Cafourek

1. a) $A = \begin{pmatrix} 2 & 2 \\ 3 & 4 \end{pmatrix}$

mod $\mathbb{Z}_5$ $\begin{vmatrix} 2 & 2 \\ 3 & 4 \end{vmatrix} = 2$

$$\left(\begin{array}{cc|cc} 2 & 2 & 1 & 0 \\ 3 & 4 & 0 & 1 \end{array}\right) \sim \left(\begin{array}{cc|cc} 2 & 2 & 1 & 0 \\ 0 & 1 & 1 & 1 \end{array}\right)\begin{array}{l} R_1 \\ R_1+R_2 \end{array} \sim \left(\begin{array}{cc|cc} 1 & 1 & 3 & 0 \\ 0 & 1 & 1 & 1 \end{array}\right)\begin{array}{l} 3R_1 \\ R_2 \end{array}$$

$$\sim \left(\begin{array}{cc|cc} 1 & 0 & 2 & 4 \\ 0 & 1 & 1 & 1 \end{array}\right)\begin{array}{l} R_1-R_2 \\ R_2 \end{array}$$

$$A^{-1} = \begin{pmatrix} 2 & 4 \\ 1 & 1 \end{pmatrix}$$

b) $A = \begin{pmatrix} 1 & 2 & 2 \\ 2 & 1 & 2 \\ 2 & 2 & 1 \end{pmatrix}$

mod $\mathbb{R}$

$\begin{vmatrix} 1 & 2 & 2 \\ 2 & 1 & 2 \\ 2 & 2 & 1 \end{vmatrix} = 5$

$$\left(\begin{array}{ccc|ccc} 1 & 2 & 2 & 1 & 0 & 0 \\ 2 & 1 & 2 & 0 & 1 & 0 \\ 2 & 2 & 1 & 0 & 0 & 1 \end{array}\right) \sim \left(\begin{array}{ccc|ccc} 1 & 2 & 2 & 1 & 0 & 0 \\ 0 & -3 & -2 & -2 & 1 & 0 \\ 0 & -2 & -3 & -2 & 0 & 1 \end{array}\right)\begin{array}{l} R_1 \\ R_2-2R_1 \\ R_3-2R_1 \end{array} \sim$$

$$\sim \left(\begin{array}{ccc|ccc} 1 & 2 & 2 & 1 & 0 & 0 \\ 0 & -3 & -2 & -2 & 1 & 0 \\ 0 & 0 & -5 & -2 & -2 & 3 \end{array}\right)\begin{array}{l} R_1 \\ R_2 \\ 3R_3-2R_2 \end{array} \sim \left(\begin{array}{ccc|ccc} 3 & 0 & 2 & -1 & 2 & 0 \\ 0 & -15 & 0 & -6 & 9 & -6 \\ 0 & 0 & -5 & -2 & -2 & 3 \end{array}\right)\begin{array}{l} 3R_1+2R_2 \\ 5R_2-2R_3 \\ R_3 \end{array}$$

$$\sim \left(\begin{array}{ccc|ccc} 15 & 0 & 0 & -9 & 6 & 6 \\ 0 & -15 & 0 & -6 & 9 & -6 \\ 0 & 0 & -5 & -2 & -2 & 3 \end{array}\right)\begin{array}{l} 5R_1+2R_3 \\ R_2 \\ R_3 \end{array} \sim \left(\begin{array}{ccc|ccc} 1 & 0 & 0 & -\frac{3}{5} & \frac{2}{5} & \frac{2}{5} \\ 0 & 1 & 0 & \frac{2}{5} & -\frac{3}{5} & \frac{2}{5} \\ 0 & 0 & 1 & \frac{2}{5} & \frac{2}{5} & -\frac{3}{5} \end{array}\right)\begin{array}{l} \frac{1}{15}R_1 \\ -\frac{1}{15}R_2 \\ -\frac{1}{5}R_3 \end{array}$$

$$A^{-1} = \begin{pmatrix} -\frac{3}{5} & \frac{2}{5} & \frac{2}{5} \\ \frac{2}{5} & -\frac{3}{5} & \frac{2}{5} \\ \frac{2}{5} & \frac{2}{5} & -\frac{3}{5} \end{pmatrix}$$

c) $A = \begin{pmatrix} 1 & 2 & 2 \\ 2 & 1 & 2 \\ 2 & 2 & 1 \end{pmatrix}$

mod $\mathbb{Z}_5$

$\begin{vmatrix} 1 & 2 & 2 \\ 2 & 1 & 2 \\ 2 & 2 & 1 \end{vmatrix} = 0$

$$\left(\begin{array}{ccc|c} 1 & 2 & 2 & 0 \\ 2 & 1 & 2 & 0 \\ 2 & 2 & 1 & 0 \end{array}\right) \sim \left(\begin{array}{ccc|c} 1 & 2 & 2 & 0 \\ 0 & 2 & 3 & 0 \\ 0 & 3 & 2 & 0 \end{array}\right)\begin{array}{l} R_1 \\ R_2-2R_1 \\ R_3-2R_1 \end{array} \sim \left(\begin{array}{ccc|c} 1 & 2 & 2 & 0 \\ 0 & 2 & 3 & 0 \\ 0 & 0 & 0 & 0 \end{array}\right)\begin{array}{l} R_1 \\ R_2 \\ R_3+R_2 \end{array}$$

