34745 E24 Multiple Choice Questionnaire

Der anvendes en scoringsalgoritme, som er baseret på "One best answer"

Dette betyder følgende:

- Der er altid netop ét svar som er mere rigtigt end de andre
- Studerende kan kun vælge ét svar per spørgsmål
- Hvert rigtigt svar giver 1 point
- Hvert forkert svar giver 0 point (der benyttes IKKE negative point)

The following approach to scoring responses is implemented and is based on "One best answer"

- There is always only one correct answer – a response that is more correct than the rest
- Students are only able to select one answer per question
- Every correct answer corresponds to 1 point
- Every incorrect answer corresponds to 0 points (incorrect answers do not result in subtraction of points)

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The fifth order LTI discrete time system

$$\mathbf{x}(k+1) = \mathbf{F}\mathbf{x}(k) + \mathbf{G}\mathbf{u}(k), \quad \mathbf{x} \in \mathbb{R}^5, \mathbf{u} \in \mathbb{R}^m$$
$$y(k) = \begin{bmatrix} 1 & 1 & 1 & 1 & 1 \end{bmatrix} \mathbf{x}(k), \quad y \in \mathbb{R}$$

has the eigenvalue map shown in the following figure (the blue diamonds mark the position of the eigenvalues)

Let assume that the system has the initial condition $\mathbf{x}(0) = [x_{10}, x_{20}, x_{30}, x_{40}, x_{50}]^{\mathrm{T}}$ where $x_{i0} \neq 0 \,\forall\, i \in \{1, \ldots, 5\}$, and that the transfer function $\mathbf{G}(z)$ does not have zero-pole cancellations . Which of the following statements is correct?

- ○ The zero-input response converges to zero without oscillations as time goes to infinity.
- ○ The zero-input output response converges oscillating to zero as time goes to infinity.
- ○ The zero-input output response converges to a finite constant value as time goes to infinity.
- ✔ The zero-input output response converges to a sinusoidal oscillation with constant amplitude and frequency as time goes to infinity.
- ○ The zero-input response grows unbounded as time goes to infinity.

The state and output responses of a third order LTI continuous time system are

$$\mathbf{x}(t) = e^{\lambda_1 t}\mathbf{w}_1^T\mathbf{x}_0\mathbf{v}_1 + e^{\lambda_2 t}\mathbf{w}_2^T\mathbf{x}_0\mathbf{v}_2 + e^{\lambda_3 t}\mathbf{w}_3^T\mathbf{x}_0\mathbf{v}_3 + \mathbf{v}_2\int_0^t e^{\lambda_2(t-\tau)}\mathbf{w}_2^T\mathbf{B}\mathbf{u}(\tau)\,\mathrm{d}\tau$$
$$+ \mathbf{v}_3\int_0^t e^{\lambda_3(t-\tau)}\mathbf{w}_3^T\mathbf{B}\mathbf{u}(\tau)\,\mathrm{d}\tau$$

$$\mathbf{y}(t) = e^{\lambda_1 t}\mathbf{w}_1^T\mathbf{x}_0\mathbf{C}\mathbf{v}_1 + e^{\lambda_2 t}\mathbf{w}_2^T\mathbf{x}_0\mathbf{C}\mathbf{v}_2 + e^{\lambda_3 t}\mathbf{w}_3^T\mathbf{x}_0\mathbf{C}\mathbf{v}_3 + \mathbf{C}\mathbf{v}_3\int_0^t e^{\lambda_3(t-\tau)}\mathbf{w}_3^T\mathbf{B}\mathbf{u}(\tau)\,\mathrm{d}\tau$$

where $\mathbf{v}_i$ and $\mathbf{w}_i$ are the right and left eigenvectors of the system dynamical matrix. Which of the following statements is <u>not correct</u>?

- [x] The controllable and observable subspace is one-dimensional.
- [ ] The controllable subspace is two-dimensional.
- [ ] The unobservable subspace is one-dimensional.
- [ ] The uncontrollable subspace is one-dimensional.
- [ ] The observable subspace is three-dimensional.

