Cafourek

$\vec{o} = 0$

1. ~~$f(1 \cdot \vec{o}_1) = 1 \cdot f(\vec{o}_1) = \vec{o}_2 \neq \vec{o}$~~

$f(1 \cdot \vec{o}_1) = 1 \cdot f(\vec{o}_1) = \vec{o}_2$

$\in L_1$ $\in L_2$ $\in L_2$

2. a) $Z_5 \to Z_5$

$x \mapsto a \cdot x$

~~Není~~ Ano

$x \mapsto ax$, $A \mapsto aA$ $\Big\}$ $ax + aA$

$f(x+A) \mapsto a \cdot (x+A)$ =

$x \mapsto a \cdot x$

$\cdot t$ $a \cdot x \cdot t$

$tx \mapsto a \cdot tx$ =

b) $\mathbb{R} \to \mathbb{R}$

Není

$x \mapsto x+1$, $A \mapsto A+1$ $\Big\}$ $\to x+A+2$ ✗

$f(x+A) \mapsto x+A+1$

$\begin{pmatrix}1\\0\end{pmatrix} \to \begin{pmatrix}4\\2\end{pmatrix}$, $\begin{pmatrix}2\\0\end{pmatrix} \to \begin{pmatrix}5\\2\end{pmatrix}$ $\Big\}$ $\begin{pmatrix}2\\4\end{pmatrix}$ ≠

$\begin{pmatrix}3\\0\end{pmatrix} \to \begin{pmatrix}6\\2\end{pmatrix}$

$\begin{pmatrix}2\\0\end{pmatrix} \to \begin{pmatrix}4\\2\end{pmatrix}$, $\begin{pmatrix}3\\2\end{pmatrix} \to \begin{pmatrix}6\\4\end{pmatrix}$ $\Big\}$ $\begin{pmatrix}3\\6\end{pmatrix}$ ✗

c) $Z_7^2 \to Z_7^2$

$\begin{pmatrix}1\\0\end{pmatrix} \mapsto \begin{pmatrix}4\\2\end{pmatrix}$

$\begin{pmatrix}0\\1\end{pmatrix} \mapsto \begin{pmatrix}3\\4\end{pmatrix}$

$\begin{pmatrix}1\\1\end{pmatrix} \mapsto \begin{pmatrix}3\\5\end{pmatrix}$

$\begin{pmatrix}x\\0\end{pmatrix} \to \begin{pmatrix}4x\\2x\end{pmatrix}$, $\begin{pmatrix}A\\0\end{pmatrix} \to \begin{pmatrix}3A\\2A\end{pmatrix}$ $\Big\}$ $\begin{pmatrix}4x+4A\\2x+2B\end{pmatrix}$

$\begin{pmatrix}x+A\\0\end{pmatrix} \to \begin{pmatrix}4x+4A\\2x+2B\end{pmatrix}$

$\begin{pmatrix}4\\2\end{pmatrix} \to \begin{pmatrix}0\\4\end{pmatrix}$

$\begin{pmatrix}4\\2\end{pmatrix} \to 0$, $\begin{pmatrix}3\\3\end{pmatrix} \to 5$

$1 \to 4$

$1 \to 3$

Ano / Ne

d) $Q^2 \to Q[x]^{\leq 2}$

$\begin{pmatrix}1\\2\end{pmatrix} \to x^2 + 3$;

$\begin{pmatrix}1\\1\end{pmatrix} \to 2x - 1$

$2x - 2$

$4x - 3$

$2 \to 4x - 3$

$3 \to 6x - 5$

$\begin{pmatrix}1\\1\end{pmatrix} \to 2x-1$

$\begin{pmatrix}2\\2\end{pmatrix} \to 4x-2$

$\begin{pmatrix}3\\3\end{pmatrix} \to 6x-3$

$\begin{pmatrix}3\\4\end{pmatrix} \to x^2 + 4x + 1$ $\Big\}$ ⊕

$\begin{pmatrix}6\\8\end{pmatrix} \to 2x^2 + 8x + 1$ $3x^2 + 12x + 2$

$\begin{pmatrix}9\\12\end{pmatrix} \to 3x^2 + 12x + 1$ ✗

Ano / Ne

3. $f\begin{pmatrix}3\\5\end{pmatrix} = 2x^2 + 2x + 5$

$f\begin{pmatrix}0\\1\end{pmatrix} = x^2 - 2x + 4$

$f(\ ) = x^2 + 2$ nebo?

nebo nekonečně mnoho?

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4) $g: L \to L$

$\vec{v}_1 \to \vec{w}_1$

$\vec{v}_2 \to \vec{w}_2$

$\vec{v}_3 \to \vec{w}_3$

a) $E_3 = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}$

b) $A_g = \begin{pmatrix} 2 & 1 & 1 \\ 0 & 1 & -2 \\ 3 & 1 & 1 \end{pmatrix}$

c) $A_g = \begin{pmatrix} 5 & 1 & -2 \\ 0 & 1 & 2 \\ 2 & 1 & 1 \\ 7 & 3 & 1 \end{pmatrix}$

5) $K_4 = \left\{ \begin{pmatrix} 1 \\ 0 \\ 0 \\ 0 \end{pmatrix}, \begin{pmatrix} 0 \\ 1 \\ 0 \\ 0 \end{pmatrix}, \begin{pmatrix} 0 \\ 0 \\ 1 \\ 0 \end{pmatrix}, \begin{pmatrix} 0 \\ 0 \\ 0 \\ 1 \end{pmatrix} \right\}$

