Master's Thesis Presentation

On the Parameterized Complexity of SEMITOTAL DOMINATING SET On Graph Classes

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Creative Introduction

Our Plan for Today

1. Motivation
2. Theory
   - Intractability
   - ω_2 hardness
3. Kernel
   - Definitions
   - Rule 1
   - Rule 2
   - Rule 3
   - Kernel Size
4. Conclusions

Motivation

DOMINATING SET

Input	Graph $G = (V, E)$, $k \in \mathbb{N}$
Question	Exists $D \subseteq V$ with $|D| \leq k$ such that $N[D] = V$?

- The domination number is the minimum cardinality of a ds of G, denotes as $\gamma(G)$
- **Observation:** In connected G every $v \in D$ has another $z \in D$ with $d(v, z) \leq 3$.

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TOTAL DOMINATING SET

Input	Graph $G = (V, E)$, $k \in \mathbb{N}$
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- The total domination number is the minimum cardinality of a tds of G, denoted as $\gamma_t(G)$.
- We say d_1 witnesses d_2 (and vice versa)

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- The semitotal domination number is the minimum cardinality of an sds of G, denoted as $\gamma_{2t}(G)$.
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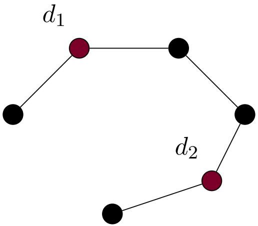
Example: $\gamma(G) < \gamma_{\mathbf{2t}}(\mathbf{G}) < \gamma_t(G)$

DOMINATING SET



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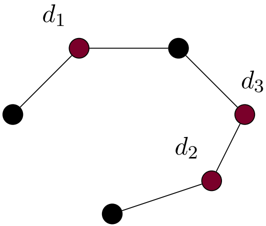

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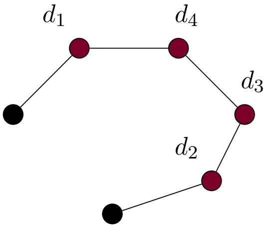

Parameterized Complexity

- NP-hard? We expect problem to be **at least** exponential
- **Idea:** Limit combinatorial explosion to some aspect of the problem
- **Goal:** Find an algorithm running in time $\mathcal{O}(f(k) \cdot n^c)$ for **some** parameter k
- In this work: by solution size
- **Techniques:** Kernelization, Bounded Search Trees, ...

If possible, the problem is **fixed-parameter tractable**.

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- Class NP corresponds to whole hierarchy $W[i]$ in parameterized setting.
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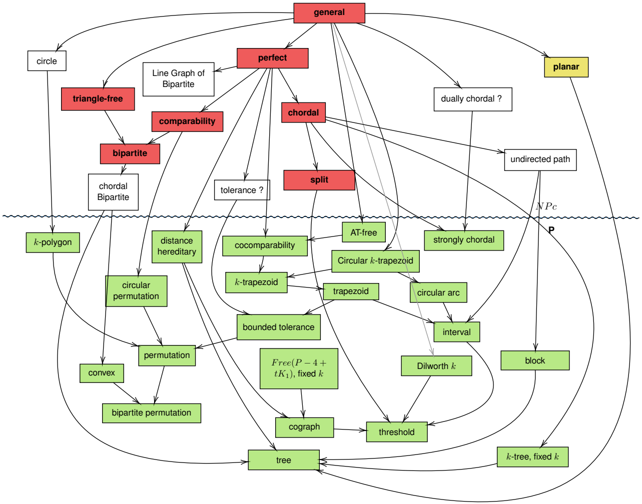
Complexity Comparison

