Final exam E19 - Questionnaire

Vis rigtige svar
Skjul rigtige svar

Side 1

Open-loop system analysis (Part I)

Spørgsmål 1

The wheel dynamics of a car travelling with linear velocity v is given by

$$\Sigma : \begin{cases} m\dot{x}_1 = -mg\mu(\lambda) \\ I\dot{x}_2 = -Bx_2 + mgR\mu(\lambda) - u \end{cases}$$

where x_1 is the linear velocity v, x_2 is the angular velocity Ω of the wheel and u is the input breaking torque. The parameters in the above equation are: *m* is the wheel mass, *I* is the wheel moment of inertia, *g* is the gravity constant, *R* is the wheel radius, *B* is the bearing friction coefficient, λ is the wheel slip and $\mu(\lambda)$ is the friction force coefficient.

The wheel slip is defined as $\lambda = (x_1 - Rx_2)/x_1$.

Let $\mathbf{x}_0 = [x_{10}, x_{20}]^{\mathrm{T}}$ be the stationary point related to the stationary input u_0. Which of the following systems is the linearized system around the point of operation?

- [x] $$\mathbf{A} = \begin{bmatrix} -g\frac{\partial\mu}{\partial\lambda}\frac{Rx_{20}}{x_{10}^2} & g\frac{\partial\mu}{\partial\lambda}\frac{R}{x_{10}} \\ \frac{mgR}{I}\frac{\partial\mu}{\partial\lambda}\frac{Rx_{20}}{x_{10}^2} & -\frac{B}{I} - \frac{mgR}{I}\frac{\partial\mu}{\partial\lambda}\frac{R}{x_{10}} \end{bmatrix} \quad \mathbf{B} = \begin{bmatrix} 0 \\ -\frac{1}{I} \end{bmatrix}$$

- [ ] $$\mathbf{A} = \begin{bmatrix} -g\frac{Rx_{20}}{x_{10}^2} & g\frac{R}{x_{10}} \\ \frac{mgR}{I}\frac{Rx_{20}}{x_{10}^2} & -\frac{B}{I} - \frac{mgR}{I}\frac{R}{x_{10}} \end{bmatrix} \quad \mathbf{B} = \begin{bmatrix} 0 \\ -\frac{1}{I} \end{bmatrix}$$

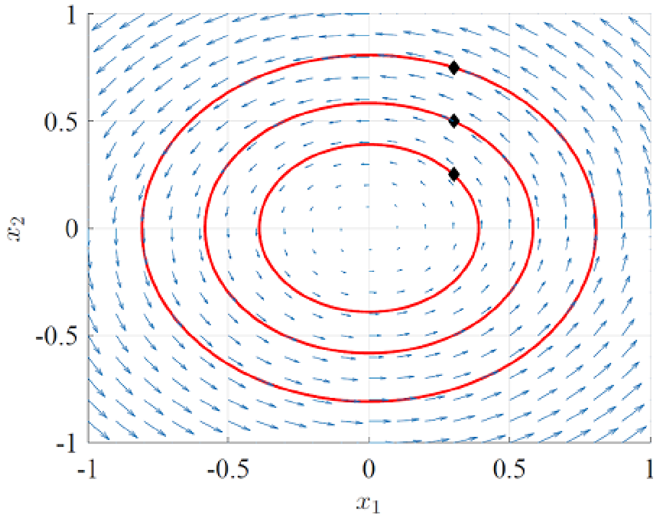
- [ ] $$\mathbf{A} = \begin{bmatrix} 0 & 1 \\ \frac{mgR}{I}\frac{Rx_{20}}{x_{10}^2} & -\frac{B}{I} - \frac{mgR}{I}\frac{R}{x_{10}} \end{bmatrix} \quad \mathbf{B} = \begin{bmatrix} 0 \\ -\frac{1}{I} \end{bmatrix}$$

- [ ] $$\mathbf{A} = \begin{bmatrix} -g\frac{\partial\mu}{\partial\lambda} & g\frac{\partial\mu}{\partial\lambda} \\ \frac{mgR}{I}\frac{\partial\mu}{\partial\lambda} & -\frac{B}{I} - \frac{mgR}{I}\frac{\partial\mu}{\partial\lambda} \end{bmatrix} \quad \mathbf{B} = \begin{bmatrix} 0 \\ -\frac{1}{I} \end{bmatrix}$$

- [ ] $$\mathbf{A} = \begin{bmatrix} -g\frac{\partial\mu}{\partial\lambda} & g\frac{\partial\mu}{\partial\lambda} \\ \frac{mgR}{I}\frac{\partial\mu}{\partial\lambda} & -\frac{B}{I} - \frac{mgR}{I}\frac{\partial\mu}{\partial\lambda} \end{bmatrix} \quad \mathbf{B} = \begin{bmatrix} 0 \\ -1 \end{bmatrix}$$

Spørgsmål 2

Consider the phase portrait shown in the following figure (in the given phase portrait each black diamond represents an initial condition $\mathbf{x} = \begin{bmatrix} x_{10} & x_{20} \end{bmatrix}^{\mathrm{T}}$ for the system; each red line is a trajectory of the system originated from the initial condition; the blue arrows represent the direction of the vector field in the neighborhood of the origin).



Which of the following linear systems describes the dynamical behavior shown in the figure when initialized at $\mathbf{x}_0 \neq 0$?

- [ ] $\dot{\mathbf{x}} = \begin{bmatrix} -\alpha & 0 \\ 0 & -\beta \end{bmatrix} \mathbf{x}, \qquad \alpha, \beta \in \mathbb{R}_+$
- [ ] $\dot{\mathbf{x}} = \begin{bmatrix} -\alpha & \gamma \\ -\gamma & -\alpha \end{bmatrix} \mathbf{x}, \qquad \alpha, \gamma \in \mathbb{R}_+$
- [ ] $\dot{\mathbf{x}} = \begin{bmatrix} -\alpha & 0 \\ 0 & 0 \end{bmatrix} \mathbf{x}, \qquad \alpha \in \mathbb{R}_+$
- [x] $\dot{\mathbf{x}} = \begin{bmatrix} 0 & \gamma \\ -\gamma & 0 \end{bmatrix} \mathbf{x}, \qquad \gamma \in \mathbb{R}_+$
- [ ] $\dot{\mathbf{x}} = \begin{bmatrix} -\alpha & 1 \\ 0 & -\alpha \end{bmatrix} \mathbf{x}, \qquad \alpha \in \mathbb{R}_+$

Spørgsmål 3

Consider the n-th order continuous time linear system

$$\Sigma : \begin{cases} \dot{\mathbf{x}} = \mathbf{A}\mathbf{x} + \mathbf{B}\mathbf{u} & \mathbf{x} \in \mathbb{R}^n,\ \mathbf{u} \in \mathbb{R}^m \\ \mathbf{y} = \mathbf{C}\mathbf{x} & \mathbf{y} \in \mathbb{R}^p \end{cases}$$

where $m > 1$. Which of the following statements is **not correct**?

