

Master's Thesis Presentation

On the Parameterized Complexity of SEMITOTAL DOMINATING SET On Graph Classes

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Creative Introduction

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Theory

Intractability
 ω_2 hardness

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- 2 Theory
 - Intractability
 - ω_2 hardness
- 3 Kernel
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DOMINATING SET

Input

Graph $G = (V, E)$, $k \in \mathbb{N}$

Question

Exists $D \subseteq V$ with $|D| \leq k$ such that $N[D] = V$?

- The domination number is the minimum cardinality of a ds of G , denotes as $\gamma(G)$
- **Observation:** In connected G every $v \in D$ has another $z \in D$ with $d(v, z) \leq 3$.

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TOTAL DOMINATING SET

Input

Graph $G = (V, E)$, $k \in \mathbb{N}$

Question

Exists $D \subseteq V$ with $|D| \leq k$ such that

$\forall d_1 \in X : \exists d_2 \in D \setminus \{d_1\}$ with $d(d_1, d_2) \leq 1$?

- The total domination number is the minimum cardinality of a tds of G , denoted as $\gamma_t(G)$.
- We say d_1 witnesses d_2 (and vice versa)

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SEMITOTAL DOMINATING SET

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- The semitotal domination number is the minimum cardinality of a sds of G , denoted as $\gamma_{2t}(G)$.
- **Observation:** $\gamma(G) \leq \gamma_{2t}(G) \leq \gamma_t(G)$
- We say d_1 witnesses d_2 (and vice versa)

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Example: $\gamma(G) < \gamma_{2t}(G) < \gamma_t(G)$

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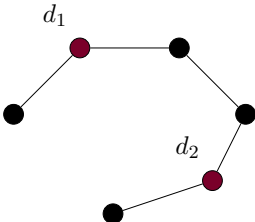
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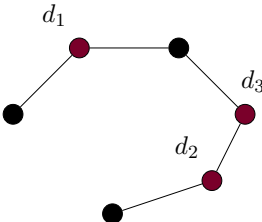
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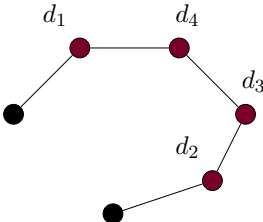
DOMINATING SET



SEMITOTAL DOMINATING SET



TOTAL DOMINATING SET



Parameterized Complexity

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- NP-hard? We expect problem to be **at least** exponential
- **Idea:** Limit combinatorial explosion to some aspect of the problem
- **Goal:** Find an algorithm running in time $\mathcal{O}(f(k) \cdot n^c)$ for **some** parameter k
- In this work: by solution size
- **Techniques:** Kernelization, Bounded Search Trees, ...

If possible, the problem is **fixed-parameter tractable**.

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Fixed-Parameter Intractability

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- Class NP corresponds to whole hierarchy $W[i]$ in parameterized setting.
- Problems at least $W[1]$ -hard considered **fixed-parameter intractable**
- DOMINATING SET is $W[2]$ -complete
- **Tool for Proving Hardness:** FPT Reductions, preserving the parameter

