34745 E22 Multiple Choice Questionnaire

Der anvendes en scoringsalgoritme, som er baseret på "One best answer"

Dette betyder følgende:
Der er altid netop ét svar som er mere rigtigt end de andre
Studerende kan kun vælge ét svar per spørgsmål
Hvert rigtigt svar giver 1 point
Hvert forkert svar giver 0 point (der benyttes IKKE negative point)

The following approach to scoring responses is implemented and is based on "One best answer"
There is always only one correct answer – a response that is more correct than the rest
Students are only able to select one answer per question
Every correct answer corresponds to 1 point
Every incorrect answer corresponds to 0 points (incorrect answers do not result in subtraction of points)

The Keynesian model of economic growth is used to describe the dynamics of the expenditure and revenue part of the economy of a nation, and it utilizes the following variables:

- Y is the gross national product,
- G is the government expenditure,
- C is the consumption expenditure,
- I is the investment expenditure.

In dynamic equilibrium at time k the gross national product Y equals the total expenditure E, i.e.

$$Y(k) = E(k)$$

where $E(k) = C(k) + I(k) + G(k)$. Let us assume that the consumption expenditure at time k is given by a fraction of the gross national product at the previous time $k-1$, i.e.

$$C(k) = \alpha Y(k-1)$$

where $0 < \alpha < 1$ is the multiplier factor. Further, let us assume that the investment expenditure at time k is proportional to the rate of change of the gross national product, i.e.

$$I(k) = \beta[Y(k-1) - Y(k-2)]$$

where $\beta > 0$.

Let $\mathbf{x} = [C, I]^T$ be the state vector, $u = G$ the input, and $y = Y$ the output. Which of the following discrete time models describe the Keynesian model of economic growth?

Choose one answer

- $$\Sigma : \begin{cases} \mathbf{x}(k+1) = \begin{bmatrix} \alpha & \alpha \\ \alpha - 1 & \beta \end{bmatrix} \mathbf{x}(k) + \begin{bmatrix} \alpha \\ \beta \end{bmatrix} u(k) \\ y(k) = [1 \quad 0] \mathbf{x}(k) + \beta u(k) \end{cases}$$

- $$\Sigma : \begin{cases} \mathbf{x}(k+1) = \begin{bmatrix} \alpha & \alpha \\ \beta & \beta \end{bmatrix} \mathbf{x}(k) + \begin{bmatrix} \alpha \\ \beta \end{bmatrix} u(k) \\ y(k) = [1 \quad 1] \mathbf{x}(k) + u(k) \end{cases}$$

- $$\Sigma : \begin{cases} \mathbf{x}(k+1) = \begin{bmatrix} \alpha & \alpha \\ \frac{\beta}{\alpha}(\alpha - 1) & \beta \end{bmatrix} \mathbf{x}(k) + \begin{bmatrix} \alpha \\ \beta \end{bmatrix} u(k) \\ y(k) = [1 \quad 1] \mathbf{x}(k) \end{cases}$$

○ $$\Sigma : \begin{cases} \mathbf{x}(k+1) = \begin{bmatrix} \alpha & \alpha \\ \frac{\beta}{\alpha}(\alpha-1) & \beta \end{bmatrix} \mathbf{x}(k) + \begin{bmatrix} \alpha \\ \beta \end{bmatrix} u(k) \\ y(k) = \begin{bmatrix} 1 & 1 \end{bmatrix} \mathbf{x}(k) + u(k) \end{cases}$$

○ $$\Sigma : \begin{cases} \mathbf{x}(k+1) = \begin{bmatrix} \alpha & \alpha \\ 0 & \beta \end{bmatrix} \mathbf{x}(k) + \begin{bmatrix} \alpha \\ \beta \end{bmatrix} u(k) \\ y(k) = \begin{bmatrix} 1 & 1 \end{bmatrix} \mathbf{x}(k) + u(k) \end{cases}$$

Consider the 2nd order nonlinear system

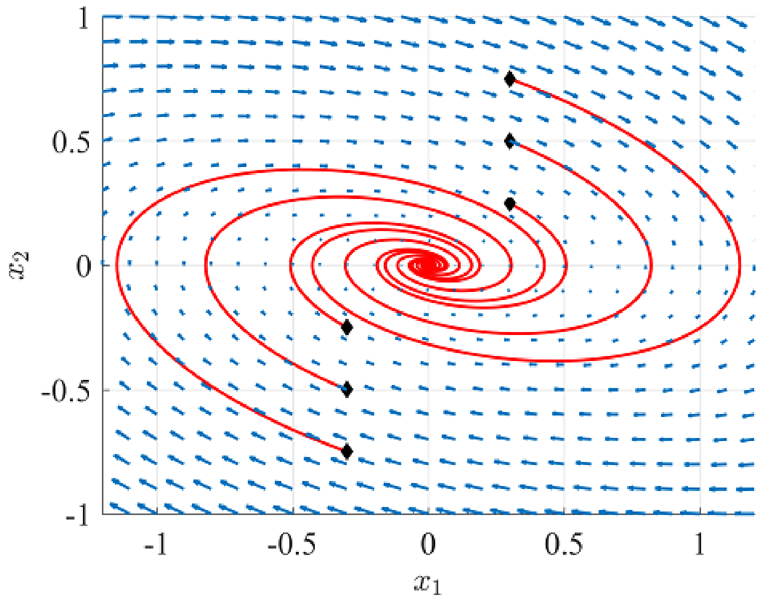
$$\Sigma : \begin{cases} \dot{x}_1 = \dfrac{\alpha x_2^2}{1 + k x_2^2} + \alpha_0 - \gamma x_1 \\ \dot{x}_2 = \beta x_1 - \delta x_2 \end{cases}$$

where the parameters $\alpha_0, \alpha, \beta, \gamma, \delta$ are all real and positive. Let $\mathbf{x}^e = [x_1^e, x_2^e]^T \neq [0, 0]^T$ be an equilibrium point of Σ. What is the linear system obtained by linearizing Σ around the point of equilibrium $\mathbf{x}^e$?

Choose one answer

- $\dot{\Delta \mathbf{x}} = \begin{bmatrix} -\gamma & 2\alpha \\ \beta & -\delta \end{bmatrix} \Delta \mathbf{x}$
- $\dot{\Delta \mathbf{x}} = \begin{bmatrix} -\gamma & 0 \\ \beta & -\delta \end{bmatrix} \Delta \mathbf{x}$
- $\dot{\Delta \mathbf{x}} = \begin{bmatrix} -\gamma & \frac{2\alpha x_2^e (1 + k(x_2^e)^2)}{(1 + k(x_2^e)^2)^2} \\ \beta & -\delta \end{bmatrix} \Delta \mathbf{x}$
- $\dot{\Delta \mathbf{x}} = \begin{bmatrix} -\gamma & \frac{2\alpha x_2^e}{(1 + k(x_2^e)^2)^2} \\ \beta & -\delta \end{bmatrix} \Delta \mathbf{x}$
- $\dot{\Delta \mathbf{x}} = \begin{bmatrix} -\gamma & 0 \\ 0 & -\delta \end{bmatrix} \Delta \mathbf{x}$

Consider the phase portrait shown in the following figure, where each black diamond is an initial condition of the system $\mathbf{x}(0) = [x_{10}, x_{20}]^T$, each red line is a trajectory of the system originating from the initial condition, and the blue arrows show the direction of the vector field describing the system.



Which of the following statements is correct?

Choose one answer

- The equilibrium point is a centre.
- The equilibrium point is a saddle point.
- The equilibrium point is a stable focus.
- The equilibrium point is an unstable focus.
- The equilibrium point is a stable node.

Consider the 3rd order LTI system

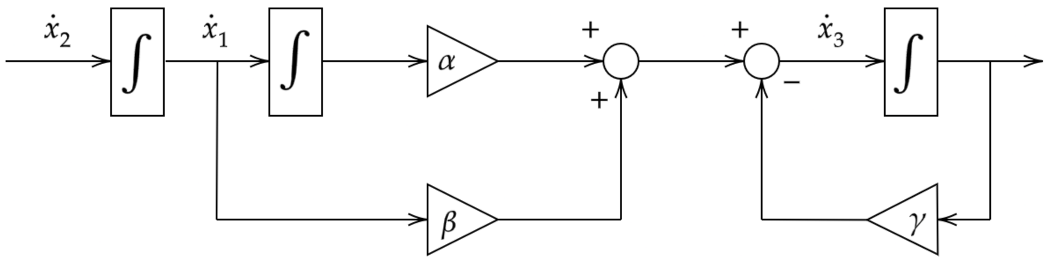
$$\dot{\mathbf{x}} = \mathbf{A}\mathbf{x} = \begin{bmatrix} 0 & 0 & 0 \\ \alpha & -\beta & 0 \\ -\alpha & \beta & 0 \end{bmatrix} \mathbf{x}, \quad \mathbf{x} \in \mathbb{R}^3$$

where $\alpha > 0$ and $\beta > 0$. $\lambda(\mathbf{A})$ is the set of all eigenvalues of the system dynamical matrix $\mathbf{A}$ and λ_i denotes a specific eigenvalue in $\lambda(\mathbf{A})$, which of the following statements is correct?