- [ ] Σ is controllable if and only if the controllability gramian
$$\mathbf{W}_c = \int_0^{+\infty} e^{-\mathbf{A}t}\mathbf{B}\mathbf{B}^{\mathrm{T}}e^{-\mathbf{A}^{\mathrm{T}}t}\,\mathrm{d}t$$
has determinant different from zero.

- [ ] Σ is controllable if and only the controllability matrix
$$\mathbf{M}_c = \begin{bmatrix}\mathbf{B} & \mathbf{A}\mathbf{B} & \mathbf{A}^2\mathbf{B} & \dots & \mathbf{A}^{n-1}\mathbf{B}\end{bmatrix}$$
is full rank.

- [x] Σ is controllable if and only the controllability matrix
$$\mathbf{M}_c = \begin{bmatrix}\mathbf{B} & \mathbf{A}\mathbf{B} & \mathbf{A}^2\mathbf{B} & \dots & \mathbf{A}^{n-1}\mathbf{B}\end{bmatrix}$$
has determinant different from zero.

- [ ] Σ is controllable if and only if no left eigenvector $\mathbf{w}_i$ of $\mathbf{A}$ exists such that $\mathbf{w}_i^{\mathrm{T}}\mathbf{B} = 0$.

- [ ] Let $\mathbf{M}$ be the modal matrix associated with $\mathbf{A}$. Σ is controllable if and only if the diagonalized system
$$\dot{\mathbf{z}} = \mathbf{\Lambda}\mathbf{z} + \mathbf{\Gamma}\mathbf{u}$$
has the input matrix $\mathbf{\Gamma}$ with no zero rows; where $\mathbf{\Lambda} = \mathbf{M}^{-1}\mathbf{A}\mathbf{M}$, $\mathbf{\Gamma} = \mathbf{M}^{-1}\mathbf{B}$ and $\mathbf{z} = \mathbf{M}^{-1}\mathbf{x}$.

Spørgsmål 4

Consider the n-th order continuous time linear system

$$\Sigma : \begin{cases} \dot{\mathbf{x}} = \mathbf{A}\mathbf{x} + \mathbf{B}\mathbf{u} & \mathbf{x} \in \mathbb{R}^n,\ \mathbf{u} \in \mathbb{R}^m \\ \mathbf{y} = \mathbf{C}\mathbf{x} & \mathbf{y} \in \mathbb{R}^p \end{cases}$$

where $p > 1$. Which of the following statements is correct?

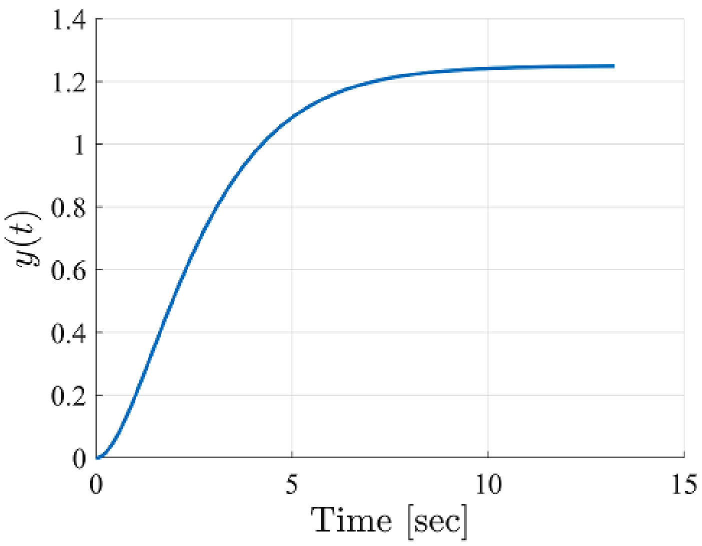
- [ ] Σ is observable if and only if the observability gramian $\mathbf{W}_o = \int_0^{+\infty} e^{\mathbf{A}^{\mathrm{T}}t}\mathbf{C}^{\mathrm{T}}\mathbf{C}e^{\mathbf{A}t}\,\mathrm{d}t$ has determinant equal to zero.
- [ ] Σ is observable if and only if the observability matrix $\mathbf{M}_o = \begin{bmatrix} \mathbf{C} \\ \mathbf{C}\mathbf{A} \\ \mathbf{C}\mathbf{A}^2 \\ \vdots \\ \mathbf{C}\mathbf{A}^{n-1} \end{bmatrix}$ is rank deficient, i.e. $\mathrm{rank}\{\mathbf{M}_o\} < n$.
- [ ] Σ is observable if and only if no left eigenvector $\mathbf{w}_i$ of $\mathbf{A}$ exists such that $\mathbf{w}_i^{\mathrm{T}}\mathbf{B} = 0$.
- [x] Σ is observable if and only if no right eigenvector $\mathbf{v}_i$ of $\mathbf{A}$ exists such that $\mathbf{C}\mathbf{v}_i = 0$.
- [ ] Let $\mathbf{M}$ be the modal matrix associated with $\mathbf{A}$. Σ is observable if and only if the diagonalized system $\begin{cases} \dot{\mathbf{z}} = \mathbf{\Lambda}\mathbf{z} + \mathbf{\Gamma}\mathbf{u} \\ \mathbf{y} = \mathbf{\Xi}\mathbf{z} \end{cases}$ has the output matrix $\mathbf{\Xi}$ with at least one zero column; where $\mathbf{\Lambda} = \mathbf{M}^{-1}\mathbf{A}\mathbf{M},\ \mathbf{\Gamma} = \mathbf{M}^{-1}\mathbf{B},\ \mathbf{\Xi} = \mathbf{C}\mathbf{M}$ and $\mathbf{z} = \mathbf{M}^{-1}\mathbf{x}$.

