Master's Thesis Presentation

On the Parameterized Complexity of Semitotal Dominating Set On Graph Classes

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Creative Introduction

Our Plan for Today

Motivation

DOMINATING SET

Input	Graph $G = (V, E)$, $k \in \mathbb{N}$
Question	Is there a set $D \subseteq V$ of size at most k such that $N[D] = V$?

- The domination number is the minimum cardinality of a ds of G, denotes as $\gamma(G)$
- **Observation:** In connected G every $v \in D$ has another $z \in D$ with $d(v, z) \leq 3$.

Motivation

TOTAL DOMINATING SET

Input	Graph $G = (V, E)$, $k \in \mathbb{N}$
Question	Is there a set $D \subseteq V$ of size at most k such that for all $d_1 \in X$ exists $d_2 \in X \setminus \{d_1\}$ s.t. $d(d_1, d_2) \leq 1$?

- The total domination number is the minimum cardinality of a tds of G, denoted as $\gamma_t(G)$.

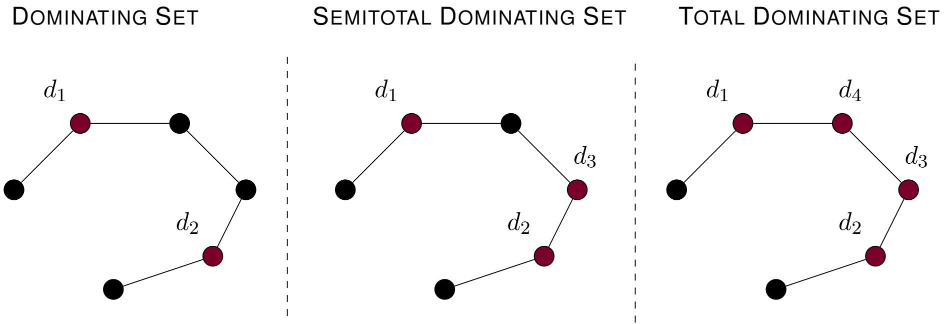
Motivation

SEMITOTAL DOMINATING SET

Input	Graph $G = (V, E)$, $k \in \mathbb{N}$
Question	Is there a subset $D \subseteq V$ with $\lvert D\rvert \leq k$ such that $N[D] = V$ and for all $d_1 \in X$ there exists another $d_2 \in X$ such that $d(d_1, d_2) \leq 2$?

- The semitotal domination number is the minimum cardinality of a sds of G, denoted as $\gamma_{2t}(G)$.
- **Observation**: $\gamma(G) \leq \gamma_{\mathbf{2t}}(\mathbf{G}) \leq \gamma t(G)$

Example: $\gamma(G) < \gamma_{2t}(G) < \gamma_t(G)$



Parameterized Complexity

- NP-hard? We expect problem to be **at least** exponential
- **Idea:** Limit combinatorial explosion to some aspect of the problem
- **Goal:** Find an algorithm running in time $\mathcal{O}(f(k) \cdot n^c)$ for **some** parameter k
- In this work: by solution size
- **Techniques:** Kernelization, Bounded Search Trees, ...

If possible, the problem is **fixed-parameter tractable**.

Fixed-Parameter Intractability

- Class NP corresponds to whole hierarchy $W[i]$ in parameterized setting.
- Problems at least $W[1]$-hard considered **fixed-parameter intractable**
- DOMINATING SET is $W[2]$-complete
- **Tool for Proving Hardness**: FPT Reductions, preserving the parameter

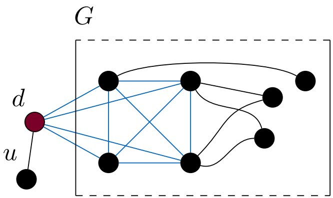
Warmup: Intractability Results

ω_2 hard on split, chordal and bipartite graphs

- **Split Graph:** $G = \texttt{Clique} + \texttt{IndependentSet}$
- Assuming *parameterization by solution size*

Split Graphs

SEMITOTAL DOMINATING SET on *split* and *chordal* graphs is ω_2-hard

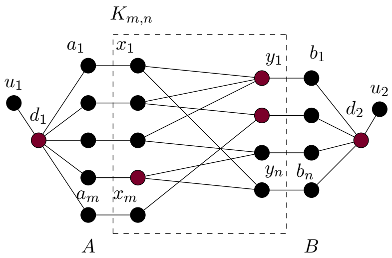


Proof by fpt-reduction from PLANAR DOMINATING SET on split graphs:

1. **Construct** G* by adding v with pendant z to clique. G* split
2. If ds D in G, $D* = D \cup \{v\}$ is sds $D*$.
3. If sds $D*$ in $G*$, $D \setminus \{v\}$ is D in G
4. Parameter k only changed by constant

Bipartite Graphs

SEMITOTAL DOMINATING SET on *bipartite* graphs is ω_2-hard



Proof by fpt-reduction from PLANAR DOMINATING SET on bipart. graphs:

1. **Construct** Add new neighbor to each vertex and add d_1, d_2, u_1, u_2
2. If ds D in G, then $D* = D \cup \{d_1, d_2\}$ is sds in $G*$
3. Assume sds $D*$ in $G*$. If $a_i \in D*$ (b_i), flip. $D = D * \setminus \{d_1, d_2\}$ is ds in G

Kernelization

- **Idea:** Preprocess an instance using *Reduction Rules* until hard *kernel* is found.