Choose one answer

- The system is unstable because the eigenvalue $\lambda = 0$ has geometric multiplicity $m_g = 1$.
- The system is asymptotically stable because $\mathrm{Re}\{\lambda_i\} < 0, \forall \lambda_i \in \lambda(\mathbf{A})$
- The system is unstable because the eigenvalue $\lambda = 0$ has algebraic multiplicity $m_a = 2$.
- The system is unstable because $\exists \lambda_i \in \lambda(\mathbf{A})$ such that $\mathrm{Re}\{\lambda_i\} > 0$.
- The system is marginally stable because the eigenvalue $\lambda = 0$ has geometric multiplicity equal to the algebraic multiplicity, i.e. $m_g = m_a$.

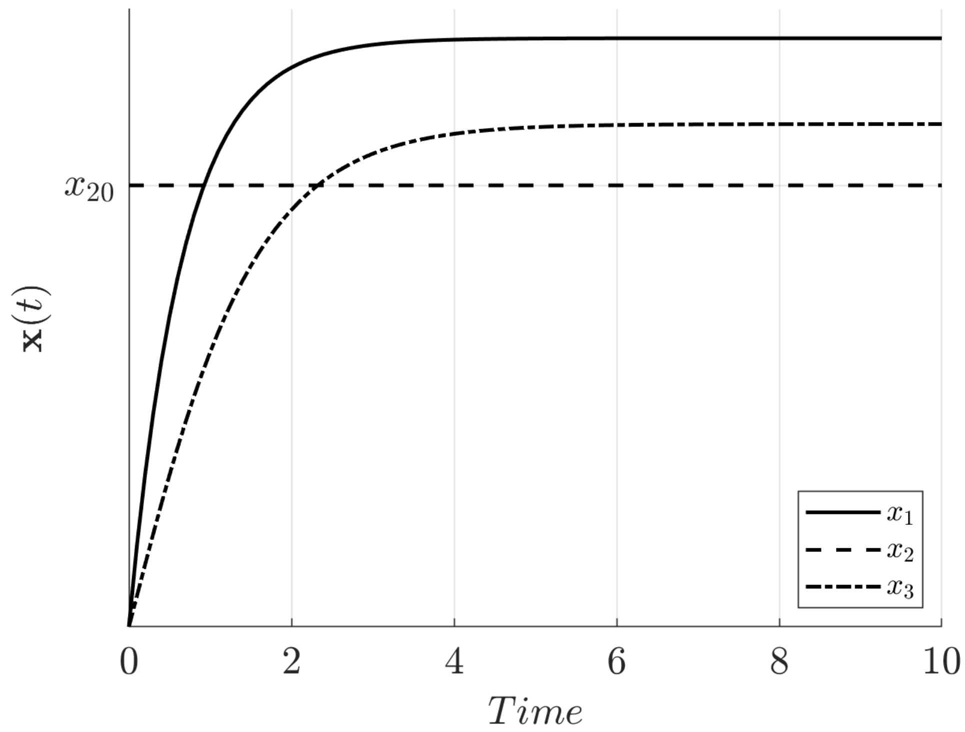
Consider the 3rd order system shown in the block diagram below



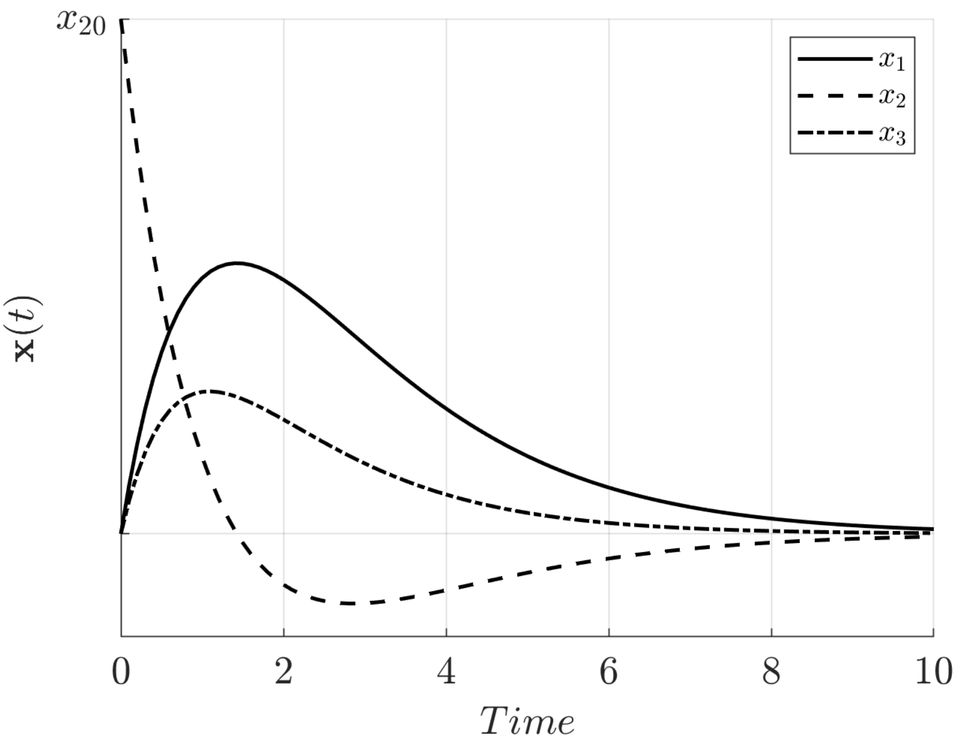
Which of the following zero-input responses is correct when the system is initialized with $\mathbf{x}_0 = [0, x_{20}, 0]^T$?

