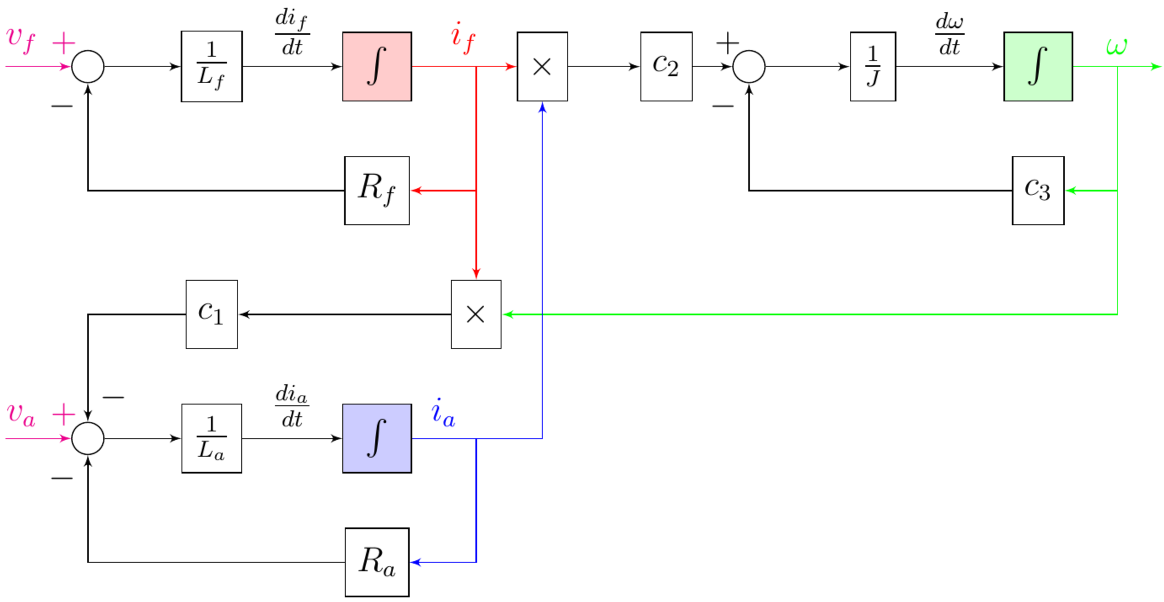
Exam 31310 E21 - Multiple Choice Questionnaire

The multiple choice questionnaire includes 12 questions. Each question is provided with 5 answers. There is only one correct choice per question. All questions are equally scored. You can only give one answer for each question.

The block diagram shown in the following figure provides a graphical representation of the dynamics of a DC motor.



The constants L_a, L_f, R_a, R_f, J, c_1, c_2 and c_3 are all positive. The signals v_a and v_f are the armature and field voltages, respectively. The signals i_a and i_f are the armature and field currents, respectively. The signal ω is the motor's shaft angular velocity.

What is the nonlinear state space model associated with this block diagram?

Choose one answer

- ○ $$\Sigma : \begin{cases} \frac{d\omega}{dt} = \frac{1}{J}(-c_3\omega + c_2 i_a i_f) \\ \frac{di_f}{dt} = \frac{1}{L_f}(-R_f i_f + v_f) \\ \frac{di_a}{dt} = \frac{1}{L_a}(-R_a i_a - c_1 i_f \omega + v_a) \end{cases}$$

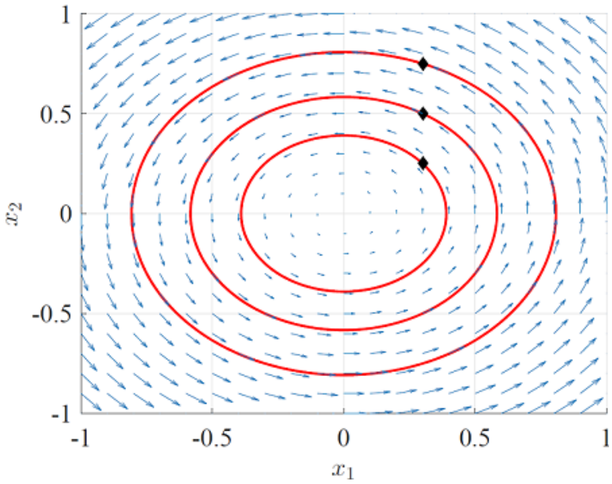
- ○ $$\Sigma : \begin{cases} \frac{d\omega}{dt} = \frac{1}{J}(-c_3\omega + c_2 i_a i_f) \\ \frac{di_f}{dt} = -\frac{R_f}{L_f} i_f + v_f \\ \frac{di_a}{dt} = -\frac{R_a}{L_a} - c_1 i_f \omega + v_a \end{cases}$$

- ○ $$\Sigma : \begin{cases} \frac{di_f}{dt} = \frac{1}{L_f}(-R_f i_f + v_f) \\ \frac{di_a}{dt} = \frac{1}{L_a}(-R_a i_a - c_1 i_f \omega + v_a) \end{cases}$$

- ○ $$\Sigma : \begin{cases} \frac{d\omega}{dt} = \frac{1}{J}(-c_3\omega + c_2 i_a i_f) \\ \frac{di_f}{dt} = \frac{1}{L_f}(-R_f i_f - c_1 i_a \omega + v_f) \\ \frac{di_a}{dt} = \frac{1}{L_a}(-R_a i_a + v_a) \end{cases}$$

- ○ $$\Sigma : \begin{cases} \frac{d\theta}{dt} = \omega \\ \frac{d\omega}{dt} = \frac{1}{J}(-c_3\omega + c_2 i_a i_f) \\ \frac{di_f}{dt} = \frac{1}{L_f}(-R_f i_f + v_f) \\ \frac{di_a}{dt} = \frac{1}{L_a}(-R_a i_a - c_1 i_f \omega + v_a) \end{cases}$$

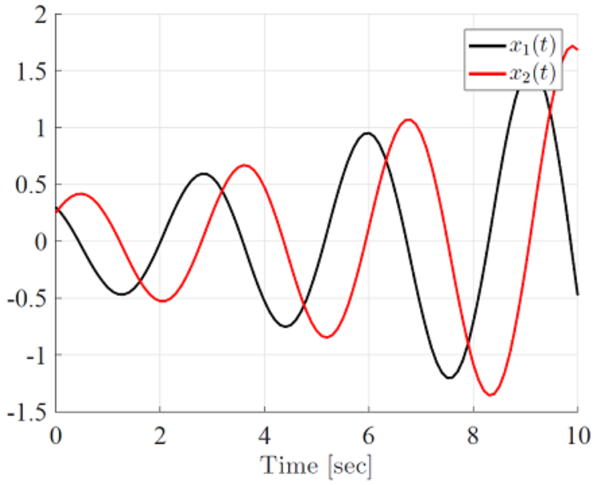
The phase portrait associated with the dynamics of a second order LTI system Σ_x is shown in the following figure