$$\ker(A) = \operatorname{span}\left(\begin{pmatrix} 1 \\ 1 \\ 1 \end{pmatrix}\right)$$

2) $\frac{1}{5} \cdot \begin{pmatrix} -3 & 2 & 2 \\ 2 & -3 & 2 \\ 2 & 2 & -3 \end{pmatrix} \cdot \begin{pmatrix} 3 \\ 3 \\ 4 \end{pmatrix} = \begin{pmatrix} 1 \\ 1 \\ 0 \end{pmatrix}\begin{array}{l} x \\ y \\ z \end{array}$

$\frac{1}{5} \cdot \begin{pmatrix} -3 & 2 & 2 \\ 2 & -3 & 2 \\ 2 & 2 & -3 \end{pmatrix} \cdot \begin{pmatrix} 1 \\ 1 \\ 3 \end{pmatrix} = \begin{pmatrix} 1 \\ 1 \\ -1 \end{pmatrix}\begin{array}{l} x \\ y \\ z \end{array}$

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3) a) $A=\begin{pmatrix}2&2\\3&4\end{pmatrix}$

nad $\mathbb{Z}_5$ $\quad \operatorname{adj}A=\begin{pmatrix}4&2\\3&2\end{pmatrix}^T=\begin{pmatrix}4&3\\2&2\end{pmatrix}$

b) $A=\begin{pmatrix}1&2&2\\2&1&2\\2&2&1\end{pmatrix}$

nad $\mathbb{R}$

$$\operatorname{adj}A=\begin{pmatrix}\begin{vmatrix}1&2\\2&1\end{vmatrix} & -\begin{vmatrix}2&2\\2&1\end{vmatrix} & \begin{vmatrix}2&1\\2&2\end{vmatrix}\\ -\begin{vmatrix}2&2\\2&1\end{vmatrix} & \begin{vmatrix}1&2\\2&1\end{vmatrix} & -\begin{vmatrix}1&2\\2&2\end{vmatrix}\\ \begin{vmatrix}2&2\\1&2\end{vmatrix} & -\begin{vmatrix}1&2\\2&2\end{vmatrix} & \begin{vmatrix}1&2\\2&1\end{vmatrix}\end{pmatrix}^T=\begin{pmatrix}-3&2&2\\2&-3&2\\2&2&-3\end{pmatrix}^T=$$

$$=\begin{pmatrix}-3&2&2\\2&-3&2\\2&2&-3\end{pmatrix}$$

c) $A=\begin{pmatrix}1&2&2\\2&1&2\\2&2&1\end{pmatrix}$

nad $\mathbb{Z}_5$ $\qquad$ nad $\mathbb{Z}_5$

$$\operatorname{adj}A=\begin{pmatrix}2&2&2\\2&2&2\\2&2&2\end{pmatrix}^T=\begin{pmatrix}2&2&2\\2&2&2\\2&2&2\end{pmatrix}$$

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4) $P=\begin{pmatrix}1&1&2&1\\0&1&1&2\\0&1&1&1\\1&2&2&1\end{pmatrix}$

nad $\mathbb{Z}_3$

$$\det P=(-1)^{1+1}\cdot 1\cdot\begin{vmatrix}1&1&2\\1&1&1\\2&2&1\end{vmatrix}+(-1)^{4+1}\cdot 1\cdot\begin{vmatrix}1&2&1\\1&1&2\\1&1&1\end{vmatrix}$$

