Master's Thesis Presentation

On the Parameterized Complexity of Semitotal Dominating Set On Graph Classes

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Creative Introduction

Our Plan for Today

Motivation

DOMINATING SET

Input	Graph $G = (V, E)$, $k \in \mathbb{N}$
Question	Is there a set $D \subseteq V$ of size at most k such that $N[D] = V$?

- The domination number is the minimum cardinality of a ds of G, denotes as $\gamma(G)$
- **Observation:** In connected G every $v \in D$ has another $z \in D$ with $d(v, z) \leq 3$.

Motivation

TOTAL DOMINATING SET

Input	Graph $G = (V, E)$, $k \in \mathbb{N}$
Question	Is there a set $D \subseteq V$ of size at most k such that for all $d_1 \in X$ exists $d_2 \in X \setminus \{d_1\}$ s.t. $d(d_1, d_2) \leq 1$?

- The total domination number is the minimum cardinality of a tds of G, denoted as $\gamma_t(G)$.

Motivation

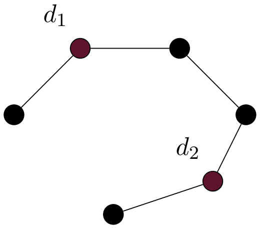
Semitotal Dominating Set

Input	Graph $G = (V, E)$, $k \in \mathbb{N}$
Question	Is there a subset $D \subseteq V$ with $\|D\| \leq k$ such that $N[D] = V$ and for all $d_1 \in X$ there exists another $d_2 \in X$ such that $d(d_1, d_2) \leq 2$?

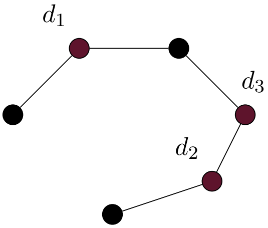
- The semitotal dominatinon number is the minimum cardinality of a sds of G, denoted as $\gamma_{2t}(G)$.
- **Observation**: $\gamma(G) \leq \gamma_{\mathbf{2t}}(\mathbf{G}) \leq \gamma t(G)$

Example: $\gamma(G) < \gamma_{\mathbf{2t}}(\mathbf{G}) < \gamma_t(G)$

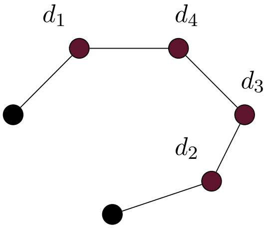
DOMINATING SET



SEMITOTAL DOMINATING SET



TOTAL DOMINATING SET



Parameterized Complexity

- Developed by Downey and Fellows
- **Idea:** Limit combinatorial explosion to some aspect of the problem
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Fixed-parameter tractability

Kernelization

- **Idea:** Preprocess an instance using *Reduction Rules* until hard *kernel* is found.

Kernelization

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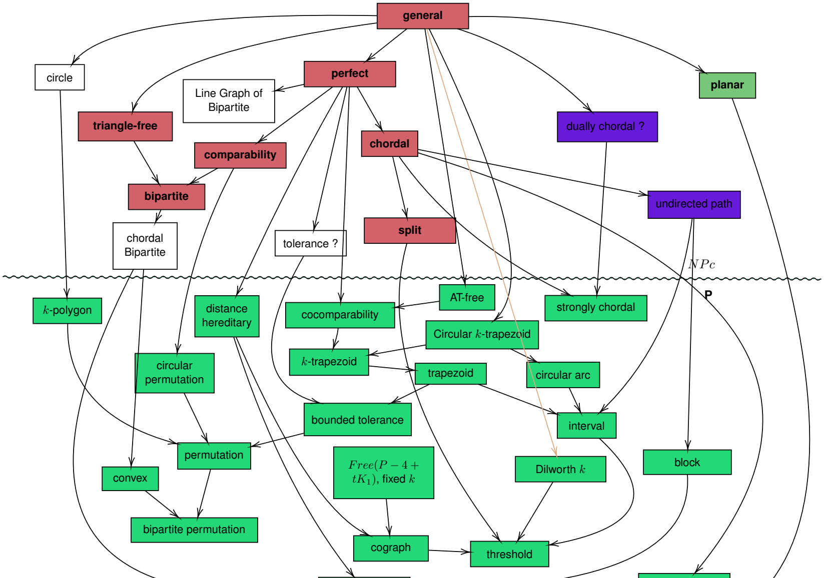
$$\overbrace{(I,\ k)\ \in Q}^{n}$$