Graph Class	Dominating Set		Semitotal Dominating Set		Total Dominating Set	
	classical	Parameterized	classical	Parameterized	classical	Parameterized
bipartite	NPc [4]	$W[2]$ [39]	NPc [25]	$W[2]$ (this)	NPc [32]	$W[2]$ (cite!)
line graph of bipartite	NPc [28]	?	NPc [19]	? (?)	NPc [35]	?
circle	NPc [26]	$W[1]$ [7]	NPc [27]	? (?)	NPc [35]	$W[1]$ [7]
chordal	NPc [6]	$W[2]$ [39]	NPc [25]	$W[2]$ (this)	NPc [37]	$W[1]$ [11] by *split*
s-chordal , $s > 3$	NPc [33]	$W[2]$ [33]	? (?)	? (?)	NPc [33]	$W[1]$ [33]
split	NPc [4]	$W[2]$ [39]	NPc [25]	$W[2]$ **this**	NPc [37]	$W[1]$ [11]
3-claw-free	NPc [14]	FPT [14]	Prob. Unk	Prob. Unk	NPc [35]	Unknown
t-claw-free, $t > 3$	NPc [14]	$W[2]$ [14]	Prob. Unknown	Unknown	NPc [35]	Prob. Unknown
chordal bipartite	NPc [36]	? (?)	NPc [25]	?		P [15]
planar	NPc (Sources!)	FPT [2]	NPc	**FPT (this)**	NPc	FPT [20]
undirected path	NPc [6]	FPT [18]	NPc [24]	?	NPc [31]	?
dually chordal	P [8]		? (attempted [19])		P [30]	
strongly chordal	P [17]		P [40]		NPc [17]	
AT-free	P [29]		P [27]		P [29]	
tolerance	P [22]		?		?	
block	P [17]		P [24]		P [10]	
interval	P [12]		P [38]		P [5]	
bounded clique-width	P [13]		P [13]		P [13]	
bounded mim-width	P [3, 9]		P [19]		P [3, 9]	

Status Semitotal Dominating Set



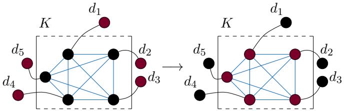
Warmup: Intractability Results

ω_2 hard on split, chordal and bipartite graphs

- **Split Graph:** $G = \mathtt{Clique} + \mathtt{IndependentSet}$

Split Graphs

SEMITOTAL DOMINATING SET on *split* and *chordal* graphs is ω_2-hard



Proof by fpt-reduction from PLANAR DOMINATING SET on split graphs:

1. **Construct** G* by adding v with pendant z to clique. G* split
2. If ds D in G, $D' = D \cup \{v\}$ is sds D'.
3. If sds D' in G', $D \setminus \{v\}$ is D in G
4. Parameter k only changed by constant

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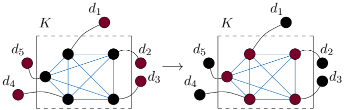

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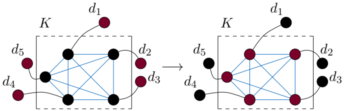

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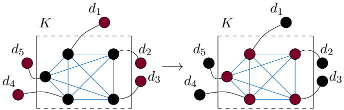

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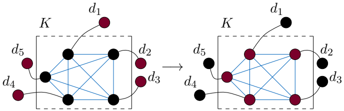

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Bipartite Graphs

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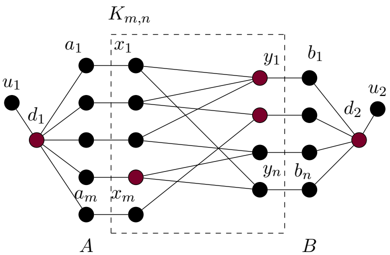

Proof by fpt-reduction from PLANAR DOMINATING SET on bipart. graphs:

1. **Construct** Add new neighbor to each vertex and add d_1, d_2, u_1, u_2
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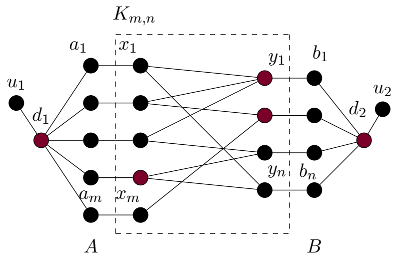

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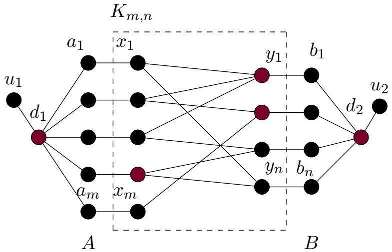

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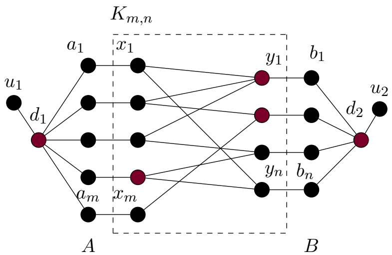

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A Linear Kernel for PLANAR SEMITOTAL DOMINATING SET

Another Explicit kernel for a Dominating Problem

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- **Idea:** Preprocess an instance using *Reduction Rules* until hard *kernel* is found.

