Master's Thesis Presentation

On the Parameterized Complexity of SEMITOTAL DOMINATING SET On Graph Classes

Lukas Retschmeier

Theoretical Foundations of Artificial Intelligence
School of Computation
Technical University of Munich

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Motivation
Theory
Intractability
ω_2 hardness
Kernel
Definitions
Rule 1
Rule 2
Rule 3
Kernel Size
Conclusions
References

Creative Introduction

Our Plan for Today

1. Motivation
2. Theory
   - Intractability
   - ω_2 hardness
3. Kernel
   - Definitions
   - Rule 1
   - Rule 2
   - Rule 3
   - Kernel Size
4. Conclusions

Motivation

DOMINATING SET

Input	Graph $G = (V, E)$, $k \in \mathbb{N}$
Question	Exists $D \subseteq V$ with $\lvert D \rvert \leq k$ such that $N[D] = V$?

- The domination number is the minimum cardinality of a ds of G, denotes as $\gamma(G)$
- **Observation:** In connected G every $v \in D$ has another $z \in D$ with $d(v, z) \leq 3$.

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- The total domination number is the minimum cardinality of a tds of G, denoted as $\gamma_t(G)$.
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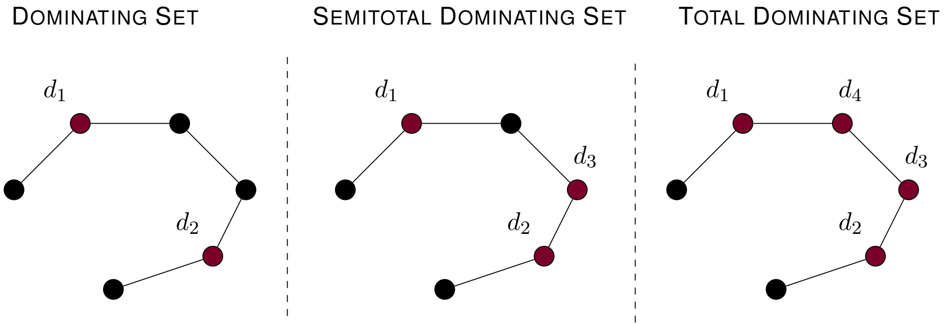

Parameterized Complexity

- NP-hard? We expect problem to be **at least** exponential
- **Idea:** Limit combinatorial explosion to some aspect of the problem
- **Goal:** Find an algorithm running in time $\mathcal{O}(f(k) \cdot n^c)$ for **some** parameter k
- In this work: by solution size
- **Techniques:** Kernelization, Bounded Search Trees, ...

If possible, the problem is **fixed-parameter tractable**.

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Fixed-Parameter Intractability

- Class NP corresponds to whole hierarchy $W[i]$ in parameterized setting.
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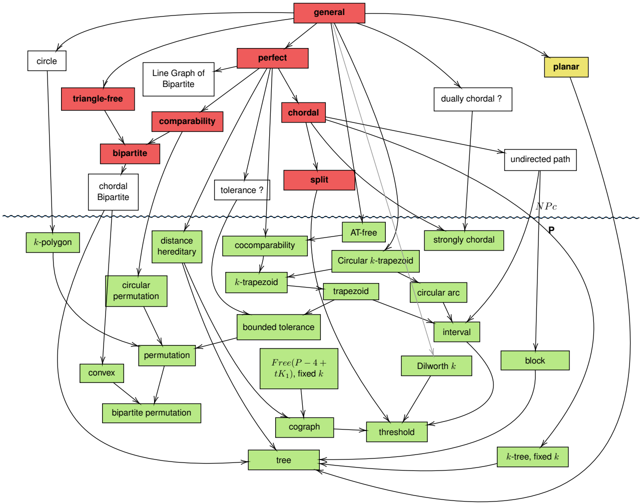
Complexity Comparison