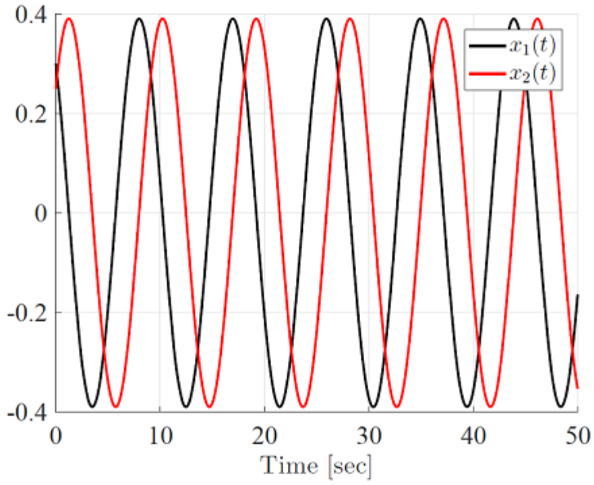


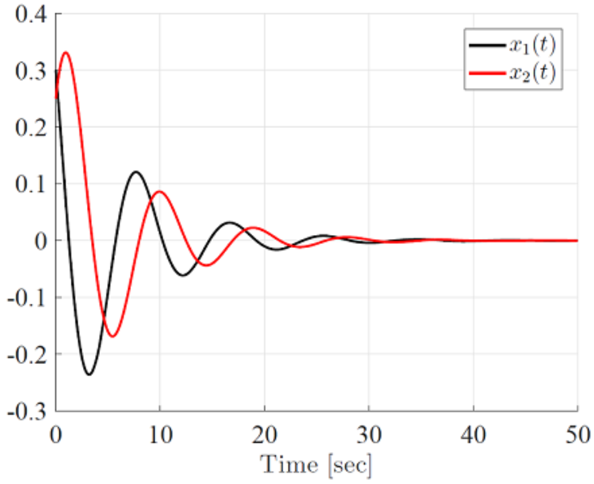
where each black diamond represents an initial condition $\mathbf{x}(0) = [x_{10}, x_{20}]^{\mathrm{T}}$ for the system Σ_x; each red line is a trajectory of the system's solution originated from the initial condition; the blue arrows represent the direction of the vector field.

What is the zero-input response associated with the given phase portrait and that originates from an arbitrary initial condition $\mathbf{x}(0) = [x_{10}, x_{20}]^{\mathrm{T}} \neq [0, 0]^{\mathrm{T}}$?

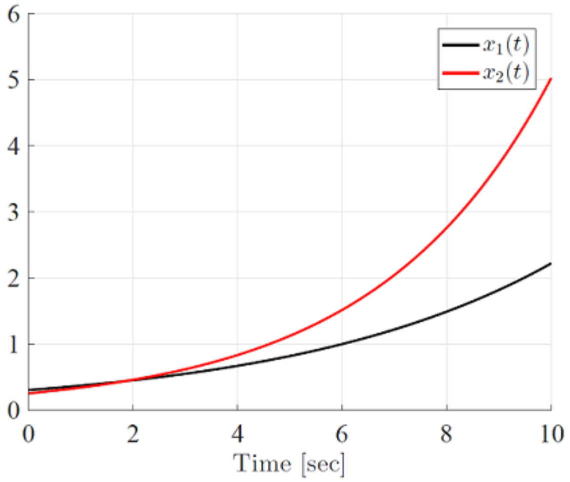
Choose one answer



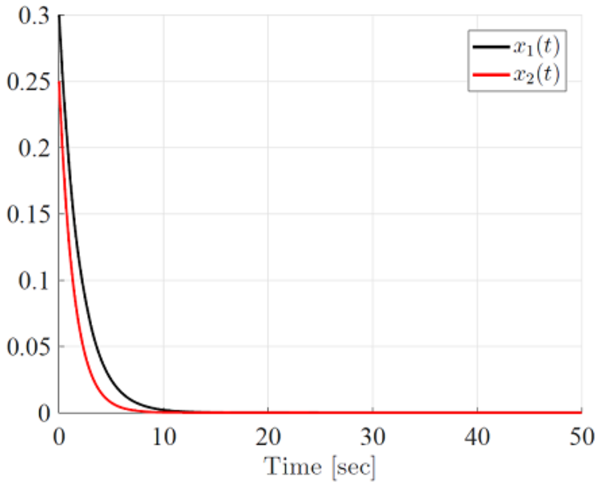




0.4
0.3
0.2
0.1
0
-0.1
-0.2
-0.3
0
10
20
30
40
50
Time [sec]
$x_1(t)$
$x_2(t)$

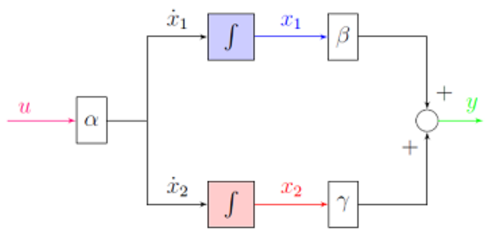
○ 

6
5
4
3
2
1
0
0
2
4
6
8
10
Time [sec]
$x_1(t)$
$x_2(t)$

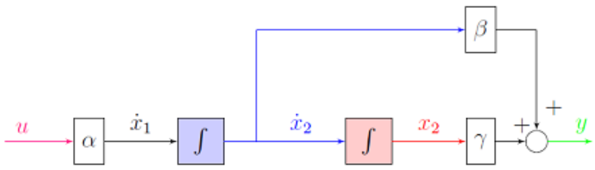
○ 

0.3
0.25
0.2
0.15
0.1
0.05
0
0
10
20
30
40
50
Time [sec]
$x_1(t)$
$x_2(t)$

Consider the two second order LTI SISO systems shown in the following figure, where the constants α, β and γ are positive.



Block diagram of system Σ_1



Block diagram of system Σ_2

Which of the following statements is **not correct**?

