Cafourek

1) a) $\begin{pmatrix}1\\1\end{pmatrix}, \begin{pmatrix}-1\\-1\end{pmatrix}$, [crossed out], $\left(\begin{pmatrix}2\\-3\end{pmatrix}, \begin{pmatrix}-2\\3\end{pmatrix}\right)$

b) [crossed out] všechny

c) žádné – kromě $\vec{o}$ $\left(\begin{pmatrix}1\\0\end{pmatrix}, \begin{pmatrix}-1\\0\end{pmatrix}\right)$

d) žádné – kromě $\vec{o}$

2) a) $|\lambda|>1$ $\lambda \oplus$ .. b) $|\lambda|>1$ $\lambda\oplus$ c) $|\lambda|<1$ $\lambda\ominus$

$|\lambda|<1$ $\lambda\ominus$

3) a) na $\mathbb{R}^2$: $\begin{pmatrix}3 & 2\\0 & 3\end{pmatrix}$ $\begin{vmatrix}3-x & 2\\0 & 3-x\end{vmatrix} = (3-x)^2$

$\hookrightarrow \lambda_1 = 3$ (nás. 2)

b) na $\mathbb{R}^2$: $\begin{pmatrix}4 & 1\\3 & 6\end{pmatrix}$ $\begin{vmatrix}4-x & 1\\3 & 6-x\end{vmatrix} = (4-x)(6-x)-3 = 24-10x+x^2-3$

$= x^2-10x+21 =$

$(x-7)(x-3)$

$\lambda_1 = 7$ (nás 1) $\lambda_2 = 3$ (nás. 1)

c) na $\mathbb{R}^3$: $\begin{pmatrix}4 & -3 & 0\\3 & 4 & 0\\5 & 10 & 10\end{pmatrix}$

$\begin{vmatrix}4-x & -3 & 0\\3 & 4-x & 0\\5 & 10 & 10-x\end{vmatrix} = (4-x)^2(10-x) - 3\cdot(-3)(10-x) = (x^2-8x+16)(10-x)-(-90+9x) =$

$= -x^3+10x^2+8x^2-80x-16x+160+90-9x =$

$= -x^3+18x^2-105x+250 =$

$= -(x^3-18x^2+105x-250)$ $\lambda_1 = 10$

$\lambda_2 = 4+3i$

$\lambda_2 = 4-3i$

nemá řešení na $\mathbb{R}^3$

4) a) $\lambda=3$ (nás 2)

$\begin{pmatrix}0 & 2\\0 & 0\end{pmatrix}$ ~~Ker A~~ $=$ span$\left(\begin{pmatrix}1\\0\end{pmatrix}\right)$ = eigen$(3,A)$

b) $\lambda = 7$ (nás 1) $\lambda = 3$ (nás 1)

$\begin{pmatrix}-3 & 1\\3 & -1\end{pmatrix}$ ~~Ker A~~ $=$ span$\left(\begin{pmatrix}1/3\\1\end{pmatrix}\right)$ $\begin{pmatrix}1 & 1\\3 & 3\end{pmatrix}$ ~~Ker A~~ $=$ span$\left(\begin{pmatrix}-1\\1\end{pmatrix}\right)$

$=$ span$\left(\begin{pmatrix}1\\3\end{pmatrix}\right)$

eigen$(7,A)$ $\not\!\!/$ eigen$(3,A)$ //

c) $\lambda = 10$ (nás 1)

$\begin{pmatrix}-6 & -3 & 0\\3 & -6 & 0\\5 & 10 & 0\end{pmatrix} \sim \begin{pmatrix}2 & 1 & 0\\1 & -2 & 0\\1 & 2 & 0\end{pmatrix} \sim \begin{pmatrix}2 & 1 & 0\\0 & -5 & 0\\0 & 3 & 0\end{pmatrix} \sim \begin{pmatrix}2 & 1 & 0\\0 & 1 & 0\\0 & 0 & 0\end{pmatrix}$

eigen$(10,A)$ ~~Ker A~~ $=$ span$\left(\begin{pmatrix}0\\0\\1\end{pmatrix}\right)$

