Cafourek

1. a) $2x+3y=0$
$-4x-6y=0$

$\vec{u}(-3;2)$
$x=-3-3t \quad , t\in\mathbb{Z}$
$y=2+2t$

b) $2x+y+3z=6$
$x+z=2$
$5x+2y+7z=14$

$\vec{u}(1;1;-1)$
$x=1+t \quad , t\in\mathbb{Z}$
$y=1+t$
$z=1-t$

c) $2x+y+3z=6$
$x+z=2$
$5x+2y+7z=13$

Ø

2. $2\alpha+\beta+3\gamma=6$
$\alpha+\gamma=2$
$5\alpha+2\beta+7\gamma=14$

$\alpha=1+t$
$\beta=1+t$
$\gamma=1-t$

3. $\vec{v}_1$ leží
$\vec{v}_2$ neleží

$$\sum_{i=1}^{3} a_1\cdot\begin{pmatrix}2\\1\\5\end{pmatrix}+a_2\begin{pmatrix}1\\0\\2\end{pmatrix}+a_3\begin{pmatrix}3\\1\\7\end{pmatrix}$$

$$\begin{pmatrix}6\\2\\14\end{pmatrix}=\sum_{i=1}^{3}1\begin{pmatrix}2\\1\\5\end{pmatrix}+1\begin{pmatrix}1\\0\\2\end{pmatrix}+1\begin{pmatrix}3\\1\\7\end{pmatrix}$$

4. a) $\check{T}=\left\{\begin{pmatrix}2\\-4\end{pmatrix};\begin{pmatrix}3\\-6\end{pmatrix}\right\}$ nad $\mathbb{R}$
span $(\check{T})\supseteq\left\{\begin{pmatrix}0\\0\end{pmatrix}\right\}$

b) $\check{R}=\left\{\begin{pmatrix}2\\1\\5\end{pmatrix};\begin{pmatrix}1\\0\\2\end{pmatrix};\begin{pmatrix}3\\1\\7\end{pmatrix}\right\}$ nad $\mathbb{R}$
span $(\check{R})\supseteq\left\{\begin{pmatrix}0\\0\\0\end{pmatrix};\begin{pmatrix}6\\2\\14\end{pmatrix}\right\}$

5. span $\left(\left\{\begin{pmatrix}2\\1\end{pmatrix};\begin{pmatrix}3\\1\end{pmatrix}\right\}\right)\subseteq\mathbb{R}^2$

$\vec{w}_1=\begin{pmatrix}5\\2\end{pmatrix}$

$\vec{w}_2=\begin{pmatrix}1\\0\\4\end{pmatrix}$

$2\alpha+3\beta=A$
$\alpha+\beta=B \quad |\cdot(-2)$

$\beta=A-2B$
$\alpha=3B-A$

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6. a) $6\alpha + 6\beta + \gamma = 0$

$2\alpha + 2\beta = 0 \rightarrow \alpha = -\beta$

$14\alpha + 13\beta + 2\gamma = 0$

$\gamma = 0 \rightarrow \beta = 0 \rightarrow \alpha = 0$ lin. nez.

$-\beta + 2\gamma = 0$

b) $2\alpha + \beta + 3\gamma = 0$

$\alpha + \gamma = 0 \rightarrow \alpha = -\gamma$

$5\alpha + 2\beta + 7\gamma = 0$

$\beta + \gamma = 0 \quad \beta = -\gamma$

$2\beta + 2\gamma = 0$

$\beta + \gamma = 0$

$0 = 0$

$\alpha = -t, \beta = -t, \gamma = t, t \in$

lin. záv.

c) $6\alpha + 6\beta = 0$

$2\alpha + 2\beta = 0 \rightarrow \alpha = -\beta$

$14\alpha + 13\beta = 0$

$0 = 0$

$-\beta = 0 \rightarrow \alpha = 0$ lin. nez.

d) $5\alpha + 10\beta = 0$

$2\alpha + 4\beta = 0 \rightarrow \alpha = -2\beta$

$0 = 0$ $\alpha = -2t$ lin. záv.

$\beta = t$

e) $\alpha x + \beta(x^2 - x) + \gamma(1 + x^2) + \delta \cdot 2 = 0$

$x^2(\beta + \gamma) + x(\alpha - \beta) + \gamma + 2\delta = 0$

$\beta + \gamma = 0 \quad \gamma = 0 \quad \alpha = \beta = \gamma = \delta = 0$ lin. nez.

$\alpha - \beta = 0 \quad \delta = 0$

7. b) odebrat $\begin{pmatrix}3\\1\\7\end{pmatrix}$ Coloured

d) odebrat $\begin{pmatrix}10\\4\end{pmatrix}$

8 a) vektor $\begin{pmatrix}6\\2\\-23\end{pmatrix}$ neleží ve span(M)

b) není lin. nez. a není generující množina

c) přidat $\begin{pmatrix}0\\0\\1\end{pmatrix}$ ✱

9. $x+y=a$
$y+z=b$
$x+z=c$

[redacted] ← [redacted]

Ano

$y=b-z$

$x-z=a-b$

$2x=a-b+c \quad x=\frac{a-b+c}{2}$

$z=\frac{c-a+b}{2} \quad y=\frac{a+b-c}{2}$

P ✱ $\begin{cases}\alpha+\beta=0\\ \beta+\gamma=0\\ \alpha+\gamma=0\end{cases}$ ← funguje pouze pro 0

[redacted]

↓ lin. nez.

10. [redacted] Ano

[redacted] (P ano)

11. $B=$ $2\alpha+\beta+3\gamma=0$

$\alpha+\gamma=0 \quad \alpha=-\gamma$

$\alpha+\beta+\gamma=0$

$\beta+\gamma=0$

$\beta=0 \Rightarrow \gamma=0 \Rightarrow \alpha=0$ lin. nez.

12. $(\vec{v}_1, \vec{v}_2, \vec{v}_3)$ lin. záv. seznam → Ano gen. prostor

(Dk)

↳ $a_1\vec{v}_1+a_2\vec{v}_2+a_3\vec{v}_3=0$ [redacted] alespoň jedno $a_i \neq 0$.

↳ [redacted] $\vec{v}_1=(-a_1)^{-1}a_2\vec{v}_2+(-a_1)^{-1}a_3\vec{v}_3$

↳ [redacted] $-a_1\vec{v}_1=a_2\vec{v}_2+a_3\vec{v}_3 \rightarrow \vec{v}_1=(-a)^{-1}a_2\vec{v}_2+(-a)^{-1}a_3\vec{v}_3$

$-a_1 \neq 0 \rightarrow$ [redacted] $(-a)^{-1}$