Fixed-Parameter Intractability

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Complexity Comparison

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Graph Class	DOMINATING SET		SEMITOTAL DOMINATING SET		TOTAL DOMINATING SET	
	classical	Parameterized	classical	Parameterized	classical	Parameterized
bipartite	NPc [4]	W_2 [39]	NPc [25]	W_2 (this)	NPc [32]	W_2 (cite!)
line graph of bipartite	NPc [28]	?	NPc [19]	? (?)	NPc [35]	?
circle	NPc [26]	W_1 [7]	NPc [27]	? (?)	NPc [35]	W_1 [7]
chordal	NPc [6]	W_2 [39]	NPc [25]	W_2 (this)	NPc [37]	W_1 [11] by <i>split</i>
s -chordal , $s > 3$	NPc [33]	W_2 [33]	? (?)	? (?)	NPc [33]	W_1 [33]
split	NPc [4]	W_2 [39]	NPc [25]	W_2 this	NPc [37]	W_1 [11]
3-claw-free	NPc [14]	FPT [14]	Prob. Unk	Prob. Unk	NPc [35]	Unknown
t -claw-free, $t > 3$	NPc [14]	W_2 [14]	Prob. Unknown	Unknown	NPc [35]	Prob. Unknown
chordal bipartite	NPc [36]	? (?)	NPc [25]	?		P [15]
planar	NPc (Sources!)	FPT [2]	NPc	FPT (this)	NPc	FPT [20]
undirected path	NPc [6]	FPT [18]	NPc [24]	?	NPc [31]	?
dually chordal		P [8]		? (attempted [19])		P [30]
strongly chordal		P [17]		P [40]	NPc [17]	
AT-free		P [29]		P [27]		P [29]
tolerance		P [22]		?		?
block		P [17]		P [24]		P [10]
interval		P [12]		P [38]		P [5]
bounded clique-width		P [13]		P [13]		P [13]
bounded mim-width		P [3, 9]		P [19]		P [3, 9]

Status SEMITOTAL DOMINATING SET

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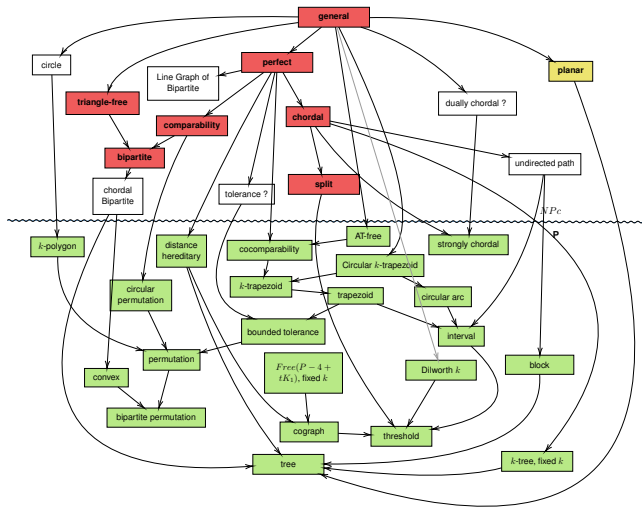
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Warmup: Intractability Results

ω_2 hard on split, chordal and bipartite graphs

- **Split Graph:** $G = \text{Clique} + \text{IndependentSet}$

Split Graphs

SEMITOTAL DOMINATING SET on *split* and *chordal* graphs is ω_2 -hard

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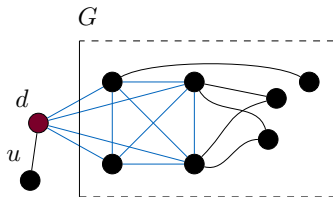
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Proof by fpt-reduction from PLANAR DOMINATING SET on split graphs:

- 1 Construct G^* by adding v with pendant z to clique. G^* split
- 2 If ds D in G , $D' = D \cup \{v\}$ is sds D' .
- 3 If sds D' in G' , $D \setminus \{v\}$ is D in G
- 4 Parameter k only changed by constant

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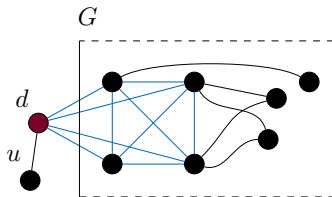
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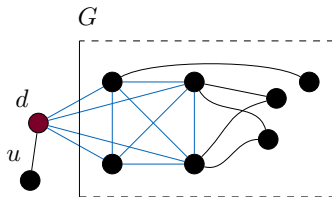
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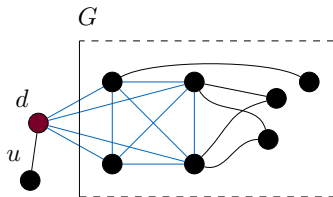
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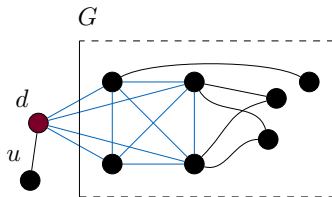
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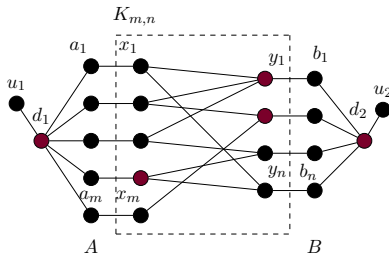