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Related Works

Problem	Size	Source
PLANAR DOMINATING SET	$67k$	[16]
PLANAR TOTAL DOMINATING SET	$410k$	[20]
PLANAR SEMITOTAL DOMINATING SET	$359k$	**This work**
PLANAR EDGE DOMINATING SET	$14k$	[23]
PLANAR EFFICIENT DOMINATING SET	$84k$	[23]
PLANAR RED-BLUE DOMINATING SET	$43k$	[21]
PLANAR CONNECTED DOMINATING SET	$130k$	[34]
PLANAR DIRECTED DOMINATING SET	Linear	[1]

Main Theorem

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PLANAR SEMITOTAL DOMINATING SET parameterized by solution size admits a linear kernel. There exists a polynomial-time algorithm that, given a planar graph (G, k), either correctly reports that (G, k) is a NO-instance or returns an equivalent instance (G', k) such that $|V(G')| \leq 359 \cdot k$.

The Big Picture

Given a planar graph $G = (V, E)$, we will:

1. Split the neighborhoods of the graph;
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The Basic Principle: Regions

Region (Simplified)

Given plane G and $v, w \in V$, a region is a closed subset, such that

- there are two non-crossing (but possibly overlapping) boundary paths
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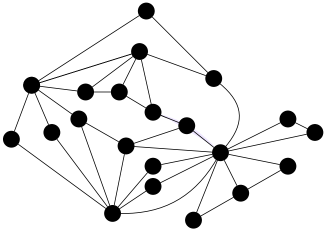

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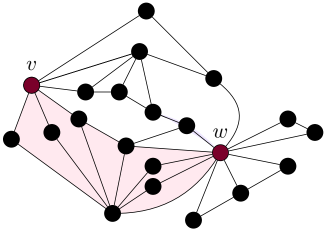

D-region decomposition

D-region decomposition [2]

Given $G = (V, W)$ and $D \subseteq V$, a *D-region decomposition* is a set $\mathfrak{R}$ with poles in D such that:

- for any vw-region $R \in \mathfrak{R}$: $D \cap V(R) = \{v, w\}$
- Regions are disjunct, but can share border vertices

A region is **maximal**, if no $R \in \mathfrak{R}$ such that $\mathfrak{R}' = \mathfrak{R} \cup \{R\}$ is a *D-region decomposition* with $V(\mathfrak{R}) \subsetneq V(\mathfrak{R}')$.

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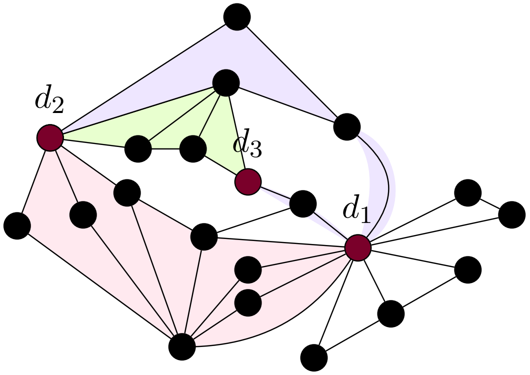
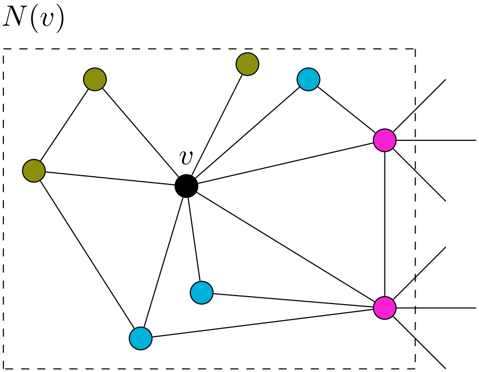
Maximal D-region decomposition



