TECHNICAL UNIVERSITY OF DENMARK

Written test, date **December 16, 2016**

Course name: **Robotics** Course number: **31383**

Aids allowed: **All according to DTU regulations.**

Exam duration: **4 Hours**

Weighting: **Problem 1 counts for 15 %, Problem 2 counts for 10 %, Problem 3 counts for 25 %, Problem 4 counts for 25 % and Problem 5 counts for 25 %.**

Problem 1

Please fill in the D-H parameters of the PUMA 560 manipulator shown in Fig. 1. The manipulator consists of six revolute joints.

joint	a	d	α	θ
1				
2				
3				
4				
5				
6				

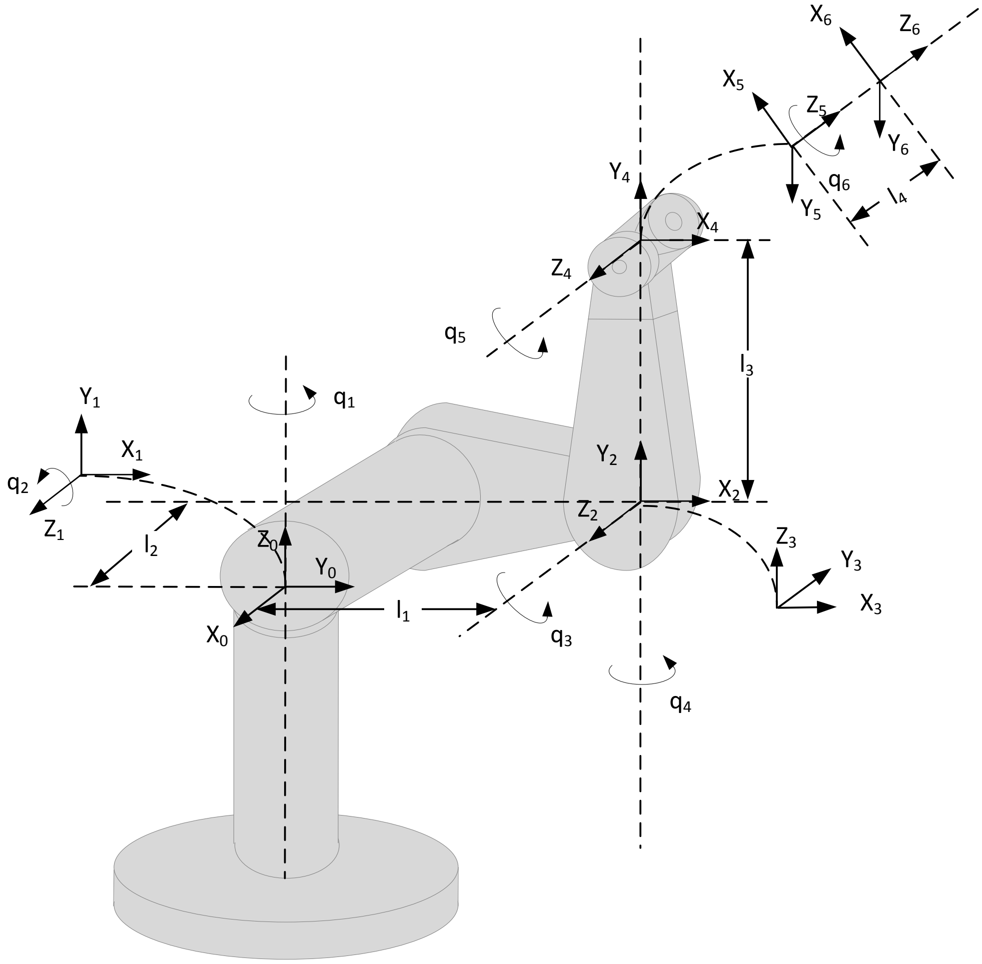


Figure 1: PUMA 560 Manipulator.

Problem 2

Identify the D-H parameters from the matrix. Assume all the angles are between 0 and 2π.

$$A = \begin{bmatrix} 1 & 0 & 0 & 3 \\ 0 & \frac{1}{\sqrt{2}} & -\frac{1}{\sqrt{2}} & 0 \\ 0 & \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

Problem 3

We seek a path plan, $q(t)$, that fulfills the following requirements

$$\begin{aligned} q(0) &= 0, & \dot{q}(0) &= 0, & \ddot{q}(0) &= 0 \\ q(10) &= 100, & \dot{q}(10) &= 0, & \ddot{q}(10) &= 0 \end{aligned}$$

The maximum allowed jerk for the path is $|\dddot{q}(t)| \leq 10$. We want a path that fulfills the requirements and at the same time has the lowest possible maximum speed $\dot{q}_{\max}$. This path is called the constant jerk path because the jerk is constant in the different segments of the path as indicated in Fig. 2.

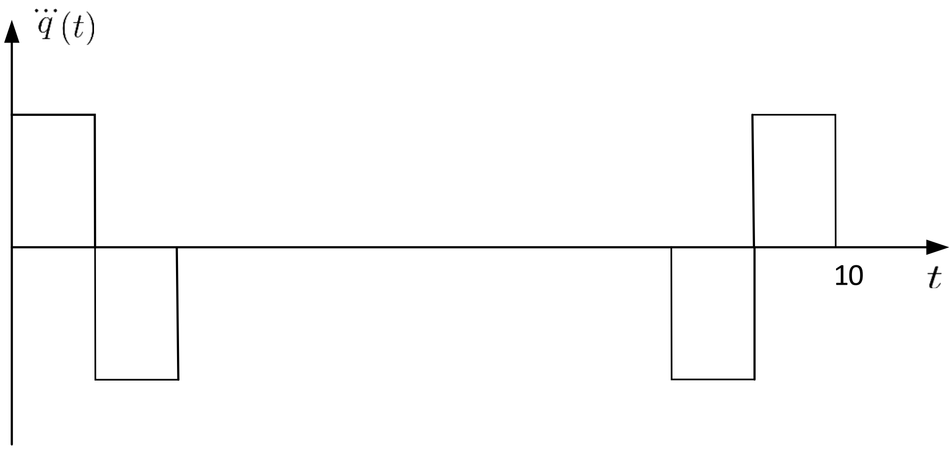


Figure 2: Schematic contant jerk curve.