Graph Class	Dominating Set		Semitotal Dominating Set		Total Dominating Set	
	classical	Parameterized	classical	Parameterized	classical	Parameterized
bipartite	NPc [4]	W_2 [39]	NPc [25]	W_2 (this)	NPc [32]	W_2 (cite!)
line graph of bipartite	NPc [28]	?	NPc [19]	? (?)	NPc [35]	?
circle	NPc [26]	W_1 [7]	NPc [27]	? (?)	NPc [35]	W_1 [7]
chordal	NPc [6]	W_2 [39]	NPc [25]	W_2 (this)	NPc [37]	W_1 [11] by *split*
s-chordal , $s > 3$	NPc [33]	W_2 [33]	? (?)	? (?)	NPc [33]	W_1 [33]
split	NPc [4]	W_2 [39]	NPc [25]	W_2 **this**	NPc [37]	W_1 [11]
3-claw-free	NPc [14]	FPT [14]	Prob. Unk	Prob. Unk	NPc [35]	Unknown
t-claw-free, $t > 3$	NPc [14]	W_2 [14]	Prob. Unknown	Unknown	NPc [35]	Prob. Unknown
chordal bipartite	NPc [36]	? (?)	NPc [25]	?	P [15]	
planar	NPc (Sources!)	FPT [2]	NPc	**FPT (this)**	NPc	FPT [20]
undirected path	NPc [6]	FPT [18]	NPc [24]	?	NPc [31]	?
dually chordal	P [8]		? (attempted [19])		P [30]	
strongly chordal	P [17]		P [40]		NPc [17]	
AT-free	P [29]		P [27]		P [29]	
tolerance	P [22]		?		?	
block	P [17]		P [24]		P [10]	
interval	P [12]		P [38]		P [5]	
bounded clique-width	P [13]		P [13]		P [13]	
bounded mim-width	P [3, 9]		P [19]		P [3, 9]	

Status Semitotal Dominating Set



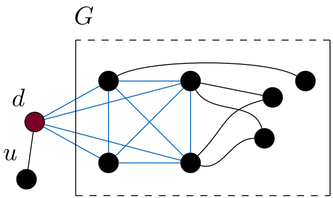
Warmup: Intractability Results

ω_2 hard on split, chordal and bipartite graphs

- **Split Graph:** $G = \texttt{Clique} + \texttt{IndependentSet}$

Split Graphs

SEMITOTAL DOMINATING SET on *split* and *chordal* graphs is ω_2-hard



Proof by fpt-reduction from PLANAR DOMINATING SET on split graphs:

1. **Construct** G* by adding v with pendant z to clique. G* split
2. If ds D in G, $D' = D \cup \{v\}$ is sds D'.
3. If sds D' in G', $D \setminus \{v\}$ is D in G
4. Parameter k only changed by constant

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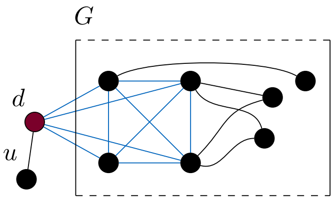

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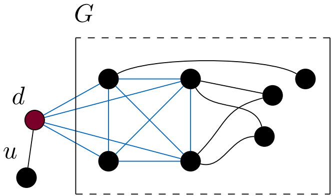

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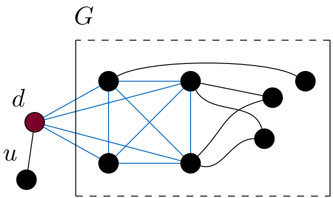

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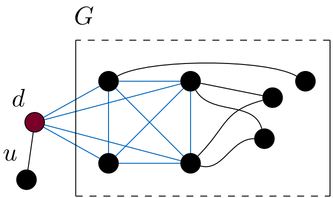

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Bipartite Graphs

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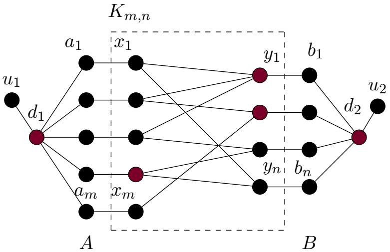

Proof by fpt-reduction from PLANAR DOMINATING SET on bipart. graphs:

1. **Construct** Add new neighbor to each vertex and add d_1, d_2, u_1, u_2
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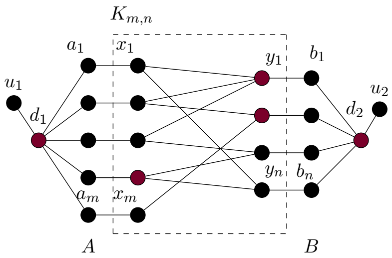

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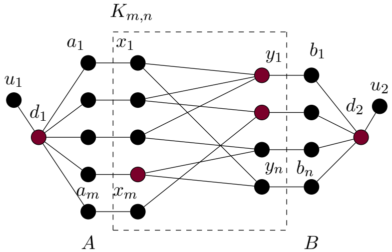

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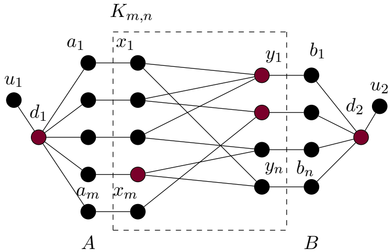

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A Linear Kernel for Planar Semitotal Dominating Set