Splitting Up $N(v)$



Splitting Up $N(v)$

We split $N(v)$ into three subsets:

$$N_1(v) = \{u \in N(v) : N(u) \setminus N[v] \neq \emptyset\} \tag{1}$$

$$N_2(v) = \{u \in N(v) \setminus N_1(v) : N(u) \cap N_1(v) \neq \emptyset\} \tag{2}$$

$$N_3(v) = N(v) \setminus (N_1(v) \cup N_2(v)) \tag{3}$$

For $i, j \in [1, 3]$, we denote $N_{i,j}(v) := N_i(v) \cup N_j(v)$.

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Rule 1: Shrinking $N_3(v)$

Let $G = (V, E)$ be a graph and let $v \in V$. If $|N_3(v)| \geq 1$:

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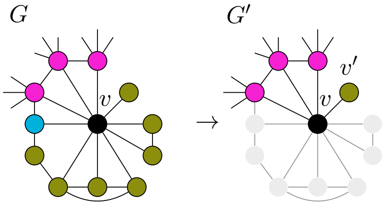

- **Idea:** v better choice than $N_{2,3}$

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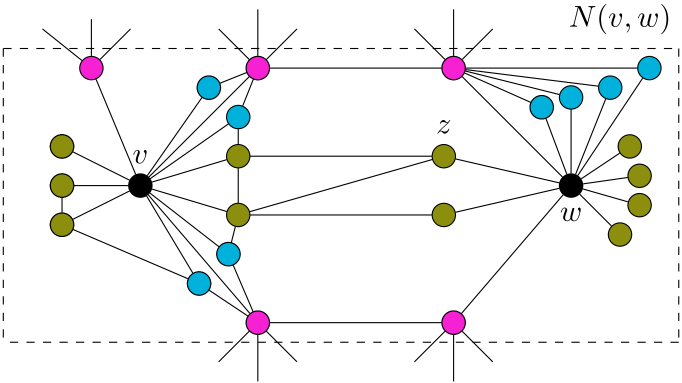

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Rule 2: Setting Up Our Weapons

Key Idea: $N_{2,3}(v,w)$ can **always** be semitotally dominated with 4 vertices.

$$\mathcal{D} = \{\tilde{D} \subseteq N_{2,3}(v,w) \mid N_3(v,w) \subseteq \bigcup_{v \in \tilde{D}} N(v),\ |\tilde{D}| \leq 3\} \tag{7}$$

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If $\mathcal{D} = \emptyset$ we apply the following:

Case 1: if $\mathcal{D}_v = \emptyset$ and $D_w = \emptyset$

- Remove $N_{2,3}(v, w)$
- Add vertices v' and w' and two edges $\{v, v'\}$ and $\{w, w'\}$
- Preserve $d(v, w)$

Case 2: if $\mathcal{D}_v \neq \emptyset$ and $D_w = \emptyset$

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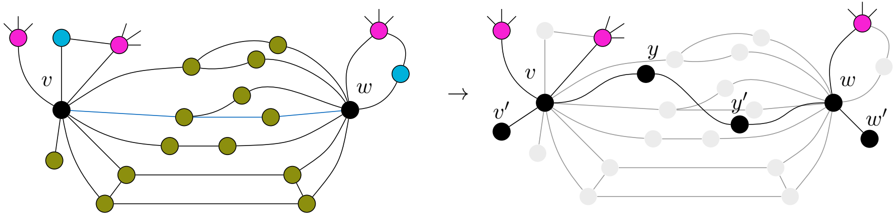
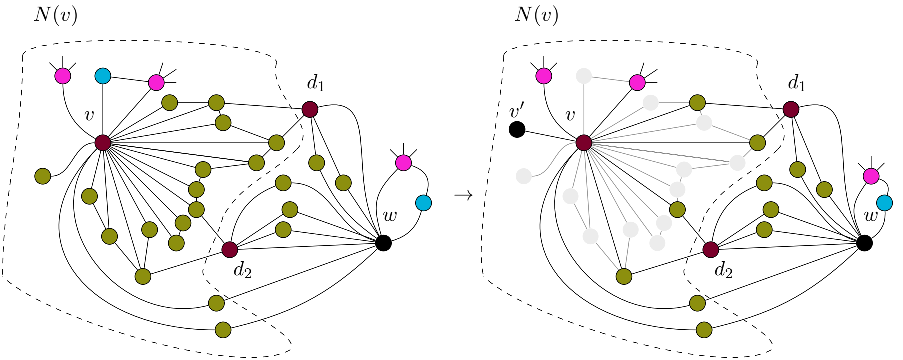