Consider the two second order LTI SISO systems shown in the following figure, where the constants $\alpha,\ \beta$ and γ are positive.

Which of the following statements is correct?

- [ ] Σ_1 and Σ_2 are two asymptotically stable systems.
- [ ] Σ_1 has both eigenvalues equal to zero, while Σ_2 has one eigenvalue equal to zero and one eigenvalue with real part larger than zero.
- [x] The zero-state output responses for $u = u_0$ for all $t \geq 0$ are

  $\Sigma_1 : y(t) = \alpha(\beta+\gamma)u_0 t$

  $\Sigma_2 : y(t) = \alpha\left(\beta + \frac{1}{2}\gamma t\right)u_0 t$
- [ ] Σ_1 and Σ_2 are both unstable systems.
- [ ] The zero-input output responses for the given equal initial condition $\mathbf{x}_0 = [x_{10}, x_{20}]^{\mathrm{T}} \neq [0, 0]^{\mathrm{T}}$ are

  $\Sigma_1 : y(t) = (\beta x_{10} + \gamma x_{20})t$

  $\Sigma_2 : y(t) = \beta x_{10} + \gamma(x_{10}t + x_{20})$

Given the third order LTI system

$$\Sigma : \begin{cases} \dot{\mathbf{x}} = \begin{bmatrix} \alpha & 0 & 0 \\ \alpha & 0 & 0 \\ 0 & 0 & -\beta \end{bmatrix} \mathbf{x} + \begin{bmatrix} \gamma_1 & \gamma_2 \\ 0 & 0 \\ \gamma_3 & 0 \end{bmatrix} \mathbf{u}, \quad \mathbf{x} \in \mathbb{R}^3, \mathbf{u} \in \mathbb{R}^2 \\ y = \begin{bmatrix} \alpha & \beta & \alpha \end{bmatrix} \mathbf{x}, \quad y \in \mathbb{R} \end{cases}$$

where α, β, are real and positive coefficients, while γ_1, γ_2, and γ_3 are real. Which of the following statements is correct?

- ○ The system is controllable <u>if and only if</u> $(\gamma_1 \neq 0 \wedge \gamma_3 \neq 0) \vee (\gamma_2 \neq 0 \wedge \gamma_3 \neq 0)$.
- ○ The system is controllable <u>if and only if</u> $\gamma_1 \neq 0 \wedge \gamma_2 \neq 0 \wedge \gamma_3 \neq 0$.
- ○ There is no triple $(\gamma_1, \gamma_2, \gamma_3) \in \mathbb{R}^3$ that makes the system controllable.
- ✔ The system is controllable <u>if and only if</u> $\gamma_2 \neq 0 \wedge \forall \gamma_1, \gamma_3 \in \mathbb{R}$.
- ○ The system is controllable <u>for any</u> triple $(\gamma_1, \gamma_2, \gamma_3) \in \mathbb{R}^3$.

Consider the third order LTI system

$$\Sigma : \begin{cases} \dot{\mathbf{x}} = \begin{bmatrix} -\alpha & 1 & 0 \\ 0 & -\beta & 0 \\ \gamma & 0 & -\delta \end{bmatrix} \mathbf{x}, \quad \mathbf{x} \in \mathbb{R}^3 \\ \mathbf{y} = \begin{bmatrix} 0 & \epsilon_1 & 0 \\ \epsilon_2 & 0 & \epsilon_3 \end{bmatrix} \mathbf{x}, \quad \mathbf{y} \in \mathbb{R}^2 \end{cases}$$

where α, β, γ, δ, ϵ_1, ϵ_2 and ϵ_3 are positive constants. Which of the following statements is correct?