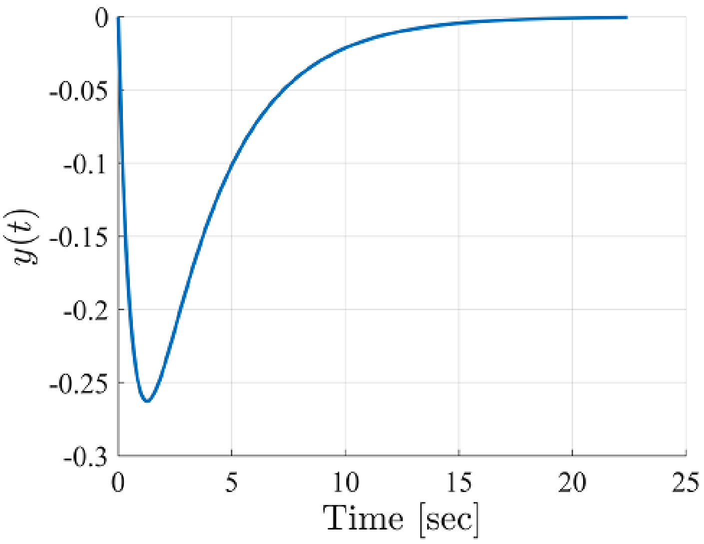
Spørgsmål 5

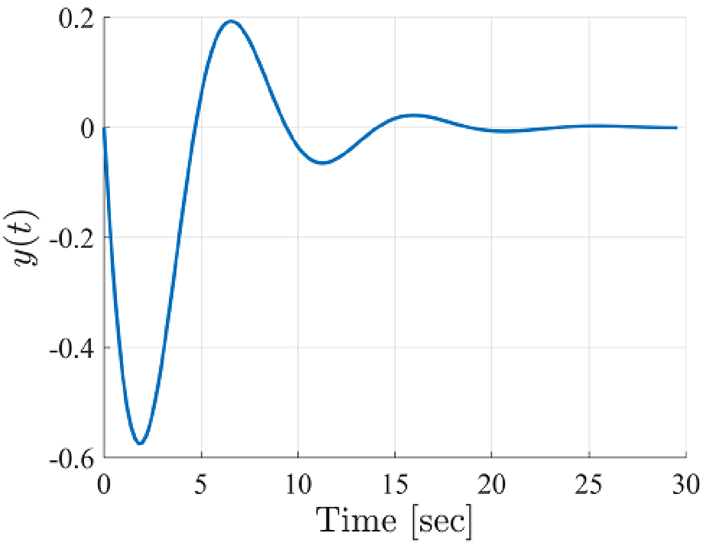
Consider the 2nd order LTI continuous time system

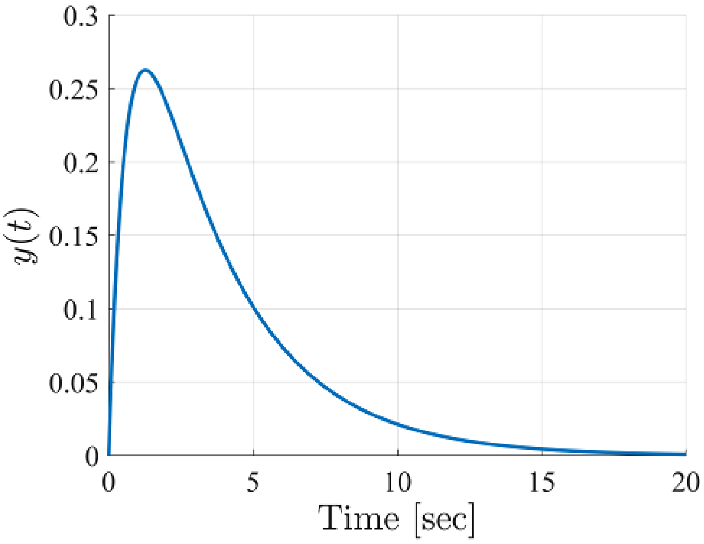
$$\dot{\mathbf{x}}(t) = \begin{bmatrix} 0 & 1 \\ -\alpha & \beta \end{bmatrix} \mathbf{x}(t) + \begin{bmatrix} 0 \\ \gamma \end{bmatrix} u(t), \quad \mathbf{x} \in \mathbb{R}^2, u \in \mathbb{R}$$
$$y(t) = \begin{bmatrix} 0 & \delta \end{bmatrix} \mathbf{x}(t), \quad y \in \mathbb{R}$$

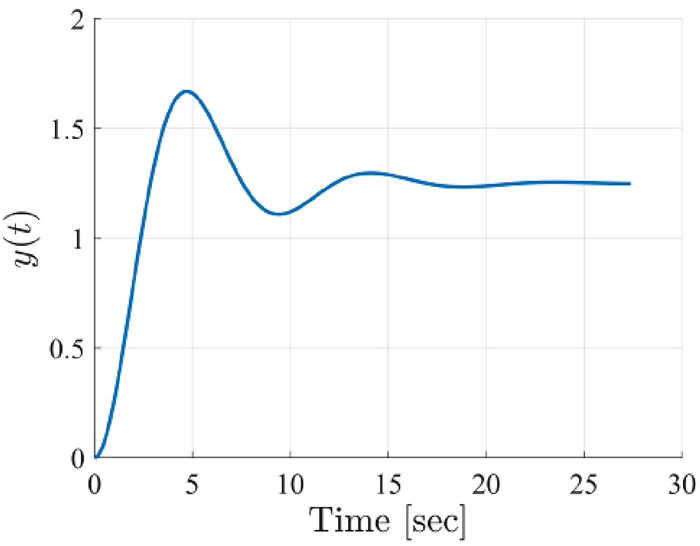
Assume that $\alpha > 0,\ \gamma > 0,\ \delta < 0$ and that $\beta < -2\sqrt{\alpha}\ \vee\ \beta > 2\sqrt{\alpha}$. Which of the following plots shows the step response associated with the system?

- [ ] 

- [x] 

☐ 
0.2
0
-0.2
-0.4
-0.6
y(t)
0
5
10
15
20
25
30
Time [sec]

☐ 
0.3
0.25
0.2
0.15
0.1
0.05
0
y(t)
0
5
10
15
20
Time [sec]

☐ 
2
1.5
1
0.5
0
y(t)
0
5
10
15
20
25
30
Time [sec]

Side 2

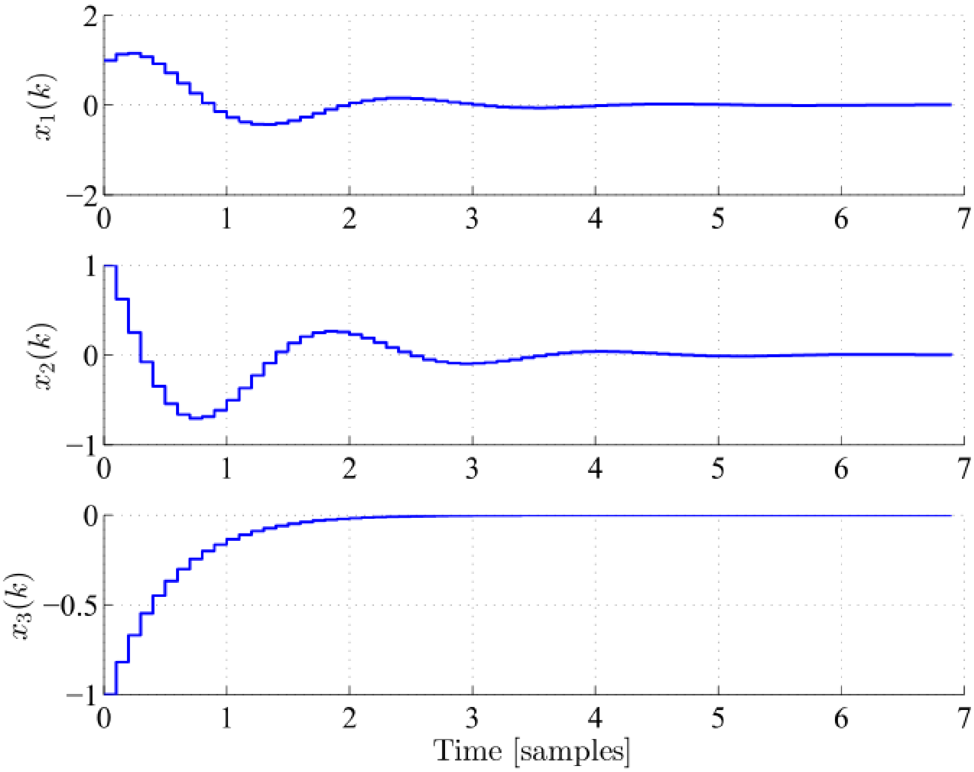
Open-loop analysis (Part II)

