Linear Control Design 2 E18 - Theoretical Questionnaire

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Analysis of open loop systems (Part 1)

Question 1

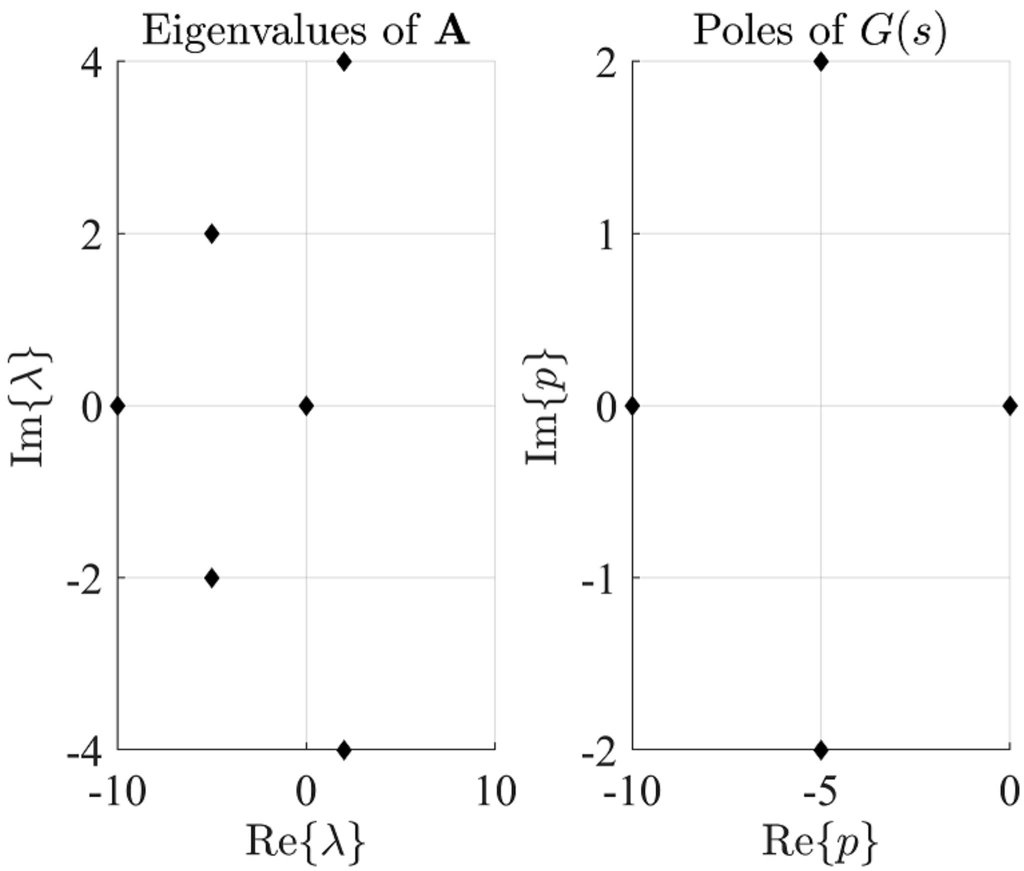
Consider the sixth order continuous time LTI SISO system

$$\dot{\mathbf{x}} = \mathbf{A}\mathbf{x} + \mathbf{B}u, \qquad \mathbf{x} \in \mathbb{R}^n, u \in \mathbb{R}$$
$$y = \mathbf{C}\mathbf{x} \qquad y \in \mathbb{R}$$

with the associated transfer function

$$G(s) = \mathbf{C}(s\mathbf{I} - \mathbf{A})^{-1}\mathbf{B}$$

The following figure shows in the complex plane the position of the eigenvalues of the system dynamical matrix $\mathbf{A}$ as well as the position of the poles of the transfer function $G(s)$.



Based on the eigenvalues and poles maps, which of the following statements is correct?

- [ ] The system is internally unstable and BIBO stable.
- [ ] The system is internally marginally stable and not BIBO stable.
- [ ] The system is internally asymptotically stable and BIBO stable.
- [x] The system is internally unstable and not BIBO stable.
- [ ] The system is internally marginally stable and BIBO stable.

Question 2

Consider the third order continuous time LTI system

$$\dot{\mathbf{x}} = \begin{bmatrix} \lambda_1 & 0 & 0 \\ 0 & \lambda_2 & 1 \\ 0 & 0 & \lambda_2 \end{bmatrix} \mathbf{x}, \qquad \mathbf{x}(0) = \mathbf{x}_0 \neq \mathbf{0}$$

where λ_i are the system eigenvalues. Said c_i a constant dependent on the initial condition and $\mathbf{v}_i$ the right eigenvector associated with the eigenvalue λ_i, what is the zero-input response of the system to the given initial condition?