Rule 2: Case 2



Simple Regions

The Main Theorem

A simple vw-region is a vw-region such that:

1. its boundary paths have length at most 2, and
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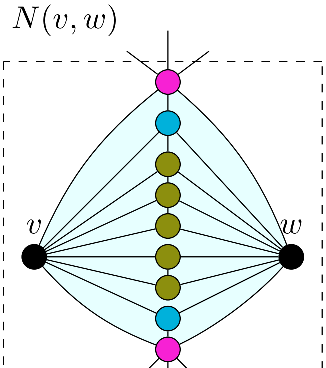

Rule 3: Shrinking the Size of Simple Regions

Let $G = (V, E)$ be a plane graph, $v, w \in V$ and R be a simple region between v and w. If $|V(R) \setminus \{v, w\}| \geq 5$ apply the following:

Case 1: If $G[R \setminus \partial R] \cong P_3$, then:

- remove $V(R \setminus \partial R)$
- add vertex y with edges $\{v, y\}$ and $\{y, w\}$

Case 2: If $G[R \setminus \partial R] \ncong P_3$, then

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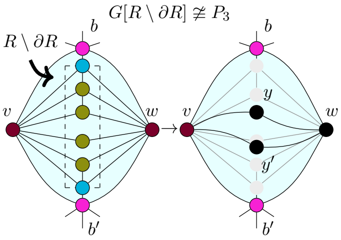

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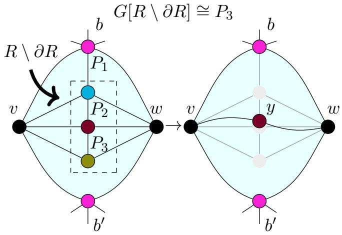

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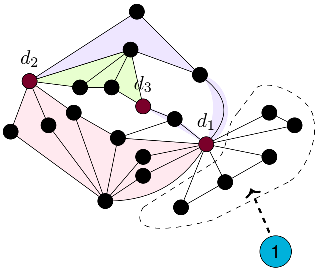
Bounding the Kernel: Vertices Outside any Region



For each d in sds D:

1. $|N_1(v) \setminus V(\mathfrak{R})| = 0$ [2], On Border
2. $|N_2(v) \setminus V(\mathfrak{R})| = 96$ [2]: TODO Reasoning
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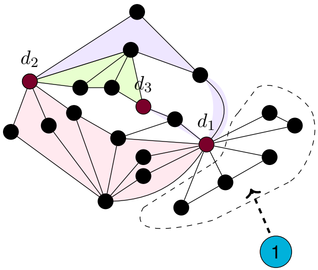

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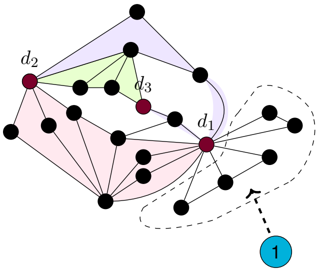

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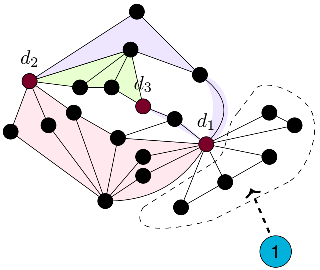

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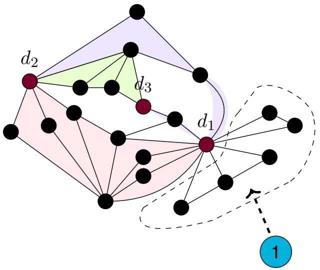