Choose one answer

- The zero-input output responses for the given equal initial condition $\mathbf{x}_0 = [x_{10}, x_{20}]^\mathrm{T} \neq [0,0]^\mathrm{T}$ are

$$\Sigma_1 : y(t) = \beta x_{10} + \gamma x_{20}$$
$$\Sigma_2 : y(t) = \beta x_{10} + \gamma(x_{10}t + x_{20})$$

- Σ_1 and Σ_2 are both unstable systems.
- Σ_1 and Σ_2 have both two eigenvalues equal to zero.
- Σ_1 is a marginally stable system and Σ_2 is an unstable system.
- The zero-state output responses for $u = u_0$ for all $t \geq 0$ are

$$\Sigma_1 : y(t) = \alpha(\beta + \gamma)u_0 t$$
$$\Sigma_2 : y(t) = \alpha\left(\beta + \frac{1}{2}\gamma t\right) u_0 t$$

Given the third order LTI system

$$\Sigma : \begin{cases} \dot{\mathbf{x}} = \begin{bmatrix} \alpha & 0 & 0 \\ \alpha & 0 & 0 \\ 0 & 0 & -\beta \end{bmatrix} \mathbf{x} + \begin{bmatrix} \gamma_1 & \gamma_2 \\ 0 & 0 \\ \gamma_3 & 0 \end{bmatrix} \mathbf{u}, \quad \mathbf{x} \in \mathbb{R}^3, \mathbf{u} \in \mathbb{R}^2 \\ y = \begin{bmatrix} \alpha & \beta & \alpha \end{bmatrix} \mathbf{x}, \quad y \in \mathbb{R} \end{cases}$$

where $\alpha,\ \beta$, are real and positive coefficients, while $\gamma_1,\ \gamma_2$, and γ_3 are real. Which of the following statements is correct?

Choose one answer

- The system is controllable if and only if $\gamma_2 \neq 0 \wedge \forall \gamma_1, \gamma_3 \in \mathbb{R}$.
- The system is controllable if and only if $(\gamma_1 \neq 0 \wedge \gamma_3 \neq 0) \vee (\gamma_2 \neq 0 \wedge \gamma_3 \neq 0)$.
- There is no triple $(\gamma_1, \gamma_2, \gamma_3) \in \mathbb{R}^3$ that makes the system controllable.
- The system is controllable for any triple $(\gamma_1, \gamma_2, \gamma_3) \in \mathbb{R}^3$.
- The system is controllable if and only if $\gamma_1 \neq 0 \wedge \gamma_2 \neq 0 \wedge \gamma_3 \neq 0$.

Consider the third order LTI system

$$\Sigma: \begin{cases} \dot{\mathbf{x}} = \begin{bmatrix} -\alpha & 1 & 0 \\ 0 & -\beta & 0 \\ \gamma & 0 & -\delta \end{bmatrix} \mathbf{x}, & \mathbf{x} \in \mathbb{R}^3 \\ \mathbf{y} = \begin{bmatrix} 0 & \epsilon_1 & 0 \\ \epsilon_2 & 0 & \epsilon_3 \end{bmatrix} \mathbf{x}, & \mathbf{y} \in \mathbb{R}^2 \end{cases}$$

where $\alpha,\ \beta,\ \gamma,\ \delta,\ \epsilon_1,\ \epsilon_2$ and ϵ_3 are positive constants. Which of the following statements is correct?

Choose one answer

- The system is observable for any triple $(\epsilon_1, \epsilon_2, \epsilon_3) \in \mathbb{R}^3$.
- There exists no triple $(\epsilon_1, \epsilon_2, \epsilon_3) \in \mathbb{R}^3$ such that the system is observable.
- The system is observable if and only if $\epsilon_3 \neq 0$ and for any $(\epsilon_1, \epsilon_2) \in \mathbb{R}^2$.
- The system is observable if and only if $(\epsilon_1, \epsilon_2, \epsilon_3) \neq (0, 0, 0)$.
- The system is observable if and only $\epsilon_1 = 0$ and for any pair $(\epsilon_2, \epsilon_3) \neq (0, 0)$.

Consider the second order LTI-SISO system

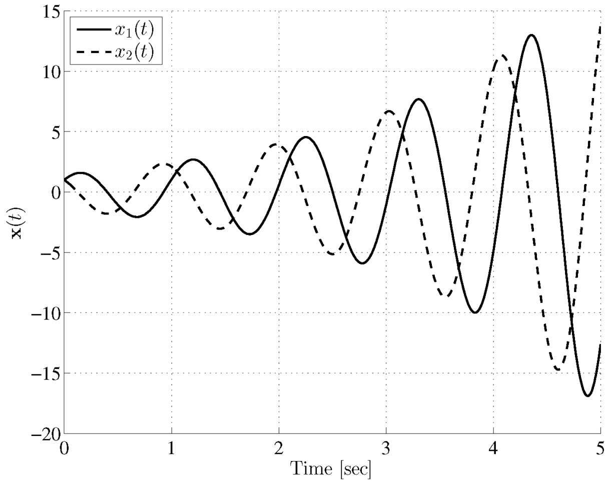
$$\Sigma : \begin{cases} \dot{\mathbf{x}} = \begin{bmatrix} -\beta & \gamma \\ -\gamma & 0 \end{bmatrix} \mathbf{x} + \begin{bmatrix} \delta \\ 0 \end{bmatrix} u, & \mathbf{x} \in \mathbb{R}^2, u \in \mathbb{R} \\ y = [\varepsilon \quad 0]\,\mathbf{x}, & y \in \mathbb{R} \end{cases}$$

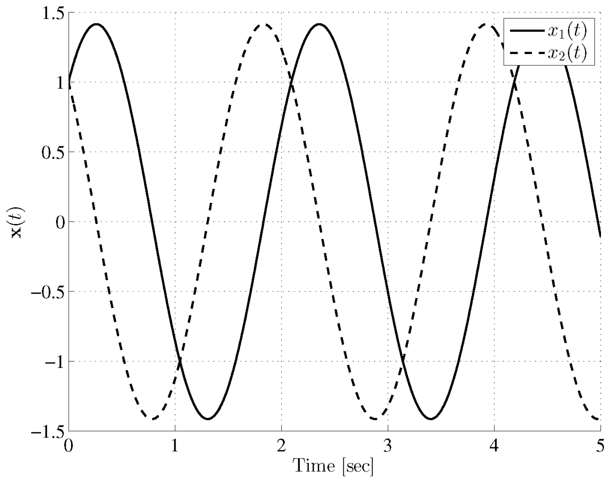
where β, δ, and ε are real and positive coefficients. Further the coefficients β and γ satisfy the following inequality

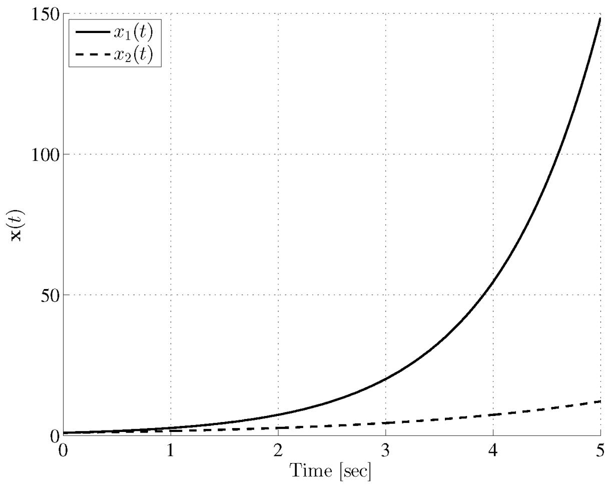
$$\beta > 2|\gamma| > 0, \quad \forall \gamma \in \mathbb{R}, \gamma \neq 0$$

Given an arbitrary initial condition $\mathbf{x}_0 > 0$, which is the zero-input response of the system?