Another Explicit kernel for a Dominating Problem

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Related Works

Problem	Size	Source
PLANAR DOMINATING SET	$67k$	[16]
PLANAR TOTAL DOMINATING SET	$410k$	[20]
PLANAR SEMITOTAL DOMINATING SET	$359k$	**This work**
PLANAR EDGE DOMINATING SET	$14k$	[23]
PLANAR EFFICIENT DOMINATING SET	$84k$	[23]
PLANAR RED-BLUE DOMINATING SET	$43k$	[21]
PLANAR CONNECTED DOMINATING SET	$130k$	[34]
PLANAR DIRECTED DOMINATING SET	Linear	[1]

Main Theorem

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SEMITOTAL DOMINATING SET parameterized by solution size admits a linear kernel on planar graphs. There exists a polynomial-time algorithm that, given a planar graph (G, k), either correctly reports that (G, k) is a NO-instance or returns an equivalent instance (G', k) such that $|V(G')| \leq 359 \cdot k$.

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Given a planar graph $G = (V, E)$, we will:

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The Basic Principle: Regions

Region (Simplified)

Given plane G and $v, w \in V$, a region is a closed subset, such that

- there are two non-crossing (but possibly overlapping) boundary paths
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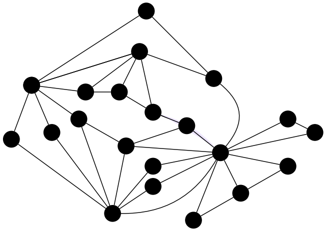

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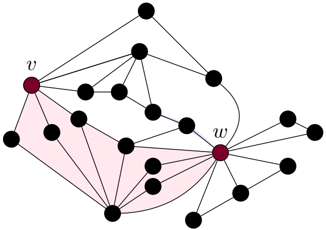

D-region decomposition

D-region decomposition [2]

Given $G = (V, W)$ and $D \subseteq V$, a *D-region decomposition* is a set $\mathfrak{R}$ with poles in D such that:

- for any vw-region $R \in \mathfrak{R}$: $D \cap V(R) = \{v, w\}$
- Regions are disjunct, but can share border vertices

A region is **maximal**, if no $R \in \mathfrak{R}$ such that $\mathfrak{R}' = \mathfrak{R} \cup \{R\}$ is a *D-region decomposition* with $V(\mathfrak{R}) \subsetneq V(\mathfrak{R}')$.

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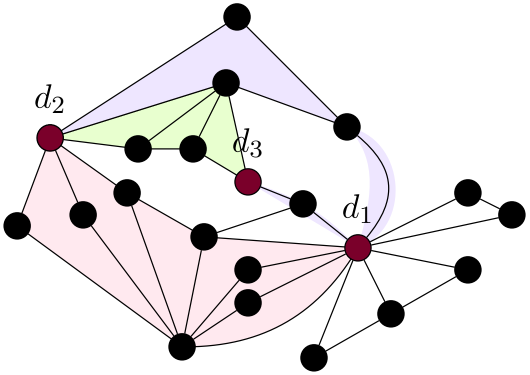
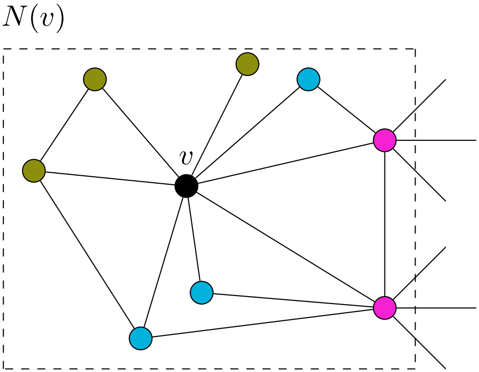


Splitting Up $N(v)$



Splitting Up $N(v)$

We split $N(v)$ into three subsets:

$$N_1(v) = \{u \in N(v) : N(u) \setminus N[v] \neq \emptyset\} \tag{1}$$

$$N_2(v) = \{u \in N(v) \setminus N_1(v) : N(u) \cap N_1(v) \neq \emptyset\} \tag{2}$$

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For $i, j \in [1, 3]$, we denote $N_{i,j}(v) := N_i(v) \cup N_j(v)$.