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Bipartite Graphs

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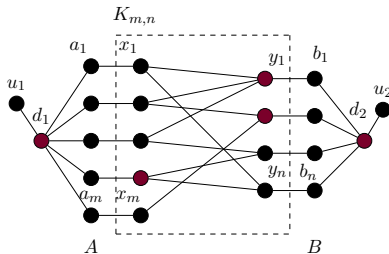


Proof by fpt-reduction from PLANAR DOMINATING SET on bipart. graphs:

- 1 **Construct** Add new neighbor to each vertex and add d_1, d_2, u_1, u_2
- 2 If ds D in G , then $D' = D \cup \{d_1, d_2\}$ is sds in G'
- 3 Assume sds D' in G' . If $a_i \in D'$ (b_i), flip. $D = D' \setminus \{d_1, d_2\}$ is ds in G

Bipartite Graphs

SEMITOTAL DOMINATING SET on *bipartite* graphs is ω_2 -hard

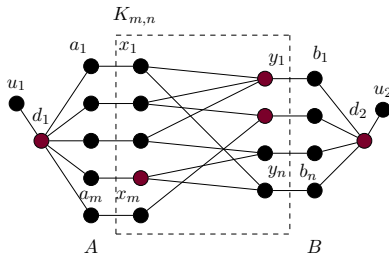


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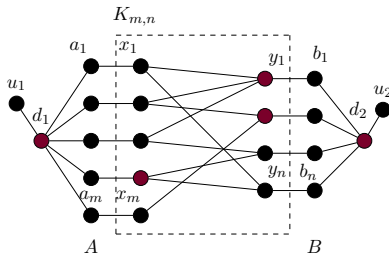


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A Linear Kernel for PLANAR SEMITOTAL DOMINATING SET

Another Explicit kernel for a Dominating Problem

Kernelization

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- **Idea:** Preprocess an instance using *Reduction Rules* until hard *kernel* is found.



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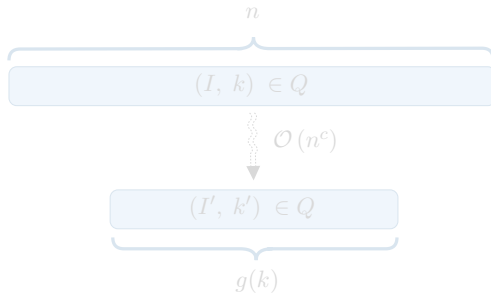
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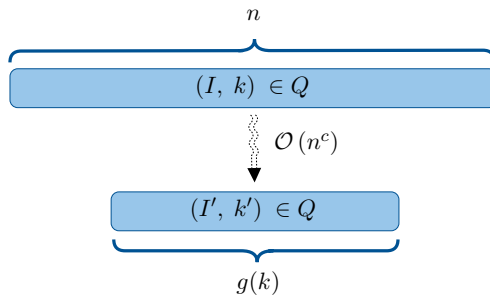
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Related Works

Problem	Size	Source
PLANAR DOMINATING SET	$67k$	[16]
PLANAR TOTAL DOMINATING SET	$410k$	[20]
PLANAR SEMITOTAL DOMINATING SET	$359k$	This work
PLANAR EDGE DOMINATING SET	$14k$	[23]
PLANAR EFFICIENT DOMINATING SET	$84k$	[23]
PLANAR RED-BLUE DOMINATING SET	$43k$	[21]
PLANAR CONNECTED DOMINATING SET	$130k$	[34]
PLANAR DIRECTED DOMINATING SET	Linear	[1]

Main Theorem

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The Main Theorem

SEMITOTAL DOMINATING SET parameterized by solution size admits a linear kernel on planar graphs. There exists a polynomial-time algorithm that, given a planar graph (G, k) , either correctly reports that (G, k) is a NO-instance or returns an equivalent instance (G', k) such that $|V(G')| \leq 359 \cdot k$.