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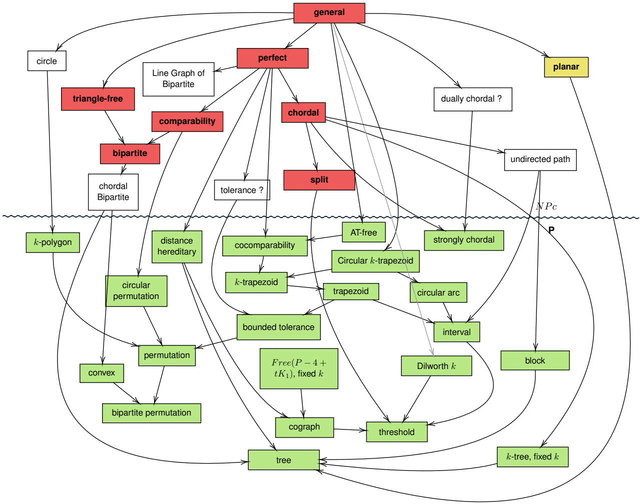
$$\overbrace{(I,\ k)\ \in Q}^{n}$$

$$\Big\downarrow\ \mathcal{O}\left(n^{c}\right)$$

$$\underbrace{(I',\ k')\ \in Q}_{g(k)}$$

Complexity Status



A Linear Kernel for PLANAR SEMITOTAL DOMINATING SET

The main result of the thesis

Related Works

Problem	Size	Source
Planar Dominating Set	$67k$	Diekert and Durand 2005
Planar Total Dominating Set	$410k$	Garnero and Sau 2018
Planar Semitotal Dominating Set	$xxxxk$	**This work**
Planar Edge Dominating Set	$14k$	Guo and Niedermeier 2007
Planar Efficient Dominating Set	$84k$	Guo and Niedermeier 2007
Planar Red-Blue Dominating Set	$43k$	Garnero, Sau, and Thilikos 2017
Planar Connected Dominating Set	$130k$	Luo et al. 2013
Planar Directed Dominating Set	Linear	Alber, Dorn, and Niedermeier 2006

Main Theorem

The Main Theorem

The SEMITOTAL DOMINATING SET problem parameterized by solution size admits a linear kernel on planar graphs. There exists a polynomial-time algorithm that, given a planar graph (G, k), either correctly reports that (G, k) is a NO-instance or returns an equivalent instance (G', k) such that $|V(G')| \leq xxx \cdot k$.

Introducing D-region decomposition

D-region decomposition

Given $G = (V, W)$ and $D \subseteq V$, a *D-region decomposition* is a set $\mathfrak{R}$ with poles in D such that:

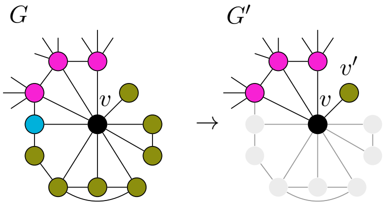
- for any vw-region $R \in \mathfrak{R}$, it holds that $D\cap$

Splitting up $N(v)$

Rule 1, Appetizer: Shrinking $N_3(v)$

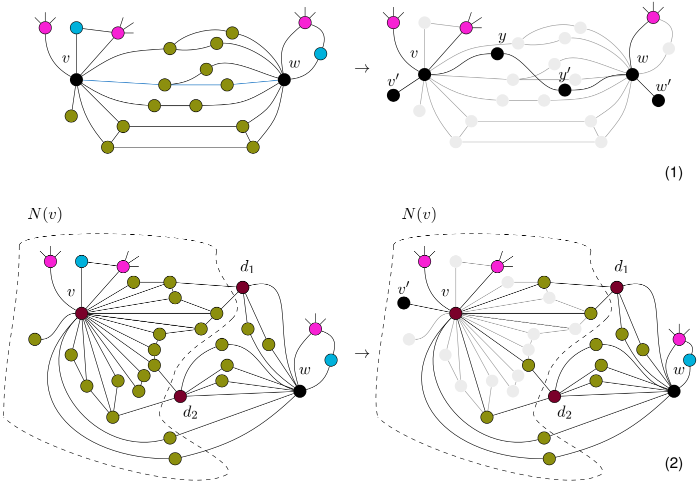
Let $G = (V, E)$ be a graph and let $v \in V$. If $|N_3(v)| \geq 1$:

- remove $N_{2,3}(v)$ from G,
- add a vertex v' and an edge $\{v, v'\}$.