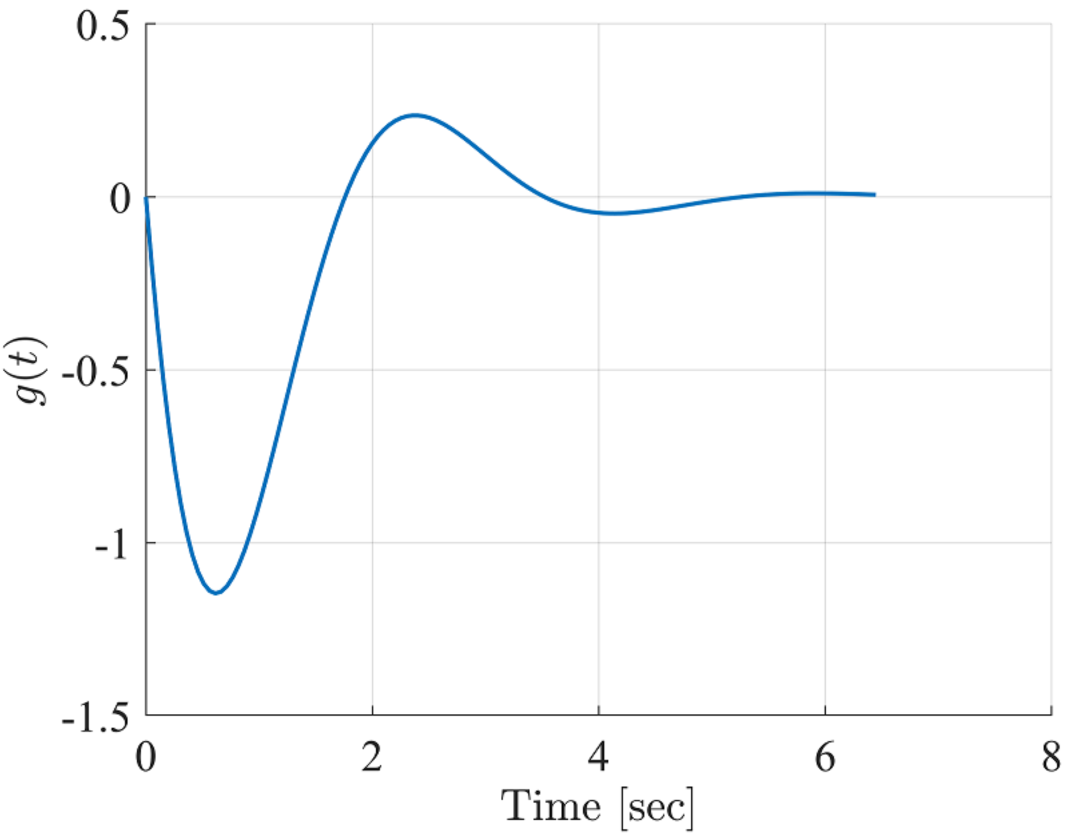
- [ ] $\mathbf{x}(t) = c_1\mathbf{v}_1 e^{\lambda_1 t} + c_2\mathbf{v}_2 e^{\lambda_2 t} + c_3\mathbf{v}_3 e^{\lambda_2 t}$
- [ ] $\mathbf{x}(t) = c_1\mathbf{v}_1 e^{\lambda_1 t} + 2c_2\mathbf{v}_2 e^{\lambda_2 t}$
- [x] $\mathbf{x}(t) = c_1\mathbf{v}_1 e^{\lambda_1 t} + c_2\mathbf{v}_2 e^{\lambda_2 t} + c_3(\mathbf{v}_2 t e^{\lambda_2 t} + \mathbf{v}_3 e^{\lambda_2 t})$
- [ ] $\mathbf{x}(t) = c_1\mathbf{v}_1 e^{\lambda_1 t} + c_2\mathbf{v}_2 e^{\lambda_2 t} + c_3\mathbf{v}_2 t e^{\lambda_2 t}$
- [ ] $\mathbf{x}(t) = c_1\mathbf{v}_1 e^{\lambda_1 t} + (c_2\mathbf{v}_2 e^{\lambda_2 t})^2$

Question 3

The second order continuous time LTI SISO system

$$\dot{\mathbf{x}} = \mathbf{A}\mathbf{x} + \begin{bmatrix} 0 \\ b \end{bmatrix} u, \qquad \mathbf{x} \in \mathbb{R}^2, u \in \mathbb{R}$$
$$y = \begin{bmatrix} c & 0 \end{bmatrix} \mathbf{x}, \qquad y \in \mathbb{R}$$

has the following impulse response function in response to a positive impulse (the coefficients b and c are real numbers different from zero)



Said α_{ss} the steady state gain of the system, what is the zero-state output response of the system if the input is

$$u(t) = \begin{cases} 0 & 0 \leq t < t_0 \\ u_0 & t \geq t_0 \end{cases}$$

with $u_0 < 0$?