The Big Picture

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Given a planar graph $G = (V, E)$, we will:

- 1 Split the neighborhoods of the graph;
- 2 Define reduction Rules
- 3 Use the region decomposition to analyse size of each region

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Given a planar graph $G = (V, E)$, we will:

- 1 Split the neighborhoods of the graph;
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- 3 Use the region decomposition to analyse size of each region

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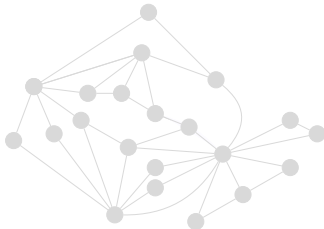
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The Basic Principle: Regions

Region (Simplified)

Given plane G and $v, w \in V$, a region is a closed subset, such that

- there are two non-crossing (but possibly overlapping) boundary paths
- Every vertex in R belongs to $N(v, w)$

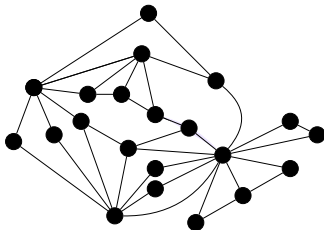


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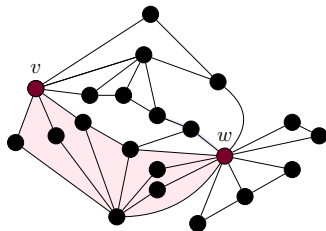
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D-region decomposition [2]

Given $G = (V, W)$ and $D \subseteq V$, a *D-region decomposition* is a set \mathfrak{R} with poles in D such that:

- for any vw -region $R \in \mathfrak{R}$: $D \cap V(R) = \{v, w\}$
- Regions are disjunct, but can share border vertices

A region is **maximal**, if no $R \in \mathfrak{R}$ such that $\mathfrak{R}' = \mathfrak{R} \cup \{R\}$ is a *D-region decomposition* with $V(\mathfrak{R}) \subsetneq V(\mathfrak{R}')$.

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Maximal D -region decomposition

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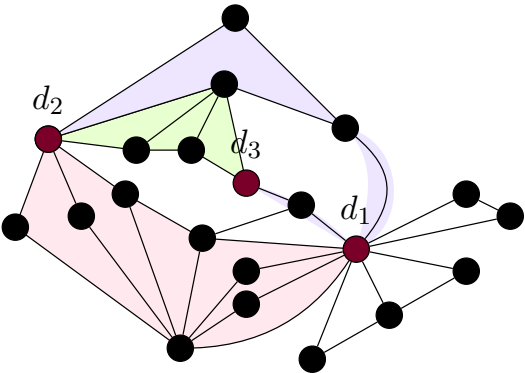
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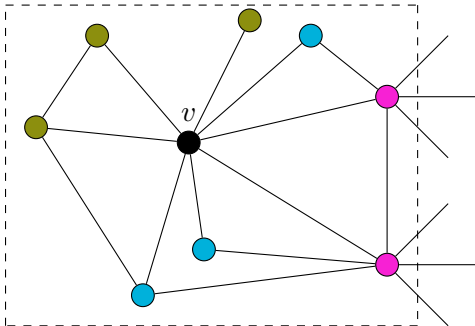
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Splitting Up $N(v)$

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We split $N(v)$ into three subsets:

$$N_1(v) = \{u \in N(v) : N(u) \setminus N[v] \neq \emptyset\} \quad (1)$$

$$N_2(v) = \{u \in N(v) \setminus N_1(v) : N(u) \cap N_1(v) \neq \emptyset\} \quad (2)$$

$$N_3(v) = N(v) \setminus (N_1(v) \cup N_2(v)) \quad (3)$$

For $i, j \in [1, 3]$, we denote $N_{i,j}(v) := N_i(v) \cup N_j(v)$.

