# Master's Thesis Presentation

## On the Parameterized Complexity of SEMITOTAL DOMINATING SET On Graph Classes

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February 28th, 2023

# Creative Introduction

# Our Plan for Today

# Motivation

**DOMINATING SET**

| | |
|---|---|
| **Input** | Graph $G = (V, E)$, $k \in \mathbb{N}$ |
| **Question** | Exists $D \subseteq V$ with $\|D\| \leq k$ such that $N[D] = V$? |

- The domination number is the minimum cardinality of a ds of $G$, denotes as $\gamma(G)$
- **Observation:** In connected $G$ every $v \in D$ has another $z \in D$ with $d(v, z) \leq 3$.

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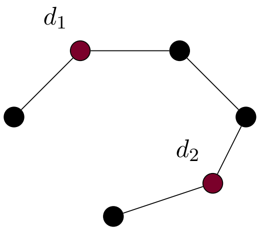
# Example: $\gamma(G) < \gamma_{2t}(G) < \gamma_t(G)$

DOMINATING SET

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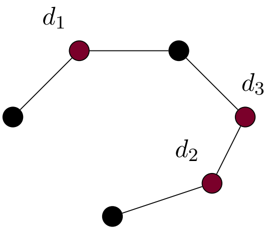


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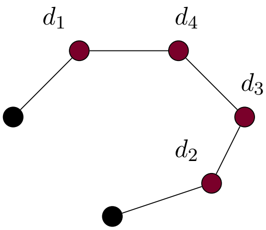

# Parameterized Complexity

- NP-hard? We expect problem to be **at least** exponential
- **Idea:** Limit combinatorial explosion to some aspect of the problem
- **Goal:** Find an algorithm running in time $\mathcal{O}(f(k) \cdot n^c)$ for **some** parameter k
- In this work: by solution size
- **Techniques:** Kernelization, Bounded Search Trees, ...

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# Fixed-Parameter Intractability

- Class NP corresponds to whole hierarchy $W[i]$ in parameterized setting.
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- Dominating Set is $W[2]$-complete
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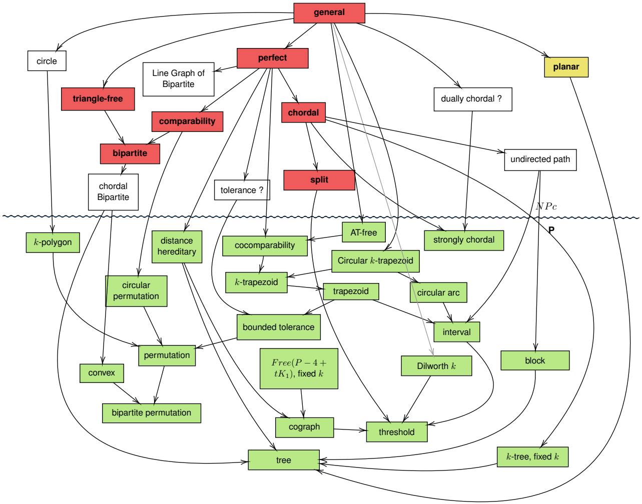
# Complexity Comparison