- ○ The system is observable <u>if and only if</u> $\epsilon_1 = 0$ and <u>for any</u> pair $(\epsilon_2, \epsilon_3) \neq (0, 0)$.
- ○ There exists <u>no triple</u> $(\epsilon_1, \epsilon_2, \epsilon_3) \in \mathbb{R}^3$ such that the system is observable.
- ○ The system is observable <u>for any</u> triple $(\epsilon_1, \epsilon_2, \epsilon_3) \in \mathbb{R}^3$.
- ✔ The system is observable <u>if and only if</u> $\epsilon_3 \neq 0$ and <u>for any</u> $(\epsilon_1, \epsilon_2) \in \mathbb{R}^2$.
- ○ The system is observable <u>if and only if</u> $(\epsilon_1, \epsilon_2, \epsilon_3) \neq (0, 0, 0)$.

Consider the second order LTI-SISO system

$$\Sigma : \begin{cases} \dot{\mathbf{x}} = \begin{bmatrix} -\beta & \gamma \\ -\gamma & 0 \end{bmatrix} \mathbf{x} + \begin{bmatrix} \delta \\ 0 \end{bmatrix} u, & \mathbf{x} \in \mathbb{R}^2, u \in \mathbb{R} \\ y = \begin{bmatrix} \varepsilon & 0 \end{bmatrix} \mathbf{x}, \quad y \in \mathbb{R} \end{cases}$$

where β, δ, and ε are real and positive coefficients. Further the coefficients β and γ satisfy the following inequality

$0 < \beta < 2\gamma, \quad \forall \gamma > 0$

Given an arbitrary initial condition $\mathbf{x}_0 > 0$, which is the zero-input response of the system?

Consider the fifth order LTI continuous time system

$$\Sigma : \begin{cases} \dot{\mathbf{x}} = \begin{bmatrix} -\alpha & 0 & 0 & 0 & 1 \\ 0 & -\beta & -\gamma & 0 & 0 \\ 0 & \gamma & -\beta & 0 & 0 \\ 0 & 0 & 0 & 0 & -\delta \\ 0 & 0 & 0 & \delta & 0 \end{bmatrix} \mathbf{x} + \begin{bmatrix} 0 \\ 1 \\ 0 \\ 0 \\ 1 \end{bmatrix} u, \quad \mathbf{x} \in \mathbb{R}^5, \ u \in \mathbb{R} \\ \mathbf{y} = \begin{bmatrix} 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 \end{bmatrix} \mathbf{x}, \quad \mathbf{y} \in \mathbb{R}^2 \end{cases}$$

where $\alpha, \ \beta, \ \gamma, \delta$ and ϵ are real positive constant coefficients. Which plot shows the correct zero-state output response for $u(t) = u_0, \ \forall t \geq 0$?

Consider the second order LTI SISO system

$$\dot{\mathbf{x}} = \begin{bmatrix} 0 & 1 \\ -\omega_n^2 & -2\zeta\omega_n \end{bmatrix} \mathbf{x} + \begin{bmatrix} 0 \\ \alpha \end{bmatrix} u + \begin{bmatrix} 0 \\ \beta \end{bmatrix} d$$
$$y = \begin{bmatrix} \gamma & 0 \end{bmatrix} \mathbf{x}$$

where $\alpha,\ \beta,\ \gamma,\ \omega_n$, and ζ are all positive constants, and d is an unknown constant disturbance. Further, consider the three control architectures:

$$\mathrm{CA}_1 : u = -\mathbf{K}\mathbf{x} + Nr$$
$$\mathrm{CA}_2 : u = -\mathbf{K}\mathbf{x} + K_i x_i, \quad x_i = \int_0^t [r(\tau) - y(\tau)]\, \mathrm{d}\tau$$
$$\mathrm{CA}_3 : u = -\mathbf{K}\hat{\mathbf{x}} + Nr - K_d\hat{d}$$
$$\begin{bmatrix} \dot{\hat{\mathbf{x}}} \\ \dot{\hat{d}} \end{bmatrix} = \begin{bmatrix} 0 & 1 & 0 \\ -\omega_n^2 & -2\zeta\omega_n & \beta \\ 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} \hat{\mathbf{x}} \\ \hat{d} \end{bmatrix} + \begin{bmatrix} 0 \\ \alpha \\ 0 \end{bmatrix} u + \begin{bmatrix} l_1 \\ l_2 \\ l_3 \end{bmatrix} (y - \hat{y})$$
$$\hat{y} = \begin{bmatrix} \gamma & 0 & 0 \end{bmatrix} \begin{bmatrix} \hat{\mathbf{x}} \\ \hat{d} \end{bmatrix}$$

