CAFOUREK

4.1. $x=(1,2,3)$, $y=(-1,0,1)$

a) $\|x\| = \sqrt{x^T x} = (1^2+2^2+3^2)^{\frac{1}{2}} = \underline{\sqrt{14}}$

b) $\|x-y\| = (2^2+2^2+2^2)^{\frac{1}{2}} = \sqrt{12} = \underline{2\sqrt{3}}$

$x-y=(2,2,2)$

c) $\cos\varphi = \frac{x^T y}{\|x\|\,\|y\|} = \frac{(-1+0+3)}{\sqrt{14}\cdot\sqrt{2}} = \frac{2}{\sqrt{28}} = \frac{1}{\sqrt{7}} = \frac{\sqrt{7}}{7}$

$\arccos\frac{\sqrt{7}}{7}$
$\|$
$\underline{\varphi \doteq 68^\circ \doteq 1{,}18}$

$\|y\| = ((-1)^2+0^2+1^2)^{\frac{1}{2}} = \sqrt{2}$

4.3. $X = \operatorname{span}\{(0,1,1),(1,2,3)\}$

$Ax = 0$

$A = \begin{bmatrix} 0 & 1 & 1 \\ 1 & 2 & 3 \end{bmatrix} \cdot \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$

$x_2 + x_3 = 0 \Rightarrow x_2 = -x_3$

$x_1 + 2x_2 + 3x_3 = 0$

$x_1 - 2x_3 + 3x_3 = 0$

$x_1 = -x_3 \Rightarrow x_3 = 1 \Rightarrow x_2 = -1 \Rightarrow x_1 = -1$

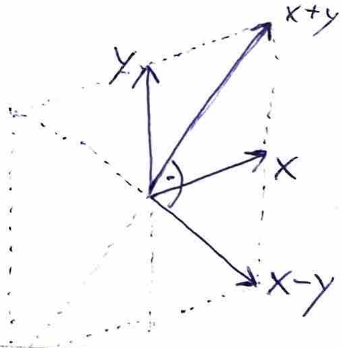
báze $X^\perp$ např. $(-1,-1,1)$

4.5. a) $\|x\| = \|y\| \Rightarrow (x+y) \perp (x-y)$

Dokázat $(x+y)^T(x-y) = 0 = 0 \Leftarrow y^T x = x^T y$

$(x+y)^T(x-y) = x^T x - x^T y + y^T x - y^T y = x^T x - y^T y = 0$ ✓

$\|x\| = \|y\| \Rightarrow \sqrt{x^T x} = \sqrt{y^T y} \Rightarrow x^T x = y^T y$



4.5. b) $x \perp y \Rightarrow \|x\|^2 + \|y\|^2 = \|x-y\|^2$

$x^T y = 0$

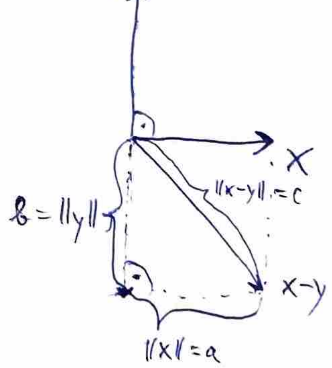
$x^T x + y^T y = (x-y)^T(x-y)$

$x^T x + y^T y = x^T x \underbrace{- x^T y - y^T x}_{=0} + y^T y$

$\Downarrow$

$x^T x + y^T y = x^T x + y^T y$ ✓

$\|x\|^2 + \|y\|^2 = \|x\|^2 + \|y\|^2$



$\Rightarrow a^2 + b^2 = c^2$

Pythagorova věta

CAFOURKEK

4.10. $f: \mathbb{R}^3 \to \mathbb{R}^4$, kde $f(1,-1,2) = (1,2,-1,1)$ a
$f(1,1,0) = (0,1,-1,0)$?

Existuje isometrie

isometrie zachovává eukleidovskou normu a skalární součin

$\|f(x)\| = \|x\|$

$f(x)^T f(y) = x^T y$

ověření:

$\|(1,-1,2)\| \overset{?}{=} \|(1,2,-1,1)\|$

$(1^2+(-1)^2+2^2)^{\frac{1}{2}} \overset{?}{=} (1^2+2^2+(-1)^2+1^2)^{\frac{1}{2}}$

$\sqrt{6} \neq \sqrt{7}$

$\to$ zobrazení není isometrie

4.13. $\text{span}\{x, y\} = \text{span}\{(0,1,1),(1,2,3)\}$

$x = (0,1,1)$

$y = (1,2,3) - \underbrace{[(x^T(1,2,3)) \cdot x / \|x\|^2]}_{\text{„jen" ortogonalizace}}$

GS pro ortonormalizaci by to bylo

$x' = (0,1,1)$ $\quad x = x'/\|x'\|$

$y' = (1,2,3) - (x^T(1,2,3))\,x$ $\quad y = y'/\|y'\|$

$[(x')^T(1,2,3)) \cdot x'] / \|x'\|^2 = [(\frac{x'}{\|x'\|})^T \cdot (1,2,3)] \cdot (\frac{x'}{\|x'\|})$

$y = (1,2,3) - [(0,1,1) \cdot \begin{pmatrix}1\\2\\3\end{pmatrix} \cdot (0,1,1)] / 2$

$y = (1,2,3) - (0, \frac{5}{2}, \frac{5}{2})$

$y = (1; -0{,}5; 0{,}5) \sim (2,-1,1)$

$\Rightarrow \text{span}\{x,y\} = \text{span}\{(0,1,1),(2,-1,1)\}$

$x = (0,1,1)$

$y = (2,-1,1)$