Master's Thesis Presentation

On the Parameterized Complexity of Semitotal Dominating Set On Graph Classes

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Creative Introduction

Our Plan for Today

1. Motivation
2. Theory
3. Kernel
   - Definitions
   - Reduction Rules

Motivation

DOMINATING SET

Input	Graph $G = (V, E)$, $k \in \mathbb{N}$
Question	Is there a set $D \subseteq V$ of size at most k such that $N[D] = V$?

- The domination number is the minimum cardinality of a ds of G, denotes as $\gamma(G)$
- **Observation:** In connected G every $v \in D$ has another $z \in D$ with $d(v, z) \leq 3$.

Motivation

Total Dominating Set

Input	Graph $G = (V, E)$, $k \in \mathbb{N}$
Question	Is there a set $D \subseteq V$ of size at most k such that for all $d_1 \in X$ exists $d_2 \in X \setminus \{d_1\}$ s.t. $d(d_1, d_2) \leq 1$?

- The total domination number is the minimum cardinality of a tds of G, denoted as $\gamma_t(G)$.

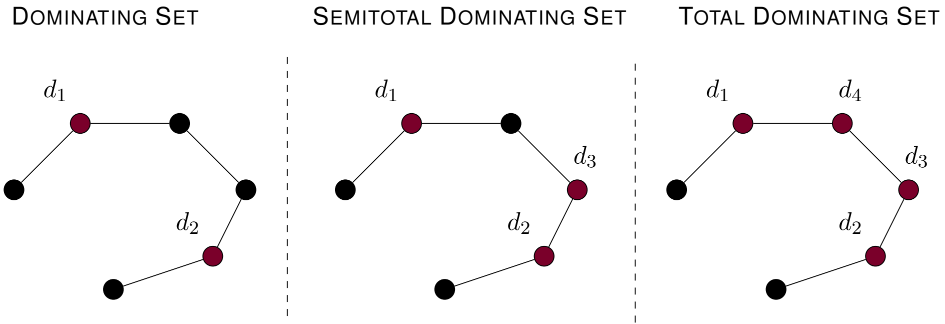
Motivation

SEMITOTAL DOMINATING SET

Input	Graph $G = (V, E)$, $k \in \mathbb{N}$
Question	Is there a subset $D \subseteq V$ with $\|D\| \leq k$ such that $N[D] = V$ and for all $d_1 \in X$ there exists another $d_2 \in X$ such that $d(d_1, d_2) \leq 2$?

- The semitotal domination number is the minimum cardinality of a sds of G, denoted as $\gamma_{2t}(G)$.
- **Observation**: $\gamma(G) \leq \gamma_{\mathbf{2t}}(\mathbf{G}) \leq \gamma t(G)$

Example: $\gamma(G) < \gamma_{2t}(G) < \gamma_t(G)$



Parameterized Complexity

- Developed by Downey and Fellows
- **Idea:** Limit combinatorial explosion to some aspect of the problem
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Fixed-Parameter Tractability

Kernelization

- **Idea:** Preprocess an instance using *Reduction Rules* until hard *kernel* is found.

Kernelization

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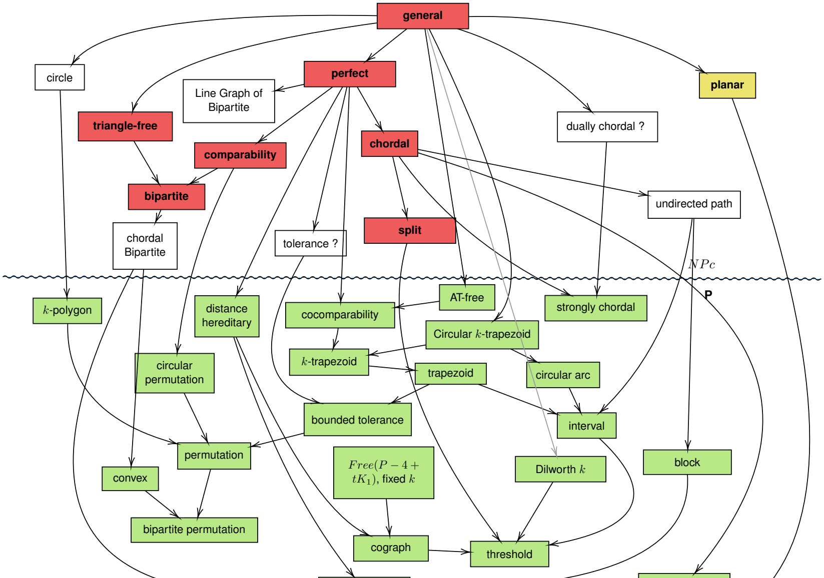
$$\overbrace{(I,\ k)\ \in Q}^{n}$$

$$\Big\downarrow\ \mathcal{O}\left(n^{c}\right)$$

$$\underbrace{(I',\ k')\ \in Q}_{g(k)}$$

Complexity Status



A Linear Kernel for PLANAR SEMITOTAL DOMINATING SET

The main result of the thesis

Related Works

Problem	Size	Source
PLANAR DOMINATING SET	$67k$	Diekert and Durand 2005
PLANAR TOTAL DOMINATING SET	$410k$	Garnero and Sau 2018
PLANAR SEMITOTAL DOMINATING SET	$xxxxk$	**This work**
PLANAR EDGE DOMINATING SET	$14k$	Guo and Niedermeier 2007
PLANAR EFFICIENT DOMINATING SET	$84k$	Guo and Niedermeier 2007
PLANAR RED-BLUE DOMINATING SET	$43k$	Garnero, Sau, and Thilikos 2017
PLANAR CONNECTED DOMINATING SET	$130k$	Luo et al. 2013
PLANAR DIRECTED DOMINATING SET	Linear	Alber, Dorn, and Niedermeier 2006

Main Theorem

Introducing Region Decompositions

Splitting up $N(v)$

Rule 1: Shrinking $N_3(v)$

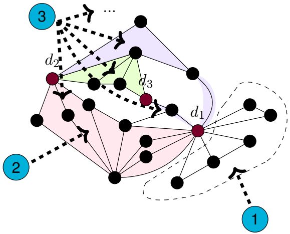
Rule 2

Rule 3: Shrinking the size of simple regions

Bounding Kernel Size

Let D be sds of size k. There exists a maximal D*-region decomposition* $\mathfrak{R}$ such that

1. $\mathfrak{R}$ has only at most $3k - 6$ regions (Alber, Fellows, and Niedermeier 2004);
2. There are at most $97 \cdot k$ vertices outside of any region;
3. Each region $R \in \mathfrak{R}$ contains at most 87 vertices.



Hence: $87 \cdot (3k - 6) + 97 \cdot k + k < 359 \cdot k$

Future Work

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Garnero, Valentin, Ignasi Sau, and Dimitrios M. Thilikos (2017). "A linear kernel for planar red-blue dominating set". In: *Discret. Appl. Math.* 217, pp. 536–547. DOI: 10.1016/j.dam.2016.09.045.

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