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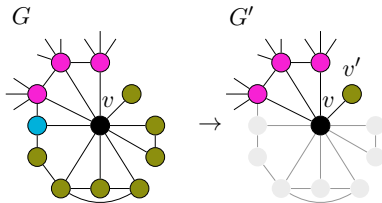
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For $i, j \in [1, 3]$, we denote $N_{i,j}(v) := N_i(v) \cup N_j(v)$.

Rule 1: Shrinking $N_3(v)$

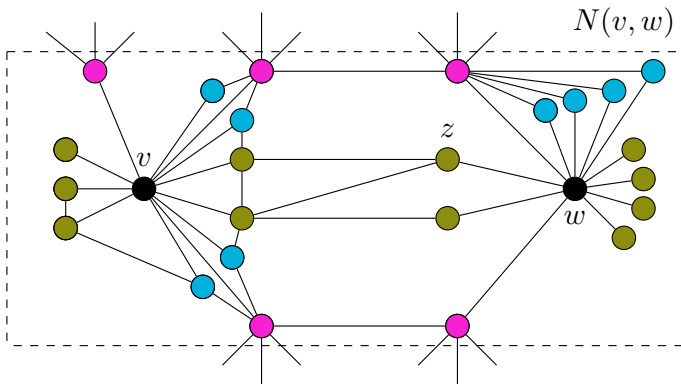
Let $G = (V, E)$ be a graph and let $v \in V$. If $|N_3(v)| \geq 1$:

- remove $N_{2,3}(v)$ from G ,
- add a vertex v' and an edge $\{v, v'\}$.



- **Idea:** v better choice than $N_{2,3}$

Splitting up $N(v, w)$



Splitting up $N(v, w)$

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$$N_1(v, w) = \{u \in N(v, w) \mid N(u) \setminus (N(v, w) \cup \{v, w\}) \neq \emptyset\} \quad (4)$$

$$N_2(v, w) = \{u \in N(v, w) \setminus N_1(v, w) \mid N(u) \cap N_1(v, w) \neq \emptyset\} \quad (5)$$

$$N_3(v, w) = N(v, w) \setminus (N_1(v, w) \cup N_2(v, w)) \quad (6)$$

For $i, j \in [1, 3]$, we denote $N_{i,j}(v, w) = N_i(v, w) \cup N_j(v, w)$.

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For $i, j \in [1, 3]$, we denote $N_{i,j}(v, w) = N_i(v, w) \cup N_j(v, w)$.

Rule 2: Setting Up Our Weapons

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Key Idea: $N_{2,3}(v, w)$ can **always** be semitotally dominated with 4 vertices.

$$\mathcal{D} = \{\tilde{D} \subseteq N_{2,3}(v, w) \mid N_3(v, w) \subseteq \bigcup_{v \in \tilde{D}} N(v), |\tilde{D}| \leq 3\} \quad (7)$$

$$\mathcal{D}_v = \{\tilde{D} \subseteq N_{2,3}(v, w) \cup \{v\} \mid N_3(v, w) \subseteq \bigcup_{v \in \tilde{D}} N(v), |\tilde{D}| \leq 3, v \in \tilde{D}\} \quad (8)$$

$$\mathcal{D}_w = \{\tilde{D} \subseteq N_{2,3}(v, w) \cup \{w\} \mid N_3(v, w) \subseteq \bigcup_{v \in \tilde{D}} N(v), |\tilde{D}| \leq 3, w \in \tilde{D}\} \quad (9)$$

Rule 2: Setting Up Our Weapons

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Key Idea: $N_{2,3}(v, w)$ can **always** be semitotally dominated with 4 vertices.