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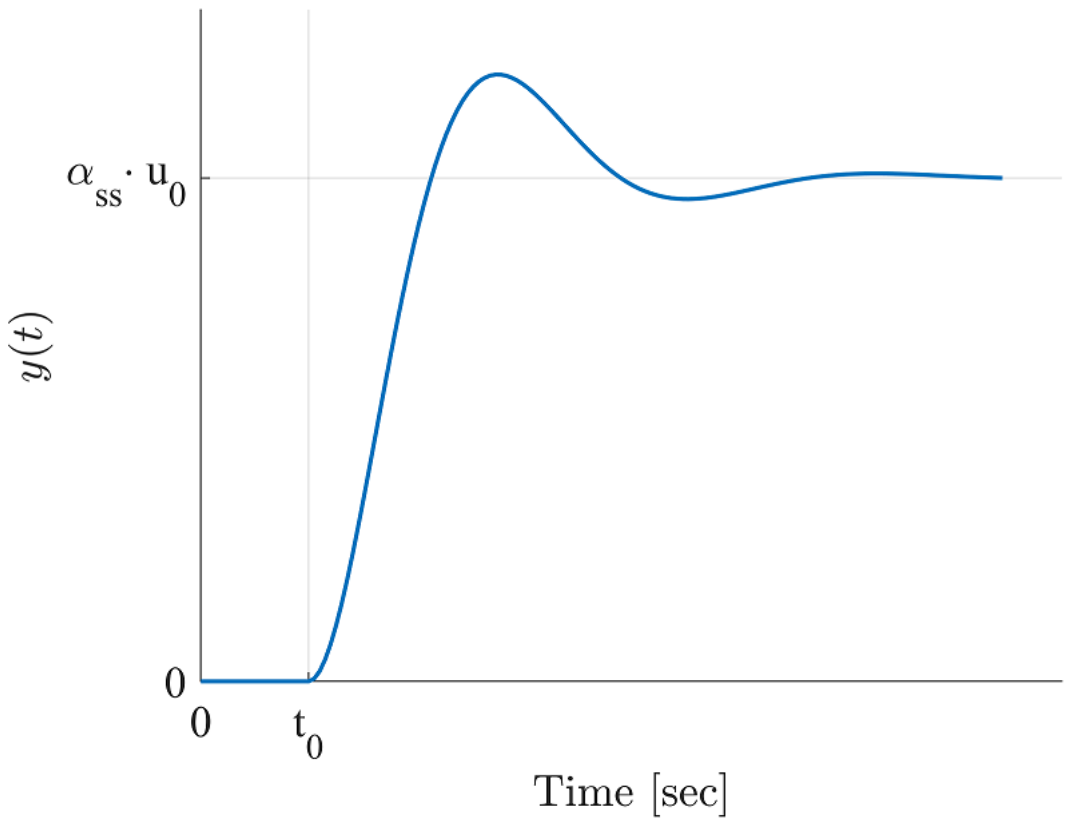
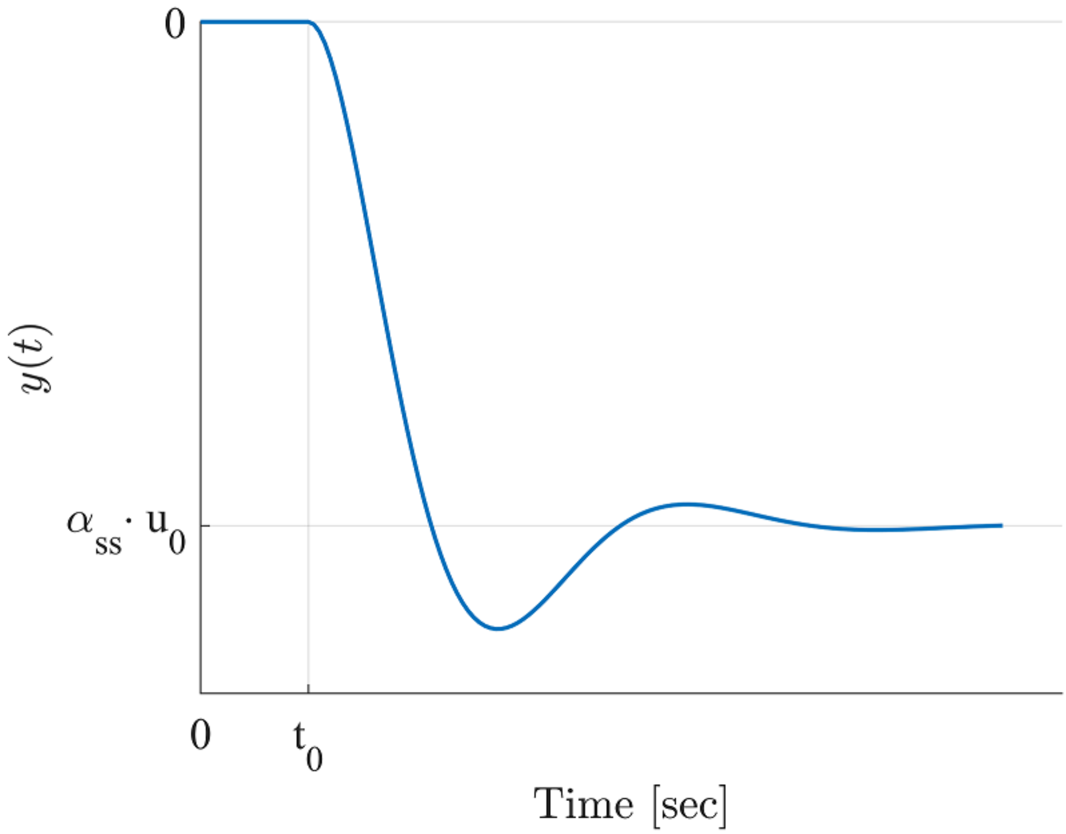
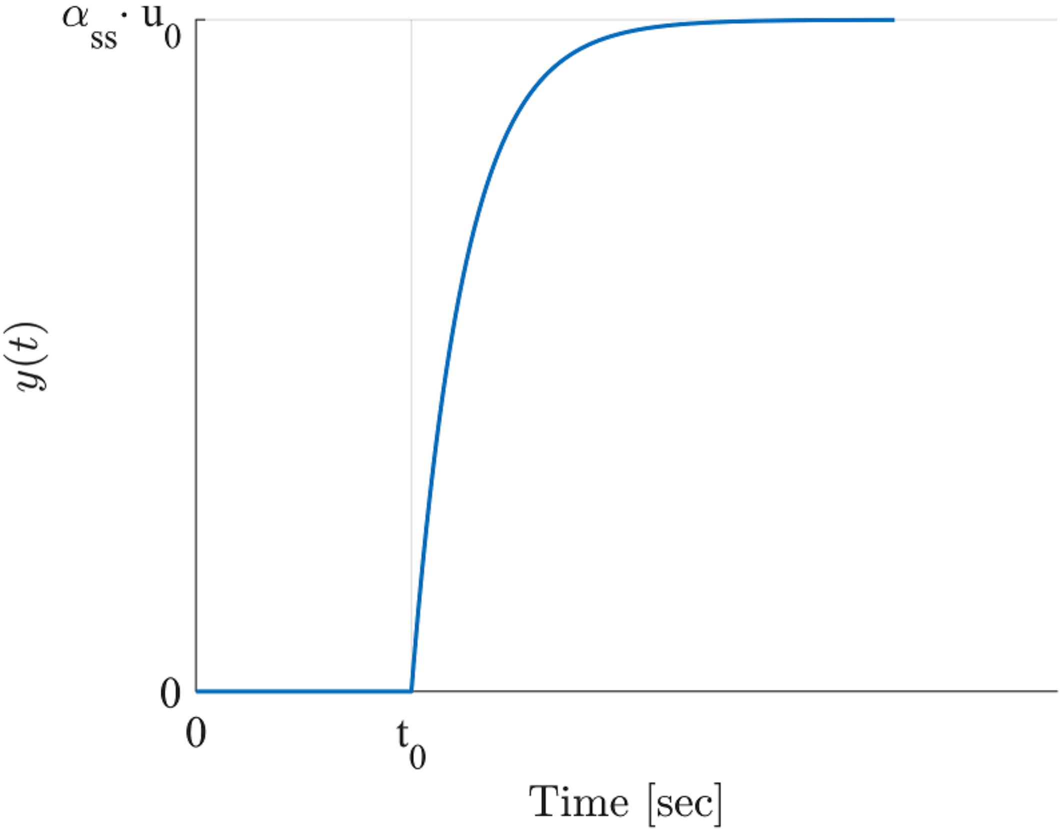
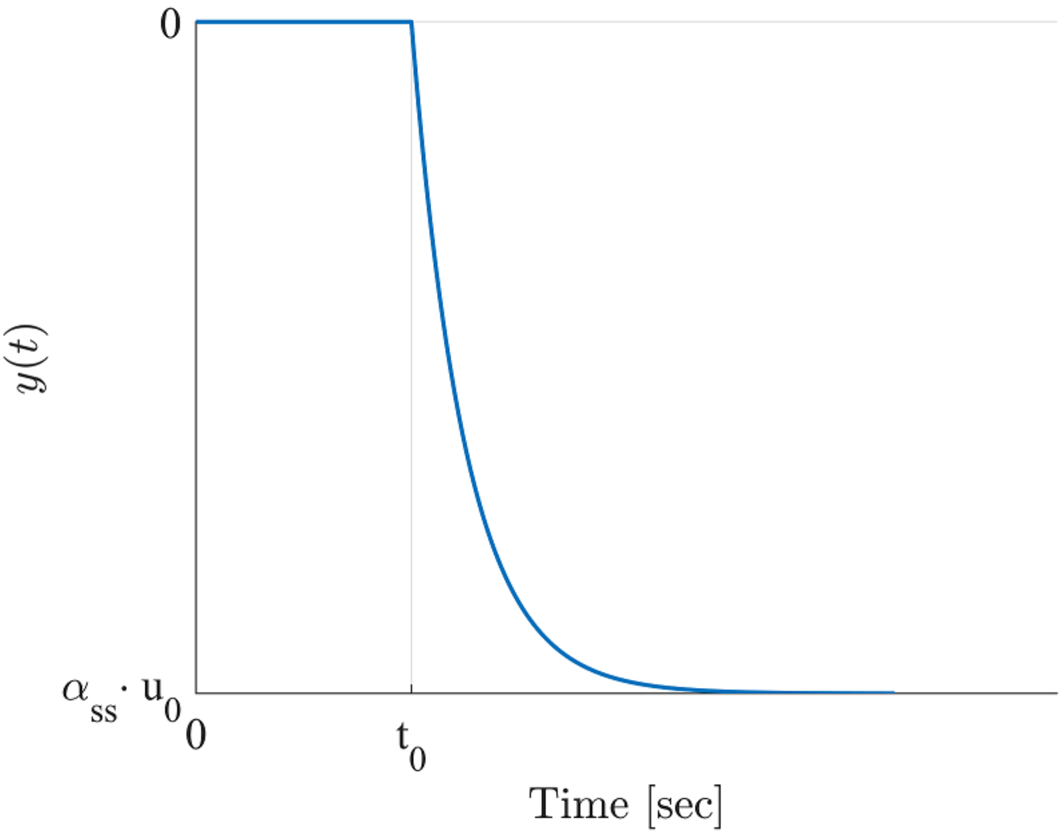

