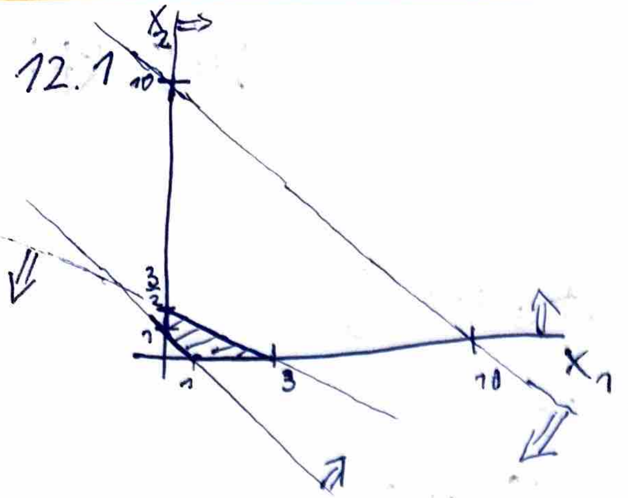
CAFOUREK

12.1




a) $c=(-1,0,1)$

min $-x_1+x_3$

směr $\rightarrow$ posouváme na bod $(3,0)$

b) $c=(0,1,0)$

min $x_2$

směr $\downarrow$ posouváme na úsečku

$(1,0)$ — $(3,0)$

c) $c=(0,0,-1)$

min $-x_3$

ale je směr $x_1, x_2$ jedno, ale s větším $x_3$ se bude minimum zlepšovat

je to tedy neomezené.

12.2 a) min $2x_1 \quad -3x_3+x_4$

z.p.

$$x_1-x_2-x_3 \geq 0$$
$$-x_1+2x_2-3x_3 \leq 5$$
$$2x_1-x_2-x_3+2x_4=6$$
$$x_i \geq 0 \quad i=1,2,3,4$$

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min $2x_1-3x_3+x_4$

$$x_1-x_2-x_3-y=0$$
$$-x_1+2x_2-3x_3+z=5$$
$$2x_1-x_2-x_3+2x_4=6$$
$$x_i, y, z \geq 0 \quad i=1,2,3,4$$

$\rightarrow$ min $\{r^T u \mid Pu=q,\ u\geq 0\}$

$$r^T=(2,0,-3,1,0,0)$$

$$u=\begin{pmatrix}x_1\\x_2\\x_3\\x_4\\y\\z\end{pmatrix} \quad P=\begin{bmatrix}1&-1&-1&0&-1&0\\-1&2&-3&0&0&1\\2&-1&-1&2&0&0\end{bmatrix}$$

$$q=\begin{pmatrix}0\\5\\6\end{pmatrix}$$

b) min $\{\langle C,X\rangle \mid X\in\mathbb{R}^{m\times n},\ X1=a,\ X^T1=b,\ X\geq 0\}$

$r^T u$

$C \Rightarrow r^T=(c_{11}\dots c_{1n}, c_{21}\dots c_{2n}, c_{m1}\dots c_{mn})$

$\overset{\shortparallel}{[c_{ij}]}$

$X \Rightarrow u=\begin{pmatrix}x_{11}\\ \vdots\\ x_{1n}\\ \vdots\\ x_{m1}\\ \vdots\\ x_{mn}\end{pmatrix}$

$\overset{\shortparallel}{[x_{ij}]}$

$$P=\begin{bmatrix}\overbrace{1^T}^{n} & 0^T & \dots & 0^T\\ 0^T & 1^T & & \vdots\\ \vdots & & \ddots & \\ 0^T & \dots & & 1^T\\ \underbrace{I_n}_{n} & \dots & & I_n\end{bmatrix}\begin{matrix}\}m\\ \\ \\ \\ \}n\end{matrix} \qquad q=\begin{bmatrix}a\\b\end{bmatrix}$$

$a=a_1\dots a_m$

$b=b_1\dots b_n$

12.3. a) $c \in \mathbb{R}^n$, $k \in \{1, \dots, n\}$ CAFOUREK

$\max\{c^T x \mid x \in \mathbb{R}^n, 0 \leq x \leq 1\}$

$\hookrightarrow \max\{\sum_i c_i x_i \mid x_i \in \mathbb{R}, 0 \leq x_i \leq 1, \forall x_i \; i = 1 \dots n\}$

pro jedno $i$ je optimum $\max\{c_i, 0\}$

$\Rightarrow$ proto pro sumu je optimum $\sum_i \max\{c_i, 0\}$

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12.4 a) $\min\{|x_1| + |x_2| \mid x_1, x_2 \in \mathbb{R}, 2x_1 - x_2 \geq 1, -x_1 + 2x_2 \geq 1\}$

$\min\{y_1 + y_2 \mid x_1, x_2, y_1, y_2 \in \mathbb{R}, 2x_1 - x_2 \geq 1, -x_1 + 2x_2 \geq 1, x_1 \leq y_1, -x_1 \leq y_1,$

$x_2 \leq y_2, -x_2 \leq y_2\}$

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12.9. $z_i$ .... zaměstnanec $i = 1, \dots, 120$ každý 30 hodin

$s$ .... standard kanoe $s_i = 7$ EUR

$l$ .... luxus kanoe $l_i = 10$ EUR

$\max\ 7s + 10l$ $\rightarrow$ maximalizace zisku

z.p. $\overbrace{4{,}5}^{\text{hodiny}}s + 5l \leq \overbrace{60}^{\text{počet zaměstnanců}} \cdot 30$ $\rightarrow$ truhlářská dílna

$2s + 1l \leq 20 \cdot 30$ $\rightarrow$ dílna na plasty

$2s + 4l \leq 40 \cdot 30$ $\rightarrow$ kompletační dílna

$s, l \geq 0$, $s, l \in \mathbb{Z}$

$(s+l)/3 \leq l \leq 2(s+l)/3$