

1. a) F_3

(i)

+	0	1	2
0	0	1	2
1	1	2	0
2	2	0	1

(ii)

x	0	1	2
0	0	0	0
1	0	1	2
2	0	2	1

b) F_5

(i)

+	0	1	2	3	4
0	0	1	2	3	4
1	1	2	3	4	0
2	2	3	4	0	1
3	3	4	0	1	2
4	4	0	1	2	3

(ii)

x	0	1	2	3	4
0	0	0	0	0	0
1	0	1	2	3	4
2	0	2	4	1	3
3	0	3	1	4	2
4	0	4	3	2	1

2.

x	0	1	2	3	4	5
0	0	0	0	0	0	0
1	0	1	2	3	4	5
2	0	2	4	0	2	4
3	0	3	0	3	0	3
4	0	4	2	0	4	2
5	0	5	4	3	2	1

→ 6 není prvočíslo

~~neplati~~ invertibilita

např. $4^{-1} \cdot (4 \cdot 3) = 4^{-1} \cdot 0$

↓
 $3 \neq 0$

3. a) $[(2+1) \cdot 4 - 4]^{-1}$

F_5 \downarrow
 $12 - 4 = 8 \quad 8 \bmod 5 = 3 \quad \underline{\underline{3^{-1} = 2}}$

b) F_7 $\rightarrow 12 - 4 = 8 \quad 8 \bmod 7 = 1 \quad \underline{\underline{1^{-1} = 1}}$

4. a) $\overline{F_5}$ $3x+4=2$ $|+1$ $x \in F_5$

$$3x=3 \quad 1 \cdot 3^{-1} \cdot 3$$

$$\downarrow$$

$$1 \cdot 2$$

$$\underline{\underline{x=1}}$$

b) $4x+3y+1=4$

$$x+3=4y$$



$$x+3=4y \quad | +2$$

$$x=4y+2$$

$$4 \cdot (4y+2) + 3y + 1 = 4$$

$$y+3+3y+1=4$$

$$4y+4=4 \quad | +1$$

$$4y=5 \quad | \cdot 4$$

$$\underline{\underline{y=0}}$$

$$\rightarrow x=4 \cdot 0 + 2$$

$$\underline{\underline{x=2}}$$

c) $2x+3y+3=2$ $|+2$ $x \in F_5$

$$2x+3y=4 \quad | +2y$$

$$2x=2y+4 \quad | \cdot 3$$

$$\underline{\underline{x=y+2}}$$

Cafourek

5. a) X
b) ✓
c) X
d) ✓
e) X
f) ✓

možná X

$$\begin{array}{rcl} g) & \begin{array}{l} x+2y=4 \\ 2x+y=1 \end{array} & \left. \begin{array}{l} \text{~~1.(-2)~~} \\ 1.(-2) \end{array} \right\} \oplus \\ & \hline & \begin{array}{l} -3x=2 \\ x=-\frac{2}{3} \\ y=\frac{7}{3} \end{array} \end{array}$$

X

$$\begin{array}{rcl} h) & \begin{array}{l} x+2y=4 \\ 2x+4y=8 \end{array} & \\ & \hline & \vec{m}(-2;1) \\ & \begin{array}{l} x=2-2t \\ y=1+t \end{array} \end{array}$$

✓

$$\begin{array}{rcl} i) & \begin{array}{l} x+2y=0 \\ 2x+4y=0 \end{array} & \\ & \hline & \vec{m}(-2;1) \\ & \begin{array}{l} x=2-2t \\ y=-1+t \end{array} \end{array}$$

✓

6. $\begin{pmatrix} 1 \\ 2 \end{pmatrix} \in \mathbb{R}^2$
 $\left(\begin{pmatrix} 2 \\ 3 \end{pmatrix}, \begin{pmatrix} 3 \\ 5 \end{pmatrix} \right)$

$$\sum_{i=1}^2 a_i \cdot \vec{x}_i = \vec{x} \Rightarrow \vec{x}_1 \text{ ~~is not in the span~~ } \vec{x}_2 \text{ ~~is not in the span~~ }$$

$$\begin{pmatrix} 1 \\ 2 \end{pmatrix} = a_1 \cdot \begin{pmatrix} 2 \\ 3 \end{pmatrix} + a_2 \cdot \begin{pmatrix} 3 \\ 5 \end{pmatrix}$$

$$\begin{array}{rcl} 2a_1 + 3a_2 = 1 & 1.(-3) & \\ 3a_1 + 5a_2 = 2 & 1.(-2) & \} \oplus \end{array}$$

~~$$-3a_1 - 5a_2 = -3$$~~

7. ~~$p(x) = x^2 + 3x + 2$~~
LP $\mathbb{R}[x]$ mod \mathbb{R}

~~$$a_2 = 1 \Rightarrow a_1 = -1$$~~

a) $(x^2; x; 1)$ $x^2 + 3x + 2 = a_1 \cdot x^2 + a_2 \cdot x + a_3 \cdot 1$

b) $(x^2+1; 3x-1; 1)$ $a_1=1, a_2=3, a_3=2$

$$x^2 + 3x + 2 = (x^2+1)a_1 + (3x-1)a_2 + 1 \cdot a_3$$

$$a_1=1, a_2=1, a_3=2$$