~~[illegible]~~ Caforrek

5) $A=\begin{pmatrix}1&1\\4&-2\end{pmatrix}$ $\begin{vmatrix}1-x&1\\4&-2-x\end{vmatrix}=(1-x)(-2-x)-4=-2-x+2x+x^2-4=$

$=x^2+x-6=(x+3)(x-2)$

$\lambda_1=-3 \quad \lambda_2=2$

$\lambda_1=-3$

$\begin{pmatrix}4&1\\4&1\end{pmatrix}$ ~~[illegible]~~ $\operatorname{span}\left(\begin{pmatrix}-1/4\\1\end{pmatrix}\right)=\operatorname{span}\left(\begin{pmatrix}-1\\4\end{pmatrix}\right)$

$\quad\quad\quad\quad\quad\quad\quad\quad\quad\quad\quad\quad\quad\quad\quad\quad\quad\quad\quad\quad\quad \parallel$

$\quad\quad\quad\quad\quad\quad\quad\quad\quad\quad\quad\quad\quad\quad\quad\quad\quad\quad \operatorname{eigen}(-3,A)$

$\lambda_2=2$

$\begin{pmatrix}-1&1\\4&-4\end{pmatrix}$ ~~[illegible]~~ $\operatorname{span}\left(\begin{pmatrix}1\\1\end{pmatrix}\right)=\operatorname{eigen}(2,A)$

6) $B=\left\{\begin{pmatrix}-1\\4\end{pmatrix},\begin{pmatrix}1\\1\end{pmatrix}\right\}$ $T_{B\to K_2}=\begin{pmatrix}-1&1\\4&1\end{pmatrix}$

$M=T_{K_2\to B}\cdot A\cdot T_{B\to K_2}$ $\quad A=\begin{pmatrix}1&1\\4&-2\end{pmatrix}$

$(T_{B\to K_2})^{-1}$

$\operatorname{adj}(T_{B\to K_2})=\begin{pmatrix}1&-4\\-1&-1\end{pmatrix}^T=\begin{pmatrix}1&-1\\-4&-1\end{pmatrix}\longrightarrow(T_{B\to K_2})^{-1}=T_{K_2\to B}=-\frac{1}{5}\cdot\begin{pmatrix}1&-1\\-4&-1\end{pmatrix}$

$\det(T_{B\to K_2})=-5\to\det^{-1}=-\frac{1}{5}$

$M=\left(-\frac{1}{5}\right)\cdot\begin{pmatrix}1&-1\\-4&-1\end{pmatrix}\begin{pmatrix}1&1\\4&-2\end{pmatrix}\begin{pmatrix}-1&1\\4&1\end{pmatrix}=\left(-\frac{1}{5}\right)\cdot\begin{pmatrix}1&-1\\-4&-1\end{pmatrix}\begin{pmatrix}3&2\\-12&2\end{pmatrix}=\left(-\frac{1}{5}\right)\cdot\begin{pmatrix}15&0\\0&-10\end{pmatrix}=$

$=\underline{\underline{\begin{pmatrix}-3&0\\0&2\end{pmatrix}}}$

7) a) na $\mathbb{R}^2$: $\begin{pmatrix}3&2\\2&6\end{pmatrix}$

$\begin{vmatrix}3-x&2\\2&6-x\end{vmatrix}=(3-x)(6-x)-4=18-3x-6x+x^2-4=x^2-9x+14=$

$=(x-2)(x-7)$

$\lambda_1=2,\ \lambda_2=7$

$\operatorname{eigen}(2,A)=\operatorname{span}\left(\begin{pmatrix}-2\\1\end{pmatrix}\right)$

$\operatorname{eigen}(7,A)=\operatorname{span}\left(\begin{pmatrix}1/2\\1\end{pmatrix}\right)=\operatorname{span}\left(\begin{pmatrix}1\\2\end{pmatrix}\right)$

$M=\left(-\frac{1}{5}\right)\begin{pmatrix}2&-1\\-1&-2\end{pmatrix}\begin{pmatrix}3&2\\2&6\end{pmatrix}\begin{pmatrix}-2&1\\1&2\end{pmatrix}=\underline{\underline{\begin{pmatrix}2&0\\0&7\end{pmatrix}}}$