1. Find the minimum $\dot{q}_{\max}$.

2. Specify the path for $t \in [0; 5]$ (because it is symmetric).

Problem 4

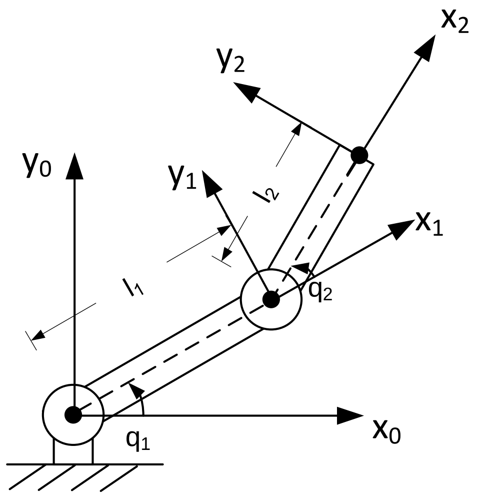


Figure 3: Two-link planar manipulator.

The *T-matrices* of this two-link planar manipulator (two revolute joints, see Fig. 3) are given by

$$T_1^0 = \begin{bmatrix} c_1 & -s_1 & 0 & l_1 c_1 \\ s_1 & c_1 & 0 & l_1 s_1 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}, \qquad T_2^0 = \begin{bmatrix} c_{12} & -s_{12} & 0 & l_1 c_1 + l_2 c_{12} \\ s_{12} & c_{12} & 0 & l_1 s_1 + l_2 s_{12} \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix},$$

with $c_{12} = cos(q_1 + q_2)$, $s_{12} = sin(q_1 + q_2)$.

1. What is the meaning of the fourth column of these two matrix?

2. Assume $l_1 = l_2 = 0.1\,\text{m}$, $q_1 = q_2 = \pi/3$. A particle has velocity $v_2 = [0.5, 0.5, 0]^T$ m/s relative to frame $o_2x_2y_2z_2$. What is the velocity of the particle in frame $o_0x_0y_0z_0$?

3. The two joint manipulator is used to grab a ball on a table. A camera is used to detect the ball placed on the $x - y$ plane. Assume the ball detected in the image has the coordinates $[470, 290]$ pixel. Considering a pinhole camera model, and the intrinsic parameters of the camera:

$$o_r\,\cancel{p_x} = 320\,\text{pixel}, \quad o_c\,\cancel{p_y} = 240\,\text{pixel}, \quad f_x = 400\,\text{pixel}, \quad f_y = 400\,\text{pixel},$$

denoting the principle point and the focal length, calculate the 3D position $[x_c, y_c, z_c]$ of the ball in the camera frame when the distance from the ball to the camera is known $z_c = 0.8\,\text{m}$.

***Problem 4 is continued on page 5.**

4. Calculate the 3D position $[x_r, y_r, z_r]$ of the ball in the robot base frame $x_0y_0z_0$. The homogeneous transformations relating the robot base frame, the world frame and camera frame are given by

$$T_w^0 = \begin{bmatrix} 0 & 1 & 0 & 0 \\ 1 & 0 & 0 & 0.3 \\ 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}, \qquad T_w^c = \begin{bmatrix} 1 & 0 & 0 & 0.1 \\ 0 & 1 & 0 & -0.1 \\ 0 & 0 & 1 & 0.5 \\ 0 & 0 & 0 & 1 \end{bmatrix}.$$

5. What joint angles are required for placing the end-effector of the manipulator on the position $[x_r, y_r]$ calculated in the previous step? Draw the resulting arm configuration. (q_1 and q_2 have the value between $-\pi$ and π, $l_1 = l_2 = 0.1\,\text{m}$).

Problem 5

The closed-loop system shown in Fig. 4 is applied to control the joint angle of the two-link manipulator in Problem 4.

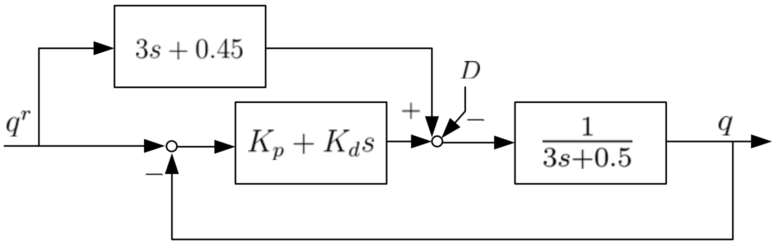


Figure 4: Block diagram of the close-loop control system.

1. Derive the transfer function in the form $q(s) = \frac{F(s)q^r(s) - G(s)D(s)}{N(s)}$.

2. For a step reference input with step value 2, and constant disturbance 0.2, determine the K_p value for the system with steady-state error $|e_{\text{ss}}| \le 0.01$.