- **Idea:** Removing isolated vertices
- Correctness: Omitted

Rule 2



Rule 3: Shrinking the size of simple regions

Let $G = (V, E)$ be a plane graph, $v, w \in V$ and R be a simple region between v and w. If $|V(R) \setminus \{v, w\}| \geq 5$ apply the following:

Case 1: If $G[R \setminus \partial R] \cong P_3$, then:

- remove $V(R \setminus \partial R)$
- add vertex y with edges $\{v, y\}$ and $\{y, w\}$

Case 2: If $G[R \setminus \partial R] \ncong P_3$, then

- remove $V(R \setminus \partial R)$
- add vertices y, y' and four edges $\{v, y\}$, $\{v, y'\}$, $\{y, w\}$ and $\{y', w\}$

Rule 3: Shrinking the size of simple regions

Case 1: If $G[R \setminus \partial R] \cong P_3$, then:

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Case 2: If $G[R \setminus \partial R] \ncong P_3$, then

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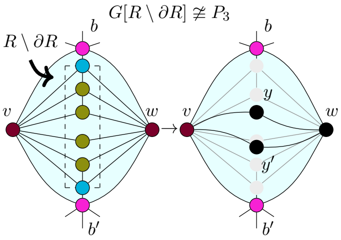

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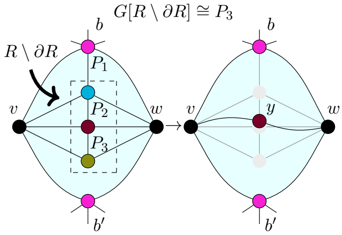
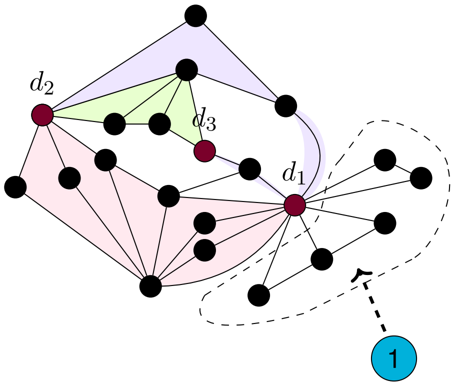


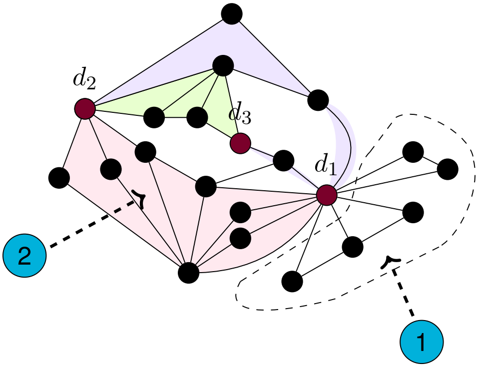
Notes

- All the rule are sound
- and only change the solution size by a constant factor
- they can be applied in pplynomial-time
- Rule 3 is a swiss-army-knife to be found on many surprising places

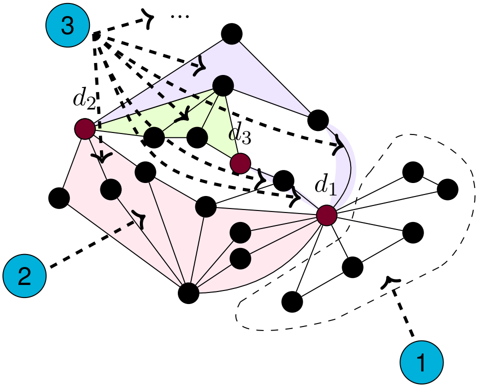
Bounding the Kernel: Idea 1



Bounding the Kernel: Idea 2



Bounding the Kernel: Idea 3



Summary: Bounding Kernel Size

Let D be sds of size k. There exists a maximal D*-region decomposition* $\mathfrak{R}$ such that:

1. $\mathfrak{R}$ has only at most $3k - 6$ regions (Alber, Fellows, and Niedermeier 2004);
2. There are at most $97 \cdot k$ vertices outside of any region;
3. Each region $R \in \mathfrak{R}$ contains at most 87 vertices.

Hence: $87 \cdot (3k - 6) + 97 \cdot k + k < 359 \cdot k$

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Proof: Add Proof here.

Conclusions

Results:

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Future Work:

- Improve Kernel Size
- Solve complexities for...

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