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Rule 2

If $\mathcal{D} = \emptyset$ we apply the following:

Case 1: if $\mathcal{D}_v = \emptyset$ and $\mathcal{D}_w = \emptyset$

- Remove $N_{2,3}(v, w)$
- Add vertices v' and w' and two edges $\{v, v'\}$ and $\{w, w'\}$
- Preserve $d(v, w)$

Case 2: if $\mathcal{D}_v \neq \emptyset$ and $\mathcal{D}_w = \emptyset$

- Remove $N_{2,3}(v)$
- Add $\{v, v'\}$

Case 3: if $\mathcal{D}_v = \emptyset$ and $\mathcal{D}_w \neq \emptyset$

Symmetric

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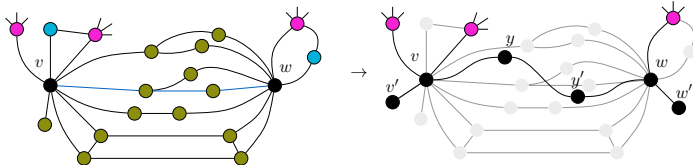
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Symmetric

Rule 2: Case 1



Rule 2: Case 2

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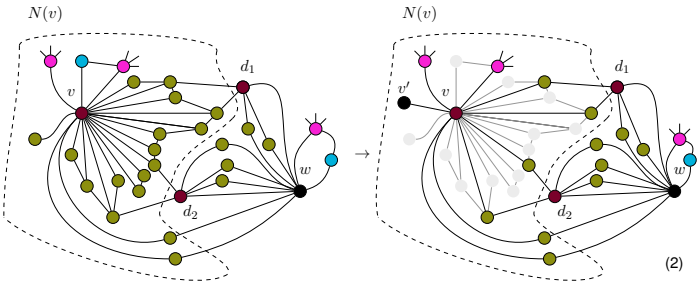
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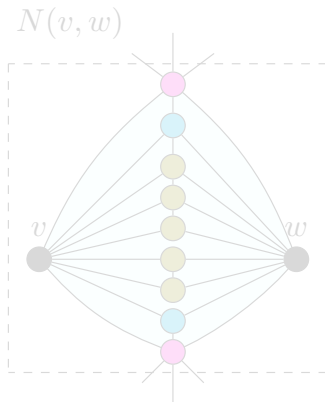
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Simple Regions

A simple vw -region is a vw -region such that:

- 1 its boundary paths have length at most 2, and
- 2 $V(R) \setminus \{v, w\} \subseteq N(v) \cap N(w)$.

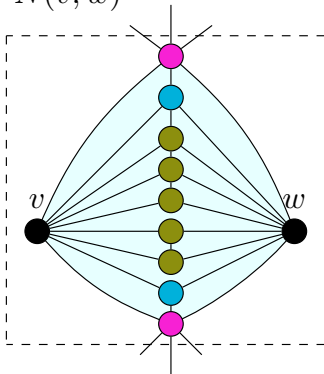


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$N(v, w)$



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Rule 3: Shrinking the Size of Simple Regions

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Let $G = (V, E)$ be a plane graph, $v, w \in V$ and R be a simple region between v and w . If $|V(R) \setminus \{v, w\}| \geq 5$ apply the following:

Case 1: If $G[R \setminus \partial R] \cong P_3$, then:

- remove $V(R \setminus \partial R)$
- add vertex y with edges $\{v, y\}$ and $\{y, w\}$

Case 2: If $G[R \setminus \partial R] \not\cong P_3$, then

- remove $V(R \setminus \partial R)$
- add vertices y, y' and four edges $\{v, y\}, \{v, y'\}, \{y, w\}$ and $\{y', w\}$

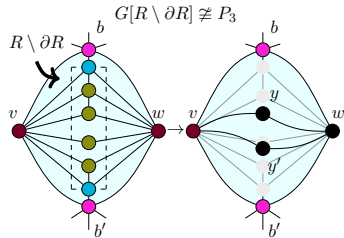
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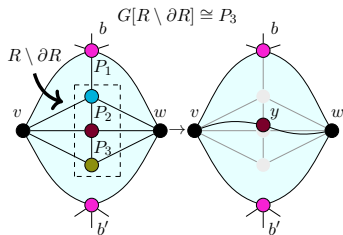
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- We proved that all these rules are sound,
- change the solution size by only a constant factor
- and can be applied in poly-time.