****

Solution to problem 1.

joint	a	d	α	θ
1	0	0	$\pi/2$	q_1
2	l_1	$-l_2$	0	q_2
3	0	0	$-\pi/2$	q_3
4	0	l_3	$\pi/2$	q_4
5	0	0	$\pi/2$	q_5
6	0	l_4	0	q_6

****

Solution to problem 2.

$$\alpha = atan2(A(3,2), A(3,3)) = atan2(1/\sqrt{2}, 1/\sqrt{2}) = \pi/4$$

$$d = A(3,4) = 0$$

$$\theta = atan2(A(2,1), A(1,1)) = atan2(0,1) = 0$$

$$a = A(1,4) * \cos(\theta) + A(2,4) * \sin(\theta) = 3$$

****

Solution to problem 3.

The first half of the path is divided in three, $t \in [0 : t_1]$, $t \in [t_1 : 2t_1]$ and $[2t_1 : 5]$.

For $t \in [0 : t_1]$ the path is given by

$$\begin{aligned}
&\dddot{q} = 10 \Rightarrow \ddot{q} = 10t + C_1\\
&\ddot{q}(0) = 0 \Rightarrow C_1 = 0\\
&\ddot{q} = 10t \Rightarrow \dot{q} = 5t^2 + C_2\\
&\dot{q}(0) = 0 \Rightarrow C_2 = 0\\
&\dot{q} = 5t^2 \Rightarrow q = \frac{5}{3}t^3 + C_3\\
&q(0) = 0 \Rightarrow C_3 = 0\\
&q = \frac{5}{3}t^3\\
&q(t_1) = \frac{5}{3}t_1^3, \quad \dot{q}(t_1) = 5t_1^2, \quad \ddot{q}(t_1) = 10t_1
\end{aligned}$$

For $t \in [t_1 : 2t_1]$ the path is given by

$$\begin{aligned}
&\dddot{q} = -10 \Rightarrow \ddot{q} = -10t + C_1\\
&\ddot{q}(t_1) = 10t_1 \Rightarrow C_1 = 20t_1\\
&\ddot{q} = -10t + 20t_1 \Rightarrow \dot{q} = -5t^2 + 20t_1 t + C_2\\
&\dot{q}(t_1) = 5t_1^2 \Rightarrow C_2 = -10t_1^2\\
&\dot{q} = -5t^2 + 20t_1 t - 10t_1^2 \Rightarrow q = -\frac{5}{3}t^3 + 10t_1 t^2 - 10t_1^2 t + C_3\\
&q(t_1) = \frac{5}{3}t_1^3 \Rightarrow C_3 = \frac{10}{3}t_1^3\\
&q = -\frac{5}{3}t^3 + 10t_1 t^2 - 10t_1^2 t + \frac{10}{3}t_1^3\\
&q(2t_1) = 10t_1^3, \quad \dot{q}(2t_1) = 10t_1^2
\end{aligned}$$

It is given that $q(5) = 50$ which gives

$$50 = q(2t_1) + (5 - 2t_1)\dot{q}(2t_1) = 10t_1^3 + (5 - 2t_1)10t_1^2 = \cancel{10t_1^2} \Rightarrow t_1 = 1.137805202$$

We therefore have now that

$\dot{q}_{\max} = 10t_1^2 = 12.94600678$

$$q(t) = \begin{cases} \frac{5}{3}t^3 & , \quad t \in [0 : 1.137805202] \\ -\frac{5}{3}t^3 + 11.37805202t^2 - 12.94600678t + 4.910011286 & , \quad t \in [1.137805202 : 2.275610404] \\ 12.94600677t - 14.73003384 & , \quad t \in [2.275610404 : 5] \end{cases}$$

****

Solution to problem 4

1. The forth column of T_1^0 denotes the position of the origin of frame 1 in relative to the frame 0. The forth column of T_2^0 denotes the position of the origin of frame 2 in relative to the frame 0.

2.

$$T_2^0 = \begin{bmatrix} 0.5000 & -0.8660 & 0 & 0.0000 \\ 0.8660 & -0.5000 & 0 & 0.1732 \\ 0 & 0 & 1.0000 & 0 \\ 0 & 0 & 0 & 1.0000 \end{bmatrix}$$

$$v = R * v_2 = \begin{bmatrix} 0.5000 & -0.8660 & 0 \\ 0.8660 & -0.5000 & 0 \\ 0 & 0 & 1.0000 \end{bmatrix} \begin{bmatrix} 0.5 \\ 0.5 \\ 0 \end{bmatrix} = \begin{bmatrix} -0.6830 \\ 0.1830 \\ 0 \end{bmatrix}$$

3.

$$\cancel{x_c = (u - p_x) * z_c / fx = -0.3}$$
$$\cancel{y_c = (v - p_y) * z_c / fy = -0.1}$$

x_c=(o_r - r) * z_c/f_x = (320-470)*0.8/400 =-0.3 m

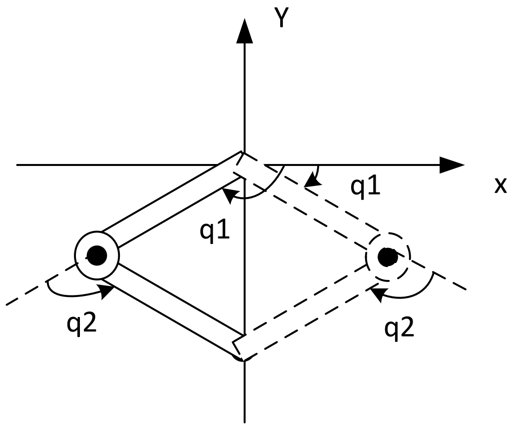
y_c=(o_c - c) * z_c/f_y = (240-290)*0.8/400 =-0.1 m