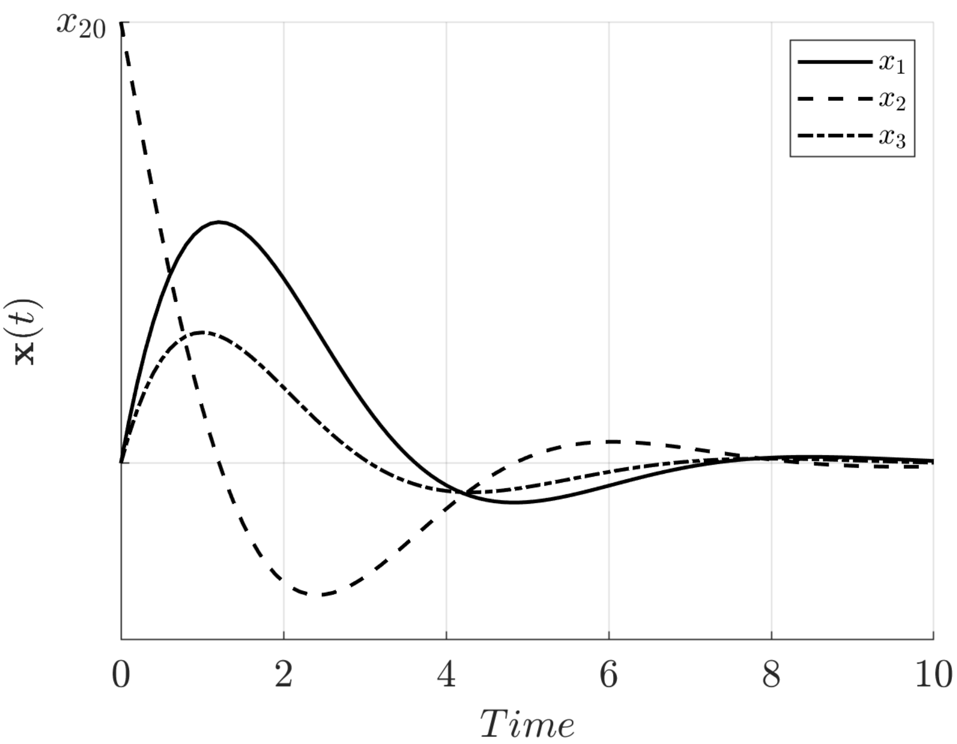
Choose one answer

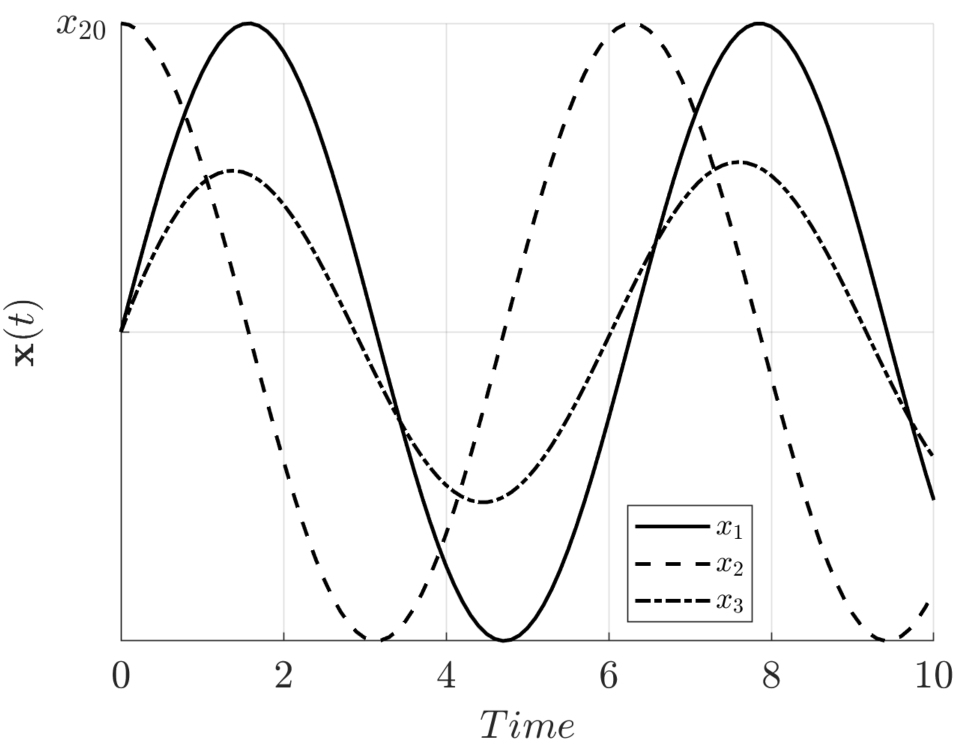
○



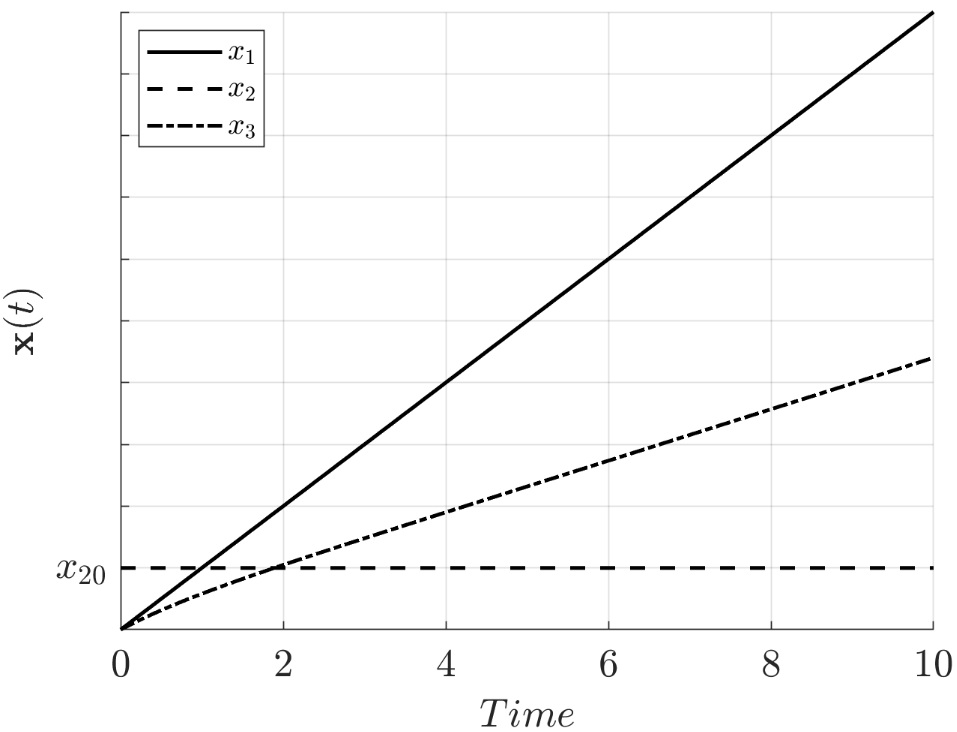
○


x_{20}
$\mathbf{x}(t)$
x_1
x_2
x_3
0
2
4
6
8
10
$Time$


x_{20}
$\mathbf{x}(t)$
x_1
x_2
x_3
0
2
4
6
8
10
$Time$



x_{20}
$\mathbf{x}(t)$
x_1
x_2
x_3
0
2
4
6
8
10
$Time$



x_1
x_2
x_3
$\mathbf{x}(t)$
x_{20}
0
2
4
6
8
10
$Time$

Consider a 3rd order LTI discrete time SISO system

$$\mathbf{x}(k+1) = \begin{bmatrix} \alpha & \beta & 0 \\ -\beta & \alpha & 1 \\ 0 & 0 & -\gamma \end{bmatrix} \mathbf{x}(k) + \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix} u(k), \quad \mathbf{x} \in \mathbb{R}^3,\ u \in \mathbb{R}$$

$$\mathbf{y}(k) = \begin{bmatrix} 1 & 0 & 0 \end{bmatrix} \mathbf{x}(k), \quad y \in \mathbb{R}$$

where α, β, and γ are real and positive coefficients such that

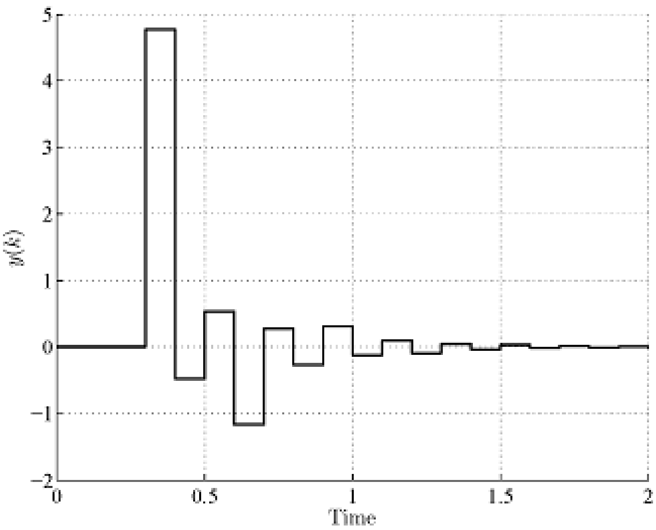
$$|\alpha| < 1 \wedge \beta = \sqrt{1-\alpha^2} \wedge |\gamma| < 1$$

The sampling time is $T_s = 0.1$ s. Which of the following plots represents the unit pulse response of the system when a unit pulse

$$u(k) = \begin{cases} 1 & \text{for } k \in [0,1] \subset \mathbb{Z} \\ 0 & \text{for } k > 1 \end{cases}$$

is sent through the input channel?