Splitting Up $N(v)$

We split $N(v)$ into three subsets:

$$N_1(v) = \{u \in N(v) : N(u) \setminus N[v] \neq \emptyset\} \tag{1}$$

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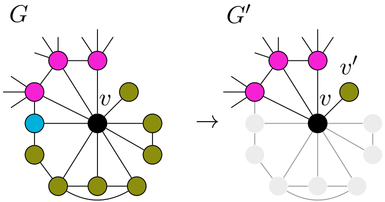
Rule 1: Shrinking $N_3(v)$

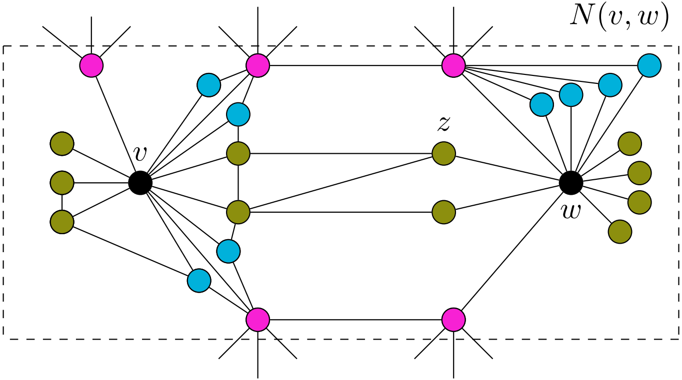
Let $G = (V, E)$ be a graph and let $v \in V$. If $|N_3(v)| \geq 1$:

- remove $N_{2,3}(v)$ from G,
- add a vertex v' and an edge $\{v, v'\}$.



- **Idea:** v better choice than $N_{2,3}$

Splitting up $N(v, w)$



Splitting up $N(v, w)$

$$N_1(v,w) = \{u \in N(v,w) \mid N(u) \setminus (N(v,w) \cup \{v,w\}) \neq \emptyset\} \tag{4}$$

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Rule 2: Setting Up Our Weapons

Key Idea: $N_{2,3}(v,w)$ can **always** be semitotally dominated with 4 vertices.

$$\mathcal{D} = \{\tilde{D} \subseteq N_{2,3}(v,w) \mid N_3(v,w) \subseteq \bigcup_{v \in \tilde{D}} N(v),\ |\tilde{D}| \leq 3\} \quad (7)$$

$$\mathcal{D}_v = \{\tilde{D} \subseteq N_{2,3}(v,w) \cup \{v\} \mid N_3(v,w) \subseteq \bigcup_{v \in \tilde{D}} N(v),\ |\tilde{D}| \leq 3,\ v \in \tilde{D}\} \quad (8)$$

$$\mathcal{D}_w = \{\tilde{D} \subseteq N_{2,3}(v,w) \cup \{w\} \mid N_3(v,w) \subseteq \bigcup_{v \in \tilde{D}} N(v),\ |\tilde{D}| \leq 3,\ w \in \tilde{D}\} \quad (9)$$

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Rule 2

If $\mathcal{D} = \emptyset$ we apply the following:

Case 1: if $\mathcal{D}_v = \emptyset$ and $D_w = \emptyset$

- Remove $N_{2,3}(v, w)$
- Add vertices v' and w' and two edges $\{v, v'\}$ and $\{w, w'\}$
- Preserve $d(v, w)$

Case 2: if $\mathcal{D}_v \neq \emptyset$ and $D_w = \emptyset$

- Remove $N_{2,3}(v)$
- Add $\{v, v'\}$

Case 3: if $\mathcal{D}_v = \emptyset$ and $D_w \neq \emptyset$

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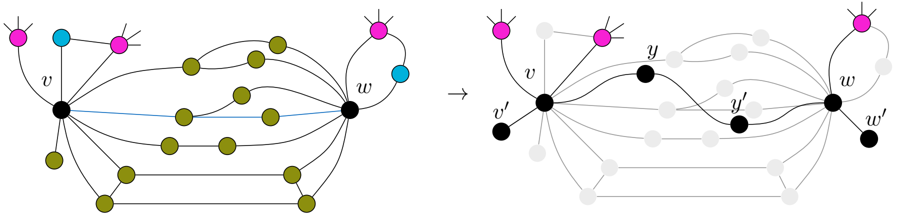
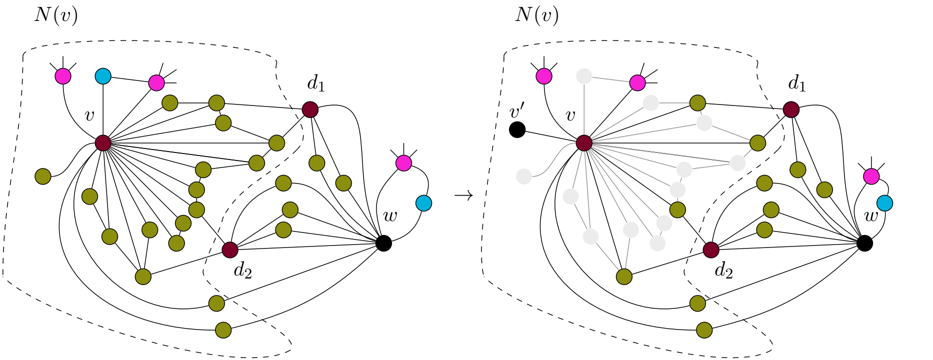
Rule 2: Case 1