| Graph Class | Dominating Set | | Semitotal Dominating Set | | Total Dominating Set | |
|---|---|---|---|---|---|---|
| | classical | Parameterized | classical | Parameterized | classical | Parameterized |
| bipartite | NPc [4] | $W_2$ [39] | NPc [25] | $W_2$ (this) | NPc [32] | $W_2$ (cite!) |
| line graph of bipartite | NPc [28] | ? | NPc [19] | ? (?) | NPc [35] | ? |
| circle | NPc [26] | $W_1$ [7] | NPc [27] | ? (?) | NPc [35] | $W_1$ [7] |
| chordal | NPc [6] | $W_2$ [39] | NPc [25] | $W_2$ (this) | NPc [37] | $W_1$ [11] by *split* |
| $s$-chordal , $s > 3$ | NPc [33] | $W_2$ [33] | ? (?) | ? (?) | NPc [33] | $W_1$ [33] |
| split | NPc [4] | $W_2$ [39] | NPc [25] | $W_2$ **this** | NPc [37] | $W_1$ [11] |
| 3-claw-free | NPc [14] | FPT [14] | Prob. Unk | Prob. Unk | NPc [35] | Unknown |
| $t$-claw-free, $t > 3$ | NPc [14] | $W_2$ [14] | Prob. Unknown | Unknown | NPc [35] | Prob. Unknown |
| chordal bipartite | NPc [36] | ? (?) | NPc [25] | ? | P [15] | |
| planar | NPc (Sources!) | FPT [2] | NPc | **FPT (this)** | NPc | FPT [20] |
| undirected path | NPc [6] | FPT [18] | NPc [24] | ? | NPc [31] | ? |
| dually chordal | P [8] | | ? (attempted [19]) | | P [30] | |
| strongly chordal | P [17] | | P [40] | | NPc [17] | |
| AT-free | P [29] | | P [27] | | P [29] | |
| tolerance | P [22] | | ? | | ? | |
| block | P [17] | | P [24] | | P [10] | |
| interval | P [12] | | P [38] | | P [5] | |
| bounded clique-width | P [13] | | P [13] | | P [13] | |
| bounded mim-width | P [3, 9] | | P [19] | | P [3, 9] | |

# Status SEMITOTAL DOMINATING SET



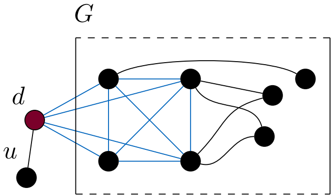
## Warmup: Intractability Results

*$\omega_2$ hard on split, chordal and bipartite graphs*

- **Split Graph:** $G = \texttt{Clique} + \texttt{IndependentSet}$

# Split Graphs

SEMITOTAL DOMINATING SET on *split* and *chordal* graphs is $\omega_2$-hard




**Proof by fpt-reduction from PLANAR DOMINATING SET on split graphs:**

1. **Construct** G* by adding $v$ with pendant $z$ to clique. G* split
2. If ds $D$ in G, $D' = D \cup \{v\}$ is sds $D'$.
3. If sds $D'$ in $G'$, $D \setminus \{v\}$ is $D$ in $G$
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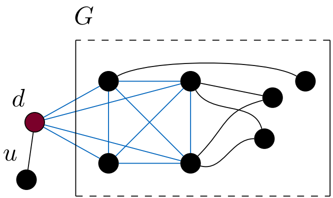


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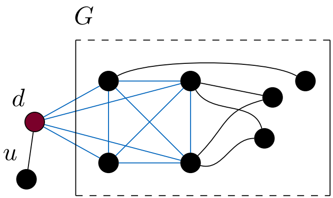


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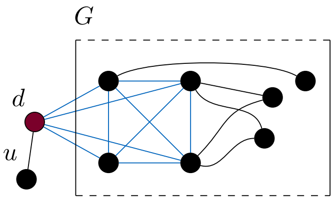


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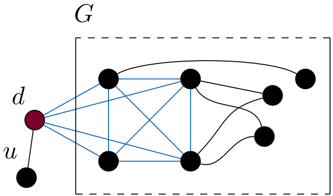


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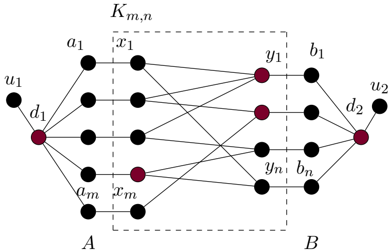
SEMITOTAL DOMINATING SET on *bipartite* graphs is $\omega_2$-hard




**Proof by fpt-reduction from PLANAR DOMINATING SET on bipart. graphs:**

1. **Construct** Add new neighbor to each vertex and add $d_1, d_2, u_1, u_2$
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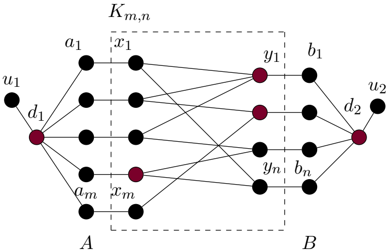


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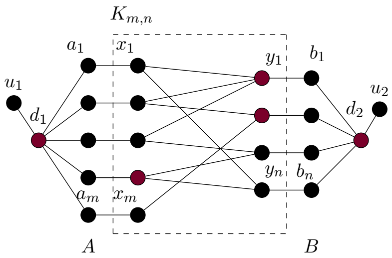