Spørgsmål 6

Consider a 3rd order LTI discrete time SISO system

$$\Sigma : \begin{cases} \mathbf{x}(k+1) = \begin{bmatrix} \mathbf{F}_1 & \mathbf{0} \\ \mathbf{0} & F_2 \end{bmatrix} \mathbf{x}(k) + \mathbf{G}u(k), & \mathbf{x} \in \mathbb{R}^3,\ u \in \mathbb{R} \\ y(k) = \mathbf{C}\mathbf{x}(k), & y \in \mathbb{R} \end{cases}$$

where $\mathbf{F}_1$ is a 2x2 matrix and F_2 is a scalar. The zero input response of the system to the initial condition $\mathbf{x}_0 = [x_{10}, x_{20}, x_{30}]^T$ with $x_{10} > 0,\ x_{20} > 0$, and $x_{30} < 0$ is shown in the following figure.



Which of the following statements is **not correct** ?

- ☐ The system is asymptotically stable.
- ☐ The system has one real eigenvalue whose magnitude is less than one.
- ☐ The continuous time system associated with Σ has a pair of natural modes which oscillate with frequency
$$\omega_n = \frac{|\ln(\lambda_1(\mathbf{F}_1))|}{T_s}$$
and asymptotically decay to zero ($\lambda_1(\mathbf{F}_1)$ is one of the eigenvalues of the submatrix $\mathbf{F}_1$ and T_s is the sampling time).
- ☑ The natural mode associated with the dynamics of the state variable x_3 is unstable.
- ☐ The eigenvalues of the system have magnitude less than one.

Spørgsmål 7

Consider the 4th order LTI continuous time system

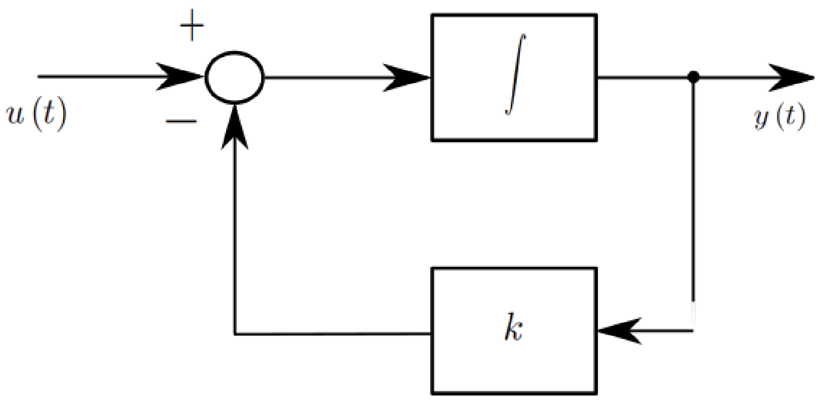
$$\dot{\mathbf{x}} = \begin{bmatrix} 0 & 1 & 0 & 0 \\ -\omega_1^2 & -2\zeta_1\omega_1 & \gamma & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & -\omega_2^2 & -2\zeta_2\omega_2 \end{bmatrix} \mathbf{x}$$

where ω_1 and ω_2 are real positive constants; ζ_1, ζ_2, and γ are real constants. Which of the following statement is **not correct**?

- ☐ If $0 < \zeta_1 < 1$ and $0 < \zeta_2 < 1$ then the system has two pairs of asymptotically stable complex eigenvalues.
- ☐ If $\zeta_1 = 0$, $\zeta_2 = 0$, and $\omega_1 \neq \omega_2$ then the system has two pairs of stable imaginary eigenvalues for any $\gamma \in \mathbb{R}$.
- ☐ If $\zeta_1 = 0$, $\zeta_2 = 0$, and $\omega_1 = \omega_2$ and $\gamma \neq 0$ then the system has one pair of unstable imaginary eigenvalues with algebraic multiplicity equal to two.
- ☐ If $\zeta_1 = \zeta_2 = 0$, $\omega_1 = \omega_2$, and $\gamma = 0$ then the system has two pairs of stable imaginary eigenvalues.
- ☑ If $-1 < \zeta_1 < 0$ and $-1 < \zeta_2 < 0$ then the system has two pairs of asymptotically stable real eigenvalues.

Spørgsmål 8

Consider the system shown in the following block diagram



where *u(t)* is a stochastic process whose autocorrelation function is given by

$$R_u(\tau) = \sigma_u^2 e^{-\beta|\tau|}$$

with σ_u^2 being the variance of the input signal, and $\beta > 0$. What is the variance σ_y^2 of the output signal *y(t)*?

- ☐ $\sigma_y^2 = \sigma_u^2$
- ☑ $\sigma_y^2 = \dfrac{1}{k(\beta + k)}\sigma_u^2$
- ☐ $\sigma_y^2 = \dfrac{\beta - k}{\beta + k}\sigma_u^2$
- ☐ $\sigma_y^2 = 0$
- ☐ $\sigma_y^2 = \dfrac{k}{\beta}\sigma_u^2$

Side 3

Closed-loop analysis and synthesis

Spørgsmål 9

Consider the 3rd order LTI asymptotically stable continuous time system

$$\dot{\mathbf{x}} = \begin{bmatrix} -\alpha & \beta & -\beta \\ \gamma & -2\alpha & 0 \\ 0 & \delta & -\varepsilon \end{bmatrix} \mathbf{x} + \begin{bmatrix} 0 \\ 0 \\ -\rho \end{bmatrix} u + \begin{bmatrix} 0 \\ \eta \\ 0 \end{bmatrix} d, \quad \mathbf{x} \in \mathbb{R}^3, u \in \mathbb{R}, d \in \mathbb{R}$$
$$y = \begin{bmatrix} 1 & 0 & 0 \end{bmatrix} \mathbf{x}, \quad y \in \mathbb{R}$$

with initial condition $\mathbf{x}(t_0) = \mathbf{x}_0$, where *u* is the control input and *d* is an unknown disturbance. The constants $\alpha,\ \beta,\ \gamma,\ \delta,\ \varepsilon,\ \rho,\ \text{and}\ \eta$ are positive.

A continuous time reduced order observer is designed to estimate the unmeasured states $x_2 \text{ and } x_3$. Assuming that $\mathbf{x} = \begin{bmatrix} x_1 & \mathbf{x}_2 \end{bmatrix}^{\mathrm{T}}$ then the dynamics of the reduced order observer is given by

$$\dot{\mathbf{z}}(t) = \mathbf{M}\mathbf{z}(t) + \mathbf{N}u(t) + \mathbf{P}y(t)$$
$$\hat{\mathbf{x}}_2(t) = \mathbf{z}(t) + \mathbf{L}y(t)$$

with $\mathbf{z}(t_0)$ such that $\hat{\mathbf{x}}_2(t_0) = \mathbf{x}_2(t_0)$. Let $\mathbf{e}(t) = \mathbf{x}_2(t) - \hat{\mathbf{x}}_2(t)$ be the estimation error. If

$$d(t) = \begin{cases} 0, & 0 \leq t < t_1 \\ d_0, & t \geq t_1 \end{cases}$$

is the disturbance acting on the system, what is the behavior of the estimation error?

