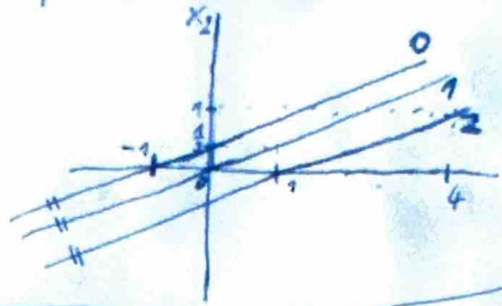
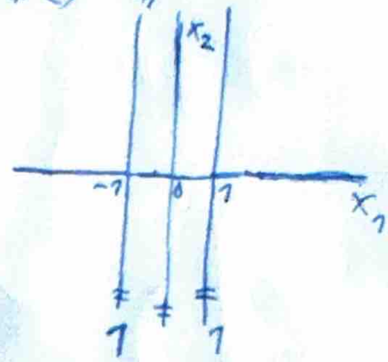
CAFOUREK

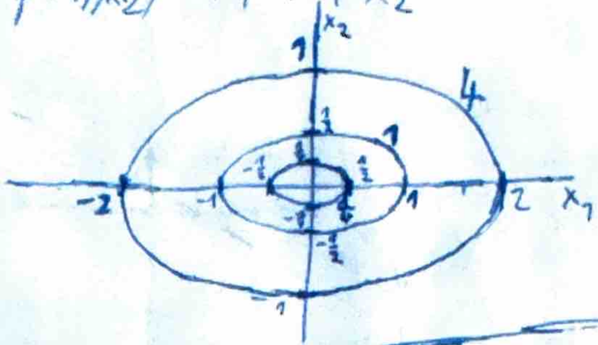
8.1. b) $f(x_1, x_2) = x_1 - 3x_2 + 1$



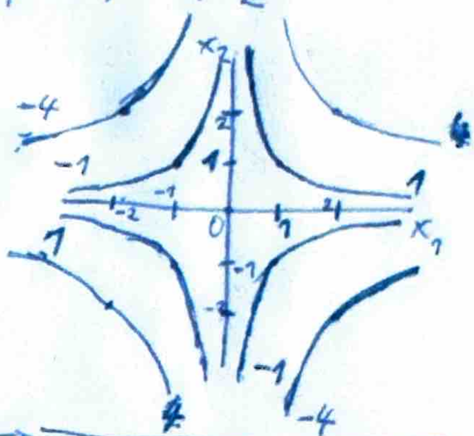
c) $f(x_1, x_2) = x_1^2$



d) $f(x_1, x_2) = x_1^2 + 4x_2^2$



f) $f(x_1, x_2) = x_1 x_2$



8.3. $f: \mathbb{R}^2 \to \mathbb{R}$: $f(x,y) = \ln(1+xy)$

$(x_0, y_0) = (1, 2)$

d) $f'(x,y) = \begin{bmatrix} y \cdot \frac{1}{1+xy} & x \cdot \frac{1}{1+xy} \end{bmatrix}$

$f'(1,2) = \begin{bmatrix} \frac{2}{3} & \frac{1}{3} \end{bmatrix}$

g) $f''(x,y) = \begin{bmatrix} -y^2 \cdot \frac{1}{(1+xy)^2} & \frac{1}{(1+xy)^2} \\ \frac{1}{(1+xy)^2} & -x^2 \cdot \frac{1}{(1+xy)^2} \end{bmatrix}$

$f''(1,2) = \begin{bmatrix} -\frac{4}{9} & \frac{1}{9} \\ \frac{1}{9} & -\frac{1}{9} \end{bmatrix}$

8.10. $h(d,s) = 2s^2 + 3sd - d^2 + 5$

$(d,s) = (-1, 1)$

a) směr nejstrmějšího stoupání → gradient

$h'(d,s) = \begin{bmatrix} 3s - 2d & 4s + 3d \end{bmatrix}$

$h'(-1,1) = \begin{bmatrix} 5 & 1 \end{bmatrix}$

$\nabla h(-1,1) = h'(-1,1)^T = \begin{bmatrix} 5 \\ 1 \end{bmatrix}$

$\|(5,1)\| = \sqrt{26}$ → zajímá nás jen směr

$\Rightarrow \begin{bmatrix} 5 \\ 1 \end{bmatrix} / \sqrt{26}$

b) JV $\Rightarrow v = (1, -1)$

strmost? $\Rightarrow h'(d,s) \cdot \frac{v}{\|v\|} = \begin{bmatrix} 5 & 1 \end{bmatrix} \cdot \sqrt{2} \begin{bmatrix} 1 \\ -1 \end{bmatrix} =$

$= 4 \cdot \sqrt{2}$

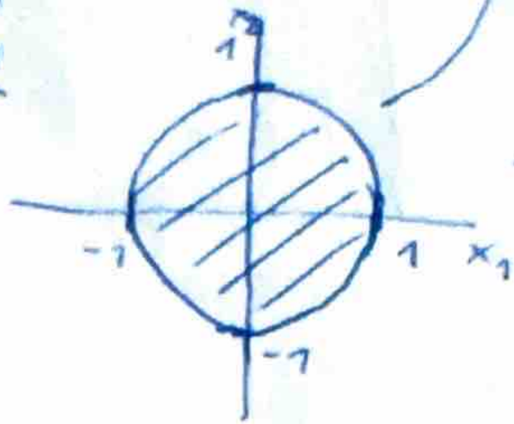
CAFOUREK

9.7. a) $f(x)=a^Tx$, $a\in\mathbb{R}^n$ $a\neq 0$

$C=\{x\in\mathbb{R}^n \mid \|x\|\leq 1\}$

extrémy $f(x)$?

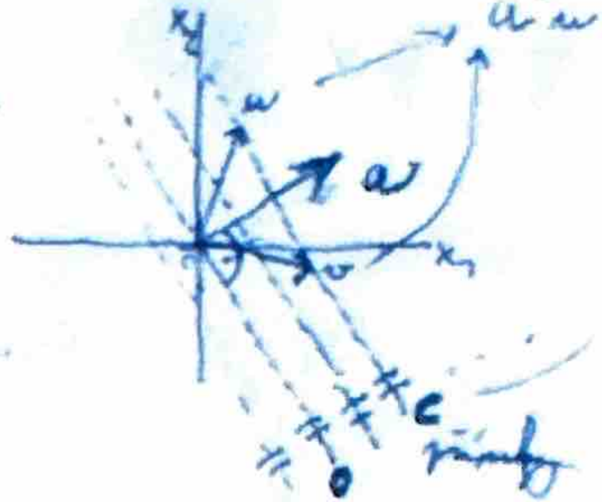
pro $n=2$



vrstevnice

příklad:

$a^Tu = a^Tv = c$



tedy maximum je ve směru a. Minimum v opačném směru k a.

max. v bodě $x=\frac{a}{\|a\|}$

min. v bodě $x=-\frac{a}{\|a\|}$

musíme normovat kvůli množině C, kde $\|x\|\leq 1$ $\forall x$