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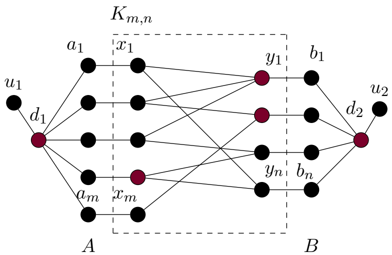


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# A Linear Kernel for PLANAR SEMITOTAL DOMINATING SET

*Another Explicit kernel for a Dominating Problem*

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# Related Works

| Problem | Size | Source |
|---|---|---|
| Planar Dominating Set | $67k$ | [16] |
| Planar Total Dominating Set | $410k$ | [20] |
| Planar Semitotal Dominating Set | $359k$ | **This work** |
| | | |
| Planar Edge Dominating Set | $14k$ | [23] |
| Planar Efficient Dominating Set | $84k$ | [23] |
| Planar Red-Blue Dominating Set | $43k$ | [21] |
| Planar Connected Dominating Set | $130k$ | [34] |
| Planar Directed Dominating Set | Linear | [1] |

# Main Theorem

## The Main Theorem

SEMITOTAL DOMINATING SET parameterized by solution size admits a linear kernel on planar graphs. There exists a polynomial-time algorithm that, given a planar graph $(G, k)$, either correctly reports that $(G, k)$ is a NO-instance or returns an equivalent instance $(G', k)$ such that $|V(G')| \leq 359 \cdot k$.

# The Big Picture

Given a planar graph $G = (V, E)$, we will:

1. Split the neighborhoods of the graph;
2. Define reduction Rules
3. Use the region decomposition to analyse size of each region

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## Region (Simplified)

Given plane $G$ and $v, w \in V$, a region is a closed subset, such that

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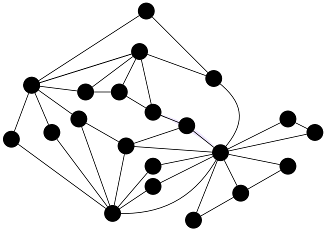

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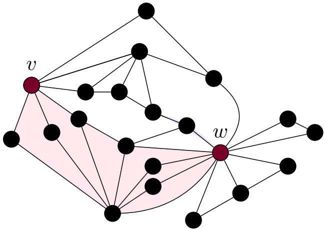

# $D$-region decomposition

**$D$-region decomposition [2]**

Given $G = (V, W)$ and $D \subseteq V$, a *$D$-region decomposition* is a set $\mathfrak{R}$ with poles in $D$ such that:

- for any $vw$-region $R \in \mathfrak{R}$: $D \cap V(R) = \{v, w\}$
- Regions are disjunct, but can share border vertices

A region is **maximal**, if no $R \in \mathfrak{R}$ such that $\mathfrak{R}' = \mathfrak{R} \cup \{R\}$ is a *$D$-region decomposition* with $V(\mathfrak{R}) \subsetneq V(\mathfrak{R}')$.

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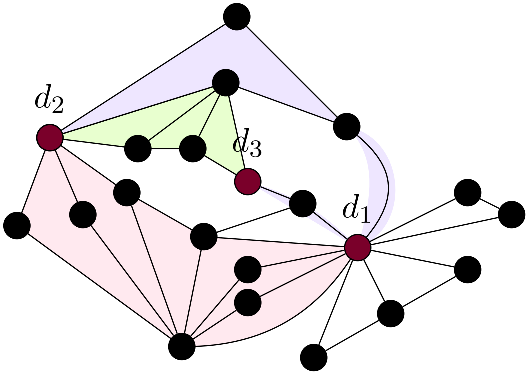
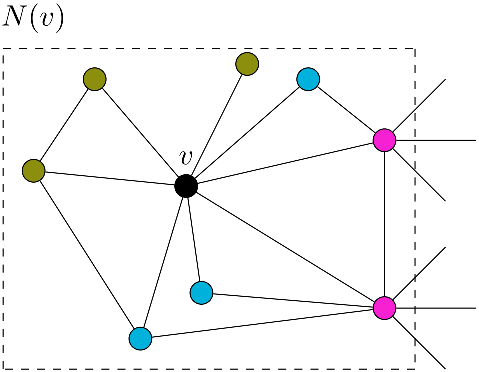
# Maximal $D$-region decomposition