Choose one answer

- 

- 

- 

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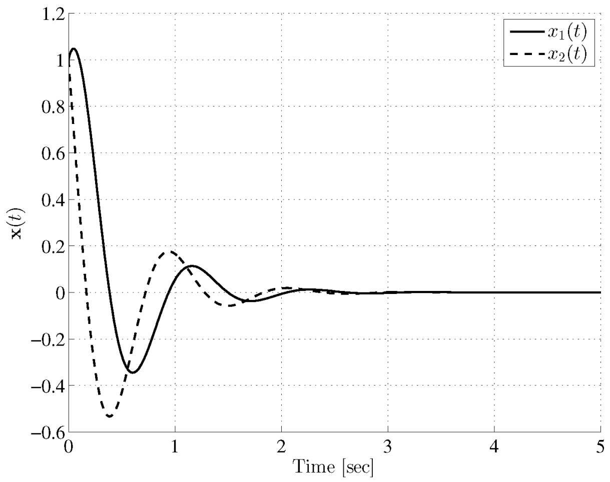
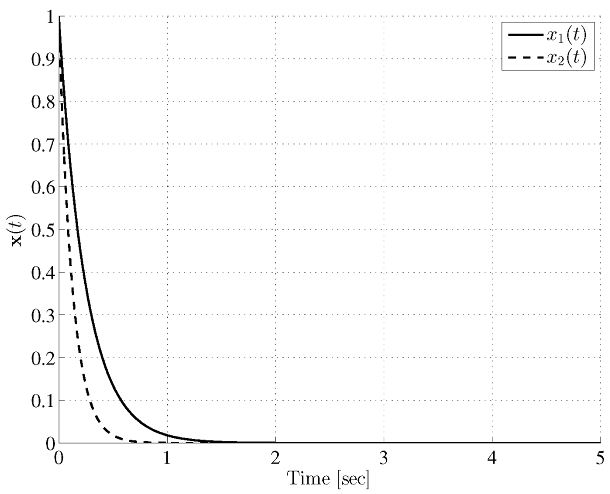


1.2
1
0.8
0.6
0.4
0.2
0
−0.2
−0.4
−0.6
0
1
2
3
4
5
x(t)
Time [sec]
$x_1(t)$
$x_2(t)$



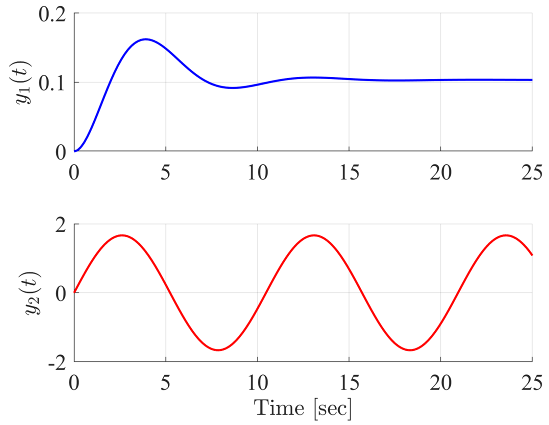
1
0.9
0.8
0.7
0.6
0.5
0.4
0.3
0.2
0.1
0
0
1
2
3
4
5
x(t)
Time [sec]
$x_1(t)$
$x_2(t)$

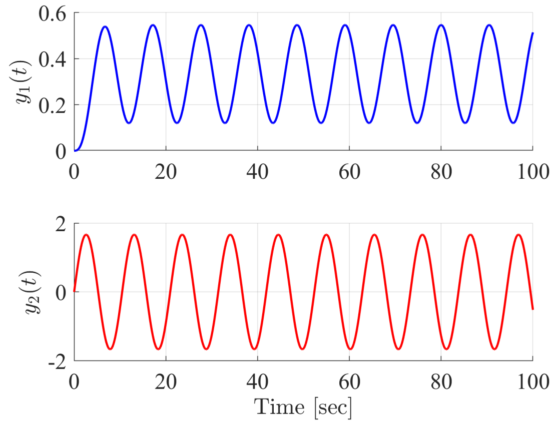
Consider the fifth order LTI continuous time system

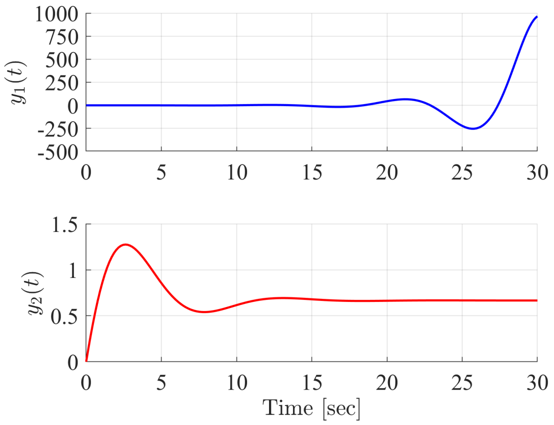
$$
\Sigma : \begin{cases} \dot{\mathbf{x}} = \begin{bmatrix} -\alpha & \varepsilon & 0 & 0 & 0 \\ 0 & -\beta & -\gamma & 0 & 0 \\ 0 & \gamma & -\beta & 0 & 0 \\ 0 & 0 & 0 & 0 & -\delta \\ 0 & 0 & 0 & \delta & 0 \end{bmatrix} \mathbf{x} + \begin{bmatrix} 0 \\ 1 \\ 0 \\ 0 \\ 1 \end{bmatrix} u, \quad \mathbf{x} \in \mathbb{R}^5,\, u \in \mathbb{R} \\ \mathbf{y} = \begin{bmatrix} 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 \end{bmatrix} \mathbf{x}, \quad \mathbf{y} \in \mathbb{R}^2 \end{cases}
$$

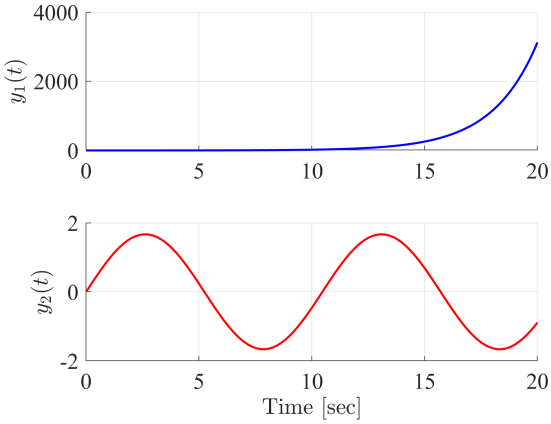
where α, β, γ and δ are real positive constant coefficients. Which plot shows the correct zero-state output response for $u(t) = u_0,\ \forall\, t \geq 0$?

