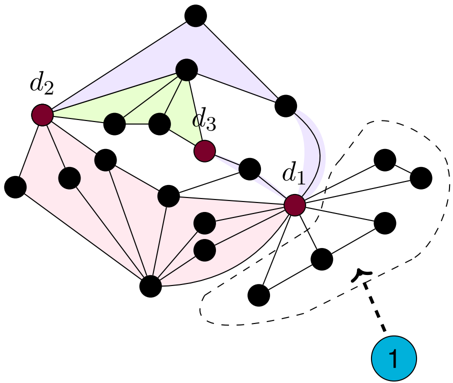
- **add vertices y, y' and four edges $\{v, y\}$, $\{v, y'\}$, $\{y, w\}$ and $\{y', w\}$**



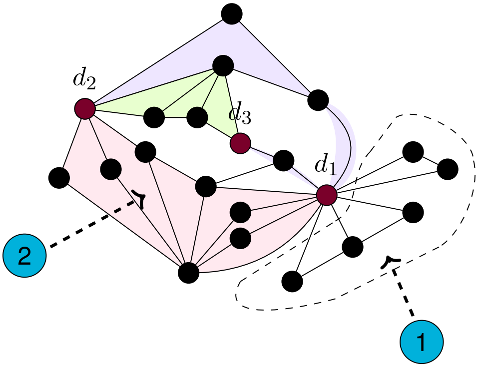
Notes

- All the rule are sound
- and only change the solution size by a constant factor
- they can be applied in pplynomial-time
- Rule 3 is a swiss-army-knife to be found on many surprising places

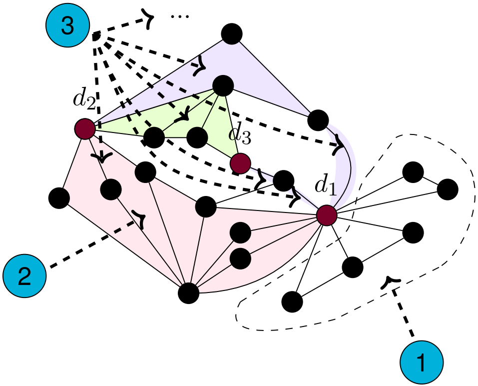
Bounding the Kernel: Idea 1



Bounding the Kernel: Idea 2



Bounding the Kernel: Idea 3



Summary: Bounding Kernel Size

Let D be sds of size k. There exists a maximal D*-region decomposition* $\mathfrak{R}$ such that:

1. $\mathfrak{R}$ has only at most $3k - 6$ regions ([2]);
2. There are at most $97 \cdot k$ vertices outside of any region;
3. Each region $R \in \mathfrak{R}$ contains at most 87 vertices.

Hence: $87 \cdot (3k - 6) + 97 \cdot k + k < 359 \cdot k$

Main Theorem

The Main Theorem

The SEMITOTAL DOMINATING SET problem parameterized by solution size admits a linear kernel on planar graphs. There exists a polynomial-time algorithm that, given a planar graph (G, k), either correctly reports that (G, k) is a NO-instance or returns an equivalent instance (G', k) such that $|V(G')| \leq xxx \cdot k$.

Proof: Add Proof here.

Conclusions

Results:

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Future Work:

- Improve Kernel Size
- Solve complexities for...

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