

$$1) a) \quad 1 \cdot 2 + 3 \cdot 1 = 5$$

$$b) \quad (-1) \cdot 1 + 1 \cdot 2 + 2 \cdot 1 = 3$$

c)

$$\begin{pmatrix} 1 & 0 \end{pmatrix} \begin{pmatrix} 5 & 1 \\ 1 & 2 \end{pmatrix} \begin{pmatrix} 0 \\ 1 \end{pmatrix} = \begin{pmatrix} 1 & 0 \end{pmatrix} \begin{pmatrix} 1 \\ 2 \end{pmatrix} = 1$$

2) a)

$$\|\vec{u}\| = \sqrt{10} \quad \|\vec{v}\| = \sqrt{5}$$

$$\cos \varphi = \frac{\langle \vec{u} | \vec{v} \rangle}{\|\vec{u}\| \cdot \|\vec{v}\|}$$

$$\varphi = 45^\circ$$

$$b) \quad \|\vec{u}\| = \sqrt{6} \quad \|\vec{v}\| = \sqrt{6}$$

$$\varphi = 60^\circ$$

$$c) \quad \|\vec{u}\| = \sqrt{5} \quad \|\vec{v}\| = \sqrt{2} \quad \begin{pmatrix} 1 & 0 \end{pmatrix} \begin{pmatrix} 5 & 1 \\ 1 & 2 \end{pmatrix} \begin{pmatrix} 1 \\ 0 \end{pmatrix} = \begin{pmatrix} 1 & 0 \end{pmatrix} \begin{pmatrix} 5 \\ 1 \end{pmatrix} = 5$$

$$\varphi = 71^\circ 34'$$

$$\begin{pmatrix} 0 & 1 \end{pmatrix} \begin{pmatrix} 5 & 1 \\ 1 & 2 \end{pmatrix} \begin{pmatrix} 0 \\ 1 \end{pmatrix} = \begin{pmatrix} 0 & 1 \end{pmatrix} \begin{pmatrix} 1 \\ 2 \end{pmatrix} = 2$$

3) a)

$$\begin{pmatrix} 1 \\ 3 \end{pmatrix} \quad x + 3y = 0$$

$$\begin{pmatrix} 1 \\ 2 \end{pmatrix} \quad 2x + y = 0$$

$$\text{span} \left(\begin{pmatrix} -3 \\ 1 \end{pmatrix} \right)$$

$$\text{span} \left(\begin{pmatrix} -1 \\ 2 \end{pmatrix} \right)$$

na oba nemí

b)

$$\begin{pmatrix} -1 \\ 1 \\ 2 \end{pmatrix} \quad -x + y + 2z = 0$$

$$\begin{pmatrix} 1 \\ 2 \\ 1 \end{pmatrix} \quad x + 2y + z = 0$$

$$\text{span} \left(\begin{pmatrix} 1 \\ 1 \\ 0 \end{pmatrix}, \begin{pmatrix} 2 \\ 0 \\ 1 \end{pmatrix} \right)$$

$$\text{span} \left(\begin{pmatrix} -2 \\ 1 \\ 0 \end{pmatrix}, \begin{pmatrix} -1 \\ 0 \\ 1 \end{pmatrix} \right)$$

$$\text{na oba span} \left(\begin{pmatrix} 1 \\ -1 \\ 1 \end{pmatrix} \right)$$

$$c) \quad \begin{pmatrix} 1 \\ 0 \end{pmatrix} \quad \begin{pmatrix} 1 & 0 \end{pmatrix} \begin{pmatrix} 5 & 1 \\ 1 & 2 \end{pmatrix} \begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} 1 & 0 \end{pmatrix} \begin{pmatrix} 5x+y \\ x+2y \end{pmatrix} \rightarrow 5x+y=0$$

$$\text{span} \left(\begin{pmatrix} -1 \\ 5 \end{pmatrix} \right)$$

$$\begin{pmatrix} 0 \\ 1 \end{pmatrix} \quad \begin{pmatrix} 0 & 1 \end{pmatrix} \begin{pmatrix} 5 & 1 \\ 1 & 2 \end{pmatrix} \begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} 0 & 1 \end{pmatrix} \begin{pmatrix} 5x+y \\ x+2y \end{pmatrix} \rightarrow x+2y=0$$

$$\text{span} \left(\begin{pmatrix} -2 \\ 1 \end{pmatrix} \right)$$

na oba nemí

4) $\|\vec{b}_1\| = 1$
 $\|\vec{b}_2\| = 1$ je ortonormalni
 $\|\vec{b}_3\| = 1$

5) $\begin{pmatrix} 2 \\ 3 \end{pmatrix}$

a) $\begin{pmatrix} 2/\sqrt{13} \\ 3/\sqrt{13} \end{pmatrix} \quad \|\begin{pmatrix} 2 \\ 3 \end{pmatrix}\| = \sqrt{13}$