Choose one answer

○ 

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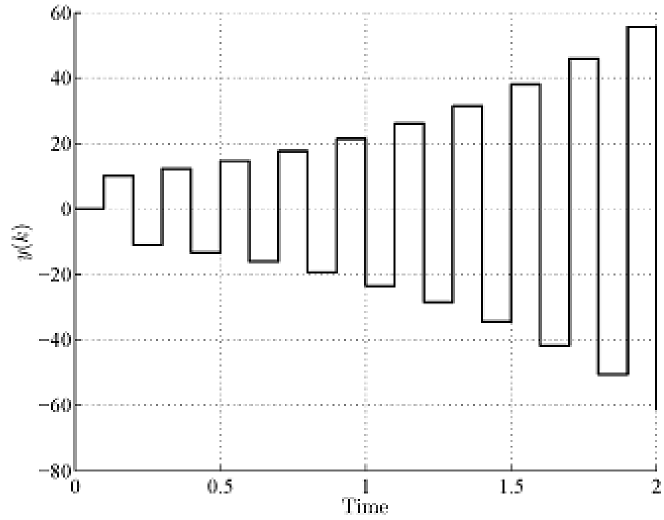
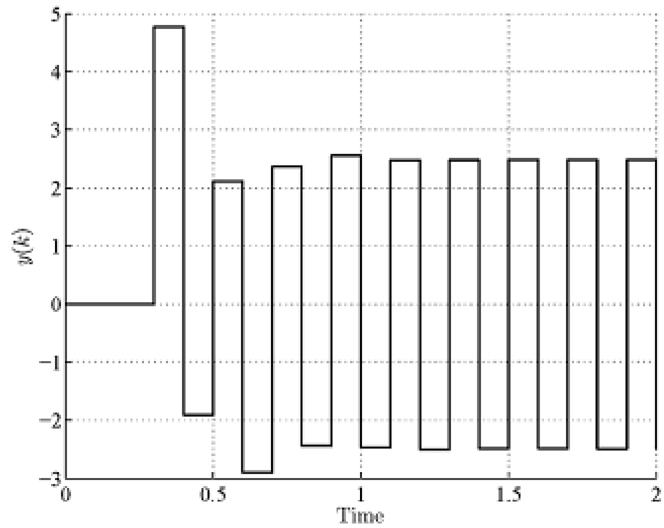
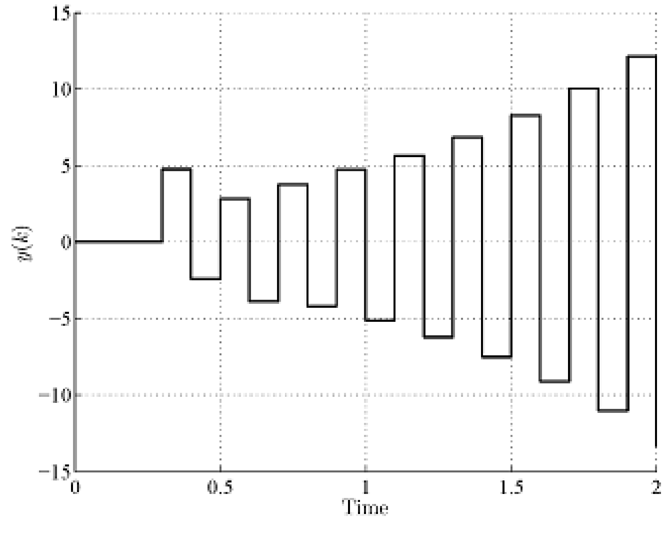
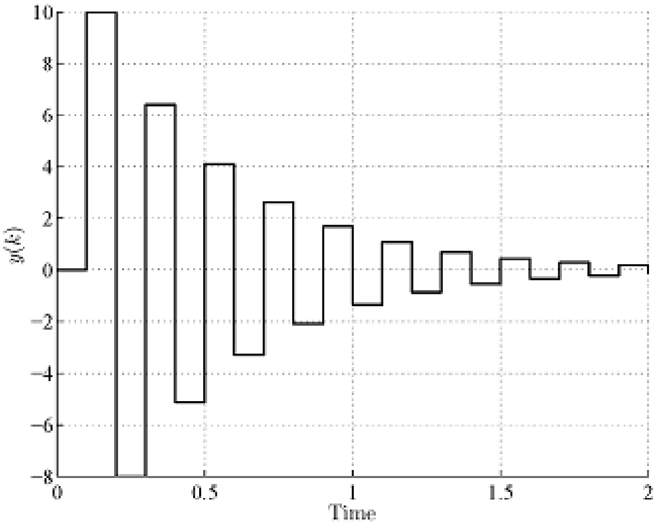

y(k)
Time


y(k)
Time


y(k)
Time



10
8
6
4
2
0
−2
−4
−6
−8
y(k)
0
0.5
1
1.5
2
Time

Consider the nth order LTI system

$$\Sigma_x : \begin{cases} \dot{\mathbf{x}} = \mathbf{A}\mathbf{x} + \mathbf{B}u + \mathbf{B}_d d, & \mathbf{x} \in \mathbb{R}^n, u \in \mathbb{R}, d \in \mathbb{R} \\ y = \mathbf{C}\mathbf{x}, & y \in \mathbb{R} \end{cases}$$

The observability matrix satisfies the rank condition $\text{rank}(\mathbf{M}_o) = n$. The system is subject to a time-varying not measurable disturbance given by

$$d(t) = d_0 + d_1 \sin(\omega_d t)$$

where d_0, d_1, ω_d are positive real constants. Which of the following observers can estimate the disturbance $d(t)$?

Choose one answer

- The dynamics of the state estimator is

$$\begin{bmatrix} \dot{\hat{\mathbf{x}}} \\ \dot{\hat{\mathbf{w}}} \end{bmatrix} = \begin{bmatrix} \mathbf{A} & \mathbf{B}_d\mathbf{C}_w \\ \mathbf{0} & \mathbf{A}_w \end{bmatrix} \begin{bmatrix} \hat{\mathbf{x}} \\ \hat{\mathbf{w}} \end{bmatrix} + \begin{bmatrix} \mathbf{B} \\ \mathbf{0} \end{bmatrix} u + \begin{bmatrix} \mathbf{L}_x \\ \mathbf{L}_w \end{bmatrix} (y - \hat{y})$$

$$\hat{y} = [\mathbf{C} \quad \mathbf{0}] \begin{bmatrix} \hat{\mathbf{x}} \\ \hat{\mathbf{w}} \end{bmatrix}$$

where

$$\mathbf{A}_w = \begin{bmatrix} 0 & 1 \\ -\omega_d^2 & -2\zeta\omega_d \end{bmatrix} \quad , \quad \mathbf{C}_w = [1 \quad 0]$$

- The dynamics of the state estimator is

$$\begin{bmatrix} \dot{\hat{\mathbf{x}}} \\ \dot{\hat{\mathbf{w}}} \end{bmatrix} = \begin{bmatrix} \mathbf{A} & \mathbf{B}_d\mathbf{C}_w \\ \mathbf{0} & \mathbf{A}_w \end{bmatrix} \begin{bmatrix} \hat{\mathbf{x}} \\ \hat{\mathbf{w}} \end{bmatrix} + \begin{bmatrix} \mathbf{B} \\ \mathbf{0} \end{bmatrix} u + \begin{bmatrix} \mathbf{L}_x \\ \mathbf{L}_w \end{bmatrix} (y - \hat{y})$$

$$\hat{y} = [\mathbf{C} \quad \mathbf{0}] \begin{bmatrix} \hat{\mathbf{x}} \\ \hat{\mathbf{w}} \end{bmatrix}$$

where

$$\mathbf{A}_w = \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix} \quad , \quad \mathbf{C}_w = [1 \quad 1]$$

- The dynamics of the state estimator is

$$
\begin{bmatrix} \dot{\hat{\mathbf{x}}} \\ \dot{\hat{\mathbf{w}}} \end{bmatrix} = \begin{bmatrix} \mathbf{A} & \mathbf{B}_d\mathbf{C}_w \\ \mathbf{0} & \mathbf{A}_w \end{bmatrix} \begin{bmatrix} \hat{\mathbf{x}} \\ \hat{\mathbf{w}} \end{bmatrix} + \begin{bmatrix} \mathbf{B} \\ \mathbf{0} \end{bmatrix} u + \begin{bmatrix} \mathbf{L}_x \\ \mathbf{L}_w \end{bmatrix} (y - \hat{y})
$$

$$
\hat{y} = [\mathbf{C} \quad \mathbf{0}] \begin{bmatrix} \hat{\mathbf{x}} \\ \hat{\mathbf{w}} \end{bmatrix}
$$

where

$$
\mathbf{A}_w = \begin{bmatrix} 0 & -\omega_d & 0 \\ \omega_d & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix} \quad , \quad \mathbf{C}_w = [1 \quad 0 \quad 1]
$$

- ◯ The dynamics of the state estimator is

$$
\begin{bmatrix} \dot{\hat{\mathbf{x}}} \\ \dot{\hat{\mathbf{w}}} \end{bmatrix} = \begin{bmatrix} \mathbf{A} & \mathbf{B}_d\mathbf{C}_w \\ \mathbf{0} & \mathbf{A}_w \end{bmatrix} \begin{bmatrix} \hat{\mathbf{x}} \\ \hat{\mathbf{w}} \end{bmatrix} + \begin{bmatrix} \mathbf{B} \\ \mathbf{0} \end{bmatrix} u + \begin{bmatrix} \mathbf{L}_x \\ \mathbf{L}_w \end{bmatrix} (y - \hat{y})
$$

$$
\hat{y} = [\mathbf{C} \quad \mathbf{0}] \begin{bmatrix} \hat{\mathbf{x}} \\ \hat{\mathbf{w}} \end{bmatrix}
$$

where

$$
\mathbf{A}_w = \begin{bmatrix} -\omega_d & 0 \\ 0 & 0 \end{bmatrix} \quad , \quad \mathbf{C}_w = [1 \quad 1]
$$

