1.

$$\underbrace{\begin{aligned} Ax + By &= a \\ Cx + Dy &= b \end{aligned}}$$

$$Pu = q$$

$$\underbrace{\begin{bmatrix} A & B \\ C & D \end{bmatrix}}_{P} \underbrace{\begin{bmatrix} x \\ y \end{bmatrix}}_{u} = \underbrace{\begin{bmatrix} a \\ b \end{bmatrix}}_{q}$$

$x, a \in \mathbb{R}^m$
$y, b \in \mathbb{R}^n$ } vyjádřit y a dosadit => najít vztah pro x

$$\begin{array}{c|cc|c} & m & n & \\ \hline m & A & B & \\ & C & D & n \end{array}$$

A, D => čtvercové?

$$Ax + By = a \Rightarrow By = a - AX$$
$$y = D^{-1}(b - Cx)$$

můžu, protože D je čtvercové

=> Substituce.

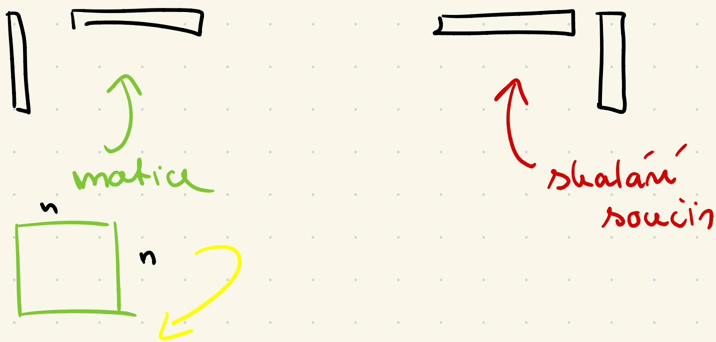
$$Ax + BD^{-1}(b - Cx) = a$$
$$(A + BD^{-1}C)x = a - BD^{-1}b$$
$$\Rightarrow x = (A - BD^{-1}C)^{-1}(a - BD^{-1}b)$$

② $(A+uv^T)^{-1} = A^{-1} - \frac{A^{-1}uv^TA^{-1}}{1+v^TA^{-1}u}$ .... Dokažte

→ Operace inverze ⇒ $O(n^3)$

→ $u,v \in \mathbb{R}^n$ $\quad A \in \mathbb{R}^{n\times n}$

→ $u \cdot v^T$ $\qquad\qquad u^Tv$



$(A+uv^T)^{-1}$ ⇒ validní součet

$A^{-1}uv^TA^{-1}$ } $O(n^3)$ operací ⇒ maticové násobení

⇒ při uzávorkování $(A^{-1}u)(v^TA^{-1})$ ⇒ $O(n^2)$!

$\underbrace{1+v^TA^{-1}u}$ → číslo

$O(n^2)$ operací

$$(A+uv^T)\left(A^{-1} - \frac{A^{-1}uv^TA^{-1}}{\dots}\right)$$

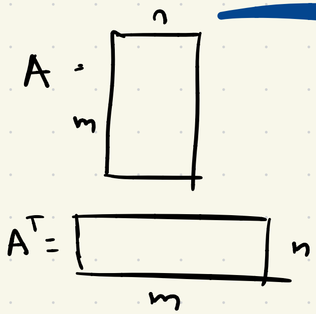
$$\left(I + uv^TA^{-1} - \frac{uv^TA^{-1} + uv^TA^{-1}uv^TA^{-1}}{1+v^TA^{-1}u}\right)$$

$$\frac{uv^TA^{-1} + u(v^TA^{-1}u)v^TA^{-1}}{1+v^TAu}$$

→ to dá číslo (můžu ho dát kam chci v součinu)

$$= I + uv^TA^{-1} - \frac{\cancel{\boxed{1+v^TA^{-1}u}}\, uv^TA}{\cancel{1+v^TA^{-1}u}}$$

Transponovaní matice



$$a^T_{ij} = a_{ji}$$

$$A = \begin{bmatrix} 2 & 3 \\ 5 & 1 \\ 8 & 9 \end{bmatrix} \qquad A^T = \begin{bmatrix} 2 & 5 & 8 \\ 3 & 1 & 9 \end{bmatrix}$$

NOTES

$$(AB)^T = B^T A^T$$

$$(AB)^{-1} = B^{-1} A^{-1}$$

$$ABB^{-1}A^{-1} = I$$

Symetrická

$\forall i,j \quad a_{ij} = a_{ji}$

$$A = A^T$$

$$\begin{bmatrix} 2 & -3 \\ -3 & 1 \end{bmatrix}$$

Antisymetrická

$a^T_{ij} = a_{ji} = -a_{ij}$

$a_{ij} = -a_{ji}$

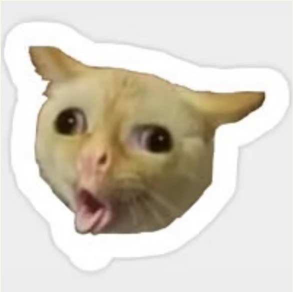
$A^T = -A$

$$\begin{bmatrix} 0 & 2 \\ -2 & 0 \end{bmatrix}$$

Opačné znam. ?