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b) na $\mathbb{R}^2$: $\begin{pmatrix} 2 & 1 \\ -4 & 6 \end{pmatrix}$

$\begin{vmatrix} 2-x & 1 \\ -4 & 6-x \end{vmatrix} = (2-x)(6-x)+4 = 12-2x-6x+x^2+4 = x^2-8x+16$

$= (x-4)^2$

$\begin{pmatrix} -2 & 1 \\ -4 & 2 \end{pmatrix}$ eigen$(4,A) = $ span$\left(\begin{pmatrix} 1/2 \\ 1 \end{pmatrix}\right)$

$\lambda_1 = 4$ (nás 2)

dim = 1

$\neq$

není diagonalisovatelná

c) na $\mathbb{Z}_3^3$: $\begin{pmatrix} 0 & 2 & 1 \\ 1 & 1 & 1 \\ 2 & 1 & 1 \end{pmatrix}$

$\begin{vmatrix} -x & 2 & 1 \\ 1 & 1-x & 1 \\ 2 & 1 & 1-x \end{vmatrix} = (-x)(1-x)^2 + 2\cdot 2 + 1 - 2(1-x) - (-x) - 2(1-x) =$

$= -x^3+2x^2-x+5+x-2+2x-2+2x = -x^3+2x^2+4x+1$

$\lambda_1 = -1$

$\lambda_2 = \ldots$

$\lambda_3 = \ldots$

není diag.

8) $\begin{pmatrix} 3 & 2 \\ 2 & 6 \end{pmatrix}^{100} = \begin{pmatrix} -2 & 1 \\ 1 & 2 \end{pmatrix} \cdot \begin{pmatrix} 2 & 0 \\ 0 & 7 \end{pmatrix}^{100} \cdot \begin{pmatrix} -\frac{2}{5} & \frac{1}{5} \\ \frac{1}{5} & \frac{2}{5} \end{pmatrix}$

$\doteq \begin{pmatrix} 6\times 10^{83} & 1\times 10^{84} \\ 1\times 10^{84} & 2\times 10^{84} \end{pmatrix}$

9) $x_0 = 0$, $x_1 = 0$, $x_2 = 1$, $x_3 = 2$, $x_4 = 5$, $x_5 = 10$, $x_6 = 21$

$x_n = 2x_{n-1} + x_{n-2} - 2x_{n-3}$ $\quad x_7 = 42$, $x_8 = 85$, $x_9 = 170$

$\begin{pmatrix} 2-x & 0 & 0 \\ 1 & -x & 0 \\ 0 & 0 & -x \end{pmatrix} = (2-x)(-x)(-x) = \cancel{\ldots} \leftarrow \begin{pmatrix} 2 & 0 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}$

$= 2x^2 - x^3 = x^2(2-x)$

$\hookrightarrow \lambda_1 = 0$ nás 2 $\quad \lambda_1 = 2$ nás 1

$A = \begin{pmatrix} 2 & 0 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}$ eigen$(0,A) = $ span$\left(\begin{pmatrix} 0 \\ 1 \\ 0 \end{pmatrix}, \begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix}\right)$

$x_{102} = 5\times 10^{30}$

$\begin{pmatrix} 0 & 0 & 0 \\ 1 & -2 & 0 \\ 0 & 0 & -2 \end{pmatrix} \sim \begin{pmatrix} 1 & -2 & 0 \\ 0 & 0 & -2 \\ 0 & 0 & 0 \end{pmatrix}$ eigen$(2,A) = $ span$\left(\begin{pmatrix} 2 \\ 1 \\ 0 \end{pmatrix}\right)$

$M = \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 2 \end{pmatrix}$

$\begin{pmatrix} x_{102} \\ x_{101} \\ x_{100} \end{pmatrix} = \begin{pmatrix} 0 & 0 & 2 \\ 1 & 0 & 1 \\ 0 & 1 & 0 \end{pmatrix} \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 2^{102} \end{pmatrix} \begin{pmatrix} 0 & 0 & 2 \\ 1 & 0 & 1 \\ 0 & 1 & 0 \end{pmatrix}^{-1} \cdot \begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix} = \begin{pmatrix} 5\times 10^{30} \\ 2\times 10^{30} \\ 0 \end{pmatrix}$