4.

$$T_w^0 * inv(T_w^c) * \begin{bmatrix} x_c \\ y_c \\ z_c \\ 1 \end{bmatrix} = \begin{bmatrix} 0 \\ -0.1 \\ -0.3 \\ 1.0 \end{bmatrix}$$

5. solution 1: solve it geometrically: $q_1 = -30\,\text{deg}(-0.5236\,\text{rad}), q_2 = -120\,\text{deg}(-2.0944\,\text{rad})$, or $q_1 = -150\,\text{deg}(-2.618\,\text{rad}), q_2 = 120\,\text{deg}(2.09\,\text{rad})$

solution 2: use solve function in matlab, solve([l1c1+l2c12==0, l1s1+l2s12==-0.1],[q1,q2]), gives same results



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Solution to problem 5:

1.

$$(q^r - q)(K_p + K_d s) + (3s + 0.45)q^r - D = q(3s + 0.5)$$

isolate q:

$$q = \frac{((K_d + 3)s + K_p + 0.45)q^r - D}{(K_d + 3)s + K_p + 0.5}$$

2.

$$E_{ss} = q^r - q = q^r - \frac{((K_d + 3)s + K_p + 0.45)q^r - D}{(K_d + 3)s + K_p + 0.5}$$

according to final value theory:

$$\begin{aligned}\lim_{s\to 0} sE_{ss} &= \lim_{s\to 0} s(q^r - q)\\ &= \lim_{s\to 0} s\frac{0.05q^r + D}{(K_d + 3)s + K_p + 0.5}\end{aligned}$$

with $q^r = 2/s$ and $D = 0.2/s$

$$\begin{aligned}\lim_{s\to 0} sE_{ss} &= \lim_{s\to 0} s\frac{0.1/s + 0.2/s}{(K_d + 3)s + K_p + 0.5} = \frac{0.3}{K_p + 0.5}\\ &\Rightarrow \frac{0.3}{K_p + 0.5} \leq 0.01\\ &\Rightarrow K_p \geq 29.5\end{aligned}$$

TECHNICAL UNIVERSITY OF DENMARK

Written test, date **December 19, 2017**

Course name: **Robotics** Course number: **31383**

Aids allowed: **All according to DTU regulations.**

Exam duration: **4 Hours**

Weighting: **Problem 1 counts for 10 %, Problem 2 counts for 15 %, Problem 3 counts for 25 %, Problem 4 counts for 25 % and Problem 5 counts for 25%**

Problem 1

A homogeneous transformation is given by

$$\begin{bmatrix} 0 & -1/2 & \sqrt{3}/2 & 0 \\ 1 & 0 & 0 & 2 \\ 0 & \sqrt{3}/2 & 1/2 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

Find the D-H parameters from the matrix (assume that all the angles are between 0 and 2π).

Problem 2

In Figure 1 three coordinate systems are show.

Find:

1. The homogeneous transformation from coordinate system 1 to 2.
2. The homogeneous transformation from coordinate system 2 to 3.
3. The homogeneous transformation from coordinate system 3 to 1.

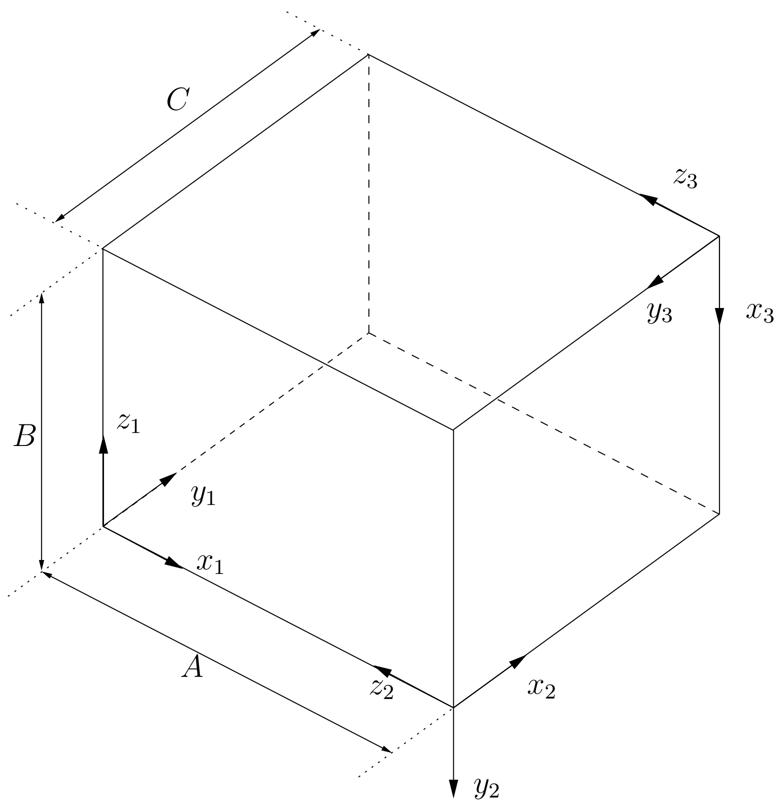


Figure 1: Three coordinate systems.