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Bounding the Kernel: Vertices Outside any Region

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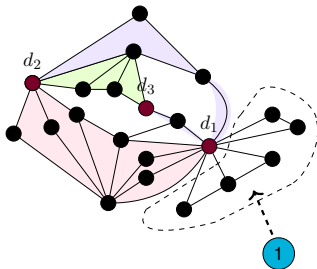
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For each d in $\text{sds } D$:

- ① $|N_1(v) \setminus V(\mathfrak{R})| = 0$ [2], On Border
- ② $|N_2(v) \setminus V(\mathfrak{R})| = 96$ [2]: TODO Reasoning
- ③ $|N_3(v) \setminus V(\mathfrak{R})| = 1$, by Rule 1

Bounding the Kernel: Vertices Outside any Region

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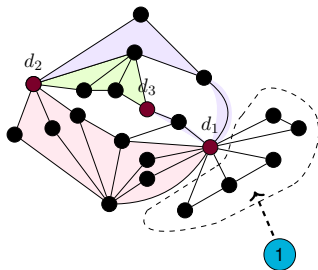
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Bounding the Kernel: Vertices Outside any Region

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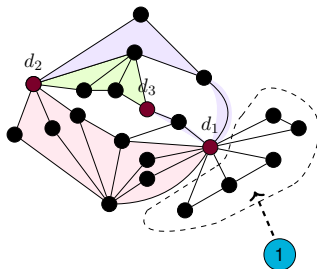
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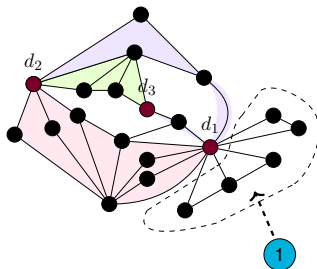
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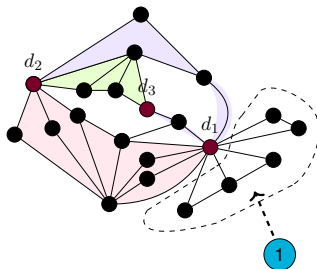
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Bounding the Kernel: Inside a region

Motivation

Theory

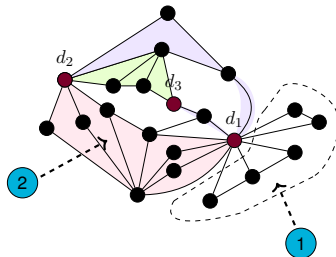
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For each vw -region, we have

- ① $|N_1(v, w)| \leq 4$ [2] (vertices on border)
 - ② $|N_2(v, w)| \leq 6 \cdot 4$ (simple regions to $N_1(v, w)$, Rule 3)
 - ③ $|N_3(v, w)| \leq \max(27, 44, 4, 57) \cdot 4$ (proof omitted depending on Rule 2)
- Total:** $|V(R)| = |\{v, w\} \cup (N_1(v, w) \cup N_2(v, w) \cup N_3(v, w))| \leq 87$