- ◯ The dynamics of the state estimator is

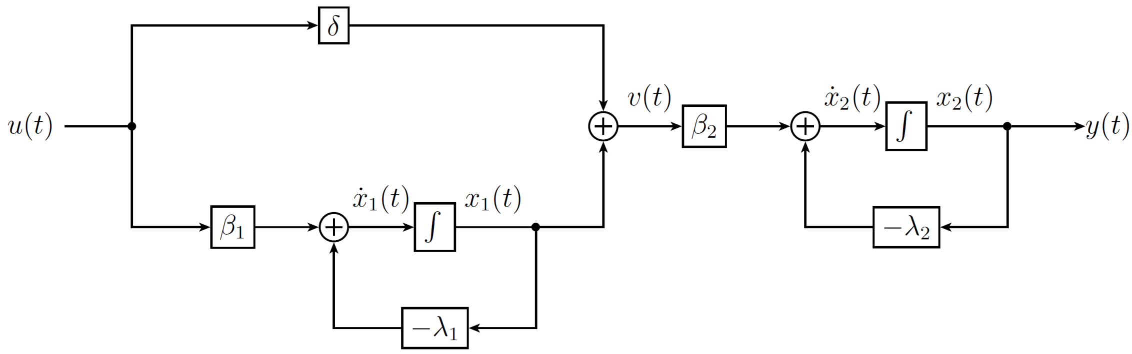
$$
\begin{bmatrix} \dot{\hat{\mathbf{x}}} \\ \dot{\hat{\mathbf{w}}} \end{bmatrix} = \begin{bmatrix} \mathbf{A} & \mathbf{B}_d\mathbf{C}_w \\ \mathbf{0} & \mathbf{A}_w \end{bmatrix} \begin{bmatrix} \hat{\mathbf{x}} \\ \hat{\mathbf{w}} \end{bmatrix} + \begin{bmatrix} \mathbf{B} \\ \mathbf{0} \end{bmatrix} u + \begin{bmatrix} \mathbf{L}_x \\ \mathbf{L}_w \end{bmatrix} (y - \hat{y})
$$

$$
\hat{y} = [\mathbf{C} \quad \mathbf{0}] \begin{bmatrix} \hat{\mathbf{x}} \\ \hat{\mathbf{w}} \end{bmatrix}
$$

where

$$
\mathbf{A}_w = \begin{bmatrix} 0 & -\omega_{d1} & 0 & 0 \\ \omega_{d1} & 0 & 0 & 0 \\ 0 & 0 & 0 & -\omega_{d2} \\ 0 & 0 & \omega_{d2} & 0 \end{bmatrix} \quad , \quad \mathbf{C}_w = [1 \quad 0 \quad 1 \quad 0]
$$

Consider the 2nd order system represented in the following block diagram below



where $\beta_1, \beta_2, \delta, \lambda_1$, and λ_2 are real and positive coefficients. Assuming that the input $u(t)$ is Gaussian white noise with zero mean and noise intensity σ_u^2, what is the variance of the output $y(t)$?

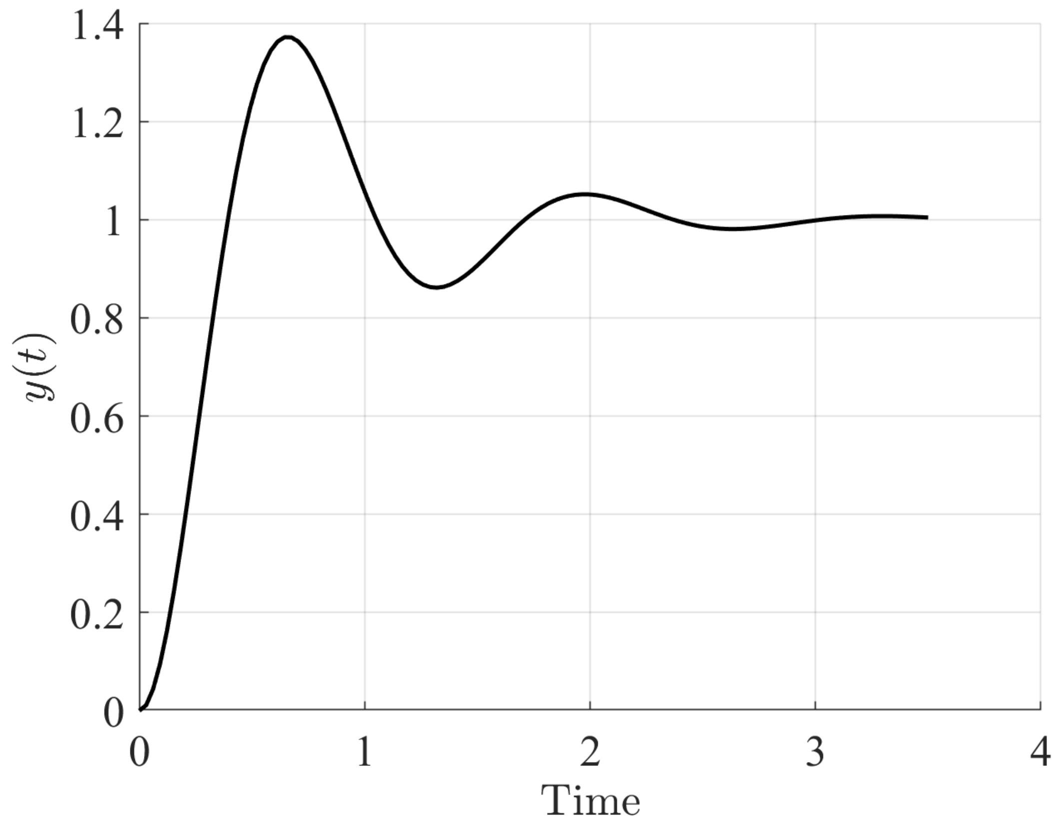
Choose one answer

- $\sigma_y^2 = \frac{\beta_2^2}{2\lambda_2}\left(\delta^2 + \frac{2\beta_1}{\lambda_1+\lambda_2}\left(\delta + \frac{\beta_1}{2\lambda_1}\right)\right)\sigma_u^2$
- $\sigma_y^2 = \frac{\beta_2^2}{2\lambda_2}\left(\delta^2 + \frac{\beta_1^2}{2\lambda_1}\right)\sigma_u^2.$
- $\sigma_y^2 = \frac{\beta_2^2}{\lambda_2(\lambda_1+\lambda_2)}\left(\beta_1\delta + \frac{\beta_1^2}{2\lambda_1}\right)\sigma_u^2.$
- $\sigma_y^2 = 0.$
- $\sigma_y^2 = \sigma_u^2.$

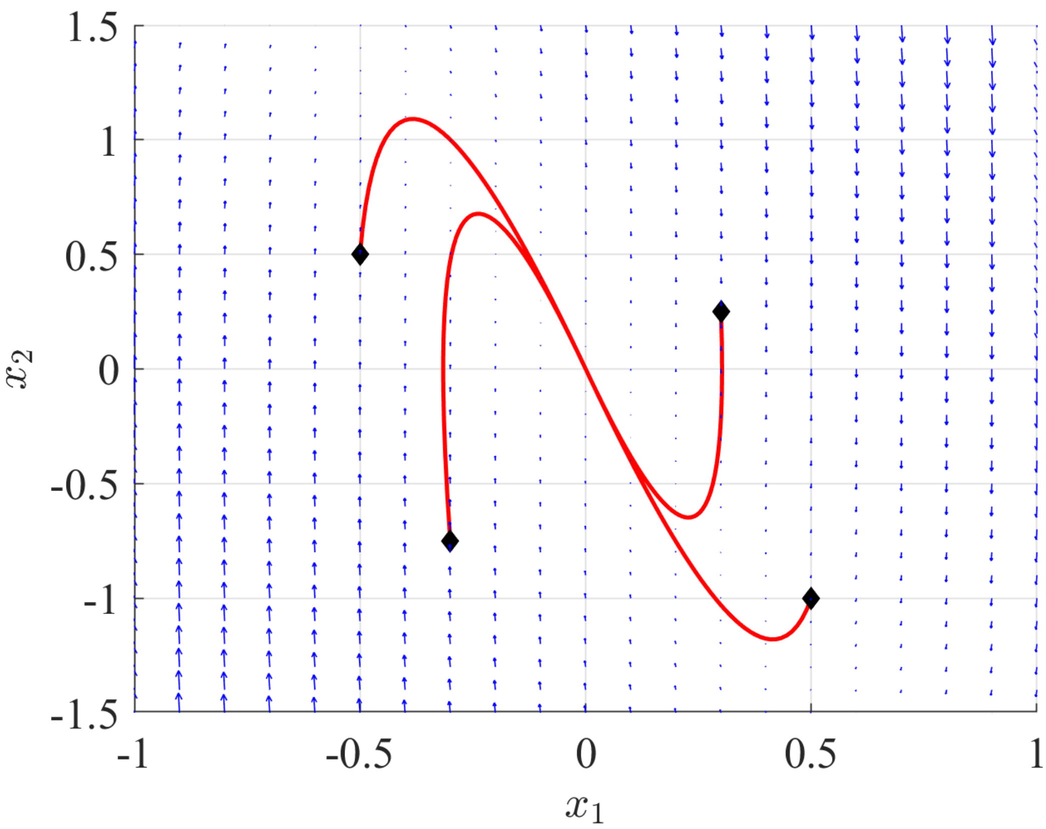
Consider the 2nd order LTI system

$$\Sigma : \begin{cases} \dot{\mathbf{x}} = \begin{bmatrix} 0 & 1 \\ -\omega_0^2 & -2\zeta\omega_0 \end{bmatrix} \mathbf{x} + \begin{bmatrix} 0 \\ \omega_0^2 \end{bmatrix} u \\ y = \begin{bmatrix} 1 & 0 \end{bmatrix} \mathbf{x} \end{cases}$$

where $\omega_0 > 0$ and $0 < \zeta < 1$. The open loop step response of the system to a unit step is shown in the following figure



A full state feedback controller $u = -\mathbf{K}\mathbf{x}$ is designed such that the dynamical behaviour shown in the phase portrait is achieved.