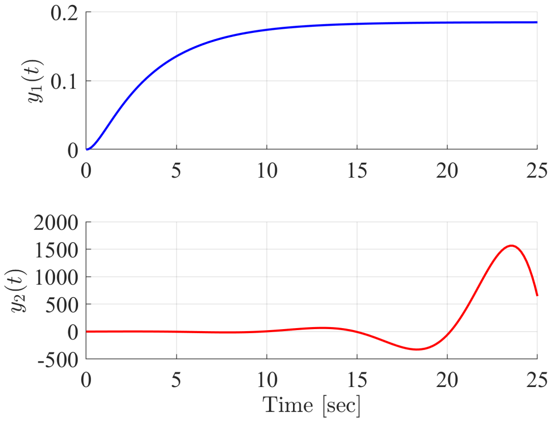
Choose one answer








4000
2000
0
$y_1(t)$
0
5
10
15
20
2
0
-2
$y_2(t)$
0
5
10
15
20
Time [sec]


0.2
0.1
0
$y_1(t)$
0
5
10
15
20
25
2000
1500
1000
500
0
-500
$y_2(t)$
0
5
10
15
20
25
Time [sec]

Consider the n-th order LTI MIMO system

$$\dot{\mathbf{x}} = \mathbf{A}\mathbf{x} + \mathbf{B}\mathbf{u} + \mathbf{B}_d\mathbf{d}, \quad \mathbf{x} \in \mathbb{R}^n\,,\, \mathbf{u} \in \mathbb{R}^m\,,\, \mathbf{d} \in \mathbb{R}^q$$
$$\mathbf{y} = \mathbf{C}\mathbf{x}, \quad \mathbf{y} \in \mathbb{R}^p$$

The system dynamical matrix $\mathbf{A}$ is characterized by $\exists\, \lambda_i \in \lambda(\mathbf{A}) \,|\, \mathrm{Re}\{\lambda_i(\mathbf{A})\} > 0$, where $\lambda(\mathbf{A})$ is the set of eigenvalues of the matrix $\mathbf{A}$. Further the system is fully observable, that is $\mathrm{rank}(\mathbf{M}_o) = n$, where $\mathbf{M}_o$ is the observability matrix.

A full order observer is designed to estimate the state $\mathbf{x}$ and the input disturbance $\mathbf{d}$

$$\begin{bmatrix} \dot{\hat{\mathbf{x}}} \\ \dot{\hat{\mathbf{w}}} \end{bmatrix} = \begin{bmatrix} \mathbf{A} & \mathbf{B}_d\mathbf{C}_w \\ \mathbf{0} & \mathbf{A}_w \end{bmatrix} \begin{bmatrix} \hat{\mathbf{x}} \\ \hat{\mathbf{w}} \end{bmatrix} + \begin{bmatrix} \mathbf{B} \\ \mathbf{0} \end{bmatrix} \mathbf{u} + \begin{bmatrix} \mathbf{L}_x \\ \mathbf{L}_w \end{bmatrix} (\mathbf{y} - \hat{\mathbf{y}})$$
$$\begin{bmatrix} \hat{\mathbf{y}} \\ \hat{\mathbf{d}} \end{bmatrix} = \begin{bmatrix} \mathbf{C} & \mathbf{0} \\ \mathbf{0} & \mathbf{C}_w \end{bmatrix} \begin{bmatrix} \hat{\mathbf{x}} \\ \hat{\mathbf{w}} \end{bmatrix}$$

where $\mathbf{L} = [\mathbf{L}_x^{\mathrm{T}}, \mathbf{L}_w^{\mathrm{T}}]^{\mathrm{T}}$ is the observer gain matrix, and $(\mathbf{A}_w, \mathbf{C}_w)$ are the dynamical and output matrices of the disturbance model

$$\dot{\mathbf{w}} = \mathbf{A}_w\mathbf{w}, \quad \mathbf{w} \in \mathbb{R}^{n_w}$$
$$\mathbf{d} = \mathbf{C}_{\mathbf{w}}\mathbf{w}$$

The disturbance model is characterized by $\mathrm{Re}\{\lambda(\mathbf{A}_w)\} = 0$, where $\lambda(\mathbf{A}_w)$ is the set of eigenvalues of the matrix $\mathbf{A}_w$.

Let $\mathbf{e}_x = \mathbf{x} - \hat{\mathbf{x}}$ and $\mathbf{e}_w = \mathbf{w} - \hat{\mathbf{w}}$ the components of the estimation error, the estimation error dynamics is given by

$$\begin{bmatrix} \dot{\mathbf{e}}_x \\ \dot{\mathbf{e}}_w \end{bmatrix} = \mathbf{A}_e \begin{bmatrix} \mathbf{e}_x \\ \mathbf{e}_w \end{bmatrix}$$
$$= \begin{bmatrix} \mathbf{A} - \mathbf{L}_x\mathbf{C} & \mathbf{B}_d\mathbf{C}_w \\ -\mathbf{L}_w\mathbf{C} & \mathbf{A}_w \end{bmatrix} \begin{bmatrix} \mathbf{e}_x \\ \mathbf{e}_w \end{bmatrix}$$

Which of the following statements is correct?

Choose one answer

- The estimation error $\mathbf{e}_x$ converges to zero because $\mathrm{Re}\{\lambda(\mathbf{A} - \mathbf{L}_x\mathbf{C})\} < 0$ by design of the observer; whereas the estimation error $\mathbf{e}_w$ remains bounded away from zero because $\mathrm{Re}\{\lambda(\mathbf{A}_w)\} = 0$.
- The estimation error $\mathbf{e}_x$ diverges to infinity because $\mathrm{Re}\{\lambda(\mathbf{A})\} > 0$; whereas the estimation error $\mathbf{e}_w$ remains bounded away from zero because $\mathrm{Re}\{\lambda(\mathbf{A}_w)\} = 0$.
- Both estimation errors, $\mathbf{e}_x$ and $\mathbf{e}_w$, converge to zero because $\mathrm{Re}\{\lambda(\mathbf{A}_e)\} < 0$ by design of the observer.
- Both estimation errors, $\mathbf{e}_x$ and $\mathbf{e}_w$, diverge to infinity because $\exists\, \lambda_i \in \lambda(\mathbf{A}) \,|\, \mathrm{Re}\{\lambda_i(\mathbf{A})\} > 0$ and $\mathrm{Re}\{\lambda(\mathbf{A}_w)\} = 0$.
- The estimation error $\mathbf{e}_x$ converges to zero because $\mathrm{Re}\{\lambda(\mathbf{A} - \mathbf{L}_x\mathbf{C})\} < 0$ by design of the observer; whereas the estimation error $\mathbf{e}_w$ diverges to infinity because $\mathrm{Re}\{\lambda(\mathbf{A}_w)\} = 0$.