$\alpha_{ss} \cdot u_0$
$y(t)$
0
0
t_0
Time [sec]

☐

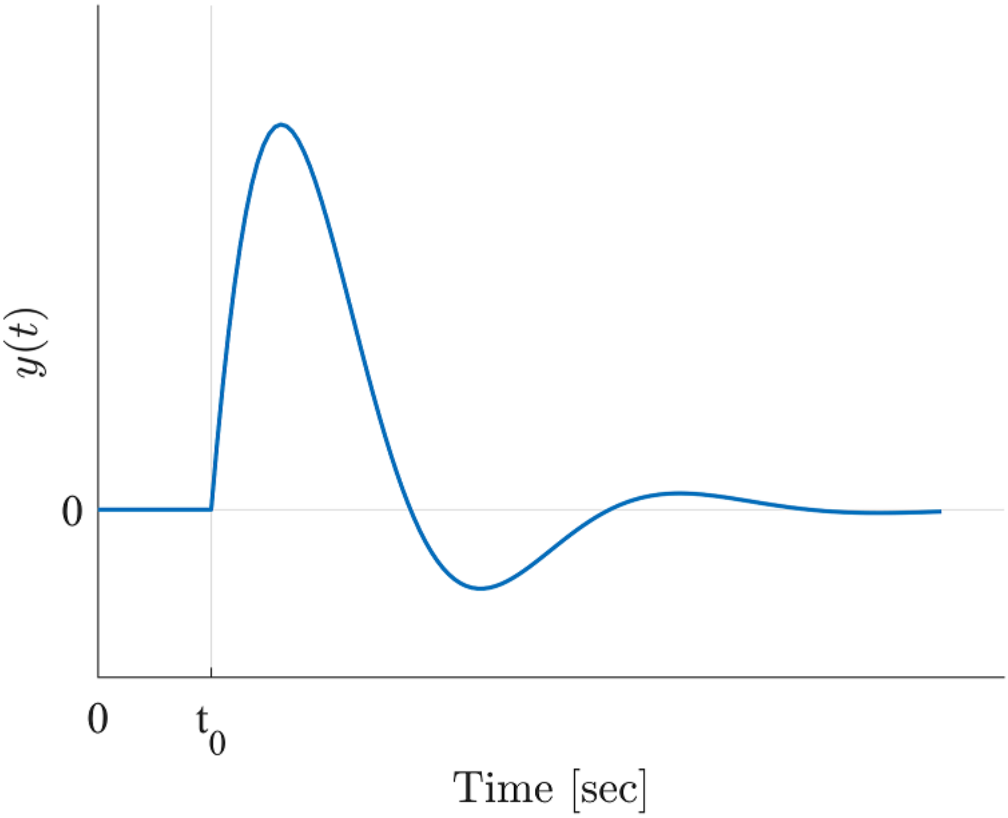

0
$\alpha_{ss} \cdot u_0$
$y(t)$
0
t_0
Time [sec]



$\alpha_{ss} \cdot u_0$
0
$y(t)$
0
t_0
Time [sec]



0
$y(t)$
$\alpha_{ss} \cdot u_0$
0
t_0
Time [sec]



$y(t)$
0
0
t_0
Time [sec]