where r is a reference signal and $\mathbf{K} = [k_1, k_2] \neq [0, 0]$ is the full state feedback controller gain vector. Which of the following statements is <u>**not correct**</u>?

- ○ If $d(t) = d_0$ and $r(t) = r_0, \forall\, t \geq 0$, then for $N = (\omega_n^2 + \alpha k_1)/(\alpha\gamma)$ and $K_d = \beta/\alpha$ all three control architectures guarantee $\lim_{t\to\infty} y(t) = r_0$.

- ○ All three control architectures change the natural frequency ω_n of the open-loop system to the value $\omega_{n,\mathrm{des}}$ if $\mathbf{K} = [(\omega_{n,\mathrm{des}}^2 - \omega_n^2)/\alpha, 2\zeta(\omega_{n,\mathrm{des}} - \omega_n)/\alpha]$.

- ✔ If $d(t) = 0$ and $r(t) = r_0, \forall\, t \geq 0$, then for $N = (\omega_n^2 + \alpha k_1)/(\alpha\gamma)$ all three control architectures guarantee $\lim_{t\to\infty} y(t) = r_0$.

- ○ If $d(t) = d_0$ and $r(t) = r_0, \forall\, t \geq 0$, then CA_1 regulates the output to $y(t) = r_0 + \frac{\beta\gamma}{\omega_n^2 + \alpha k_1} d_0$.

- ○ If $d(t) = d_0$ and $r(t) = 0$, then at steady state the integral action of CA_2 cancels the action of the disturbance, i.e. $K_i x_i = -(\beta/\alpha) d_0$.

Consider the second order LTI continuous time system

$$\Sigma : \begin{cases} \dot{\mathbf{x}} = \begin{bmatrix} 0 & 1 \\ -\omega_n^2 & -2\zeta\omega_n \end{bmatrix} \mathbf{x} + \begin{bmatrix} 0 \\ \omega_n^2 \end{bmatrix} u \\ y = \begin{bmatrix} 1 & 0 \end{bmatrix} \mathbf{x} \end{cases}$$

where $\omega_n = \omega_{n,1} > 0$ is the natural frequency and $0 < \zeta = \zeta_1 < 1$ is the damping ratio. The phase diagram of the open loop system is shown in the following figure, where each black diamond represents an initial condition $\mathbf{x}(0) = [x_{10}, x_{20}]^{\mathrm{T}}$; each red line is a trajectory of the system originated from the initial condition; the blue arrows represent the direction of the vector field.

A full state feedback control law $u = -\mathbf{K}\mathbf{x}$ is imposed on the system such that the closed-loop eigenvalues are strictly real and negative. Which of the following phase diagrams represent the closed-loop dynamics when the system is initialized with the same initial conditions?

Consider the first order LTI SISO system in the block diagram shown in the following figure

where $\alpha,\ \beta,\ \gamma$ and δ are positive constants. The input signal $u(t)$ is a stochastic process characterized by the autocorrelation function

$R_u(\tau) = \sigma_u^2 e^{-\lambda|\tau|}$

with σ_u and λ being both positive and constant. What is the variance of the output $y(t)$?