Vlastnosti

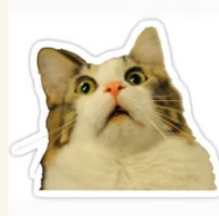
- $A^T \cdot A \Rightarrow$ · Čtvercová
  - · Symetrická matice



③ $A \in \mathbb{R}^{n \times n}$

platí? Sym. Antisym

$A = B + C \qquad B = B^T \qquad C = -C^T$



$(AB)^T = B^T A^T$

$B = \frac{1}{2}(A + A^T)$ → ze vztahu symetrie ???

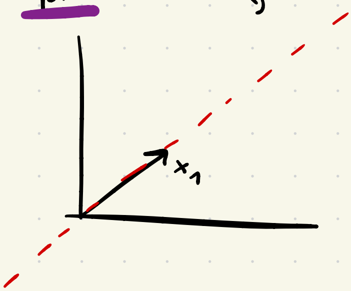
$C = \frac{1}{2}(A - A^T)$ → ze vztahu antisym

Lineární prostory

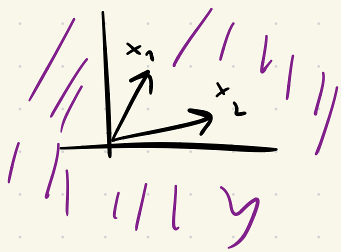
$x_1 \dots x_n \in \mathbb{R}^m$, $a_1 \dots a_n \in \mathbb{R}$ } Lineární kombinace ⇒ LK

př.

LK 1 vektoru → přímka



LK → $\mathbb{R}^2$



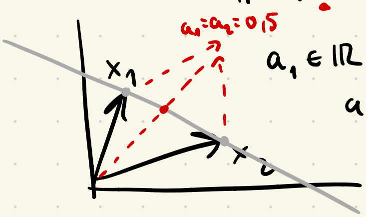
Afinní podprostory

→ afinní kombinace

→ $x_1 \dots x_n \in \mathbb{R}^n$

$a_1 \dots a_n \in \mathbb{R}$

$a_1 + \dots + a_n = 1$!

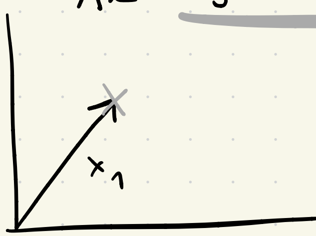


$a_1 \in \mathbb{R},\ a_2 \in \mathbb{R}$

$a_1 + a_2 = 1$

AK ⇒ Dá přímku

AK → jen bod



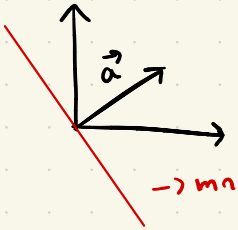
• Obdobně 3 vekt. udělají rovinu

④ $\{x \in \mathbb{R}^n \mid Ax = 0\}$ l. p.

$\{x \in \mathbb{R}^n \mid Ax = b\}$ af. p.

a) $A = a^T$ $\quad a, x \in \mathbb{R}^2$ $\quad a^T \cdot x = a \cdot x = 0$

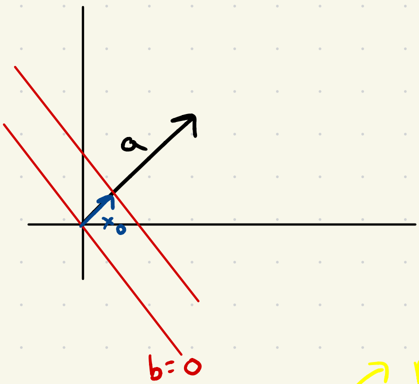
$\{a^T x = 0\}$



$\{x \in \mathbb{R}^n \mid a^T x = 0\}$

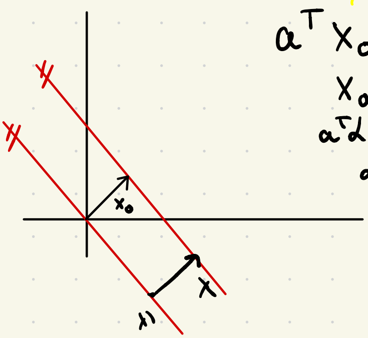
→ množina holmých vektorů

b) $\{x \in \mathbb{R}^n \mid a^T x = b\}$ $\quad a \in \mathbb{R}^n$, $b \in \mathbb{R}$



$a^T x = a x = \|a\| \cdot \|x\| \cdot \cos\alpha$

partikulární řešení



$$a^T x_0 = b$$
$$x_0 = \alpha a$$
$$a^T \alpha a = b$$
$$\alpha = \frac{b}{a^T a} \Rightarrow \alpha = \frac{b}{\|a\|^2}$$