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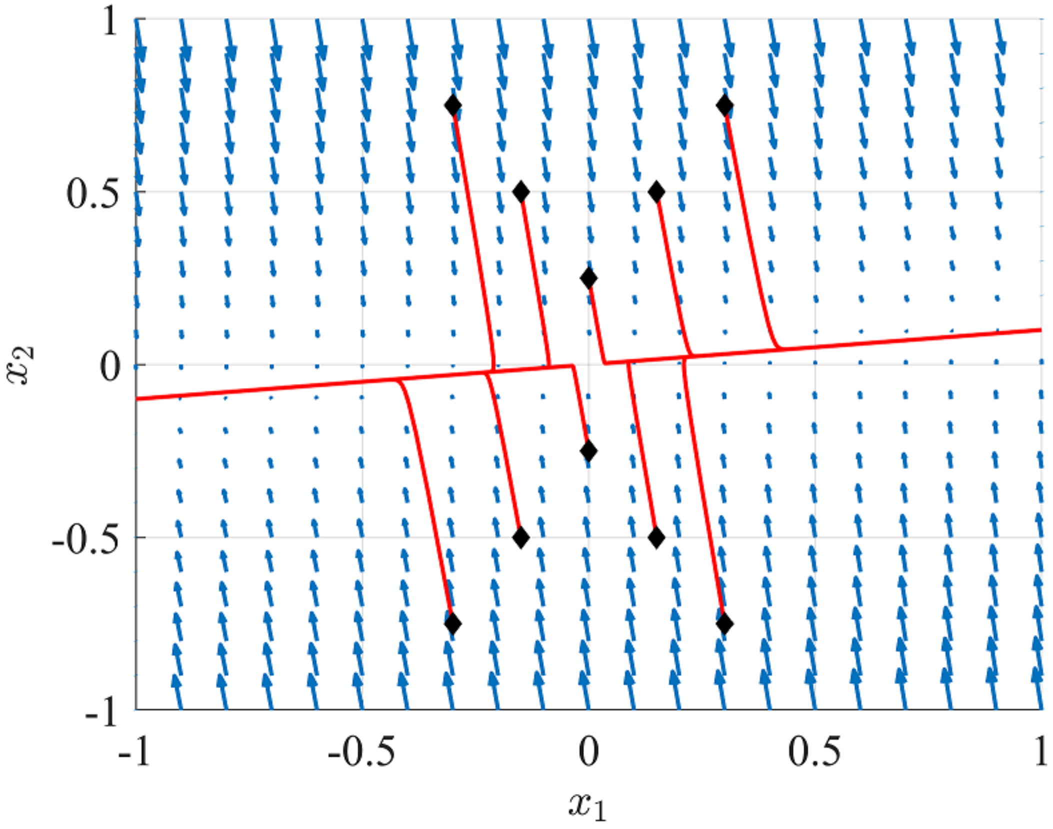
Analysis of open loop systems (Part 2)

Question 4

The phase portrait of a second order continuous time LTI system is shown in the following figure (in the given phase portraits each black diamond represents an initial condition $\mathbf{x}(0) = [x_{10}, x_{20}]^{\mathrm{T}}$ for the system; each red line is a trajectory of the system originated from the initial condition; the blue arrows represent the direction of the vector field in the neighborhood of the origin.)



Which of the following statement is correct?

- [ ] The equilibrium point is a stable node.
- [ ] The equilibrium point is an unstable focus.
- [x] The equilibrium point is a saddle point.
- [ ] The equilibrium point is an unstable node.
- [ ] The equilibrium point is a centre.

Question 5

Consider the third order discrete time LTI system

$$\mathbf{x}(k+1) = \begin{bmatrix} \alpha & \varepsilon_1 & \varepsilon_2 \\ 0 & \gamma & \beta \\ 0 & -\beta & \gamma \end{bmatrix} \mathbf{x}(k)$$

where

$$|\alpha| < 1 \wedge |\gamma \pm j\beta| = 1$$

and $\varepsilon_1, \varepsilon_2 \in \mathbb{R}$.

Which of the following statements is **not correct**?

- [ ] Given the initial condition $\mathbf{x}(0) = [x_{10}, 0, 0]^{\mathrm{T}}$, then for $k \to +\infty$ the zero-input response converges to $\mathbf{x} = [0, 0, 0]^{\mathrm{T}}$ as α^k.
- [ ] Given the initial condition $\mathbf{x}(0) = [0, x_{20}, x_{30}]^{\mathrm{T}}$ such that $x_{20}^2 + x_{30}^2 = 1$, then for $k \to +\infty$ the zero-input response converges to $\mathbf{x} = [c_1 \cos(\beta k T_s) + c_2 \sin(\beta k T_s), \cos(\beta k T_s), \sin(\beta k T_s)]^{\mathrm{T}}$.
- [ ] Given the initial condition $\mathbf{x}(0) = [0, 0, 0]^{\mathrm{T}}$, then the zero-input response stays at $\mathbf{x} = \mathbf{x}(0)$ for all future times.
- [x] Given the initial condition $\mathbf{x}(0) = [0, x_{20}, x_{30}]^{\mathrm{T}}$ such that $x_{20}^2 + x_{30}^2 = 1$, then for $k \to +\infty$ the zero-input response converges to $\mathbf{x} = [0, \cos(\beta k T_s), \sin(\beta k T_s)]^{\mathrm{T}}$.
- [ ] Given the initial condition $\mathbf{x}(0) = [x_{10}, x_{20}, x_{30}]^{\mathrm{T}}$ such that $x_{20}^2 + x_{30}^2 = 1$, then for $k \to +\infty$ the zero-input response converges to $\mathbf{x} = [c_1 \cos(\beta k T_s) + c_2 \sin(\beta k T_s), \cos(\beta k T_s), \sin(\beta k T_s)]^{\mathrm{T}}$ as α^k.

Question 6

Consider the third order continuous time LTI SISO system

$$\dot{\mathbf{x}} = \begin{bmatrix} 0 & 0 & \alpha \\ \beta & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix} \mathbf{x} + \begin{bmatrix} 0 \\ 0 \\ \gamma \end{bmatrix} u, \qquad \mathbf{x} \in \mathbb{R}^3, u \in \mathbb{R}$$

$$y = \begin{bmatrix} 0 & \delta & 0 \end{bmatrix} \mathbf{x} \qquad y \in \mathbb{R}$$

where $\alpha,\ \beta,\ \gamma$ and δ belong to the set of real numbers. Which of the following statements is correct?

- [x] The system is controllable if $\alpha \neq 0 \ \wedge \ \beta \neq 0 \ \wedge \ \gamma \neq 0$.
- [ ] The system is controllable if $\gamma \neq 0$ and $\forall\, \alpha, \beta \in \mathbb{R}$.
- [ ] The system is controllable if $\alpha \neq 0 \ \wedge \ \beta \neq 0$ and $\forall \gamma \in \mathbb{R}$.
- [ ] The system is controllable $\forall \alpha, \beta, \gamma \in \mathbb{R}$.
- [ ] The system is controllable if $\delta \neq 0$ and $\forall \alpha, \beta, \gamma \in \mathbb{R}$.