Problem 3

A trajectory is specified by the initial conditions at time $t = 0$:

Position: $s(0) = 0\mathrm{m}$
Velocity: $v(0) = 0\mathrm{m/s}$
Acceleration $a(0) = 0\mathrm{m/s^2}$

Furthermore the jerk, $j(t)$, (derivative of the acceleration with respect to time) is given by

$$j(t) = \begin{cases} 10\sin(\pi\frac{t}{\mathrm{s}})\mathrm{m/s^3} & , \quad t \in [0, 2\mathrm{s}] \\ 0 & , \quad t \in [2\mathrm{s}, T] \\ 10\sin(\pi + \pi\frac{t-T}{\mathrm{s}})\mathrm{m/s^3} & , \quad t \in [T, T+2\mathrm{s}] \end{cases}$$

Find:

1. The maximum speed.
2. The position at $t = 2\mathrm{s}$.
3. T such that the position at $t = T + 2\mathrm{s}$ is $s = 20\mathrm{m}$.

Problem 4

A UR-5 robot is mounted on a table. A task coordinate system is defined by three points on the table P_0, P_x and P_y. P_0 is the origo of the system and P_x is a point on the x-axis and P_y is a point on the y-axis. The coordinate system is right handed with the z-axis going up from the table. The coordinates of the points in the robot basesystem are found by moving the TCP to each of the points and read the coordinates from the controller. The coordinates are:

$$P_0 = [0.3000, 0.2000, 0.1000]\mathrm{m}$$
$$P_x = [0.6536, 0.5536, 0.1000]\mathrm{m}$$
$$P_y = [0.0879, 0.4121, 0.1000]\mathrm{m}$$

1. Find the transformation, T^B_{Task}, from taskspace to basespace.

A camera is mounted $1.00\mathrm{m}$ above the table with the z-axis of the camera orthogonal to the table plane. The intrinsic parameters of the camera are:

principal point: $[320, 240]\mathrm{pixel}$
focal length: $f_x = f_y = 500\mathrm{pixel}$

The image coordinates of the points above are:

$$P_0 = [270, 140]\mathrm{pixel}$$
$$P_x = [270, 390]\mathrm{pixel}$$
$$P_y = [420, 140]\mathrm{pixel}$$

2. Find the 3D positions of the points in the camera frame.
3. Find the transformation, T^C_{Task}, from taskspace to camera frame.
4. An object on the table has the image coordinates $[340, 360]\mathrm{pixel}$. Find the robot base coordinates for the object.

Problem 5

The closed-loop system shown in Figure 2 is applied to control the joint angle of a manipulator.

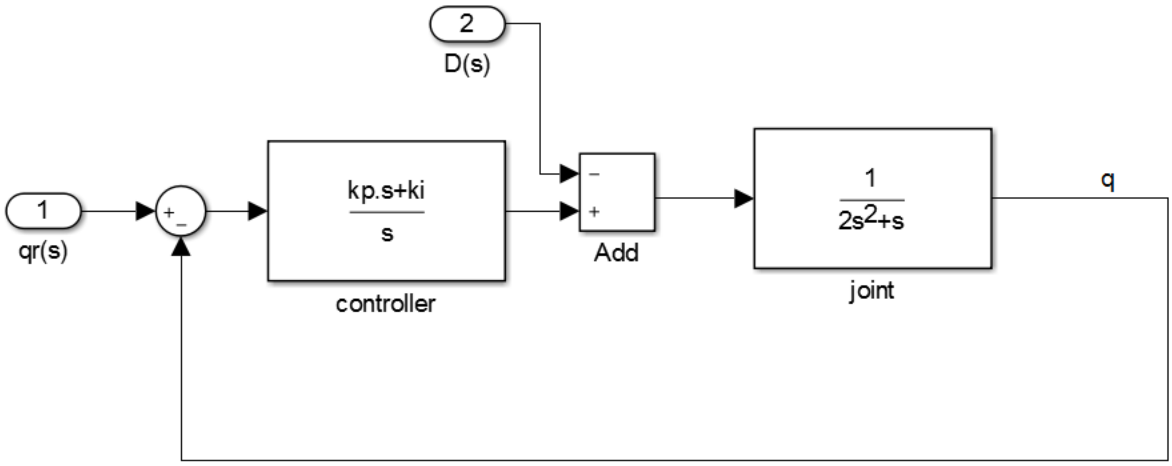


Figure 2: Block diagram of the close-loop control system.

given values are $kp = 0.3$ and $ki = 0.03$

1. Derive the transfer function in the form $q(s) = \dfrac{F(s)qr(s) - G(s)D(s)}{N(s)}$.

2. For a step reference input with step value 1.5, and constant disturbance 0.1, determine the steady-state error e_{ss}.

```
> restart; with(linalg);
```

[*BlockDiagonal*, *GramSchmidt*, *JordanBlock*, *LUdecomp*, *QRdecomp*, *Wronskian*, *addcol*, *addrow*, *adj*, *adjoint*, *angle*, *augment*, *backsub*, *band*, *basis*, *bezout*, *blockmatrix*, *charmat*, *charpoly*, *cholesky*, *col*, *coldim*, *colspace*, *colspan*, *companion*, *concat*, *cond*, *copyinto*, *crossprod*, *curl*, *definite*, *delcols*, *delrows*, *det*, *diag*, *diverge*, *dotprod*, *eigenvals*, *eigenvalues*, *eigenvectors*, *eigenvects*, *entermatrix*, *equal*, *exponential*, *extend*, *ffgausselim*, *fibonacci*, *forwardsub*, *frobenius*, *gausselim*, *gaussjord*, *geneqns*, *genmatrix*, *grad*, *hadamard*, *hermite*, *hessian*, *hilbert*, *htranspose*, *ihermite*, *indexfunc*, *innerprod*, *intbasis*, *inverse*, *ismith*, *issimilar*, *iszero*, *jacobian*, *jordan*, *kernel*, *laplacian*, *leastsqrs*, *linsolve*, *matadd*, *matrix*, *minor*, *minpoly*, *mulcol*, *mulrow*, *multiply*, *norm*, *normalize*, *nullspace*, *orthog*, *permanent*, *pivot*, *potential*, *randmatrix*, *randvector*, *rank*, *ratform*, *row*, *rowdim*, *rowspace*, *rowspan*, *rref*, *scalarmul*, *singularvals*, *smith*, *stackmatrix*, *submatrix*, *subvector*, *sumbasis*, *swapcol*, *swaprow*, *sylvester*, *toeplitz*, *trace*, *transpose*, *vandermonde*, *vecpotent*, *vectdim*, *vector*, *wronskian*] **(1)**

```
> #31383 Ex2017 Problem 1;
> # Question 1;
```