Rule 2: Case 2



Simple Regions

The Main Theorem

A simple vw-region is a vw-region such that:

1. its boundary paths have length at most 2, and
2. $V(R) \setminus \{v, w\} \subseteq N(v) \cap N(w)$.

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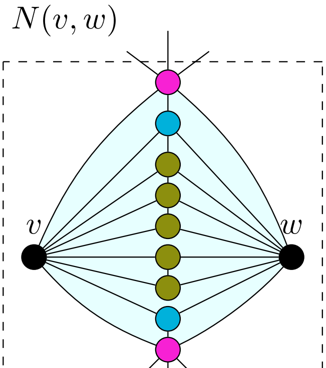

Rule 3: Shrinking the Size of Simple Regions

Let $G = (V, E)$ be a plane graph, $v, w \in V$ and R be a simple region between v and w. If $|V(R) \setminus \{v, w\}| \geq 5$ apply the following:

Case 1: If $G[R \setminus \partial R] \cong P_3$, then:

- remove $V(R \setminus \partial R)$
- add vertex y with edges $\{v, y\}$ and $\{y, w\}$

Case 2: If $G[R \setminus \partial R] \ncong P_3$, then

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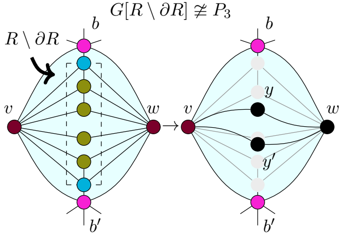

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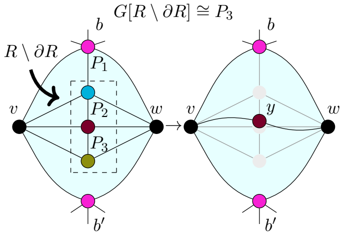

Notes

- We proved that all these rules are sound,
- change the solution size by only a constant factor
- and can be applied in poly-time.

Motivation
Theory
Intractability
ω_2 hardness
Kernel
Definitions
Rule 1
Rule 2
Rule 3
Kernel Size
Conclusions
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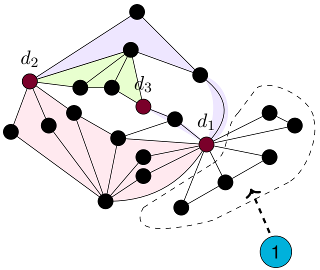
Bounding the Kernel: Vertices Outside any Region



For each d in sds D:

1. $|N_1(v) \setminus V(\mathfrak{R})| = 0$ [2], On Border
2. $|N_2(v) \setminus V(\mathfrak{R})| = 96$ [2]: TODO Reasoning
3. $|N_3(v) \setminus V(\mathfrak{R})| = 1$, by Rule 1

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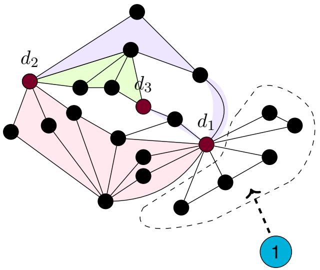

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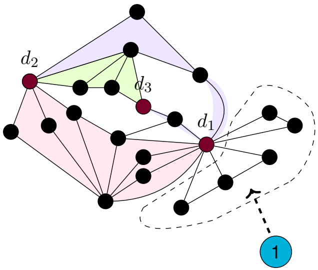

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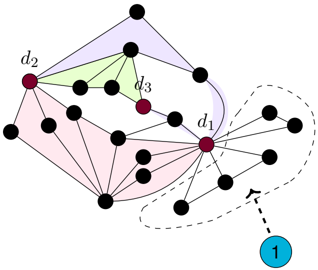

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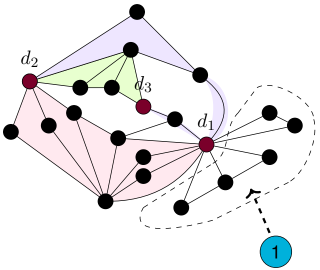