Which of the following full state feedback controller matrix $\mathbf{K} = \begin{bmatrix} k_1 & k_2 \end{bmatrix}$ achieves the closed-loop behaviour shown in the phase portrait?

Choose one answer

- The closed-loop behaviour can be achieved for $k_1 > -1$ and $-k_2^\star \leq k_2 \leq k_2^\star$, where

  $k_2^\star = \frac{2}{\omega_0}(\sqrt{1+k_1} - \zeta)$

- The closed-loop behaviour can be achieved for any $k_1, k_2 \neq 0$.
- The closed-loop behaviour can be achieved for $k_1 > 1$ and $k_2 \leq -k_2^\star$ where

  $k_2^\star = \frac{2}{\omega_0}(\sqrt{1+k_1} - \zeta)$

- The closed-loop behaviour can be achieved for any $k_1 > 0$ and any $k_2 > 0$.
- The closed-loop behaviour can be achieved for $k_1 > -1$ and $k_2 > k_2^\star$, where

  $k_2^\star = \frac{2}{\omega_0}(\sqrt{1+k_1} - \zeta)$

Consider the 2nd order LTI system

$$\Sigma : \begin{cases} \dot{\mathbf{x}} = \begin{bmatrix} \alpha & \beta \\ 0 & \gamma \end{bmatrix} \mathbf{x} + \begin{bmatrix} \delta \\ 0 \end{bmatrix} u \\ y = \begin{bmatrix} 1 & 0 \end{bmatrix} \mathbf{x} \end{cases}$$

where $\alpha, \beta, \gamma \in \mathbb{R}$ and $\delta \in \mathbb{R} \setminus \{0\}$. The full state feedback controller $u = \mathbf{K}\mathbf{x}$ with $\mathbf{K} = \begin{bmatrix} k_1 & k_2 \end{bmatrix}$, $k_1, k_2 \in \mathbb{R}_+$ is designed to stabilize the system. Which of the following statements is correct?

Choose one answer

- If $\gamma < 0$ then the control law $u = \mathbf{K}\mathbf{x}$ can stabilize the system Σ by arbitrary eigenvalue assignment only to the subsystem $\dot{x}_1 = \alpha x_1 + \beta x_2 + \delta u$.
- If $\gamma \geq 0$ then the control law $u = \mathbf{K}\mathbf{x}$ can stabilize the system Σ by arbitrary eigenvalue assignment only to the subsystem $\dot{x}_1 = \alpha x_1 + \beta x_2 + \delta u$.
- The control law $u = \mathbf{K}\mathbf{x}$ can stabilize the system Σ by arbitrary eigenvalue assignment because the open loop system is observable.
- If $\alpha < 0$ then the control law $u = \mathbf{K}\mathbf{x}$ can stabilize the system Σ by arbitrary eigenvalue assignment only to the subsystem $\dot{x}_2 = \gamma x_2$.
- The control law $u = \mathbf{K}\mathbf{x}$ can stabilize the system Σ by arbitrary eigenvalue assignment because the open loop system is controllable.

Consider the 2nd order LTI system

$$\Sigma : \begin{cases} \dot{\mathbf{x}} = \begin{bmatrix} 0 & 1 \\ -\alpha & -\beta \end{bmatrix} \mathbf{x} + \begin{bmatrix} 0 \\ \gamma \end{bmatrix} u + \begin{bmatrix} 0 \\ \delta \end{bmatrix} d \\ y = \begin{bmatrix} 1 & 0 \end{bmatrix} \mathbf{x} \end{cases}$$

where $\alpha, \beta, \in \mathbb{R}_+$ and $\gamma, \delta \in \mathbb{R} \setminus \{0\}$. The system is subject to an unknown disturbance $d(t)$.

The following observer is designed under the assumption that the disturbance acting on the system is constant, i.e.

$$\Sigma_o : \begin{cases} \dot{\hat{\mathbf{x}}}_o = \begin{bmatrix} 0 & 1 & 0 \\ -\alpha & -\beta & \delta \\ 0 & 0 & 0 \end{bmatrix} \hat{\mathbf{x}}_o + \begin{bmatrix} 0 \\ \gamma \\ 0 \end{bmatrix} u + \mathbf{L}(y - \hat{y}_o) \\ \hat{y}_o = \begin{bmatrix} 1 & 0 & 0 \end{bmatrix} \hat{\mathbf{x}}_o \end{cases}$$

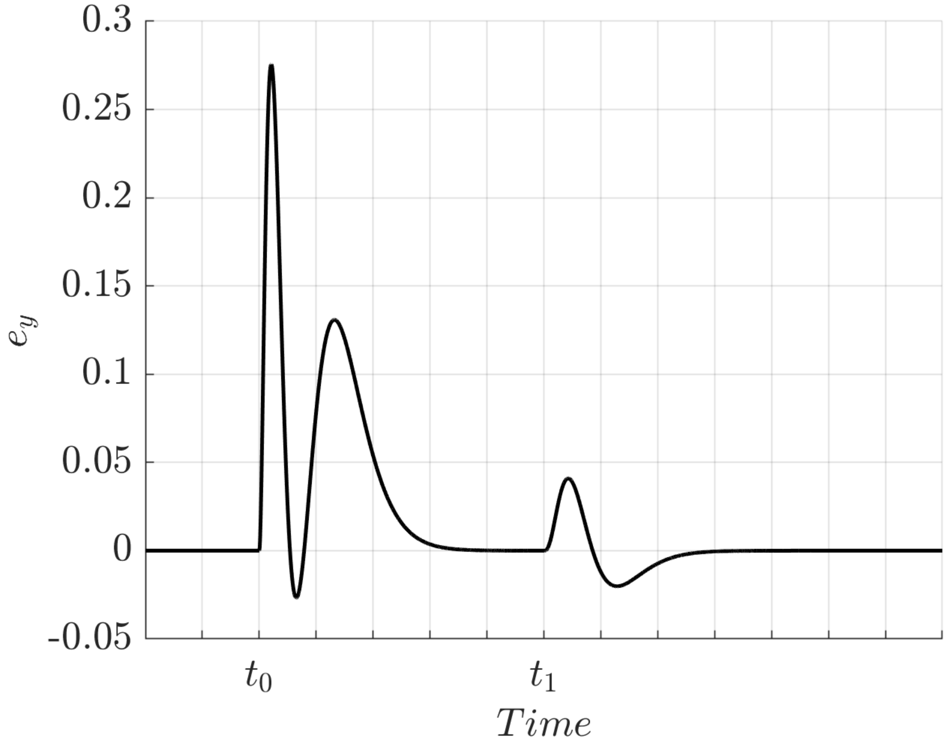
where the state of the observer is $\hat{\mathbf{x}}_o = [\hat{x}_1, \hat{x}_2, \hat{d}\,]^T$. The following disturbance profile acts on the system

$$d(t) = \begin{cases} 0 & 0 \leq t < t_0 \\ d_0 & t_0 \leq t < t_1 \\ d_0 + A_d \sin(\omega_d t + \varphi_d) & t \geq t_1 \end{cases}$$

