CAFOUREK

11.4 d) extrémy $f(x,y) = x^2 y$

v.p. $x^2+y^2=1 \Rightarrow g(x,y) = x^2+y^2-1$

$L(x,y,\lambda) = f(x,y) + \lambda g(x,y) = x^2y + \lambda(x^2+y^2-1)$

$L'(x,y,\lambda) = [2xy+2x\lambda,\ x^2+2y\lambda,\ x^2+y^2-1] = 0$

$2xy+2x\lambda = 0 \Rightarrow 2x(y+\lambda)=0$

$\Downarrow x=0 \qquad \Downarrow y=-\lambda$

$x^2+2y\lambda = 0$

$x^2+y^2-1=0$

$x^2 - 2y^2 = 0$

$x^2 = 2y^2$

$2y^2+y^2-1=0$

$y^2 = \frac{1}{3} \Rightarrow y = \pm\sqrt{\frac{1}{3}}$

$x = \pm\sqrt{\frac{2}{3}}$

$0^2+y^2-1=0$

$y = \pm 1$

body extrémů:

$(0,-1), (0,1), \left(-\sqrt{\frac{1}{3}}, -\sqrt{\frac{2}{3}}\right), \left(-\sqrt{\frac{1}{3}}, \sqrt{\frac{2}{3}}\right), \left(\sqrt{\frac{1}{3}}, -\sqrt{\frac{2}{3}}\right), \left(\sqrt{\frac{1}{3}}, \sqrt{\frac{2}{3}}\right)$

11.4. c) $x^2y = 1$ min $f(x,y) = x^2+y^2$

$\Downarrow x^2 = \frac{1}{y}$

$f(y) = \frac{1}{y} + y^2$

$f'(y) = 2y - \frac{1}{y^2} = 0$

$y^3 = \frac{1}{2}$

$y = \frac{1}{\sqrt[3]{2}} \Rightarrow x^2 = \sqrt[3]{2}$

$x = \pm\sqrt[6]{2}$

dva body: $\left(-\sqrt[6]{2}, \frac{1}{\sqrt[3]{2}}\right), \left(\sqrt[6]{2}, \frac{1}{\sqrt[3]{2}}\right)$

11.11 a)

$a - x - b$

$\Downarrow$

$\|x-a\| + \|x-b\|$

$L(x,\lambda) = \|x-a\| + \|x-b\| + \lambda g(x)$

$\frac{x-a}{\|x-a\|} + \frac{x-b}{\|x-b\|} = \lambda \nabla g(x)$ → normála k vrcholu