$$=0-1=-1$$

$$\operatorname{adj}P=\begin{pmatrix}\begin{vmatrix}1&1&2\\1&1&1\\2&2&1\end{vmatrix} & -\begin{vmatrix}0&1&2\\0&1&1\\1&2&1\end{vmatrix} & \begin{vmatrix}0&1&2\\0&1&1\\1&2&1\end{vmatrix} & -\begin{vmatrix}0&1&1\\0&1&1\\1&2&2\end{vmatrix}\\ -\begin{vmatrix}1&2&1\\1&1&1\\2&2&1\end{vmatrix} & \begin{vmatrix}1&2&1\\0&1&1\\1&2&1\end{vmatrix} & -\begin{vmatrix}1&1&1\\0&1&1\\1&2&1\end{vmatrix} & \begin{vmatrix}1&1&2\\0&1&1\\1&2&2\end{vmatrix}\\ \begin{vmatrix}1&2&1\\1&1&2\\2&2&1\end{vmatrix} & -\begin{vmatrix}1&2&1\\0&1&2\\1&2&1\end{vmatrix} & \begin{vmatrix}1&1&1\\0&1&2\\1&2&1\end{vmatrix} & -\begin{vmatrix}1&1&2\\0&1&1\\1&2&2\end{vmatrix}\\ -\begin{vmatrix}1&2&1\\1&1&2\\1&1&1\end{vmatrix} & \begin{vmatrix}1&2&1\\0&1&2\\0&1&1\end{vmatrix} & -\begin{vmatrix}1&1&1\\0&1&2\\0&1&1\end{vmatrix} & \begin{vmatrix}1&1&2\\0&1&1\\0&1&1\end{vmatrix}\end{pmatrix}^T=\begin{pmatrix}0&1&2&0\\2&0&1&2\\0&0&1&1\\2&2&1&0\end{pmatrix}^T=\begin{pmatrix}0&2&0&2\\1&0&0&2\\2&1&1&1\\0&2&1&0\end{pmatrix}$$

$$P^{-1}=-1\cdot\operatorname{adj}P=\begin{pmatrix}0&1&0&1\\2&0&0&1\\1&2&2&2\\0&1&2&0\end{pmatrix}$$

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5) více / méně ?

6) $A[1;1]$, $B[2;2]$, $C[\alpha;3]$, $\alpha \in \mathbb{R}$

$y = ax^2 + bx + c$

$$\cancel{\left(\begin{array}{ccc|c} a & b & c & 1 \\ 4a & 2b & c & 2 \\ a\alpha^2 & b\alpha & c & 3 \end{array}\right)}$$

$$\begin{array}{cccc} a & b & c & \\ \end{array}$$
$$\left(\begin{array}{ccc|c} 1 & 1 & 1 & 1 \\ 4 & 2 & 1 & 2 \\ \alpha^2 & \alpha & 1 & 3 \end{array}\right)$$

a)
$$a = \frac{\begin{vmatrix} 1 & 1 & 1 \\ 2 & 2 & 1 \\ 3 & \alpha & 1 \end{vmatrix}}{\begin{vmatrix} 1 & 1 & 1 \\ 4 & 2 & 1 \\ \alpha^2 & \alpha & 1 \end{vmatrix}} = \frac{(2\alpha+2+3)-(\alpha+6+2)}{(\alpha^2+4\alpha+2)-(2\alpha^2+\alpha+4)} =$$

$$= \frac{\alpha-3}{-\alpha^2+3\alpha-2} = \frac{\alpha-3}{-(\alpha-1)(\alpha-2)} \Leftarrow \text{pro } \alpha \in \mathbb{R}-\{1;2\}$$

$$b = \frac{\begin{vmatrix} 1 & 1 & 1 \\ 4 & 2 & 1 \\ \alpha^2 & 3 & 1 \end{vmatrix}}{-(\alpha-1)(\alpha-2)} = \frac{(\alpha^2+12+2)-(2\alpha^2+3+4)}{-(\alpha-1)(\alpha-2)} = \frac{-\alpha^2+7}{-(\alpha-1)(\alpha-2)}$$

$$c = \frac{\begin{vmatrix} 1 & 1 & 1 \\ 4 & 2 & 2 \\ \alpha^2 & \alpha & 3 \end{vmatrix}}{-(\alpha-1)(\alpha-2)} = \frac{(2\alpha^2+4\alpha+6)-(2\alpha^2+2\alpha+12)}{-(\alpha-1)(\alpha-2)} = \frac{2\alpha-6}{-(\alpha-1)(\alpha-2)}$$

b) pro $\alpha = 1$ nebo $\alpha = 2$: ~~nelze protože det ≠ 0~~ nelze, protože det = 0 → nelze dělit 0

$$\left(\begin{array}{ccc|c} 1 & 1 & 1 & 1 \\ 4 & 2 & 1 & 2 \\ 1 & 1 & 1 & 3 \end{array}\right) \sim \left(\begin{array}{ccc|c} 1 & 1 & 1 & 1 \\ 0 & -2 & -3 & -2 \\ 0 & 0 & 0 & 2 \end{array}\right)$$

↳ nemá řešení

neplatí F. věta

$$\left(\begin{array}{ccc|c} 1 & 1 & 1 & 1 \\ 4 & 2 & 1 & 2 \\ 4 & 2 & 1 & 3 \end{array}\right) \sim \left(\begin{array}{ccc|c} 1 & 1 & 1 & 1 \\ 0 & -2 & -3 & -2 \\ 0 & 0 & 0 & 1 \end{array}\right)$$

↳ nemá řešení

neplatí F. věta