$\|\begin{pmatrix} 2 \\ 3 \end{pmatrix}\| = \sqrt{29}$

b) $\begin{pmatrix} 2/\sqrt{29} \\ 3/\sqrt{29} \end{pmatrix} \quad (2 \ 3) \begin{pmatrix} 2 & 1 \\ 1 & 1 \end{pmatrix} \begin{pmatrix} 2 \\ 3 \end{pmatrix} = (2 \ 3) \begin{pmatrix} 7 \\ 5 \end{pmatrix} = 29$

c) $\begin{pmatrix} 2/\sqrt{53} \\ 3/\sqrt{53} \end{pmatrix} \quad (x_1 \ y_1) \begin{pmatrix} 5 & 2 \\ 2 & 1 \end{pmatrix} \begin{pmatrix} x_2 \\ y_2 \end{pmatrix} = (x_1 \ y_1) \begin{pmatrix} 5x_2 + 2y_2 \\ 2x_2 + y_2 \end{pmatrix} =$
 $(2 \ 3) \begin{pmatrix} 5 & 2 \\ 2 & 1 \end{pmatrix} \begin{pmatrix} 2 \\ 3 \end{pmatrix} = (2 \ 3) \begin{pmatrix} 16 \\ 7 \end{pmatrix} = 53$
 $\|\begin{pmatrix} 2 \\ 3 \end{pmatrix}\| = \sqrt{53}$
 $= 5x_1x_2 + 2x_1y_2 + 2x_2y_1 + y_1y_2$

6) $\begin{pmatrix} 2/\sqrt{3} \\ 1/\sqrt{2} \\ 5/\sqrt{6} \end{pmatrix} \quad \frac{2}{\sqrt{3}} + \frac{1}{\sqrt{3}} - \frac{1}{\sqrt{3}} \quad \frac{2}{\sqrt{6}} + \frac{1}{\sqrt{6}} + \frac{2}{\sqrt{6}}$
 $\frac{2}{\sqrt{2}} - \frac{1}{\sqrt{2}} + 0$

7) a) $(2 \ 3) \begin{pmatrix} 2 & 1 \\ 1 & 1 \end{pmatrix} \begin{pmatrix} x \\ y \end{pmatrix} = (2 \ 3) \begin{pmatrix} 2x+y \\ x+y \end{pmatrix} \Rightarrow 7x + 5y = 0$
 $(-5 \ 7) \begin{pmatrix} 2 & 1 \\ 1 & 1 \end{pmatrix} \begin{pmatrix} -5 \\ 7 \end{pmatrix} = (-5 \ 7) \begin{pmatrix} -3 \\ 2 \end{pmatrix} = 29$
 $\text{Baze} = \left\{ \begin{pmatrix} 2/\sqrt{29} \\ 3/\sqrt{29} \end{pmatrix}, \begin{pmatrix} -5/\sqrt{29} \\ 7/\sqrt{29} \end{pmatrix} \right\}$
 $\text{Span} \left(\begin{pmatrix} -5 \\ 7 \end{pmatrix} \right) = \text{Span} \left(\begin{pmatrix} -5 \\ 7 \end{pmatrix} \right)$
 $\|\begin{pmatrix} -5 \\ 7 \end{pmatrix}\| = \sqrt{29}$
 $\frac{7}{\sqrt{29}}x - \frac{5}{\sqrt{29}}y = 0$
 $x = \frac{5}{7}y$

b) $\begin{pmatrix} 5/\sqrt{29} \\ 2/\sqrt{29} \end{pmatrix}$

8) a) $\vec{b}_1 = \begin{pmatrix} 1/\sqrt{3} \\ 1/\sqrt{3} \\ 1/\sqrt{3} \end{pmatrix} \quad \vec{b}_2 = \begin{pmatrix} 1/\sqrt{2} \\ -1/\sqrt{2} \\ 0 \end{pmatrix}$
 $\vec{a} = \begin{pmatrix} 1 \\ 0 \\ 1 \end{pmatrix}$
 $\text{proj}_{\vec{b}_1}(\vec{a}) = \frac{2}{\sqrt{3}} \cdot \begin{pmatrix} 1 \\ 1 \\ 1 \end{pmatrix} = \begin{pmatrix} 2/3 \\ 2/3 \\ 2/3 \end{pmatrix}$

$\langle \vec{b}_1 | \vec{a} \rangle$

$\text{re}_{\vec{b}_1}(\vec{a}) = \vec{a} - \text{proj}_{\vec{b}_1}(\vec{a}) = \begin{pmatrix} 1/3 \\ -2/3 \\ 1/3 \end{pmatrix}$

$\text{proj}_{\text{span}(\vec{b}_1, \vec{b}_2)}(\vec{a}) = \sum_{i=1}^2 \frac{\langle \vec{b}_i | \vec{a} \rangle}{\|\vec{b}_i\|^2} \cdot \vec{b}_i$
 $\begin{pmatrix} 2/3 \\ 2/3 \\ 2/3 \end{pmatrix} + \begin{pmatrix} 1/2 \\ -1/2 \\ 0 \end{pmatrix} = \begin{pmatrix} 7/6 \\ 1/6 \\ 2/3 \end{pmatrix}$
 $\text{re}_{\text{span}(\vec{b}_1, \vec{b}_2)}(\vec{a}) = \begin{pmatrix} -1/6 \\ -1/6 \\ 1/3 \end{pmatrix}$

9) a) $\langle \text{proj} | \text{rij} \rangle$

$$\frac{2}{3} \cdot \frac{1}{3} + \frac{2}{3} \cdot \left(-\frac{2}{3}\right) + \frac{2}{3} \cdot \frac{1}{3} = 0$$

b) $\langle \text{proj} | \text{rej} \rangle$

$$\frac{7}{6} \cdot \left(-\frac{1}{6}\right) + \frac{1}{6} \cdot \left(-\frac{1}{6}\right) + \frac{2}{3} \cdot \frac{1}{3} = 0$$

10) a)

b)

11) a)

b)

c)

d)

12)

13)