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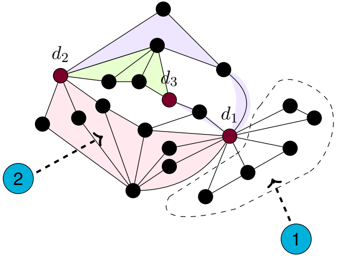

For each vw-region, we have

1. $|N_1(v,w)| \leq 4$ [2] (vertices on border)
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3. $|N_3(v,w)| \leq \max(27, 44, 4, 57) \cdot 4$ (proof omitted depending on Rule 2)

Total: $|V(R)| = |\{v,w\} \cup (N_1(v,w) \cup N_2(v,w) \cup N_3(v,w))| \leq 87$

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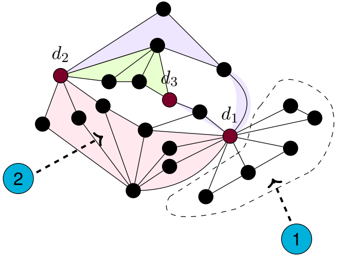

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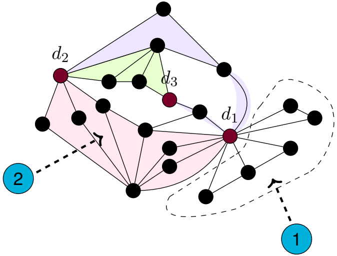

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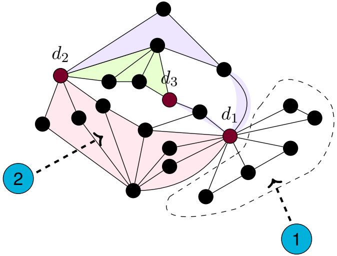

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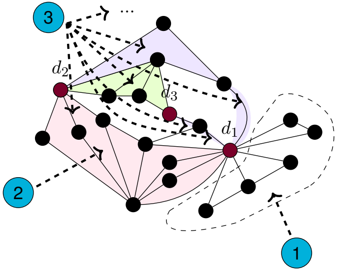

Number of Regions [2]

Let G be a plane graph and let D be a SEMITOTAL DOMINATING SET with $|D| \geq 3$. There is a maximal *D-region decomposition* of G sucht that $|\mathfrak{R}| \leq 3 \cdot |D| - 6$.

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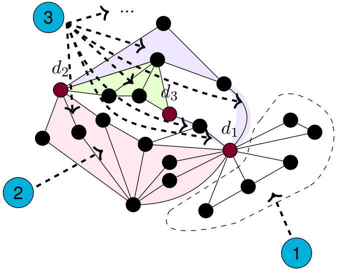

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Summary: Bounding Kernel Size

Let D be sds of size k. There exists a maximal *D-region decomposition* $\mathfrak{R}$ such that:

1. $\mathfrak{R}$ has only at most $3k - 6$ regions ([2]);
2. There are at most $97 \cdot k$ vertices outside of any region;
3. Each region $R \in \mathfrak{R}$ contains at most 87 vertices.

Hence: $|V| = \bigcup_{v \in D} N(v) = 87 \cdot (3k - 6) + 97 \cdot k + k < 359 \cdot k$

Main Theorem

All reduction rules can be applied in poly/time, hence:

The Main Theorem

The SEMITOTAL DOMINATING SET problem parameterized by solution size admits a linear kernel on planar graphs. There exists a polynomial-time algorithm that, given a planar graph (G, k), either correctly reports that (G, k) is a NO-instance or returns an equivalent instance (G', k) such that $|V(G')| \leq 359 \cdot k$.

Proof: Add Proof here.

Conclusions

Results:

- Given an overview over the status
- SEMITOTAL DOMINATING SET is $W[1]$ for *chordal, split* and *bipartite* graphs
- exists linear kernel of size $359 \cdot k$ when parameterized by solution size

Future Work:

- Improve kernel size and do empirical evaluation
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? Any Questions ?

$\cdots$ *And Thank You For Your Attention* $\cdots$

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