Question 7

Consider the first order continuous time LTI system

$$\dot{x} = \alpha x + \beta v$$

where $\alpha > 0$ and $\beta \in \mathbb{R}$. The system is driven by the stochastic process v that is white noise with zero mean and noise intensity V_1. Which of the following statements is correct?

- [ ] The variance q_x of the state x is found as the solution of the steady state Lyapunov equation $2q_x\alpha + \beta^2 V_1 = 0$.
- [x] The variance q_x of the state x is found as the solution of the time-varying Lyapunov equation $\dot{q}_x = 2q_x\alpha + \beta^2 V_1$.
- [ ] The variance q_x of the state x cannot be computed because the system is unstable.
- [ ] The variance q_x of the state x equals the noise intensity V_1 since the dynamic system has no effect onto the stochastic process v.
- [ ] The variance q_x of the state x is zero because the system is unstable.

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Control system design and closed loop system analysis

Question 8

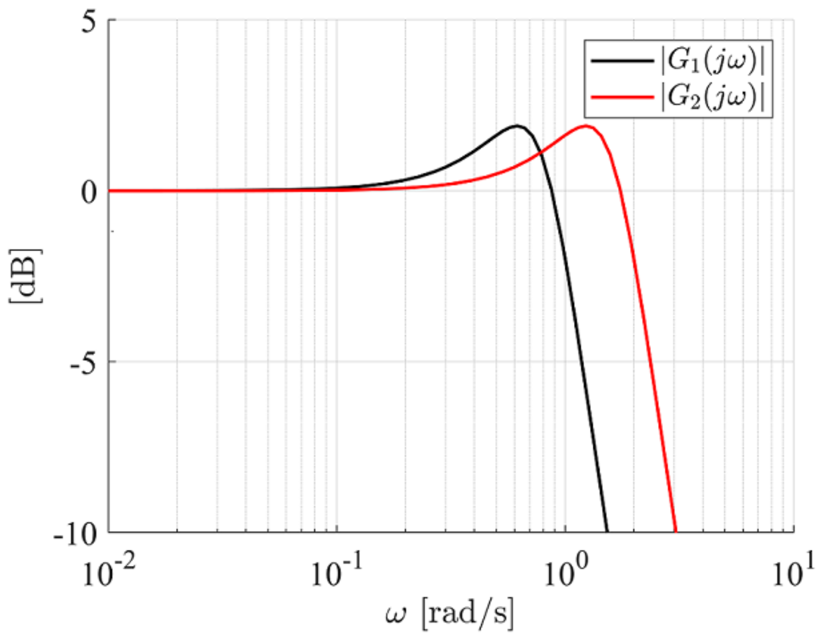
Consider the second order LTI continuous time system

$$\Sigma_x : \begin{cases} \dot{\mathbf{x}}(t) = \begin{bmatrix} 0 & 1 \\ -\omega_0^2 & -2\zeta\omega_0 \end{bmatrix} \mathbf{x}(t) + \begin{bmatrix} 0 \\ \omega_0^2 \end{bmatrix} u(t), & \mathbf{x} \in \mathbb{R}^2, u \in \mathbb{R} \\ y(t) = \begin{bmatrix} 1 & 0 \end{bmatrix} \mathbf{x}(t), & y \in \mathbb{R} \end{cases}$$

where $\omega_0 > 0$ and $0 < \zeta < 1$. The transfer function associated with open loop system reads

$$G_1(s) = \frac{\omega_0^2}{s^2 + 2\zeta\omega_0 s + \omega_0^2}$$

The figure below shows the Bode diagrams of the open loop transfer function $G_1(s)$ and the transfer function $G_2(s)$ associated with the closed-loop system.



Which of the following control architectures has been applied to the open loop system Σ_x in order to obtained the closed-loop system $G_2(s)$?

- [ ] $u(t) = -\frac{1}{\omega_0^2}\left[(\omega_1^2 - \omega_0^2) \;\; 2\zeta(\omega_1 - \omega_0)\right] \mathbf{x}$

- [x] $u(t) = -\frac{1}{\omega_0^2}\left[(\omega_1^2 - \omega_0^2) \;\; 2\zeta(\omega_1 - \omega_0)\right] \mathbf{x} + \frac{\omega_1^2}{\omega_0^2} r$
  where r is a reference signal.

- [ ] $u(t) = -\frac{1}{\omega_0^2}\left[0 \;\; 2\omega_0(\zeta_1 - \zeta)\right] \mathbf{x} + r$
  where r is a reference signal.

- [ ] $u(t) = -\frac{1}{\omega_0^2}\left[(\omega_1^2 - \omega_0^2) \;\; 2(\zeta_1\omega_1 - \zeta\omega_0)\right] \mathbf{x} + \frac{\omega_1^2}{\omega_0^2} r$
  where r is a reference signal.

- [ ] $u(t) = -\frac{1}{\omega_0^2}\left[(\omega_1^2 - \omega_0^2) \;\; 2(\zeta_1\omega_1 - \zeta\omega_0)\right] \mathbf{x} + r$
  where r is a reference signal.

Question 9

Consider the second order continuous time LTI SISO system

$$\dot{\mathbf{x}} = \begin{bmatrix} 0 & 1 \\ -\alpha_1 & -\alpha_2 \end{bmatrix} \mathbf{x} + \begin{bmatrix} 0 \\ \beta \end{bmatrix} u$$
$$y = \begin{bmatrix} 1 & 0 \end{bmatrix} \mathbf{x} + \gamma d$$

where $\alpha_1, \alpha_2, \beta$ and γ are real and positive coefficients. The signal $d(t)$ acting on the system output is an unknown constant disturbance.

Let T_s be a properly chosen sampling time and $\mathbf{e}(k) = \mathbf{x}(k) - \hat{\mathbf{x}}(k)$ be the estimation error associated with a discrete time observer. Which of the following discrete time observers achieves

$$\lim_{k \to +\infty} \mathbf{e}(k) = \mathbf{0}$$

for a step change in the disturbance $d(t)$?