- [ ] $\mathbf{e}(t) = \mathbf{0} \text{ for } 0 \leq t < t_1 \text{ and } \lim_{t \to +\infty} \mathbf{e}(t) = 0$
- [ ] $\mathbf{e}(t) = \bar{\mathbf{e}} < \infty \text{ for } 0 \leq t < t_1 \text{ and } \lim_{t \to +\infty} \mathbf{e}(t) = 0$
- [ ] $\mathbf{e}(t) = \mathbf{0} \text{ for } 0 \leq t < t_1 \text{ and } \lim_{t \to +\infty} \mathbf{e}(t) = +\infty$
- [x] $\mathbf{e}(t) = \mathbf{0} \text{ for } 0 \leq t < t_1 \text{ and } \lim_{t \to +\infty} \mathbf{e}(t) = \bar{\mathbf{e}} < \infty$
- [ ] $\mathbf{e}(t) = \bar{\mathbf{e}}_1 < \infty \text{ for } 0 \leq t < t_1 \text{ and } \lim_{t \to +\infty} \mathbf{e}(t) = \bar{\mathbf{e}}_2 < \infty$

Spørgsmål 10

Consider the 2nd order LTI-SISO system

$$\Sigma : \begin{cases} \dot{\mathbf{x}} = \begin{bmatrix} -\alpha & \beta \\ -\beta & 0 \end{bmatrix} \mathbf{x} + \begin{bmatrix} \gamma_1 \\ \gamma_2 \end{bmatrix} u + \begin{bmatrix} \delta \\ 0 \end{bmatrix} v, \quad \mathbf{x} \in \mathbb{R}^2, u \in \mathbb{R}, v \in \mathbb{R} \\ y = \begin{bmatrix} 1 & 0 \end{bmatrix} \mathbf{x}, \quad y \in \mathbb{R} \end{cases}$$

where $\alpha, \beta, \gamma_1, \gamma_2, \text{and } \delta$ are real and positive constants; *u* is the control input and *v* is a deterministic disturbance. A full state feedback controller with integral action

$$\mathbf{K} = \begin{bmatrix} \mathbf{K}_p & \mathbf{K}_i \end{bmatrix}$$

is designed for the following augmented system

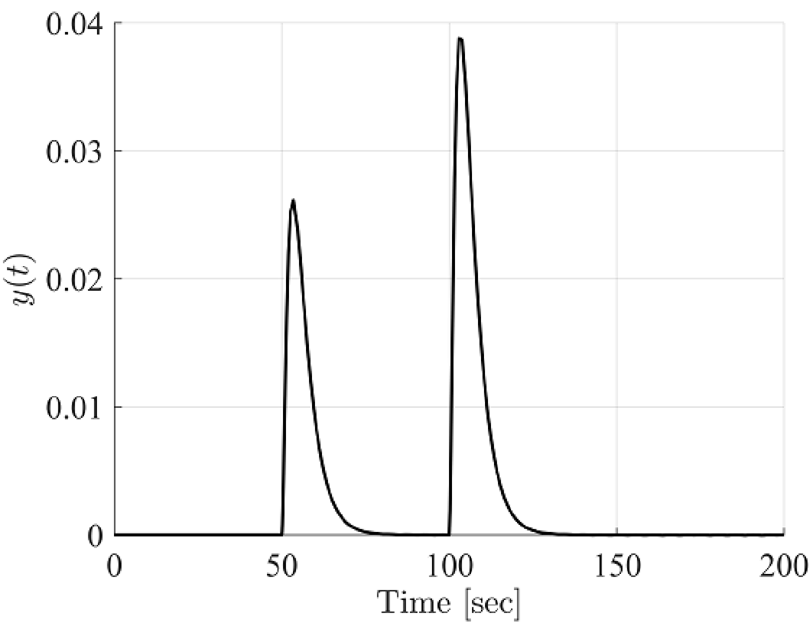
$$\dot{\mathbf{x}}_a = \begin{bmatrix} -\alpha & \beta & 0 & 0 \\ -\beta & 0 & 0 & 0 \\ -1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \end{bmatrix} \mathbf{x}_a + \begin{bmatrix} \gamma_1 \\ \gamma_2 \\ 0 \\ 0 \end{bmatrix} u + \begin{bmatrix} \delta \\ 0 \\ 0 \\ 0 \end{bmatrix} v + \begin{bmatrix} 0 \\ 0 \\ 1 \\ 0 \end{bmatrix} r, \quad \mathbf{x}_a = \begin{bmatrix} \mathbf{x} \\ \mathbf{x}_i \end{bmatrix} \in \mathbb{R}^4, u \in \mathbb{R}, v \in \mathbb{R}, r \in \mathbb{R}$$

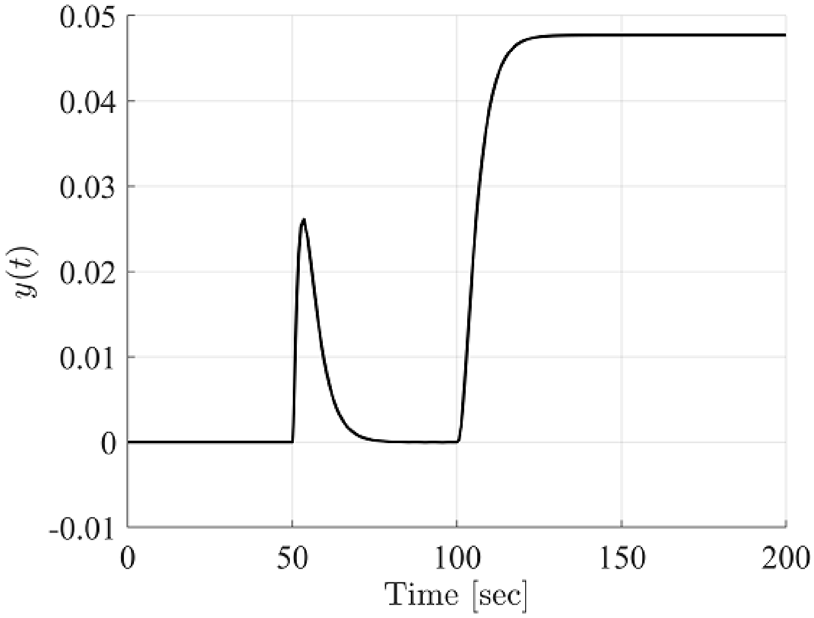
$$y = \begin{bmatrix} 1 & 0 & 0 & 0 \end{bmatrix} \mathbf{x}_a, \quad y \in \mathbb{R}$$