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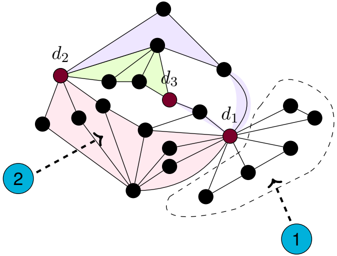
Bounding the Kernel: Inside a region



For each vw-region, we have

1. $|N_1(v,w)| \leq 4$ [2] (vertices on border)
2. $|N_2(v,w)| \leq 6 \cdot 4$ (simple regions to $N_1(v,w)$, Rule 3)
3. $|N_3(v,w)| \leq \max(27, 44, 4, 57) \cdot 4$ (proof omitted depending on Rule 2)

Total: $|V(R)| = |\{v,w\} \cup (N_1(v,w) \cup N_2(v,w) \cup N_3(v,w))| \leq 87$

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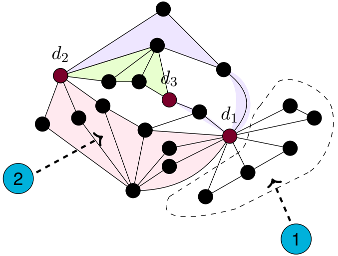

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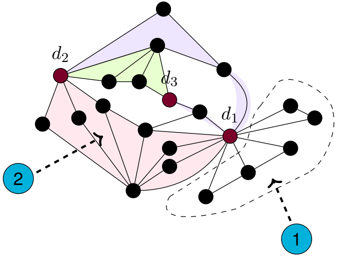

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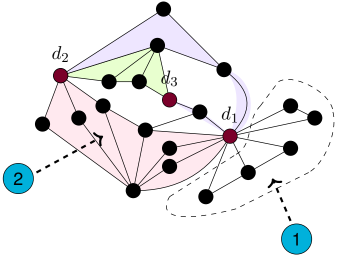

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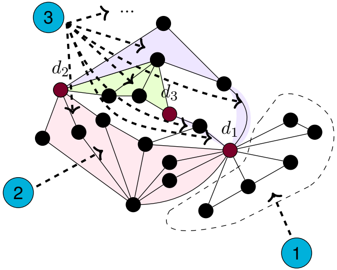
Bounding the Kernel: Number of Regions



Number of Regions [2]

Let G be a plane graph and let D be a SEMITOTAL DOMINATING SET with $|D| \geq 3$. There is a maximal D*-region decomposition* of G sucht that $|\mathfrak{R}| \leq 3 \cdot |D| - 6$.

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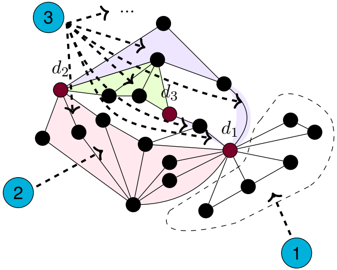

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Summary: Bounding Kernel Size

Let D be sds of size k. There exists a maximal D*-region decomposition* $\mathfrak{R}$ such that:

1. $\mathfrak{R}$ has only at most $3k-6$ regions ([2]);
2. There are at most $97 \cdot k$ vertices outside of any region;
3. Each region $R \in \mathfrak{R}$ contains at most 87 vertices.

Hence: $|V| = \bigcup_{v \in D} N(v) = 87 \cdot (3k-6) + 97 \cdot k + k < 359 \cdot k$

Main Theorem

All reduction rules can be applied in poly/time, hence:

The Main Theorem

The SEMITOTAL DOMINATING SET problem parameterized by solution size admits a linear kernel on planar graphs. There exists a polynomial-time algorithm that, given a planar graph (G, k), either correctly reports that (G, k) is a NO-instance or returns an equivalent instance (G', k) such that $|V(G')| \leq 359 \cdot k$.

Proof: Add Proof here.

Conclusions

Results:

- Given an overview over the status
- SEMITOTAL DOMINATING SET is W_1 for *chordal, split* and *bipartite* graphs
- exists linear kernel of size $359 \cdot k$ when parameterized by solution size

Future Work:

- Improve kernel size and do empirical evaluation
- Solve parameterized complexities for *Circle*, *chordal bipartite* and *undirected path graphs*

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? Any Questions ?

$\cdots$ *And Thank You For Your Attention* $\cdots$

References I

Jochen Alber, Britta Dorn, and Rolf Niedermeier. "A General Data Reduction Scheme for Domination in Graphs". In: *SOFSEM 2006: Theory and Practice of Computer Science, 32nd Conference on Current Trends in Theory and Practice of Computer Science, Merin, Czech Republic, January 21-27, 2006, Proceedings*. Ed. by Jiri Wiedermann et al. Vol. 3831. Lecture Notes in Computer Science. Springer, 2006, pp. 137–147.