> $d = 0$; theta $= \frac{\text{Pi}}{2}$; $a = 2$; alpha $= \frac{\text{Pi}}{3}$;

$$d = 0$$

$$\theta = \frac{1}{2}\pi$$

$$a = 2$$

$$\alpha = \frac{1}{3}\pi \qquad \textbf{(2)}$$

```
> #31383 Ex2017 Problem 2;
> # Question 1;
> A1 := Matrix([[0, 1, 0, 0], [0, 0,-1, 0], [-1, 0, 0, A], [0, 0, 0, 1]]);
```

$$A1 := \begin{bmatrix} 0 & 1 & 0 & 0 \\ 0 & 0 & -1 & 0 \\ -1 & 0 & 0 & A \\ 0 & 0 & 0 & 1 \end{bmatrix} \qquad \textbf{(3)}$$

```
> # Question 2;
> A2 := Matrix([[0, 1, 0, B], [-1, 0, 0, C], [0, 0, 1, 0], [0, 0, 0, 1]]);
```

$$A2 := \begin{bmatrix} 0 & 1 & 0 & B \\ -1 & 0 & 0 & C \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \qquad \textbf{(4)}$$

```
> # Question3;
> A3 := Matrix([[0, 0,-1, A], [0,-1, 0, C], [-1, 0, 0, B ], [0, 0, 0, 1]]);
```

$$A3 := \begin{bmatrix} 0 & 0 & -1 & A \\ 0 & -1 & 0 & C \\ -1 & 0 & 0 & B \\ 0 & 0 & 0 & 1 \end{bmatrix} \tag{5}$$

> #31383 Ex2017 Problem 3;

> # Question 1;

> $j := 10 \cdot \sin(\mathrm{Pi} \cdot t);$

$$j := 10 \sin(\pi t) \tag{6}$$

> $a := integrate(j, t) + a0;$

$$a := -\frac{10 \cos(\pi t)}{\pi} + a0 \tag{7}$$

> $v := integrate(a, t) + a1;$

$$v := -\frac{10 \sin(\pi t)}{\pi^2} + a0\, t + a1 \tag{8}$$

> $s := integrate(v, t) + a2;$

$$s := \frac{10 \cos(\pi t)}{\pi^3} + \frac{1}{2}\, a0\, t^2 + a1\, t + a2 \tag{9}$$

> $solve\left(\left\{t = 0, \frac{10 \cos(\pi t)}{\pi^3} + \frac{1}{2}\, a0\, t^2 + a1\, t + a2 = 0, -\frac{10 \sin(\pi t)}{\pi^2} + a0\, t + a1 = 0, -\frac{10 \cos(\pi t)}{\pi} + a0 = 0\right\}, \{t, a0, a1, a2\}\right);$

$$\left\{a0 = \frac{10}{\pi}, a1 = 0, a2 = -\frac{10}{\pi^3}, t = 0\right\} \tag{10}$$

> $-\frac{10 \sin(\pi \cdot 2)}{\pi^2} + \frac{10}{\pi} \cdot 2$

$$\frac{20}{\pi} \tag{11}$$

> # Question 2;

> $\frac{10 \cos(\pi \cdot 2)}{\pi^3} + \frac{1}{2} \frac{10}{\pi} \cdot 2^2 + -\frac{10}{\pi^3}$

$$\frac{20}{\pi} \tag{12}$$

> # Question 3;

> $solve\left(2 \cdot \frac{20}{\pi} + \frac{20}{\pi} \cdot t = 20, t\right);$

$$-2 + \pi \tag{13}$$

> $-2 + \pi + 2;$

(14)

$$\pi \qquad (14)$$

>

>

Exam 2017, problem 3

Three time intervals A [0sec,2sec], B [2sec,T] and C [T,T+2sec] are considered.

```
j_A(t):=10·sin(π·t)$
j_B(t):=0$
j_C(t):=10·sin(π+π·(t−T))$
```

```
define(a(t),a_0+integrate(j_A(t), t));
sol:solve(a(0)=0,a_0)[1];
define(a_A(t),subst(sol,a(t)));
```

$$a(t) := a_0 - \frac{10\cos(\pi t)}{\pi}$$

$$a_0 = \frac{10}{\pi}$$

$$\mathrm{a\_A}(t) := \frac{10}{\pi} - \frac{10\cos(\pi t)}{\pi}$$

```
define(a_B(t),a_A(2));
```

$$\mathrm{a\_B}(t) := 0$$

```
define(a(t),a_1+integrate(j_C(t), t));
sol:solve(a(T)=a_B(T),a_1)[1];
define(a_C(t),subst(sol,a(t)));
```