Consider the second order LTI SISO system

$$\dot{\mathbf{x}} = \begin{bmatrix} 0 & 1 \\ -\omega_n^2 & -2\zeta\omega_n \end{bmatrix} \mathbf{x} + \begin{bmatrix} 0 \\ \alpha \end{bmatrix} u + \begin{bmatrix} 0 \\ \beta \end{bmatrix} d$$
$$y = \begin{bmatrix} \gamma & 0 \end{bmatrix} \mathbf{x}$$

where $\alpha,\ \beta,\ \gamma,\ \omega_n$, and ζ are all positive constants. Further, consider the three control architectures:

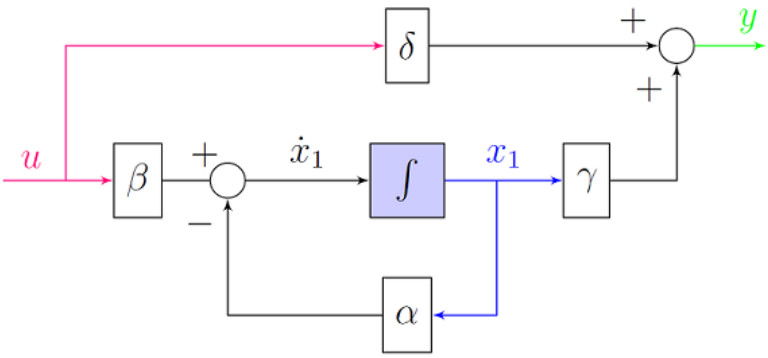
$$\mathrm{CA}_1 : u = -\mathbf{K}\mathbf{x} + Nr$$
$$\mathrm{CA}_2 : u = -\mathbf{K}\mathbf{x} + K_i x_i, \quad x_i = \int_0^t [r(\tau) - y(\tau)]\,\mathrm{d}\tau$$
$$\mathrm{CA}_3 : u = -\mathbf{K}\hat{\mathbf{x}} + Nr - K_d\hat{d}$$
$$\begin{bmatrix} \dot{\hat{\mathbf{x}}} \\ \dot{\hat{d}} \end{bmatrix} = \begin{bmatrix} 0 & 1 & 0 \\ -\omega_n^2 & -2\zeta\omega_n & \beta \\ 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} \hat{\mathbf{x}} \\ \hat{d} \end{bmatrix} + \begin{bmatrix} 0 \\ \alpha \\ 0 \end{bmatrix} u + \begin{bmatrix} l_1 \\ l_2 \\ l_3 \end{bmatrix} (y - \hat{y})$$
$$\hat{y} = \begin{bmatrix} \gamma & 0 & 0 \end{bmatrix} \begin{bmatrix} \hat{\mathbf{x}} \\ \hat{d} \end{bmatrix}$$

where r is a reference signal and $\mathbf{K} = [k_1, k_2] \neq [0, 0]$ is the full state feedback controller gain vector. Which of the following statements is **not correct**?

Choose one answer

- If $d(t) = 0$ and $r(t) = r_0\,, \forall\, t \geq 0$, then for $N = (\omega_n^2 + \alpha k_1)/(\alpha\gamma)$ all three control architectures guarantee $\lim_{t\to\infty} y(t) = r_0$.
- All three control architectures change the natural frequency ω_n of the open-loop system to the value $\omega_{n,\mathrm{des}}$ if $\mathbf{K} = [(\omega_{n,\mathrm{des}}^2 - \omega_n^2)/\alpha, 2\zeta(\omega_{n,\mathrm{des}} - \omega_n)/\alpha]$.
- If $d(t) = d_0$ and $r(t) = 0$, then at steady state the integral action of CA_2 cancels the action of the disturbance, i.e. $K_i x_i = -(\beta/\alpha) d_0$.
- If $d(t) = d_0$ and $r(t) = r_0\,, \forall\, t \geq 0$, then for $N = (\omega_n^2 + \alpha k_1)/(\alpha\gamma)$ and $K_d = \beta/\alpha$ all three control architectures guarantee $\lim_{t\to\infty} y(t) = r_0$.
- If $d(t) = d_0$ and $r(t) = r_0\,, \forall\, t \geq 0$, then CA_1 regulates the output to $y(t) = r_0 + \frac{\beta\gamma}{\omega_n^2 + \alpha k_1} d_0$.

Consider the first order LTI SISO system in the block diagram shown in the following figure



where $\alpha,\ \beta,\ \gamma$ and δ are positive constants. The input signal $u(t)$ is a stochastic process characterized by the autocorrelation function

$$R_u(\tau) = \sigma_u^2 e^{-\lambda|\tau|}$$

with σ_u and λ being both positive and constant. What is the variance of the output $y(t)$?

Choose one answer

- The variance of the output is $\sigma_y^2 = \left(\frac{\gamma^2\beta^2}{\alpha(\alpha+\lambda)} + \frac{2\gamma\delta\beta}{\alpha+\lambda} + \delta^2\right)\sigma_u^2$.
- The variance of the output is $\sigma_y^2 = \sigma_u^2$.
- The variance of the output is $\sigma_y^2 = \left(\frac{\gamma^2\beta^2}{2\alpha} + \delta^2\right)\sigma_u^2$.
- The variance of the output is $\sigma_y^2 = \frac{\beta^2}{\alpha(\alpha+\lambda)}\sigma_u^2$
- The variance of the output is $\sigma_y^2 = 0$.

Consider the three LTI-SISO systems in controllable subspace decomposition form

$$\sum^a : \begin{cases} \dot{\mathbf{x}}^a = \begin{bmatrix} \mathbf{A}_c^a & \mathbf{A}_{12}^a \\ \mathbf{0} & \mathbf{A}_{nc}^a \end{bmatrix} \mathbf{x}^a + \begin{bmatrix} \mathbf{B}_c^a \\ 0 \end{bmatrix} u^a \\ y^a = \begin{bmatrix} \mathbf{C}_c^a & \mathbf{C}_{nc}^a \end{bmatrix} \mathbf{x}^a \end{cases} , \qquad \sum^b : \begin{cases} \dot{\mathbf{x}}^b = \begin{bmatrix} \mathbf{A}_c^b & \mathbf{A}_{12}^b \\ \mathbf{0} & \mathbf{A}_{nc}^b \end{bmatrix} \mathbf{x}^b + \begin{bmatrix} \mathbf{B}_c^b \\ 0 \end{bmatrix} u^b \\ y^b = \begin{bmatrix} \mathbf{C}_c^b & \mathbf{C}_{nc}^b \end{bmatrix} \mathbf{x}^b \end{cases} ,$$

$$\sum^c : \begin{cases} \dot{\mathbf{x}}^c = \begin{bmatrix} \mathbf{A}_c^c & \mathbf{A}_{12}^c \\ \mathbf{0} & \mathbf{A}_{nc}^c \end{bmatrix} \mathbf{x}^c + \begin{bmatrix} \mathbf{B}_c^c \\ 0 \end{bmatrix} u^c \\ y^c = \begin{bmatrix} \mathbf{C}_c^c & \mathbf{C}_{nc}^c \end{bmatrix} \mathbf{x}^c \end{cases} .$$

where $\mathbf{x}^a,\ \mathbf{x}^b,\ \mathbf{x}^c \in \mathbb{R}^n\, ; u^a,\ u^b,\ u^c \in \mathbb{R}\, ; y^a,\ y^b,\ y^c \in \mathbb{R}\,$.