Jochen Alber, Michael R. Fellows, and Rolf Niedermeier. "Polynomial-time data reduction for dominating set". In: (May 2004), pp. 363–384.

Rémy Belmonte and Martin Vatshelle. "Graph Classes with Structured Neighborhoods and Algorithmic Applications". In: *Proceedings of the 37th International Conference on Graph-Theoretic Concepts in Computer Science*. WG'11. Teplá Monastery, Czech Republic: Springer-Verlag, 2011, pp. 47–58.

Alan A. Bertossi. "Dominating sets for split and bipartite graphs". English. In: *Information Processing Letters* 19 (1984), pp. 37–40.

Alan A. Bertossi. "Total domination in interval graphs". In: *Information Processing Letters* 23.3 (1986), pp. 131–134.

References II

- Kellogg S. Booth and J. Howard Johnson. “Dominating Sets in Chordal Graphs”. In: *SIAM J. Comput.* 11.1 (Feb. 1982), pp. 191–199.
- Nicolas Bousquet et al. “Parameterized Domination in Circle Graphs”. In: *Proceedings of the 38th International Conference on Graph-Theoretic Concepts in Computer Science*. WG’12. Jerusalem, Israel: Springer-Verlag, 2012, pp. 308–319.
- Andreas Brandstädt, Victor D. Chepoi, and Feodor F. Dragan. “The Algorithmic Use of Hypertree Structure and Maximum Neighbourhood Orderings”. In: *Discrete Appl. Math.* 82.1–3 (Mar. 1998), pp. 43–77.
- Binh-Minh Bui-Xuan, Jan Arne Telle, and Martin Vatshelle. “Fast Dynamic Programming for Locally Checkable Vertex Subset and Vertex Partitioning Problems”. In: *Theor. Comput. Sci.* 511 (Nov. 2013), pp. 66–76.
- Gerard J Chang. “Total domination in block graphs”. In: *Operations Research Letters* 8.1 (1989), pp. 53–57.
- Gerard J. Chang. “Algorithmic Aspects of Domination in Graphs”. In: *Handbook of Combinatorial Optimization: Volume1–3*. Ed. by Ding-Zhu Du and Panos M. Pardalos. Boston, MA: Springer US, 1998, pp. 1811–1877.

References III

Maw-Shang Chang. “Efficient Algorithms for the Domination Problems on Interval and Circular-Arc Graphs”. In: *SIAM Journal on Computing* 27.6 (1998), pp. 1671–1694. eprint: `https://doi.org/10.1137/S0097539792238431`.

Bruno Courcelle. “The Monadic Second-Order Logic of Graphs. I. Recognizable Sets of Finite Graphs”. In: *Inf. Comput.* 85.1 (Mar. 1990), pp. 12–75.

Marek Cygan et al. “Dominating set is fixed parameter tractable in claw-free graphs”. In: *Theoretical Computer Science* 412.50 (2011), pp. 6982–7000.

Peter Damaschke, Haiko Müller, and Dieter Kratsch. “Domination in Convex and Chordal Bipartite Graphs”. In: *Inf. Process. Lett.* 36.5 (Dec. 1990), pp. 231–236.

Volker Diekert and Bruno Durand, eds. *STACS 2005, 22nd Annual Symposium on Theoretical Aspects of Computer Science, Stuttgart, Germany, February 24-26, 2005, Proceedings*. Vol. 3404. Lecture Notes in Computer Science. Springer, 2005.

Martin Farber. “Domination, independent domination, and duality in strongly chordal graphs”. In: *Discrete Applied Mathematics* 7.2 (1984), pp. 115–130.

References IV

Celina M. H. de Figueiredo et al. “Parameterized Algorithms for Steiner Tree and Dominating Set: Bounding the Leafage by the Vertex Leafage”. In: *WALCOM: Algorithms and Computation: 16th International Conference and Workshops, WALCOM 2022, Jember, Indonesia, March 24–26, 2022, Proceedings*. Jember, Indonesia: Springer-Verlag, 2022, pp. 251–262.

Esther Galby, Andrea Munaro, and Bernard Ries. “Semitotal Domination: New Hardness Results and a Polynomial-Time Algorithm for Graphs of Bounded Mim-Width”. In: *Theor. Comput. Sci.* 814.C (Apr. 2020), pp. 28–48.

Valentin Garnero and Ignasi Sau. “A Linear Kernel for Planar Total Dominating Set”. In: *Discrete Mathematics & Theoretical Computer Science* Vol. 20 no. 1 (May 2018). Sometimes we explicitly refer to the arXiv preprint version: `https://doi.org/10.48550/arXiv.1211.0978`. eprint: `1211.0978`.