- [ ] $$\hat{\mathbf{x}}(k+1) = \begin{bmatrix} 1 & T_s \\ -\alpha_1 T_s & 1 - \alpha_2 T_s \end{bmatrix} \hat{\mathbf{x}}(k) + \begin{bmatrix} \beta \frac{T_s^2}{2} \\ \beta T_s - \alpha_2 \beta \frac{T_s^2}{2} \end{bmatrix} u(k) + \begin{bmatrix} l_1 \\ l_2 \end{bmatrix} (y(k) - \hat{y}(k))$$
$$\hat{y}(k) = \begin{bmatrix} 1 & 0 \end{bmatrix} \hat{\mathbf{x}}(k)$$

- [ ] $$\hat{\mathbf{x}}(k+1) = \begin{bmatrix} 1 & T_s \\ -\alpha_1 T_s & 1 - \alpha_2 T_s \end{bmatrix} \hat{\mathbf{x}}(k) + \begin{bmatrix} \beta \frac{T_s^2}{2} \\ \beta T_s - \alpha_2 \beta \frac{T_s^2}{2} \end{bmatrix} u(k) + \begin{bmatrix} l_1 \\ l_2 \end{bmatrix} (y(k) - \hat{y}(k))$$
$$\hat{y}(k) = \begin{bmatrix} 1 & 0 \end{bmatrix} \hat{\mathbf{x}}(k) + \gamma d(k)$$

- [x] $$\hat{\mathbf{x}}_a(k+1) = \begin{bmatrix} 1 & T_s & 0 \\ -\alpha_1 T_s & 1 - \alpha_2 T_s & 0 \\ 0 & 0 & 1 \end{bmatrix} \hat{\mathbf{x}}_a(k) + \begin{bmatrix} \beta \frac{T_s^2}{2} \\ \beta T_s - \alpha_2 \beta \frac{T_s^2}{2} \\ 0 \end{bmatrix} u(k) + \begin{bmatrix} l_1 \\ l_2 \\ l_3 \end{bmatrix} (y(k) - \hat{y}(k))$$
$$\hat{y}(k) = \begin{bmatrix} 1 & 0 & \gamma \end{bmatrix} \hat{\mathbf{x}}_a(k)$$
where $\hat{\mathbf{x}}_a = [\hat{\mathbf{x}}^{\mathrm{T}}, \hat{d}]^{\mathrm{T}}$.

- [ ] $$\hat{\mathbf{x}}_a(k+1) = \begin{bmatrix} 1 & T_s & \gamma \frac{T_s^2}{2} \\ -\alpha_1 T_s & 1 - \alpha_2 T_s & \gamma T_s - \alpha_2 \gamma \frac{T_s^2}{2} \\ 0 & 0 & 1 \end{bmatrix} \hat{\mathbf{x}}_a(k) + \begin{bmatrix} \beta \frac{T_s^2}{2} \\ \beta T_s - \alpha_2 \beta \frac{T_s^2}{2} \\ 0 \end{bmatrix} u(k) + \begin{bmatrix} l_1 \\ l_2 \\ l_3 \end{bmatrix} (y(k) - \hat{y}$$
$$\hat{y}(k) = \begin{bmatrix} 1 & 0 & 0 \end{bmatrix} \hat{\mathbf{x}}_a(k)$$
where $\hat{\mathbf{x}}_a = [\hat{\mathbf{x}}^{\mathrm{T}}, \hat{d}]^{\mathrm{T}}$.

- [ ] $$\hat{\mathbf{x}}_a(k+1) = \begin{bmatrix} 1 & T_s & 0 \\ -\alpha_1 T_s & 1 - \alpha_2 T_s & 0 \\ 0 & 0 & 0 \end{bmatrix} \hat{\mathbf{x}}_a(k) + \begin{bmatrix} \beta \frac{T_s^2}{2} \\ \beta T_s - \alpha_2 \beta \frac{T_s^2}{2} \\ 0 \end{bmatrix} u(k) + \begin{bmatrix} l_1 \\ l_2 \\ l_3 \end{bmatrix} (y(k) - \hat{y}(k))$$
$$\hat{y}(k) = \begin{bmatrix} 1 & 0 & \gamma \end{bmatrix} \hat{\mathbf{x}}_a(k)$$
where $\hat{\mathbf{x}}_a = [\hat{\mathbf{x}}^{\mathrm{T}}, \hat{d}]^{\mathrm{T}}$.

Question 10

Consider the second order continuous time LTI SISO system

$$\dot{\mathbf{x}} = \begin{bmatrix} 0 & 1 \\ -\omega_n^2 & -2\zeta\omega_n \end{bmatrix} \mathbf{x} + \begin{bmatrix} 0 \\ \omega_n^2 \end{bmatrix} u + \begin{bmatrix} 0 \\ \alpha \end{bmatrix} d$$
$$y = \begin{bmatrix} 1 & 0 \end{bmatrix} \mathbf{x}$$

where $\omega_n > 0, 0 < \zeta < 1$ and $\alpha \in \mathbb{R}$. The disturbance $d(t)$ acting on the state equation is time-varying and unknown, i.e. $d(t) = d_0(t - t_0)$ where d_0 is the unknown slope and t_0 is the time when the disturbance enters the system.

Which of the following control architectures (CAs) guarantees perfect tracking of the constant reference $r(t) = r_0$ in the presence of the given disturbance?

- [x] $\mathrm{CA} : \begin{cases} u = -\mathbf{K}\mathbf{x} + \mathbf{K}_i\mathbf{x}_i \\ \dot{\mathbf{x}}_i = \begin{bmatrix} 0 & 0 \\ 1 & 0 \end{bmatrix} \mathbf{x}_i + \begin{bmatrix} -1 & 0 \\ 0 & 0 \end{bmatrix} \mathbf{x} + \begin{bmatrix} 1 \\ 0 \end{bmatrix} r \end{cases}$
  where $\mathbf{K} = [K_1, K_2], \mathbf{K}_i = [K_{i,1}, K_{i,2}]$ and $\mathbf{x}_i = [x_{i,1}, x_{i,2}]^{\mathrm{T}}$.