The open loop eigenvalues of the non controllable subsystems are real and they satisfy the following inequality:

$$\lambda_{ol,\max}\left(\mathbf{A}_{nc}^a\right) < \lambda_{ol,\min}\left(\mathbf{A}_{nc}^b\right) < \lambda_{ol,\max}\left(\mathbf{A}_{nc}^b\right) < \lambda_{ol,\min}\left(\mathbf{A}_{nc}^c\right) < 0.$$

A full state feedback controller with integral action is designed for each of the given systems such that the closed loop eigenvalues of the controllable subsystems are real, negative, and satisfy the following relation:

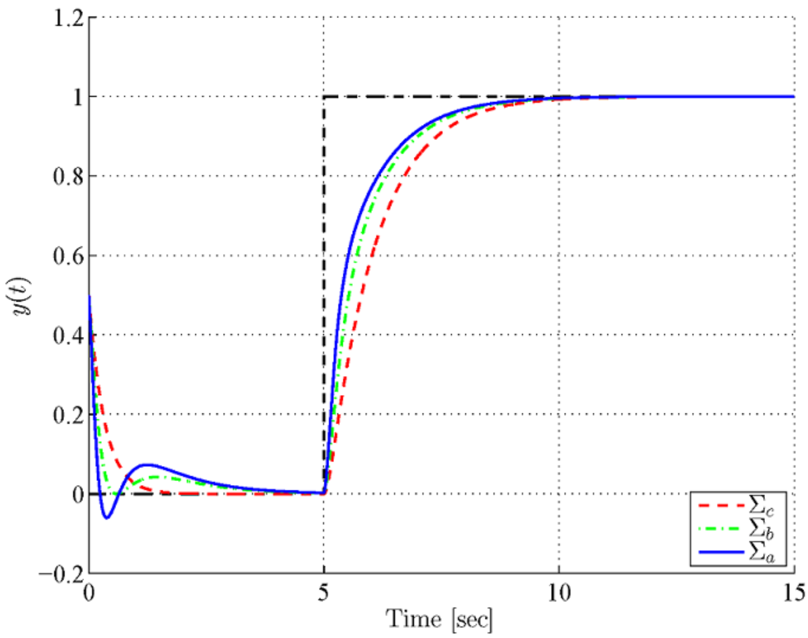
$$\lambda_{cl}\left(\begin{bmatrix} \mathbf{A}_c^a - \mathbf{B}_c^a\mathbf{K}^a & \mathbf{B}_c^a\mathbf{K}_i^a \\ \mathbf{C}_c^a & 0 \end{bmatrix}\right) = \lambda_{cl}\left(\begin{bmatrix} \mathbf{A}_c^b - \mathbf{B}_c^b\mathbf{K}^b & \mathbf{B}_c^b\mathbf{K}_i^b \\ \mathbf{C}_c^b & 0 \end{bmatrix}\right) = \lambda_{cl}\left(\begin{bmatrix} \mathbf{A}_c^c - \mathbf{B}_c^c\mathbf{K}^c & \mathbf{B}_c^c\mathbf{K}_i^c \\ \mathbf{C}_c^c & 0 \end{bmatrix}\right).$$

Let $\mathbf{x}_c^a(t_0) = \mathbf{x}_c^b(t_0) = \mathbf{x}_c^c(t_0) = \mathbf{0}$ be the initial conditions for the part of the state vectors belonging to the controllable subspace; $\mathbf{x}_{nc}^a(t_0) = \mathbf{x}_{nc}^b(t_0) = \mathbf{x}_{nc}^c(t_0) = \mathbf{x}_0 > 0$ be the initial conditions for the part of the state vectors belonging to the non controllable subspace; $\mathbf{x}_i^a(t_0) = \mathbf{x}_i^b(t_0) = \mathbf{x}_i^c(t_0) = \mathbf{0}$ be the initial conditions for the integral state vectors.

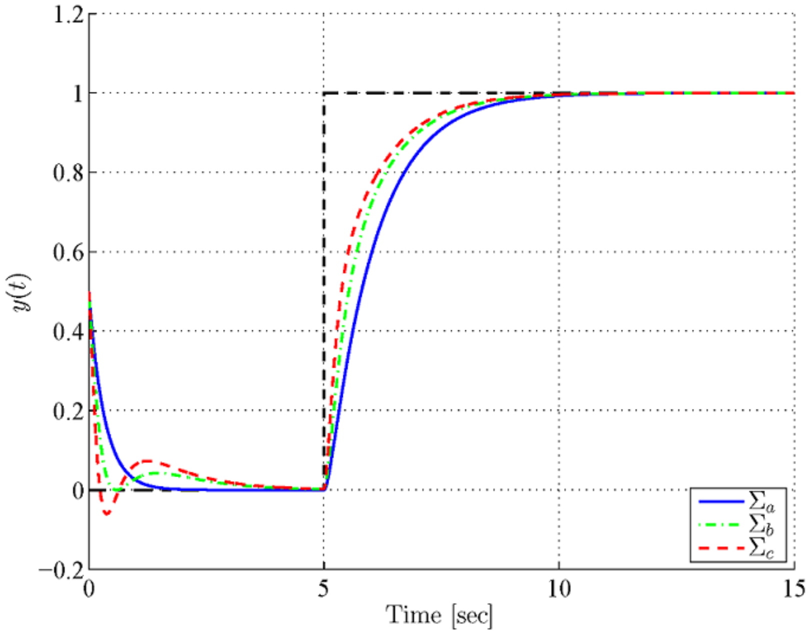
Which of the following plots represents the output response of the closed-loop systems when a reference step change is made at time $t = t_{\text{step}}$? (In the following plots $t_{\text{step}} = 5\ sec$)