# Splitting Up $N(v)$



# Splitting Up $N(v)$

We split $N(v)$ into three subsets:

$$N_1(v) = \{u \in N(v) : N(u) \setminus N[v] \neq \emptyset\} \tag{1}$$

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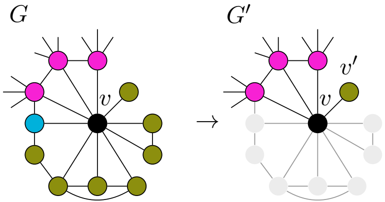
# Rule 1: Shrinking $N_3(v)$

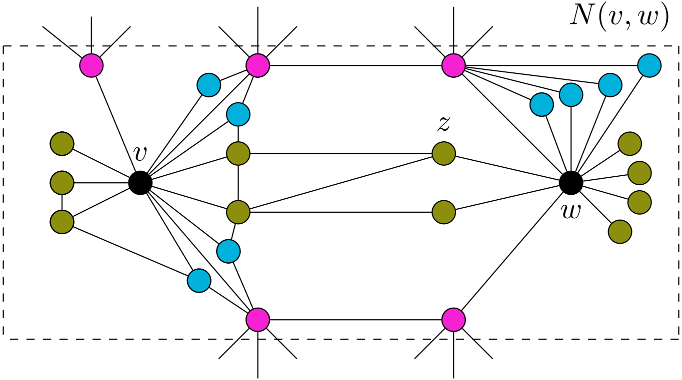
Let $G = (V, E)$ be a graph and let $v \in V$. If $|N_3(v)| \geq 1$:

- remove $N_{2,3}(v)$ from G,
- add a vertex $v'$ and an edge $\{v, v'\}$.




- **Idea:** $v$ better choice than $N_{2,3}$

# Splitting up $N(v, w)$



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# Rule 2: Setting Up Our Weapons

**Key Idea:** $N_{2,3}(v,w)$ can **always** be semitotally dominated with 4 vertices.

$$\mathcal{D} = \{\tilde{D} \subseteq N_{2,3}(v,w) \mid N_3(v,w) \subseteq \bigcup_{v \in \tilde{D}} N(v),\ |\tilde{D}| \leq 3\} \tag{7}$$

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If $\mathcal{D} = \emptyset$ we apply the following:

**Case 1:** if $\mathcal{D}_v = \emptyset$ and $D_w = \emptyset$

- Remove $N_{2,3}(v, w)$
- Add vertices $v'$ and $w'$ and two edges $\{v, v'\}$ and $\{w, w'\}$
- Preserve $d(v, w)$

**Case 2:** if $\mathcal{D}_v \neq \emptyset$ and $D_w = \emptyset$

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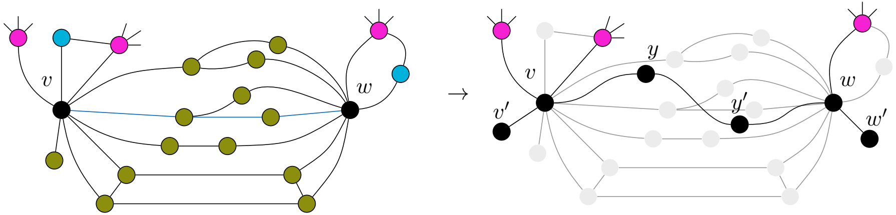
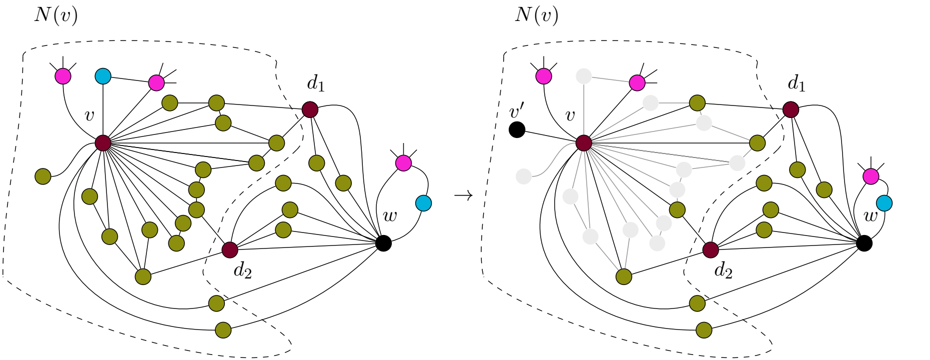