$$\Big\downarrow\ \mathcal{O}\left(n^c\right)$$

$$\underbrace{(I',\ k')\ \in Q}_{g(k)}$$

Complexity Status



A Linear Kernel for Planar Semitotal Dominating Set

The main result of the thesis

Related Work

Problem	Size	Source
Planar Dominating Set	$67k$	Diekert and Durand 2005
Planar Total Dominating Set	$410k$	Garnero and Sau 2018
Planar Semitotal Dominating Set	$xxxxk$	**This work**
Planar Edge Dominating Set	$14k$	Guo and Niedermeier 2007
Planar Efficient Dominating Set	$84k$	Guo and Niedermeier 2007
Planar Red-Blue Dominating Set	$43k$	Garnero, Sau, and Thilikos 2017
Planar Connected Dominating Set	$130k$	Luo et al. 2013
Planar Directed Dominating Set	Linear	Alber, Dorn, and Niedermeier 2006

Main Theorem

Introducing Region Decompositions

Splitting up $N(v)$

Rule 1: Shrinking $N_3(v)$

Rule 2

Rule 3: Shrinking the size of simple regions

Proof Outline

We will now prove the correctness

Future Work

References I

Alber, Jochen, Britta Dorn, and Rolf Niedermeier (2006). “A General Data Reduction Scheme for Domination in Graphs”. In: *SOFSEM 2006: Theory and Practice of Computer Science, 32nd Conference on Current Trends in Theory and Practice of Computer Science, Merin, Czech Republic, January 21-27, 2006, Proceedings*. Ed. by Jiri Wiedermann et al. Vol. 3831. Lecture Notes in Computer Science. Springer, pp. 137–147. DOI: `10.1007/11611257\_{1}{1}`. URL: `https://doi.org/10.1007/11611257_11`.

Diekert, Volker and Bruno Durand, eds. (2005). *STACS 2005, 22nd Annual Symposium on Theoretical Aspects of Computer Science, Stuttgart, Germany, February 24-26, 2005, Proceedings*. Vol. 3404. Lecture Notes in Computer Science. Springer. ISBN: 3-540-24998-2. DOI: `10.1007/b106485`.

Garnero, Valentin and Ignasi Sau (May 2018). “A Linear Kernel for Planar Total Dominating Set”. In: *Discrete Mathematics & Theoretical Computer Science* Vol.

References II

20 no. 1. Sometimes we explicitly refer to the arXiv preprint version: `https://doi.org/10.48550/arXiv.1211.0978`. DOI: `10.23638/DMTCS-20-1-14`. eprint: `1211.0978`. URL: `https://dmtcs.episciences.org/4487`.

Garnero, Valentin, Ignasi Sau, and Dimitrios M. Thilikos (2017). "A linear kernel for planar red-blue dominating set". In: *Discret. Appl. Math.* 217, pp. 536–547. DOI: `10.1016/j.dam.2016.09.045`.

Guo, Jiong and Rolf Niedermeier (2007). "Linear Problem Kernels for NP-Hard Problems on Planar Graphs". In: *Automata, Languages and Programming*. Ed. by Lars Arge et al. Berlin, Heidelberg: Springer Berlin Heidelberg, pp. 375–386. ISBN: 978-3-540-73420-8.

References III

Luo, Weizhong et al. (2013). "Improved linear problem kernel for planar connected dominating set". In: *Theor. Comput. Sci.* 511, pp. 2–12. DOI: 10.1016/j.tcs.2013.06.011.