- [ ] $\mathrm{CA} : \begin{cases} u = -\mathbf{K}\mathbf{x} + K_i x_i \\ \dot{x}_i = \begin{bmatrix} -1 & 0 \end{bmatrix} \mathbf{x} + r \end{cases}$
  where $\mathbf{K} = [K_1, K_2]$.

- [ ] $\mathrm{CA} : \{ u = -\mathbf{K}\mathbf{x} + Nr$
  where $\mathbf{K} = [K_1, K_2], N = (\omega_n^2 + K_1)/\omega_n^2$.

- [ ] $\mathrm{CA} : \{ u = -\mathbf{K}\mathbf{x} + r$
  where $\mathbf{K} = [K_1, K_2]$.

- [ ] $\mathrm{CA} : \begin{cases} u = -\mathbf{K}\hat{\mathbf{x}} - \mathbf{K}_d\hat{d} + Nr \\ \dot{\mathbf{x}}_o = \begin{bmatrix} 0 & 1 & 0 \\ -\omega_n^2 & -2\zeta\omega_n^2 & \alpha \\ 0 & 0 & 0 \end{bmatrix} \mathbf{x}_o + \begin{bmatrix} 0 \\ \omega_n^2 \\ 0 \end{bmatrix} u + \begin{bmatrix} l_1 \\ l_2 \\ l_3 \end{bmatrix} (y - \hat{y}) \\ \hat{y} = \begin{bmatrix} 1 & 0 & 0 \end{bmatrix} \mathbf{x}_o \end{cases}$
  where $\mathbf{K} = [K_1, K_2], N = (\omega_n^2 + K_1)/\omega_n^2$ and $\mathbf{x}_o = [\hat{\mathbf{x}}^{\mathrm{T}}, \hat{d}]^{\mathrm{T}}$.

Question 11

Consider the first order LTI continuous time system

$$\dot{x} = ax + b_v v$$
$$y_1 = x + w_1$$
$$y_2 = x + w_2$$

where $a \in \mathbb{R} \setminus \{0\}$, $b_v \in \mathbb{R} \setminus \{0\}$, v is white Gaussian noise with zero mean and noise intensity $\sigma_v^2 \geq 0$, w_1 and w_2 are uncorrelated white Gaussian noise sources with zero mean and noise intensity matrix

$$\mathbf{V} = \begin{bmatrix} \sigma_{w_1}^2 & 0 \\ 0 & \sigma_{w_2}^2 \end{bmatrix} > 0\,.$$

Which of the following statements is correct? (Given a matrix $\mathbf{M}$ the symbol $\|\mathbf{M}\|_\infty$ indicates the infinity norm of the matrix, which is defined as $\|\mathbf{M}\|_\infty = \max_{1\leq i\leq m} \sum_{j=1}^n |m_{ij}|$, where m_{ij} is the i-th row and j-th column entry of the matrix. The symbol $\ll$ means "much smaller than" and the symbol $\gg$ means "much larger than").

- [ ] If $\|\mathbf{V}\|_\infty \gg \sigma_v^2$ then the Kalman filter strongly relies on the measurements to estimate x.
- [ ] If $\sigma_{w1}^2 \ll \sigma_{w2}^2$ then the Kalman gain associated with the measurement y_1 is smaller than the Kalman gain associated with the measurement y_2.
- [ ] If the plant dynamics is unstable ($a > 0$) then the dynamics of the estimation error $e = x - \hat{x}$ is also unstable.
- [x] If the plant dynamics is asymptotically stable ($a < 0$) and the intensity of the process noise is zero ($\sigma_v^2 = 0$) then the solution of the Riccati equation associated with the design of the Kalman gain is zero.
- [ ] If the plant dynamics is unstable ($a > 0$) and the intensity of the process noise is zero ($\sigma_v^2 = 0$) then the solution of the Riccati equation associated with the design of the Kalman gain is zero.

Question 12

Consider the n-th order continuous time LTI system

$$\dot{\mathbf{x}} = \mathbf{A}\mathbf{x} + \mathbf{B}\mathbf{u}, \quad \mathbf{x} \in \mathbb{R}^n,\ \mathbf{u} \in \mathbb{R}^m$$
$$\mathbf{y} = \mathbf{C}\mathbf{x}, \quad \mathbf{y} \in \mathbb{R}^p$$

where $\mathbf{A}, \mathbf{B}$, and $\mathbf{C}$ are matrices with constant coefficients. A finite-time optimal regulator problem is set-up for the given system using the performance index

$$J(\mathbf{u}) = \int_{t_0}^{t_1} \mathbf{x}^{\mathrm{T}}\mathbf{Q}(t)\,\mathbf{x} + \mathbf{u}^{\mathrm{T}}\mathbf{R}(t)\,\mathbf{u}\;\mathrm{d}t + \mathbf{x}^{\mathrm{T}}(t_1)\,\mathbf{S}(t_1)\,\mathbf{x}(t_1)$$

where the weighting matrices fulfill the following inequalities

$$\mathbf{Q}(t) \geq 0, \quad \forall t \geq t_0$$
$$\mathbf{R}(t) > 0, \quad \forall t \geq t_0$$
$$\mathbf{S}(t_1) \geq 0, \quad \forall t_1$$

Which of the following statements is correct?

- [ ] The optimal controller $\mathbf{K}(t)$ is time independent when $\mathbf{Q}(t)$ and $\mathbf{R}(t)$ are constant weighting matrices.
- [ ] If the pair $(\mathbf{A}, \mathbf{B})$ is not stabilizable there is no finite solution to the finite-time optimal control problem.
- [ ] If $\mathbf{S}(t_1) = 0$ then it is possible to achieve a constant non zero control law $\mathbf{K}(t) = \bar{\mathbf{K}} \neq \mathbf{0}$.
- [x] If $\mathbf{S}(t_1) = \bar{\mathbf{P}}$ ($\bar{\mathbf{P}}$ being the solution of the algebraic Riccati equation), and $\mathbf{Q}(t)$ and $\mathbf{R}(t)$ are constant weighting matrices then it is possible to achieve a constant non zero control law $\mathbf{K}(t) = \bar{\mathbf{K}} \neq \mathbf{0}$.
- [ ] The performance index $J(\mathbf{u})$ reaches its minimum at the end of the optimization time horizon $[t_0, t_1]$.