Valentin Garnero, Ignasi Sau, and Dimitrios M. Thilikos. “A linear kernel for planar red-blue dominating set”. In: *Discret. Appl. Math.* 217 (2017), pp. 536–547.

Archontia C. Giannopoulou and George B. Mertzios. “New Geometric Representations and Domination Problems on Tolerance and Multitolerance Graphs”. In: *SIAM Journal on Discrete Mathematics* 30.3 (2016), pp. 1685–1725. eprint: `https://doi.org/10.1137/15M1039468`.

References V

Jiong Guo and Rolf Niedermeier. “Linear Problem Kernels for NP-Hard Problems on Planar Graphs”. In: *Automata, Languages and Programming*. Ed. by Lars Arge et al. Berlin, Heidelberg: Springer Berlin Heidelberg, 2007, pp. 375–386.

Michael A. Henning, Saikat Pal, and D. Pradhan. “The semitotal domination problem in block graphs”. English. In: *Discussiones Mathematicae. Graph Theory* 42.1 (2022), pp. 231–248.

Michael A. Henning and Arti Pandey. “Algorithmic aspects of semitotal domination in graphs”. In: *Theoretical Computer Science* 766 (2019), pp. 46–57.

J. Mark Keil. “The Complexity of Domination Problems in Circle Graphs”. In: *Discrete Appl. Math.* 42.1 (Feb. 1993), pp. 51–63.

Ton Kloks and Arti Pandey. “Semitotal Domination on AT-Free Graphs and Circle Graphs”. In: *Algorithms and Discrete Applied Mathematics: 7th International Conference, CALDAM 2021, Rupnagar, India, February 11–13, 2021, Proceedings*. Rupnagar, India: Springer-Verlag, 2021, pp. 55–65.

D. V. Korobitsin. “On the complexity of domination number determination in monogenic classes of graphs”. In: 2.2 (1992), pp. 191–200.

References VI

- Dieter Kratsch. “Domination and Total Domination on Asteroidal Triple-Free Graphs”. In: *Proceedings of the 5th Twente Workshop on on Graphs and Combinatorial Optimization*. Enschede, The Netherlands: Elsevier Science Publishers B. V., 2000, pp. 111–123.
- Dieter Kratsch and Lorna Stewart. “Total domination and transformation”. In: *Information Processing Letters* 63.3 (1997), pp. 167–170.
- James K. Lan and Gerard Jennhwa Chang. “On the algorithmic complexity of k-tuple total domination”. In: *Discrete Applied Mathematics* 174 (2014), pp. 81–91.
- J. Pfaff; R. Laskar and S.T. Hedetniemi. *NP-completeness of Total and Connected Domination,and Irredundance for bipartite graphs*. Technical Report 428. Department of Mathematical Sciences: Clemson University, 1983.
- Chunmei Liu and Yinglei Song. “Parameterized Complexity and Inapproximability of Dominating Set Problem in Chordal and near Chordal Graphs”. In: *J. Comb. Optim.* 22.4 (Nov. 2011), pp. 684–698.
- Weizhong Luo et al. “Improved linear problem kernel for planar connected dominating set”. In: *Theor. Comput. Sci.* 511 (2013), pp. 2–12.

References VII

Alice Anne McRae. “Generalizing NP-Completeness Proofs for Bipartite Graphs and Chordal Graphs”. UMI Order No. GAX95-18192. PhD thesis. USA, 1995.

Haiko Müller and Andreas Brandstädt. “The NP-Completeness of Steiner Tree and Dominating Set for Chordal Bipartite Graphs”. In: *Theor. Comput. Sci.* 53.2 (June 1987), pp. 257–265.

R. Laskar;J. Pfaff. *Domination and irredundance in split graphs*. Technical Report 428. Department of Mathematical Sciences: Clemson University, 1983.

D. Pradhan and Saikat Pal. “An $$O(n+m)$$time algorithm for computing a minimum semitotal dominating set in an interval graph”. In: *Journal of Applied Mathematics and Computing* 66.1 (June 2021), pp. 733–747.

Venkatesh Raman and Saket Saurabh. “Short Cycles Make W-hard Problems Hard: FPT Algorithms for W-hard Problems in Graphs with no Short Cycles”. In: *Algorithmica* 52.2 (2008), pp. 203–225.

Vikash Tripathi, Arti Pandey, and Anil Maheshwari. *A linear-time algorithm for semitotal domination in strongly chordal graphs*. 2021.