- The variance of the output is $\sigma_y^2 = \left(\frac{\gamma^2\beta^2}{\alpha(\alpha+\lambda)} + \frac{2\gamma\delta\beta}{\alpha+\lambda} + \delta^2\right)\sigma_u^2$.
- The variance of the output is $\sigma_y^2 = \frac{\beta^2}{\alpha(\alpha+\lambda)}\sigma_u^2$
- The variance of the output is $\sigma_y^2 = 0$.
- The variance of the output is $\sigma_y^2 = \sigma_u^2$.
- The variance of the output is $\sigma_y^2 = \left(\frac{\gamma^2\beta^2}{2\alpha} + \delta^2\right)\sigma_u^2$.

Consider the second order LTI SISO system

$$\dot{\mathbf{x}} = \begin{bmatrix} 0 & 1 \\ 0 & -\alpha \end{bmatrix} \mathbf{x} + \begin{bmatrix} 0 \\ \beta \end{bmatrix} u$$

where α and β are positive constants. An optimal linear quadratic regulator is designed by minimizing the performance index

$$J(u) = \int_0^\infty \mathbf{x}^{\mathrm{T}} \mathbf{Q} \mathbf{x} + \rho u^2 \, \mathrm{d}t$$

where $\mathbf{Q} = \begin{bmatrix} q_1 & 0 \\ 0 & q_2 \end{bmatrix} \geq 0$ and $\rho = 1$. Said $\mathbf{K}_\infty = [k_1, k_2]$ the steady state LQ regulator, the system in closed-loop reads

$$\dot{\mathbf{x}} = \begin{bmatrix} 0 & 1 \\ -\beta k_1 & -\alpha - \beta k_2 \end{bmatrix} \mathbf{x}$$

What is the minimum value of damping ratio ζ that the LQ regulator $\mathbf{K}_\infty$ guarantees for the closed-loop eigenvalues? (The notation $\mathbb{R}_+^a$ denotes the a-dimensional space where all the elements of the space can only assume positive values)

- ○ $\zeta = \frac{1}{\sqrt{2}}$ if $q_2 = 0$ and $q_1 \to +\infty$
- ● $\zeta = \frac{1}{\sqrt{2}}$ if $q_2 = 0$ and $\forall \, q_1 \in \mathbb{R}_+$
- ○ $\zeta = 1, \ \forall (q_1, q_2) \in \mathbb{R}_+^2$
- ○ $\zeta = \frac{1}{\sqrt{2}}$ if $q_1 = 0$ and $q_2 \to +\infty$
- ○ $\zeta = 0, \ \forall (q_1, q_2) \in \mathbb{R}_+^2$

Consider the first order continuous time LTI system

$$\dot{x}(t) = \alpha x(t) + v_1(t)$$
$$y(t) = \beta x(t) + v_2(t)$$

where α and β are real constants. The process noise $v_1(t)$ and the measurement noise $v_2(t)$ are uncorrelated white noise sources with noise intensities σ_1^2 and σ_2^2, respectively. The steady state Kalman filter

$$\dot{\hat{x}}(t) = \alpha \hat{x}(t) + l_\infty (y(t) - \hat{y}(t))$$
$$\hat{y}(t) = \beta \hat{x}(t)$$

is designed in order to reconstruct the state variable $x(t)$ based on the measurement $y(t)$. Which of the following statements is correct?

- ○ If $(\sigma_1^2/\sigma_2^2) \gg 1$ then the steady state Kalman gain is large and the filter relies heavily on the measurement.
- ○ If the intensity σ_1^2 of the process noise is zero and $\alpha < 0$, then the steady state Kalman gain is $l_\infty = 2\alpha$, which ensures the asymptotic stability of the estimation error dynamics.
- ○ The steady state Kalman gain is given by $l_\infty = \beta \frac{\sigma_1^2}{\sigma_2^2}$
- ✔ If $(\sigma_1^2/\sigma_2^2) \ll 1$ then the steady state Kalman gain is large and the filter relies heavily on the system model.
- ○ If the intensity σ_1^2 of the process noise is zero and $\alpha > 0$, then the steady state Kalman gain is $l_\infty = 0$, which ensures the asymptotic stability of the estimation error dynamics.