DEPARTMENT OF INFORMATICS

TECHNICAL UNIVERSITY MUNICH

Master Thesis

On Parametrized Semitotal Dominating Set

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A Survey and a Linear Kernel For Planar Graphs

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I confirm that this master thesis is my own work and I have documented all sources and material used.

København, August 17, 2022 Lukas Retschmeier

Acknowledgments

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Abstract all the way

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CHAPTER 2

PRELIMINARIES

We start by recapping some basic notation in Graph Theory and Parametrized Complexity.

Continuing an intensive study of parametrized complexity of that problem.

2.1. Graph Theory

We quickly state the following definitions given by [Die10, p. xxx].

Definition 1 (Graph)**.** *A graph is a pair $G = (V, E)$ of two sets where V denotes the vertices and $E \subseteq V \times C$ the edges of the graph. A vertex $v \in V$ is incident with an edge $e \in E$ if $v \in e$. Two vertices x, y are adjacent, or neighbours, if $\{x, y\} \in E$.*

Definition 2 (Special Graph Notations [Die10, p. 27])**.** *A simple Graph*

A directed Graph is a graph

A Multi Graph

A Planar Graph

Definition 3 (Adjacent Vertices)**.**

Definition 4 (Closed and Open Neighborhoods of Vertices)**.** *+ Sets*

Definition 5 (Induced Subgraph)**.** *asd*

Special Graph Classes

We call the class of graphs without any special restrictions "General Graphs".

Definition 6 (r-partite Graphs)**.** *Let $r \geq 2$ be an integer. A Graph $G = (V, E)$ is called "r-partite" if V admits a parititon into r classes such that every edge has its ends in different classes: Vertices in the same partition class must not be adjacent.*

For the case $r = 2$ we say that the G is "bipartite"

Definition 7 (Chordal Graphs)**.**

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TODO Alber et. al, Total Domination.

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Lemma 2 (Correctness of the Reduction).

4.3.3. Reduction Rule III (R3): Shrinking the Size of Simple Regions

Lemma 3 (Correctness of the Reduction).

Lemma 4 (Reduced Plane Graph under R2).

Lemma 5 (Given size of N1 and N2 regions).

4.4. Bounding the Size of the Kernel

Lemma 6. *Given a plane Graph $G = (V, E)$ reduced under R2 and a region R(v, w), if $\mathcal{D}_v \neq$ (resp. $\mathcal{D}_w \neq \emptyset$), $N_3(v, w) \cap V(R)$ can be covered by:*

- 11 *simple regions if $\mathcal{D}_w \neq$,*
- 14 *simple regions if $N_{2,3}(v) \cap N_3(v, w) =$*

Proof. □

Lemma 7 (Number of Vertices inside a Region)**.**

Proof. □

Lemma 8 (Number of Vertices outside a Region)**.**

Proof. □

Lemma 9 (Number of Regions in a Maximum Region Decomposition)**.**

Proof. □

We now have all the tools ready to proof the central theorem of this section:

Lemma 10 (Running Time of Reduction Procedure)**.**

Proof. □

Theorem 11. *Semitotal Dominating Set has a linear kernel of size XXX on planar graphs.*

Proof. □

NP Hardness on General Graphs

CHAPTER 5

OPEN QUESTIONS AND FURTHER RESEARCH

Chordal Bipartite Graphs a very interesting case.

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BIBLIOGRAPHY

[Die10] R. Diestel. *Graph Theory*. Fourth. Vol. 173. Graduate Texts in Mathematics. Heidelberg; New York: Springer, 2010. ISBN: 9783642142789 3642142788 9783642142796 3642142796.