# Rule 2: Case 2



# Simple Regions

A simple $vw$-region is a $vw$-region such that:

1. its boundary paths have length at most 2, and
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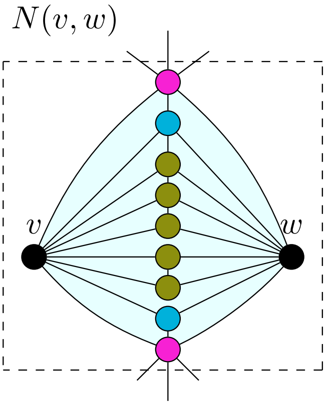

# Rule 3: Shrinking the Size of Simple Regions

Let $G = (V, E)$ be a plane graph, $v, w \in V$ and $R$ be a simple region between $v$ and $w$. If $|V(R) \setminus \{v, w\}| \geq 5$ apply the following:

**Case 1:** If $G[R \setminus \partial R] \cong P_3$, then:

- remove $V(R \setminus \partial R)$
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**Case 2:** If $G[R \setminus \partial R] \ncong P_3$, then

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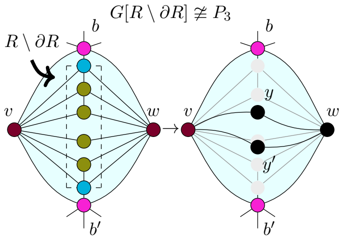

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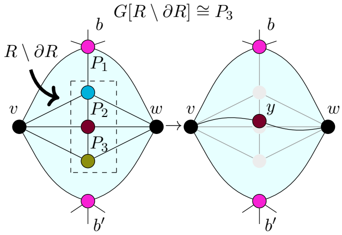

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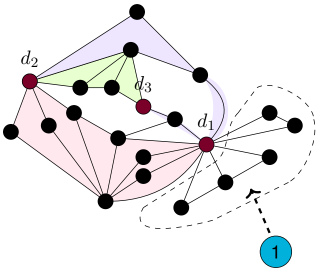
# Bounding the Kernel: Vertices Outside any Region




For each $d$ in sds $D$:

1. $|N_1(v) \setminus V(\mathfrak{R})| = 0$ [2], On Border
2. $|N_2(v) \setminus V(\mathfrak{R})| = 96$ [2]: TODO Reasoning
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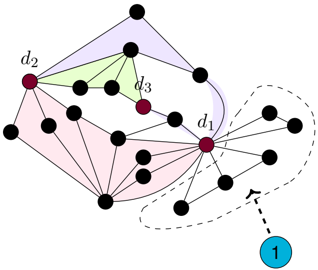


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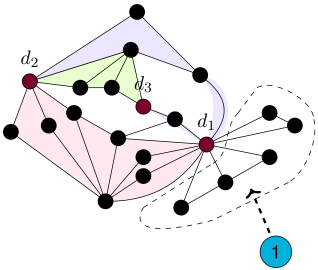


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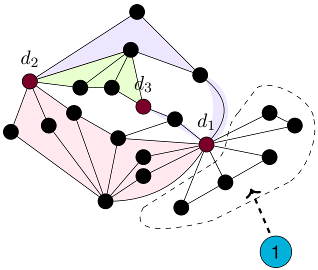


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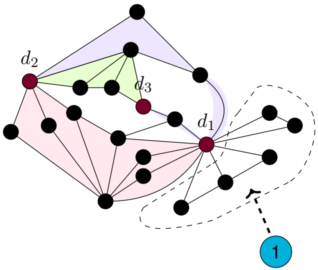