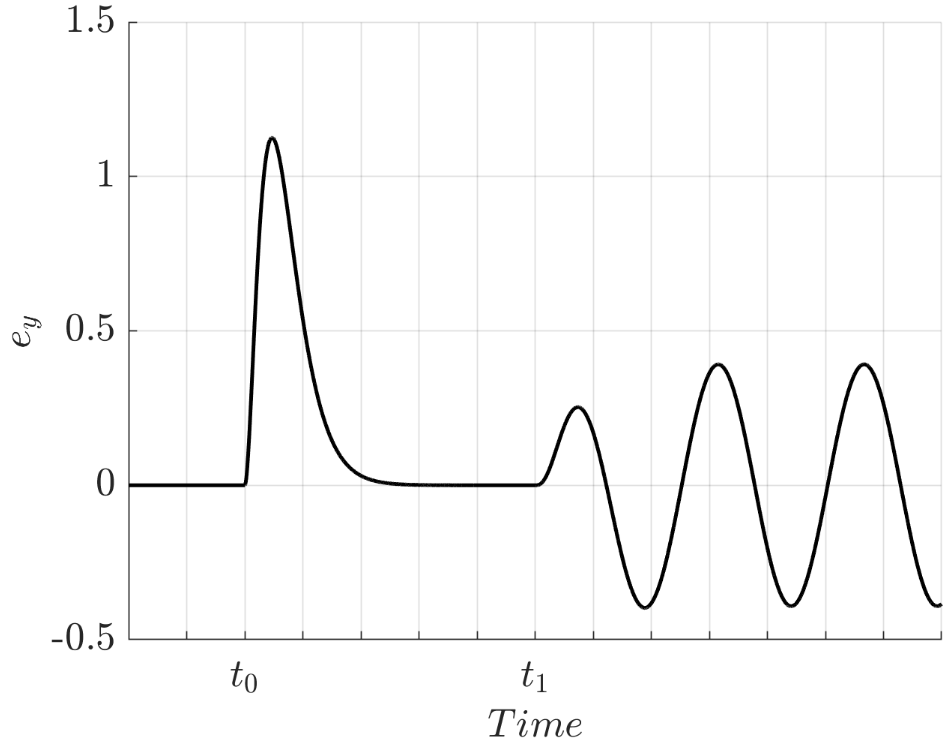
where $d_0, A_d, \omega_d, \varphi_d \in \mathbb{R}_+$. Under the assumption that the observer initial condition matches the system initial condition, i.e. $\hat{\mathbf{x}}_o(0) = [\hat{x}_1(0), \hat{x}_2(0), \hat{d}\,(0)]^T = [x_1(0), x_2(0), 0]^T$, and that $e_y = y - \hat{y}_o$ is the output estimation error, which of the following plots shows the correct behaviour of $e_y(t)$ for all $t \geq 0$?

Choose one answer

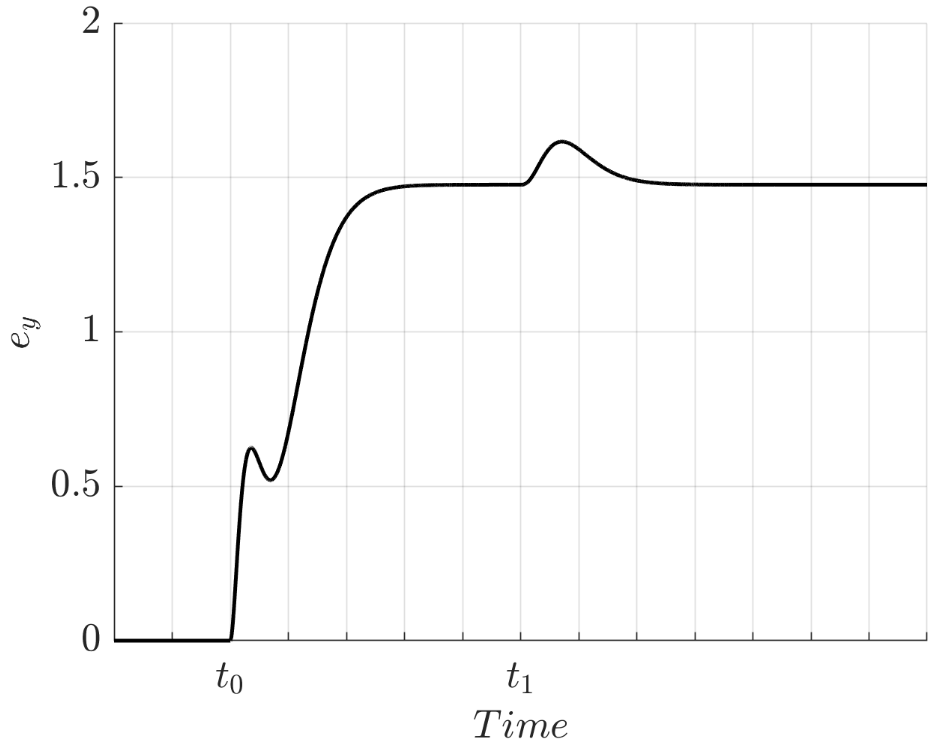
○



0.3
0.25
0.2
0.15
0.1
0.05
0
-0.05
e_y
t_0
t_1
$Time$

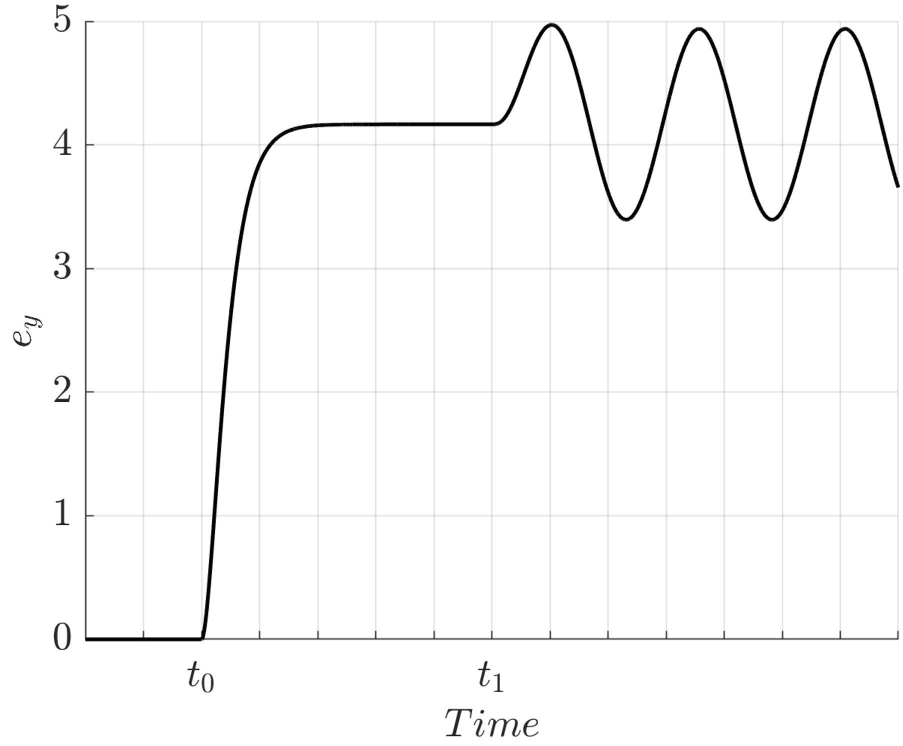


1.5
1
0.5
0
-0.5
e_y
t_0
t_1
$Time$



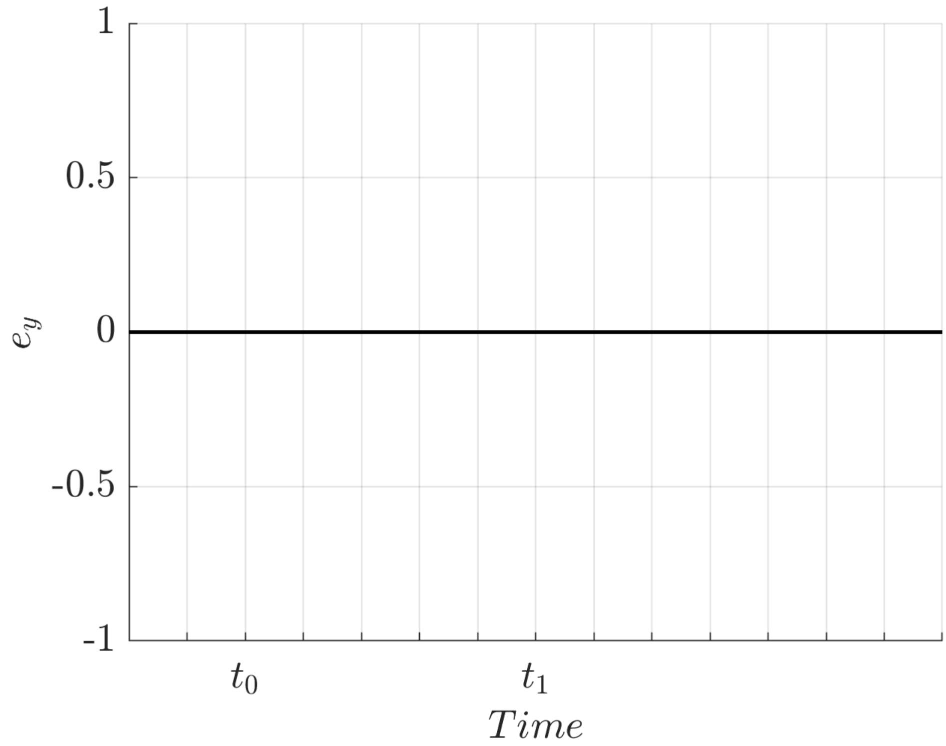
2
1.5
1
0.5
0
e_y
t_0
t_1
$Time$

○



5
4
3
2
1
0
e_y
t_0
t_1
$Time$

○



1
0.5
0
-0.5
-1
e_y
t_0
t_1
$Time$

Consider the 2nd order LTI system

$$\Sigma : \begin{cases} \dot{\mathbf{x}} = \mathbf{A}\mathbf{x} + \mathbf{B}u + \begin{bmatrix} 0 \\ 1 \end{bmatrix} g(t) \\ y = \mathbf{C}\mathbf{x} + n \end{cases} \qquad \mathbf{x} \in \mathbb{R}^2, u \in \mathbb{R}, y \in \mathbb{R}$$