$\text{coord}_{K_4}(\vec{v}) = \begin{pmatrix} 2 \\ 0 \\ 2 \\ 4 \end{pmatrix} \quad \vec{v} = \begin{pmatrix} 2 \\ 0 \\ 2 \\ 4 \end{pmatrix}$

$\begin{pmatrix} 2 & 1 & 1 \\ 0 & 1 & -2 \\ 3 & 1 & 1 \end{pmatrix} \cdot \begin{pmatrix} 2 \\ 0 \\ 0 \end{pmatrix} = \begin{pmatrix} 4 \\ 0 \\ 6 \end{pmatrix}$

není obraz

a) $\text{coord}_{B_1}(\vec{v}') = \begin{pmatrix} 4 \\ 0 \\ 6 \end{pmatrix}$

$\text{coord}_{B_1}\begin{pmatrix} 2 \\ 0 \\ 2 \\ 4 \end{pmatrix} = \begin{pmatrix} 2 \\ 0 \\ 0 \end{pmatrix} \xrightarrow{\begin{pmatrix} 2 & 1 & 1 \\ 0 & 1 & -2 \\ 3 & 1 & 1 \end{pmatrix}} \begin{pmatrix} 4 \\ 0 \\ 6 \end{pmatrix}$

$\vec{v}' = \begin{pmatrix} 10 \\ 0 \\ 4 \\ 14 \end{pmatrix}$

obraz vektoru $\begin{pmatrix} 2 \\ 0 \\ 2 \\ 4 \end{pmatrix}$

b) $\text{coord}_{B_2}(\vec{v}') = \begin{pmatrix} 2 \\ 0 \\ 0 \end{pmatrix}$

c) $\text{coord}_{K_4}(\vec{v}') = \begin{pmatrix} 10 \\ 0 \\ 4 \\ 14 \end{pmatrix}$

není potřeba vědět

$B_1 = \left\{ \begin{pmatrix} 1 \\ 0 \\ 1 \\ 2 \end{pmatrix}, \begin{pmatrix} -1 \\ 1 \\ 0 \\ 0 \end{pmatrix}, \begin{pmatrix} 1 \\ 0 \\ 0 \\ 1 \end{pmatrix} \right\}$

$\begin{pmatrix} 2 & 1 & 1 \\ 0 & 1 & -2 \\ 3 & 1 & 1 \end{pmatrix} \begin{pmatrix} 1 \\ 2 \\ 3 \end{pmatrix} = \begin{pmatrix} 7 \\ 8 \\ 2 \end{pmatrix}$

b) $\text{coord}_{B_1}\begin{pmatrix} 2 \\ 2 \\ 1 \\ 5 \end{pmatrix} = \begin{pmatrix} 1 \\ 2 \\ 3 \end{pmatrix}$

a) $\text{coord}_{B_1}\begin{pmatrix} 7 \\ 8 \\ 2 \\ 14 \end{pmatrix} = \begin{pmatrix} 7 \\ 8 \\ 2 \end{pmatrix}$

$\text{coord}_{B_1}\begin{pmatrix} 2 \\ 2 \\ 1 \\ 5 \end{pmatrix} = \begin{pmatrix} 1 \\ 2 \\ 3 \end{pmatrix} \xrightarrow{\begin{pmatrix} 2 & 1 & 1 \\ 0 & 1 & -2 \\ 3 & 1 & 1 \end{pmatrix}} \begin{pmatrix} 7 \\ 8 \\ 2 \end{pmatrix}$

není obraz

obraz vektoru $\begin{pmatrix} 2 \\ 2 \\ 1 \\ 5 \end{pmatrix}$

b) $\text{coord}_{B_2}\begin{pmatrix} 7 \\ 8 \\ 2 \\ 14 \end{pmatrix} = \begin{pmatrix} 1 \\ 2 \\ 3 \end{pmatrix}$

c) $\text{coord}_{K_4}\begin{pmatrix} 7 \\ 8 \\ 2 \\ 14 \end{pmatrix} = \begin{pmatrix} 7 \\ 8 \\ 2 \\ 14 \end{pmatrix}$

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7) Jen $\begin{pmatrix}0\\0\\0\end{pmatrix}$ a $\begin{pmatrix}0\\0\\0\\0\end{pmatrix}$

$\text{coord}_{B_1}(\vec{x}) = \begin{pmatrix}0\\0\\0\end{pmatrix}$

$\text{coord}_{K_4}(\vec{x}) = \begin{pmatrix}0\\0\\0\\0\end{pmatrix}$

8) $\ker(g) = \{\vec{0}\}$

$\text{def}(g) = 0$

9) $\text{coord}_{B_1}(\vec{x}) = \begin{pmatrix}2\\0\\3\end{pmatrix}$ $\text{coord}_{B_1}(\vec{y}) = \begin{pmatrix}3\\7\\4\end{pmatrix}$

$\text{coord}_{K_4}(\vec{x}) = \begin{pmatrix}5\\0\\2\\7\end{pmatrix}$ $\text{coord}_{K_4}(\vec{y}) = \begin{pmatrix}6\\1\\3\\10\end{pmatrix}$

10) $\text{rank}(g) = 3$ $\text{def}(g) + \text{rank}(g) = \dim(L)$

11)