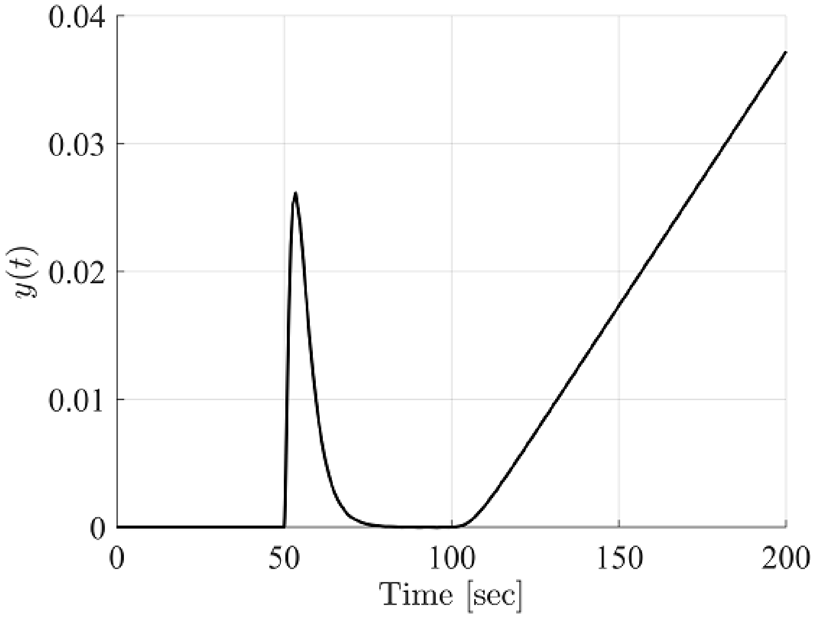
where *r* is reference set-point for the output *y*. Assume that the deterministic disturbance is given by

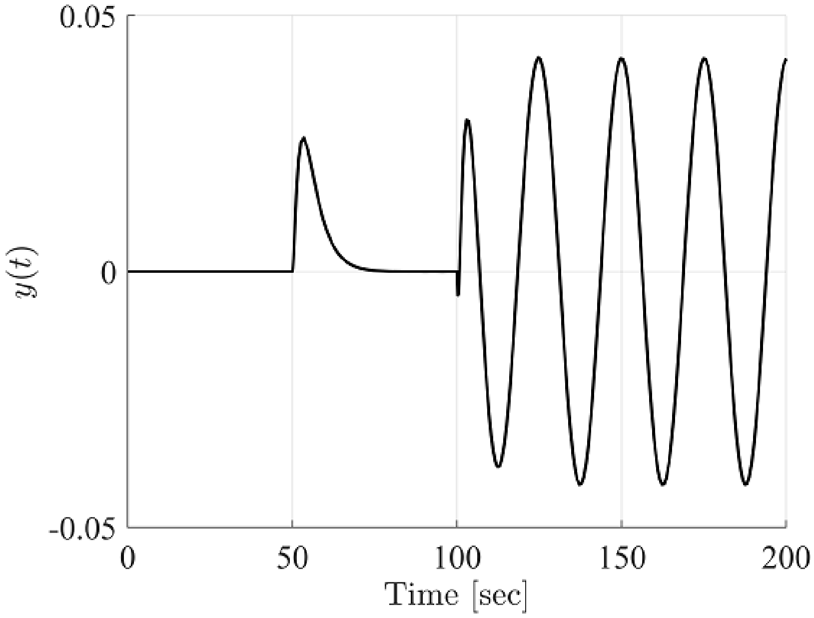
$$v(t) = \begin{cases} v_0 (t - t_0), & t_0 \le t < t_1 \\ v_0 (t - t_0) + v_1 (t - t_1)^2, & t \ge t_1 \end{cases}$$

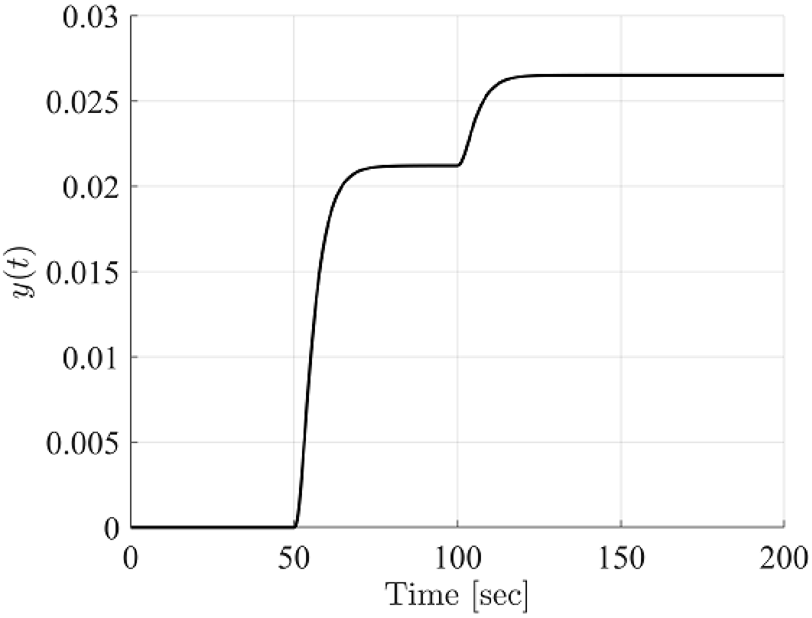
Which of the following plots shows the closed-loop output response when the system is subject to the disturbance *v(t)*?

☐ 

- [x] 
0.05
0.04
0.03
0.02
0.01
0
-0.01
$y(t)$
0
50
100
150
200
Time [sec]

- [ ] 
0.04
0.03
0.02
0.01
0
$y(t)$
0
50
100
150
200
Time [sec]

- [ ] 
0.05
0
-0.05
$y(t)$
0
50
100
150
200
Time [sec]

☐ 
0.03
0.025
0.02
0.015
0.01
0.005
0
$y(t)$
0
50
100
150
200
Time [sec]

Spørgsmål 11

Consider the three LTI-SISO systems in controllable subspace decomposition form

$$\Sigma^a : \begin{cases} \dot{\mathbf{x}} = \begin{bmatrix} \mathbf{A}_c^a & \mathbf{A}_{12}^a \\ \mathbf{0} & \mathbf{A}_{nc}^a \end{bmatrix} \mathbf{x} + \begin{bmatrix} \mathbf{B}_c^a \\ 0 \end{bmatrix} u \\ y = \begin{bmatrix} \mathbf{C}_c^a & \mathbf{C}_{nc}^a \end{bmatrix} \mathbf{x} \end{cases}$$

$$\Sigma^b : \begin{cases} \dot{\mathbf{x}} = \begin{bmatrix} \mathbf{A}_c^b & \mathbf{A}_{12}^b \\ \mathbf{0} & \mathbf{A}_{nc}^b \end{bmatrix} \mathbf{x} + \begin{bmatrix} \mathbf{B}_c^b \\ 0 \end{bmatrix} u \\ y = \begin{bmatrix} \mathbf{C}_c^b & \mathbf{C}_{nc}^b \end{bmatrix} \mathbf{x} \end{cases}$$

$$\Sigma^c : \begin{cases} \dot{\mathbf{x}} = \begin{bmatrix} \mathbf{A}_c^c & \mathbf{A}_{12}^c \\ \mathbf{0} & \mathbf{A}_{nc}^c \end{bmatrix} \mathbf{x} + \begin{bmatrix} \mathbf{B}_c^c \\ 0 \end{bmatrix} u \\ y = \begin{bmatrix} \mathbf{C}_c^c & \mathbf{C}_{nc}^c \end{bmatrix} \mathbf{x} \end{cases}$$