Choose one answer

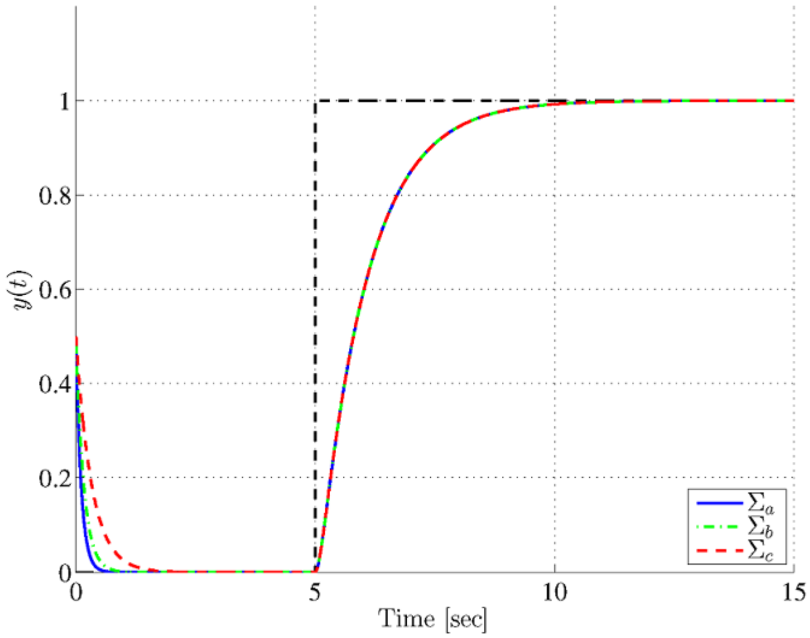
○



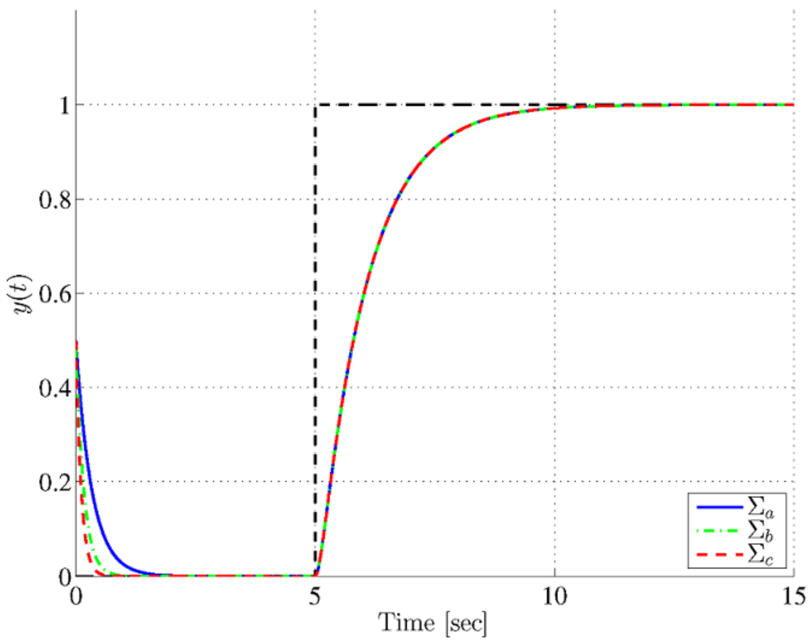
○



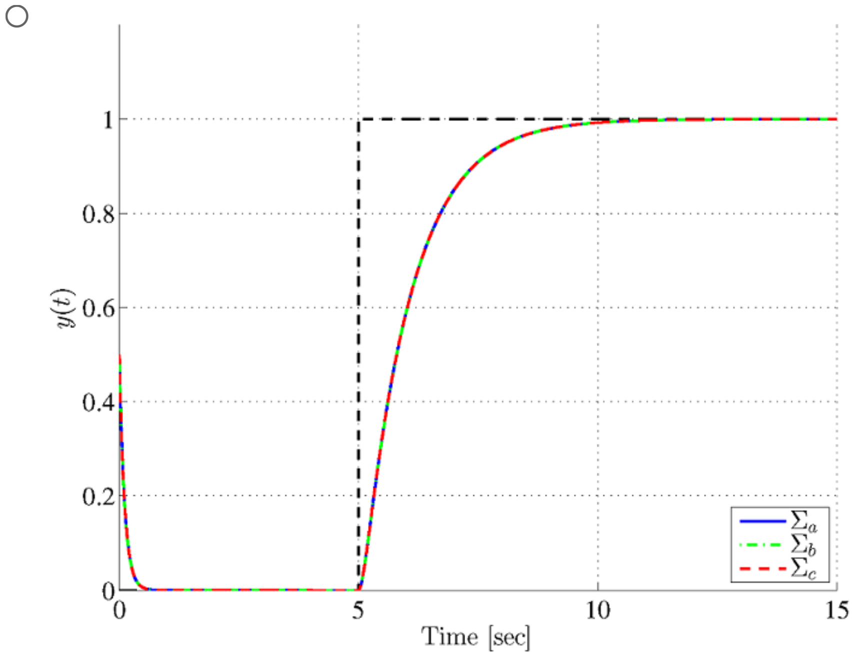
1.2
1
0.8
0.6
0.4
0.2
0
−0.2
0
5
10
15
$y(t)$
Time [sec]
Σ_a
Σ_b
Σ_c



1
0.8
0.6
0.4
0.2
0
0
5
10
15
$y(t)$
Time [sec]
Σ_a
Σ_b
Σ_c



1
0.8
0.6
0.4
0.2
0
0
5
10
15
$y(t)$
Time [sec]
Σ_a
Σ_b
Σ_c



1
0.8
0.6
0.4
0.2
0
$y(t)$
0
5
10
15
Time [sec]
Σ_a
Σ_b
Σ_c

Consider the first order continuous time LTI system

$$\dot{x}(t) = \alpha x(t) + v_1(t)$$
$$y(t) = \beta x(t) + v_2(t)$$

where α and β are real constants. The process noise $v_1(t)$ and the measurement noise $v_2(t)$ are uncorrelated white noise sources with noise intensities σ_1^2 and σ_2^2, respectively. The steady state Kalman filter

$$\dot{\hat{x}}(t) = \alpha \hat{x}(t) + l_\infty (y(t) - \hat{y}(t))$$
$$\hat{y}(t) = \beta \hat{x}(t)$$

is designed in order to reconstruct the state variable $x(t)$ based on the measurement $y(t)$. Which of the following statements is **not correct**?

Choose one answer

- The steady state Kalman gain is given by $l_\infty = \frac{1}{\beta}\left(\alpha + \sqrt{\alpha^2 + \beta^2 \frac{\sigma_1^2}{\sigma_2^2}}\right)$
- If the intensity of the process noise is zero and $\alpha > 0$, then the Kalman gain is $l_\infty = 2\alpha$, which ensures the asymptotic stability of the estimation error dynamics.
- If $\left(\sigma_1^2/\sigma_2^2\right) \gg 1$ then the steady state Kalman gain is large and the filter relies heavily on the measurement.
- If $\alpha = 0$ and the intensity of the process noise is zero, then the steady state Kalman is zero, i.e. $l_\infty = 0$.
- If $\left(\sigma_1^2/\sigma_2^2\right) \ll 1$ then the steady state Kalman gain is large and the filter relies heavily on the system model.