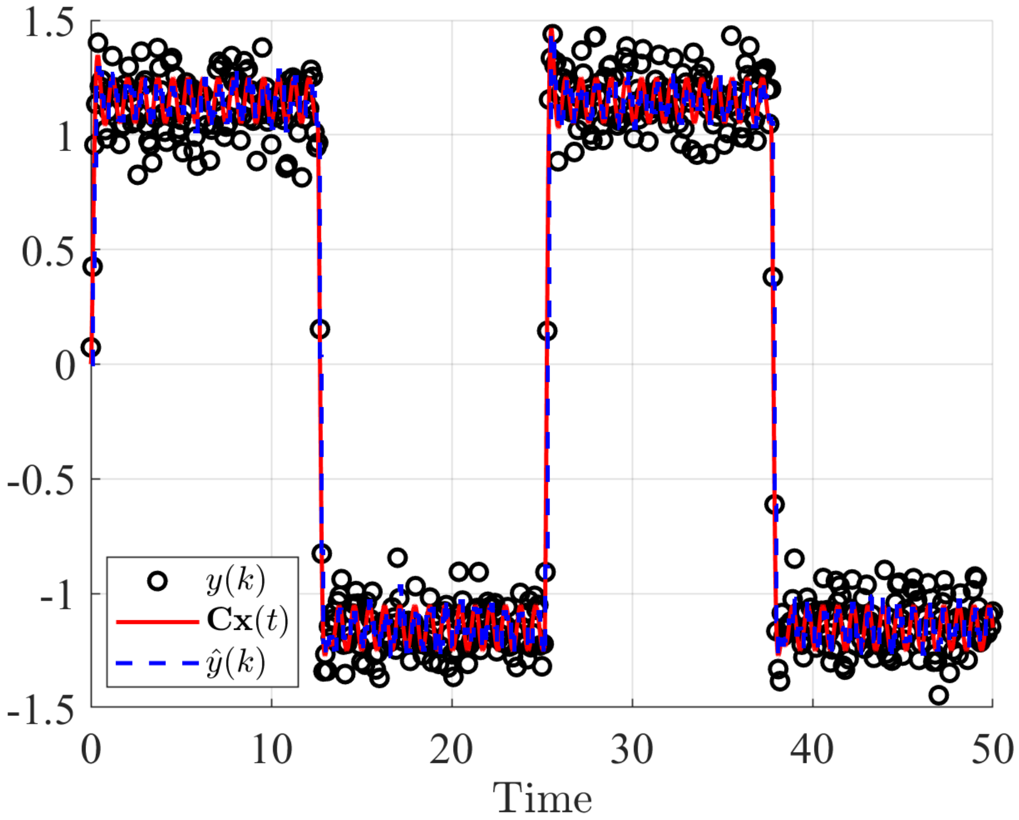
where $g(t)$ is an unknown function of time, and n is zero mean white Gaussian noise with noise intensity σ_n^2.

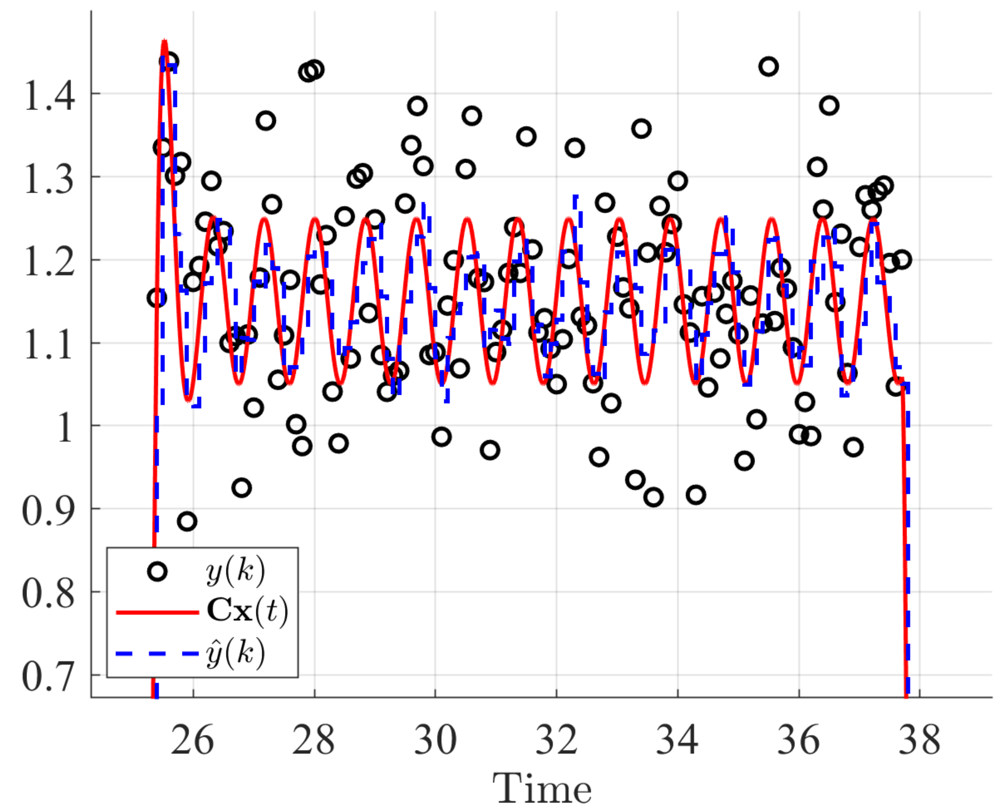
A discrete time Kalman filter is designed to reconstruct the state $\mathbf{x}$ based on noisy measurements y, i.e.

$$\Sigma_{KF} : \begin{cases} \hat{\mathbf{x}}(k+1) = \mathbf{F}\hat{\mathbf{x}}(k) + \mathbf{G}u(k) + \mathbf{L}'(y(k) - \hat{y}(k)) \\ \hat{y}(k) = \mathbf{C}\hat{\mathbf{x}}(k) \end{cases}$$

where $\mathbf{F} = e^{\mathbf{A}T_s}$, $\mathbf{G} = \int_0^{T_s} e^{\mathbf{A}t}\mathbf{B}\,\mathrm{d}t$, $\mathbf{L}' = \mathbf{F}\mathbf{L}$ is the steady state Kalman gain in predictive form, and T_s is the sampling time. The Kalman gain $\mathbf{L}$ is designed based on the variance of the measurement noise n and the variance of the process noise v. The process noise is used to account for the model uncertainty introduced by the function $g(t)$ on the second state equation. The process noise is zero mean white Gaussian noise with noise intensity σ_v^2.

The estimated output $\hat{y}$ is shown in the following figures (second figure is a zoom in), when the system Σ is excited by a square wave and subject to the function $g(t) = A_g \sin \omega_g t$, where both A_g and ω_g are unknown.





Based on the comparison of the estimated output $\hat{y}$ with the true output $\mathbf{Cx}$, which of the following statements is correct?

Choose one answer

- The Kalman filter estimates the unknown dynamics of the system Σ by tuning the process noise intensity to be equal to the measurement noise intensity, i.e. $\sigma_v^2 = \sigma_n^2$.

- ◯ The Kalman filter estimates the unknown dynamics of the system Σ by setting the process noise intensity to zero, i.e. $\sigma_v^2 = 0$.
- ◯ The Kalman filter estimates the unknown dynamics of the system Σ by tuning the process noise intensity to be much smaller than the measurement noise intensity, i.e. $\sigma_v^2 \ll \sigma_n^2$.
- ◯ The Kalman filter estimates the unknown dynamics of the system Σ by tuning the process noise intensity to be much larger than the measurement noise intensity, i.e. $\sigma_v^2 \gg \sigma_n^2$.
- ◯ The Kalman filter estimates the unknown dynamics of the system Σ regardless of the value of the process and measurement noise intensities.