Bounding the Kernel: Inside a region

Motivation

Theory

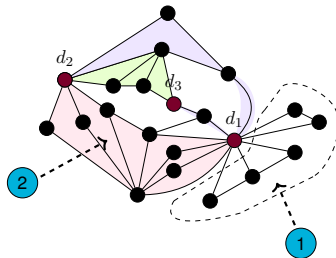
Intractability
 ω_2 hardness

Kernel

Definitions
Rule 1
Rule 2
Rule 3
Kernel Size

Conclusions

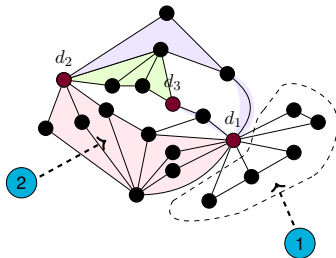
References



For each vw -region, we have

- 1 $|N_1(v, w)| \leq 4$ [2] (vertices on border)
 - 2 $|N_2(v, w)| \leq 6 \cdot 4$ (simple regions to $N_1(v, w)$, Rule 3)
 - 3 $|N_3(v, w)| \leq \max(27, 44, 4, 57) \cdot 4$ (proof omitted depending on Rule 2)
- Total:** $|V(R)| = |\{v, w\} \cup (N_1(v, w) \cup N_2(v, w) \cup N_3(v, w))| \leq 87$

Bounding the Kernel: Inside a region



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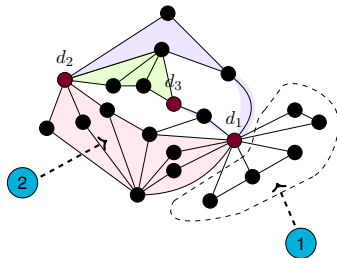
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Bounding the Kernel: Number of Regions

Motivation

Theory

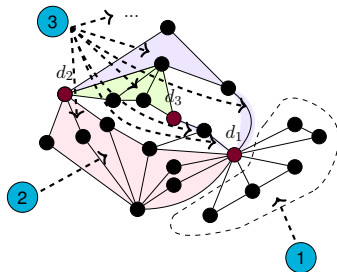
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Number of Regions [2]

Let G be a plane graph and let D be a SEMITOTAL DOMINATING SET with $|D| \geq 3$. There is a maximal D -region decomposition of G such that $|\mathfrak{R}| \leq 3 \cdot |D| - 6$.

Bounding the Kernel: Number of Regions

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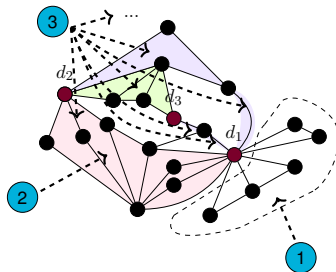
Rule 2

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Let G be a plane graph and let D be a SEMITOTAL DOMINATING SET with $|D| \geq 3$. There is a maximal D -region decomposition of G such that $|\mathfrak{R}| \leq 3 \cdot |D| - 6$.

Summary: Bounding Kernel Size

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Let D be sds of size k . There exists a maximal D -region decomposition \mathfrak{R} such that:

- 1 \mathfrak{R} has only at most $3k - 6$ regions ([2]);
- 2 There are at most $97 \cdot k$ vertices outside of any region;
- 3 Each region $R \in \mathfrak{R}$ contains at most 87 vertices.

Hence: $|V| = \bigcup_{v \in D} N(v) = 87 \cdot (3k - 6) + 97 \cdot k + k < 359 \cdot k$

Main Theorem

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All reduction rules can be applied in poly/time, hence:

The Main Theorem

The SEMITOTAL DOMINATING SET problem parameterized by solution size admits a linear kernel on planar graphs. There exists a polynomial-time algorithm that, given a planar graph (G, k) , either correctly reports that (G, k) is a NO-instance or returns an equivalent instance (G', k) such that $|V(G')| \leq 359 \cdot k$.

Proof: Add Proof here.

Conclusions

Results:

- Given an overview over the status
- SEMITOTAL DOMINATING SET is W_1 for *chordal*, *split* and *bipartite* graphs
- exists linear kernel of size $359 \cdot k$ when parameterized by solution size

Future Work:

- Improve kernel size and do empirical evaluation
- Solve parameterized complexities for *Circle*, *chordal bipartite* and *undirected path graphs*

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? Any Questions ?
... And Thank You For Your Attention ...

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