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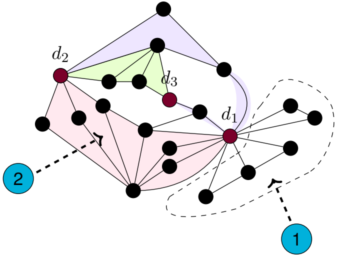


For each $vw$-region, we have

1. $|N_1(v,w)| \leq 4$ [2] (vertices on border)
2. $|N_2(v,w)| \leq 6 \cdot 4$ (simple regions to $N_1(v,w)$, Rule 3)
3. $|N_3(v,w)| \leq \max(27, 44, 4, 57) \cdot 4$ (proof omitted depending on Rule 2)

**Total:** $|V(R)| = |\{v,w\} \cup (N_1(v,w) \cup N_2(v,w) \cup N_3(v,w))| \leq 87$

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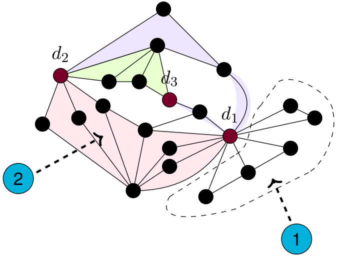


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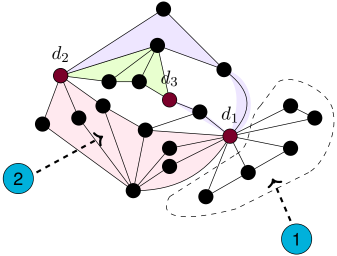


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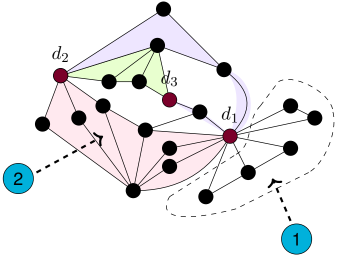


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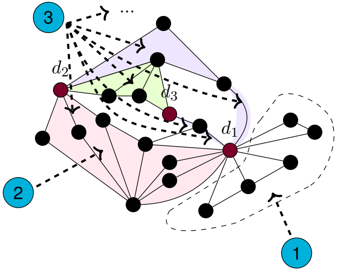


### Number of Regions [2]

Let G be a plane graph and let $D$ be a SEMITOTAL DOMINATING SET with $|D| \geq 3$. There is a maximal *$D$-region decomposition* of $G$ sucht that $|\mathfrak{R}| \leq 3 \cdot |D| - 6$.

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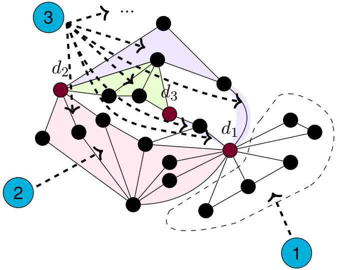


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# Summary: Bounding Kernel Size

Let D be sds of size $k$. There exists a maximal *$D$-region decomposition* $\mathfrak{R}$ such that:

1. $\mathfrak{R}$ has only at most $3k - 6$ regions ([2]);
2. There are at most $97 \cdot k$ vertices outside of any region;
3. Each region $R \in \mathfrak{R}$ contains at most $87$ vertices.

**Hence:** $|V| = \bigcup_{v \in D} N(v) = 87 \cdot (3k - 6) + 97 \cdot k + k < 359 \cdot k$

# Main Theorem

All reduction rules can be applied in poly/time, hence:

**The Main Theorem**

The SEMITOTAL DOMINATING SET problem parameterized by solution size admits a linear kernel on planar graphs. There exists a polynomial-time algorithm that, given a planar graph $(G, k)$, either correctly reports that $(G, k)$ is a NO-instance or returns an equivalent instance $(G', k)$ such that $|V(G')| \leq 359 \cdot k$.

**Proof:** Add Proof here.

# Conclusions

**Results:**

- Given an overview over the status
- SEMITOTAL DOMINATING SET is $W_1$ for *chordal, split* and *bipartite* graphs
- exists linear kernel of size $359 \cdot k$ when parameterized by solution size

**Future Work:**

- Improve kernel size and do empirical evaluation
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# ? Any Questions ?

$\cdots$ *And Thank You For Your Attention* $\cdots$

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