# The dual

Primal $\min_{w, w_0} \overbrace{\frac{1}{2}\|w\|^2}^{f(w)}$ s.t. $\overbrace{y_i(w^T x_i + w_0) \geq 1}^{h_i(w)}$ for $i \leq n$

$\mathcal{L} = f + \sum_i \alpha_i h_i$

Dual $\max_{\cancel{\lambda}, \alpha} \underbrace{\theta(\alpha)}_{\min_w \mathcal{L}(w, \alpha)}$ s.t. $\alpha_i \geq 0$

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$\max_{\lambda, \alpha} \boxed{\underbrace{\inf_w \mathcal{L}}_{\theta}} \leq f$

$$\mathcal{L}(w, w_0, \alpha) = \frac{1}{2}\|w\|^2 + \sum_{i \leq n} \alpha_i \left(1 - y_i (w^T x_i + w_0)\right) \quad \text{with } \alpha_i \geq 0$$

$$= \frac{1}{2} w^T w + \sum_i \alpha_i - \sum_i \alpha_i y_i w^T x_i - w_0 \sum_i \alpha_i y_i$$

# The dual

$$\frac{\partial \mathcal{L}}{\partial w} = w - \sum_i \alpha_i y_i x_i$$

$$w^* = \underbrace{\boxed{\sum_i \alpha_i y_i x_i}}_{w(\alpha)}$$

$$\frac{\partial \mathcal{L}}{\partial w_0} = -\sum_i \alpha_i y_i$$

$$\Downarrow$$

$$\sum_i \alpha_i y_i = 0$$

extra constraint

$$\boxed{\theta(\alpha) = \min_{w, w_0} \mathcal{L}(w, w_0, \alpha) = \sum_i \alpha_i - \frac{1}{2} \|w(\alpha)\|^2}$$

# The dual

$$\omega(\alpha) = \sum_i \alpha_i y_i x_i$$

$$\max_\alpha \quad \theta(\alpha) \qquad \sum_i \alpha_i - \frac{1}{2} \|\omega(\alpha)\|^2$$

$$\text{s.t.} \quad \alpha_j \geq 0 \quad \text{and} \quad \sum_i \alpha_i y_i = 0$$