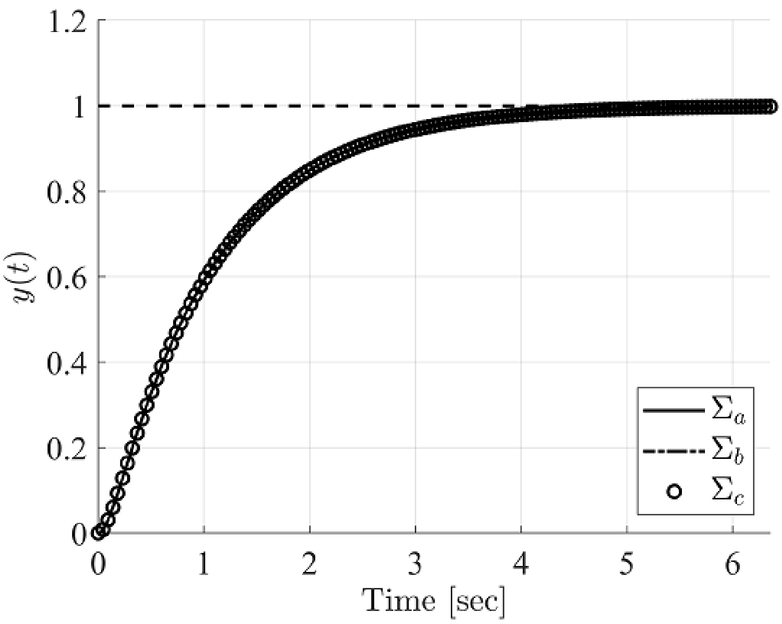
The open loop eigenvalues of the non controllable subsystems are real and they satisfy the following inequality:

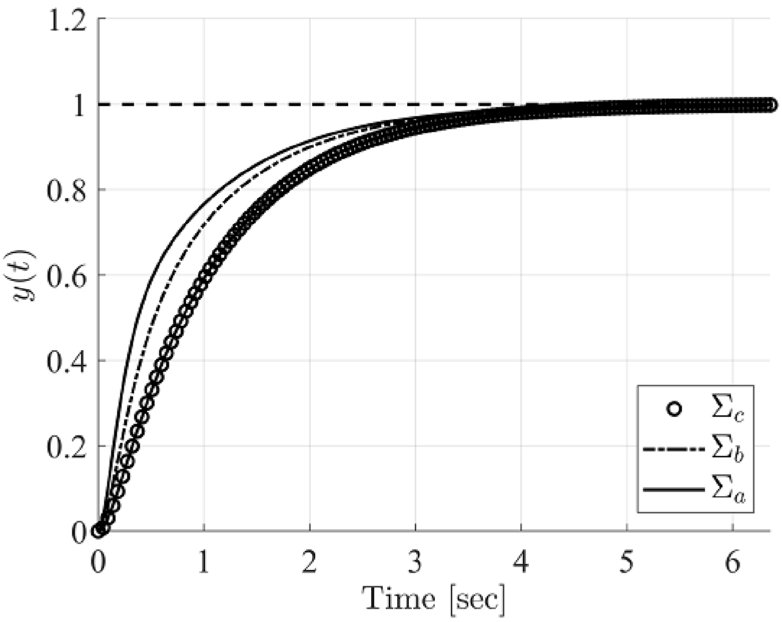
$$\lambda_{ol}\left(\mathbf{A}_{nc}^a\right) < \lambda_{ol}\left(\mathbf{A}_{nc}^b\right) < \lambda_{ol}\left(\mathbf{A}_{nc}^c\right) < 0.$$

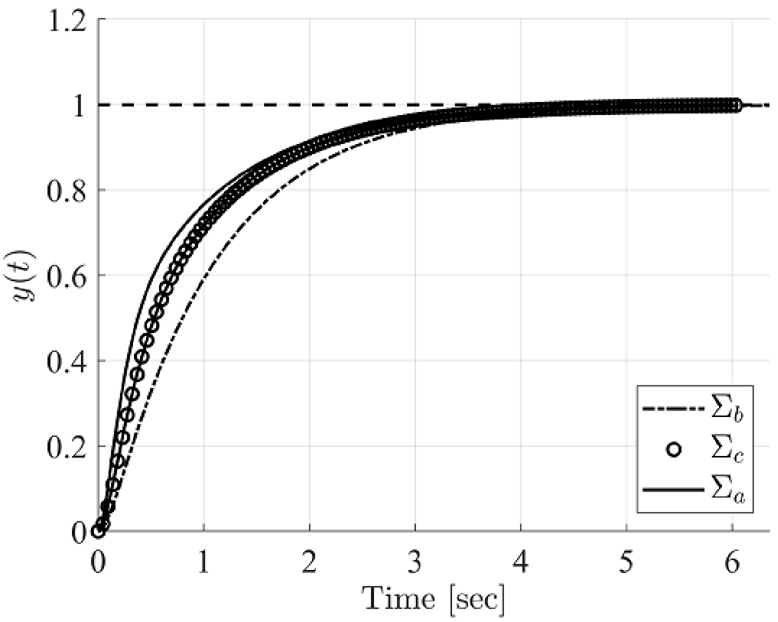
A full state feedback controller with integral action is designed for each of the given systems such that the closed loop eigenvalues of the controllable subsystems are real, negative, and satisfy the following relation:

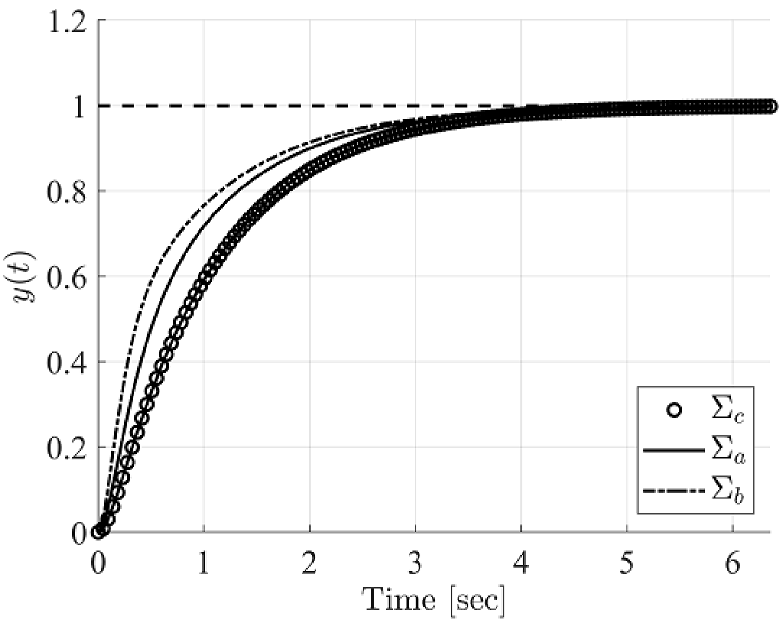
$$\lambda_{cl}\left(\begin{bmatrix} \mathbf{A}_c^a - \mathbf{B}_c^a\mathbf{K}^a & \mathbf{B}_c^a\mathbf{K}_i^a \\ \mathbf{C}_c^a & 0 \end{bmatrix}\right) = \lambda_{cl}\left(\begin{bmatrix} \mathbf{A}_c^b - \mathbf{B}_c^b\mathbf{K}^b & \mathbf{B}_c^b\mathbf{K}_i^b \\ \mathbf{C}_c^b & 0 \end{bmatrix}\right) = \lambda_{cl}\left(\begin{bmatrix} \mathbf{A}_c^c - \mathbf{B}_c^c\mathbf{K}^c & \mathbf{B}_c^c\mathbf{K}_i^c \\ \mathbf{C}_c^c & 0 \end{bmatrix}\right).$$

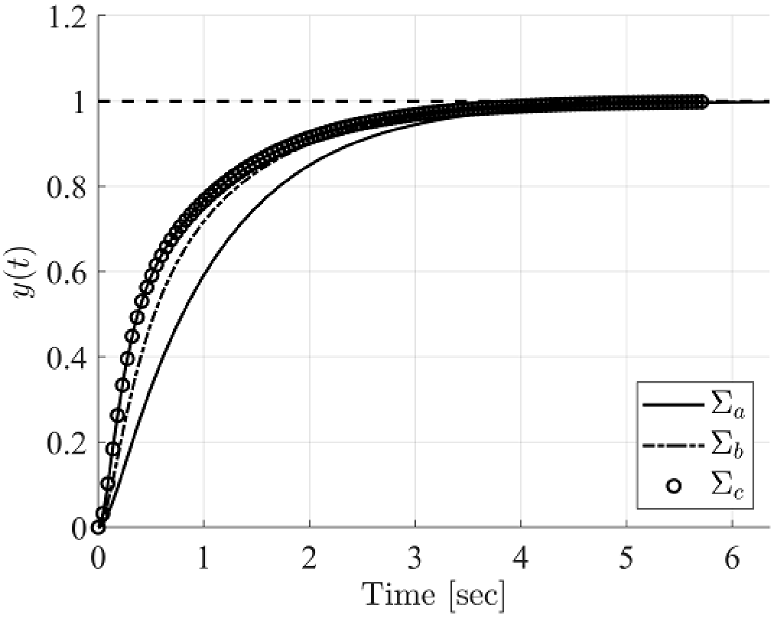
Which of the following plots represents the unit step responses of the closed-loop systems?

- [x] 

☐ 
1.2
1
0.8
0.6
0.4
0.2
0
$y(t)$
0 1 2 3 4 5 6
Time [sec]
Σ_c
Σ_b
Σ_a

☐ 
1.2
1
0.8
0.6
0.4
0.2
0
$y(t)$
0 1 2 3 4 5 6
Time [sec]
Σ_b
Σ_c
Σ_a

☐ 
1.2
1
0.8
0.6
0.4
0.2
0
$y(t)$
0 1 2 3 4 5 6
Time [sec]
Σ_c
Σ_a
Σ_b



1.2
1
0.8
0.6
0.4
0.2
0
$y(t)$
0
1
2
3
4
5
6
Time [sec]
Σ_a
Σ_b
Σ_c

Spørgsmål 12

Consider the n-th order LTI continuous time system

$$\begin{aligned}\dot{\mathbf{x}}(t) &= \mathbf{A}\mathbf{x}(t) + \mathbf{B}\mathbf{u}(t) + \mathbf{B}_n\mathbf{n}_1(t), \quad \mathbf{x} \in \mathbb{R}^n,\ \mathbf{u} \in \mathbb{R}^m,\ \mathbf{n}_1 \in \mathbb{R}^q \\ \mathbf{y}(t) &= \mathbf{C}\mathbf{x}(t) + \mathbf{n}_2(t), \quad \mathbf{y} \in \mathbb{R}^p,\ \mathbf{n}_2 \in \mathbb{R}^p\end{aligned}$$

where $\mathbf{n}_1$ is the process noise characterized as white noise with zero mean and noise intensity matrix $\mathbf{\Sigma}_1$; $\mathbf{n}_2$ is the measurement noise characterized as white noise with zero mean and noise intensity matrix $\mathbf{\Sigma}_2$. The two noise sources are uncorrelated, that is

$$E\left\{\mathbf{n}_1(t)\,\mathbf{n}_2^{\mathrm{T}}(t)\right\} = 0$$

The steady state Linear Quadratic Gaussian (LQG) regulator

$$\mathbf{u}(t) = -\mathbf{K}_\infty \hat{\mathbf{x}}(t),$$

where $\hat{\mathbf{x}}$ is the state estimate provided by a continuous time Kalman filter, is the *optimal linear solution* associated with the minimization of the performance index

$$J(\mathbf{u}) = E\left\{\mathbf{x}^{\mathrm{T}}(t)\,\mathbf{R}_1\mathbf{x}(\mathbf{t}) + \mathbf{u}^{\mathrm{T}}(t)\,\mathbf{R}_2\mathbf{u}(t)\right\}$$

where $\mathbf{R}_1$ and $\mathbf{R}_2$ are constant weighting matrices.Which of the following statements is **not correct**?

- [ ] The steady state Linear Quadratic regulator and the Kalman filter, which constitute the LQG regulator, can be designed independently of each other with guaranteed overall optimality and asymptotic stability of the closed-loop system.

- [ ] If both $\mathbf{n}_1(t)$ and $\mathbf{n}_2(t)$ are Gaussian distributed white noise processes, and the initial condition $\mathbf{x}_0 = \mathbf{x}(t_0)$ is also Gaussian distributed then the optimal linear solution is the optimal solution without qualification.

- [x] If the control input $\mathbf{u}(t)$ is not weighted at all then the minimum of the performance index is zero, that is $\lim_{\mathbf{R}_2 \to 0} J(\mathbf{u}) = 0$

- [ ] The position of the closed-loop poles is strongly influenced by the choice of the weighting matrix $\mathbf{R}_2$ and by the noise intensity matrix $\mathbf{\Sigma}_2$ of the measurement noise.

- [ ] Although there is no measurement noise affecting the output of the system, the minimum of the performance index is still larger than zero, that is

  $$\lim_{\mathbf{\Sigma}_2 \to 0} J(\mathbf{u}) \geq tr\left(\mathbf{P}_\infty \mathbf{\Sigma}_1\right)$$

  where $\mathbf{P}_\infty$ is the solution of the algebraic Riccati equation associated with the Linear Quadratic regulation problem.