$$a(t) := \frac{10\cos(\pi(t-T))}{\pi} + a_1$$

$$a_1 = -\frac{10}{\pi}$$

$$\mathrm{a\_C}(t) := \frac{10\cos(\pi(t-T))}{\pi} - \frac{10}{\pi}$$

```
define(v(t),v_0+integrate(a_A(t), t));
sol:solve(v(0)=0,v_0)[1];
define(v_A(t),subst(sol,v(t)));
```

$$v(t) := v_0 - \frac{10\sin(\pi t)}{\pi^2} + \frac{10\,t}{\pi}$$

$$v_0 = 0$$

$$\mathrm{v\_A}(t) := \frac{10\,t}{\pi} - \frac{10\sin(\pi t)}{\pi^2}$$

```
define(v_B(t),v_A(2));
```

$$\mathrm{v\_B}(t) := \frac{20}{\pi}$$

```
define(v(t),v_1+integrate(a_C(t), t));
sol:solve(v(T)=v_B(T),v_1)[1];
define(v_C(t),subst(sol,v(t)));
```

$$v(t) := v_1 + \frac{10\sin(\pi(t-T))}{\pi^2} - \frac{10\,t}{\pi}$$

$$v_1 = \frac{10\,T+20}{\pi}$$

$$\mathrm{v\_C}(t) := \frac{10\sin(\pi(t-T))}{\pi^2} - \frac{10\,t}{\pi} + \frac{10\,T+20}{\pi}$$

```
define(s(t),s_0+integrate(v_A(t), t));
sol:solve(s(0)=0,s_0)[1];
define(s_A(t),subst(sol,s(t)));
```

$$s(t):=\frac{10\cos(\pi t)}{\pi^3}+\frac{5t^2}{\pi}+s_0$$

$$s_0=-\frac{10}{\pi^3}$$

$$s\_A(t):=\frac{10\cos(\pi t)}{\pi^3}+\frac{5t^2}{\pi}-\frac{10}{\pi^3}$$

```
define(s(t),s_1+integrate(v_B(t), t));
sol:solve(s(2)=s_A(2),s_1)[1];
define(s_B(t),subst(sol,s(t)));
```

$$s(t):=\frac{20t}{\pi}+s_1$$

$$s_1=-\frac{20}{\pi}$$

$$s\_B(t):=\frac{20t}{\pi}-\frac{20}{\pi}$$

```
define(s(t),s_2+integrate(v_C(t), t));
sol:solve(s(T)=s_B(T),s_2)[1];
define(s_C(t),subst(sol,s(t)));
```

$$s(t):=-\frac{10\cos(\pi(t-T))}{\pi^3}-\frac{5t^2}{\pi}+\frac{(10T+20)t}{\pi}+s_2$$

$$s_2=-\frac{5\pi^2T^2+20\pi^2-10}{\pi^3}$$

$$s\_C(t):=-\frac{10\cos(\pi(t-T))}{\pi^3}-\frac{5t^2}{\pi}+\frac{(10T+20)t}{\pi}-\frac{5\pi^2T^2+20\pi^2-10}{\pi^3}$$

Question 1

From the results above we can see that a_A > 0, hence velocity is increasing from 0 to 2 sec, then a_B=0 means constant velocity from 2 sec until T, and finally we observe that a_C < 0, so that velocity is decreasing again after time T. Hence the maximum velocity occurs during the entire time interval B [2sec,T]:

```
v_max:v_B(t);
```

$$\frac{20}{\pi}$$

in [m/s].

Question 2

The position at t=2sec is now easy to comput:

```
s_A(2);
s_B(2);
```

$$\frac{20}{\pi}$$

$$\frac{20}{\pi}$$

in [m].

Question 3

The position at t=2sec+T is also easy to compute:

```
s_final:ratsimp(s_C(T+2));
```

$$\frac{20\,T}{\pi}$$

Then T can be found from the equation:

```
solve(s_final=20,T)[1];
```

$$T=\pi$$

in [s].

Exam 2017, problem 4

(%i3)
```
fpprintprec : 4$ kill(all)$
ratprint: false$
load("vect")$
norm(x) := sqrt(x . x)$
```

(%i6)
```
P0:[0.3,0.2,0.1];
Px:[0.6536,0.5536,0.1];
Py:[0.0879,0.4121,0.1];
```

[0.3,0.2,0.1]

[0.6536,0.5536,0.1]

[0.0879,0.4121,0.1]

1 Question 1

The x- and y-axis of the task system expressed in the base system (need to be unit vectors):

(%i8)
```
xvec:(Px−P0)/norm(Px−P0);
yvec:(Py−P0)/norm(Py−P0);
```

[0.7071,0.7071,0.0]

[−0.7071,0.7071,0.0]

z-axis is always the cross product of x and y:

(%i9)
```
zvec:express(xvec~yvec);
```

[0.0,0.0,1.0]

putting all columns together and adding a 4th row with three zeros and one:

(%i10)
```
T_task_wrt_B:append(transpose(matrix(xvec,yvec,zvec,P0)),matrix([0,0,0,1]));
```

$$\begin{pmatrix} 0.7071 & -0.7071 & 0.0 & 0.3 \\ 0.7071 & 0.7071 & 0.0 & 0.2 \\ 0.0 & 0.0 & 1.0 & 0.1 \\ 0 & 0 & 0 & 1 \end{pmatrix}$$

2 Question 2

(%i13)
```
p0:[270,140];
px:[270,390];
py:[420,140];
```

[270,140]

[270,390]

[420,140]

(%i14)
```
z:1;
```

1

(%i18)
```
o_r:320;
o_c:240;
f_x:500;
f_y:500;
```

320
240
500
500

Using Eq.(11.6) from the text book:

(%i23)
```
z0:z;
r0:p0[1];
c0:p0[2];
x0:float((o_r−r0)·z0/f_x);
y0:float((o_c−c0)·z0/f_y);
```

1
270
140
0.1
0.2

(%i28)
```
zx:z;
rx:px[1];
cx:px[2];
xx:float((o_r−rx)·zx/f_x);
yx:float((o_c−cx)·zx/f_y);
```
1
270
390
0.1
−0.3

(%i33)
```
zy:z;
ry:py[1];
cy:py[2];
xy:float((o_r−ry)·zy/f_x);
yy:float((o_c−cy)·zy/f_y);
```
1
420
140
−0.2
0.2

Hence, the 3 points P0, Px, and Py, expressed in the camera frame are:

(%i36)
```
P0c:[x0,y0,z0];
Pxc:[xx,yx,zx];
Pyc:[xy,yy,zy];
```
[0.1,0.2,1]
[0.1,−0.3,1]
[−0.2,0.2,1]

3 Question 3

(%i37)
```
kill(xx,xy,xz,yx,yy,yz,ox,oy)$
```

(%i38)
```
T_B_wrt_task:invert(T_task_wrt_B);
```
$$\begin{pmatrix} 0.7071 & 0.7071 & 0.0 & -0.3536 \\ -0.7071 & 0.7071 & -0.0 & 0.07071 \\ 0.0 & -0.0 & 1.0 & -0.1 \\ 0 & 0 & 0 & 1.0 \end{pmatrix}$$

We know that the camera z-axis points downwards, normal to the table, while the task z-axis points upwards normal to the table. We also know that the task fram is on the table, while the camera frame is 1 m above the table. Hence we can assume the following format for the camera frame with respect to the task frame

(%i39)
```
T_C_wrt_task:matrix([xx,yx,0,ox],[xy,yy,0,oy],[xz,yz,−1,1],[0,0,0,1]);
```
$$\begin{pmatrix} xx & yx & 0 & ox \\ xy & yy & 0 & oy \\ xz & yz & -1 & 1 \\ 0 & 0 & 0 & 1 \end{pmatrix}$$

Extend all point vectors with an extra 1:

(%i45)
```
P0c:append(P0c,[1]);
Pxc:append(Pxc,[1]);
Pyc:append(Pyc,[1]);
P0B:append(P0,[1]);
PxB:append(Px,[1]);
PyB:append(Py,[1]);
```
[0.1,0.2,1,1]
[0.1,−0.3,1,1]
[−0.2,0.2,1,1]
[0.3,0.2,0.1,1]
[0.6536,0.5536,0.1,1]
[0.0879,0.4121,0.1,1]

For any point P the following identity needs to hold
T_C_wrt_task.P_in_C = T_B_wrt_task.P_in_B
we apply this identity to all 3 points P0,Px,Pz that we know their coordinates, both in C (camera) and B (base) frame, and we get 9 equations:

(%i48) eq123:T_C_wrt_task.P0c−T_B_wrt_task.P0B;
eq456:T_C_wrt_task.Pxc−T_B_wrt_task.PxB;
eq789:T_C_wrt_task.Pyc−T_B_wrt_task.PyB;

$$\begin{pmatrix} 0.2\,yx+0.1\,xx+ox \\ 0.2\,yy+0.1\,xy+oy \\ 0.2\,yz+0.1\,xz \\ 0.0 \end{pmatrix}$$

$$\begin{pmatrix} -(0.3\,yx)+0.1\,xx+ox-0.5001 \\ -(0.3\,yy)+0.1\,xy+oy \\ 0.1\,xz-0.3\,yz \\ 0.0 \end{pmatrix}$$

$$\begin{pmatrix} 0.2\,yx-0.2\,xx+ox+5.551\;10^{-17} \\ 0.2\,yy-0.2\,xy+oy-0.3 \\ 0.2\,yz-0.2\,xz \\ 0.0 \end{pmatrix}$$

(%i51) eq123:args(transpose(eq123))[1]$
eq456:args(transpose(eq456))[1]$
eq789:args(transpose(eq789))[1]$

(%i52) sol:solve([eq123[1],eq123[2],eq123[3],eq456[1],eq456[2],eq456[3],eq789[1],eq789[2]],[xx,xy,xz,yx,yy,yz,ox,oy])[1];

$$[xx=\frac{5}{27021597764222976}, xy=-\left(\frac{78962243}{78974169}\right), xz=0, yx=-\left(\frac{16788817}{16786604}\right), yy=0, yz=0, ox=\frac{226830329955574324646441}{1134002152788241164533760}, oy=\frac{78962243}{789741690}]$$

(%i60) float(sol[1]);
float(sol[2]);
float(sol[3]);
float(sol[4]);
float(sol[5]);
float(sol[6]);
float(sol[7]);
float(sol[8]);

$xx=1.85\;10^{-16}$
$xy=-0.9998$
$xz=0.0$
$yx=-1.0$
$yy=0.0$
$yz=0.0$
$ox=0.2$
$oy=0.09998$

Checking that the 9th equation is also fulfilled:

(%i61) subst(sol,eq789[3]);

0

Hence the camera frame with respect to the task frame evaluates to

(%i62) T_C_wrt_task:float(subst(sol,T_C_wrt_task));

$$\begin{pmatrix} 1.85\;10^{-16} & -1.0 & 0.0 & 0.2 \\ -0.9998 & 0.0 & 0.0 & 0.09998 \\ 0.0 & 0.0 & -1.0 & 1.0 \\ 0.0 & 0.0 & 0.0 & 1.0 \end{pmatrix}$$

And finally the task frame with respect to the camera frame is

(%i63) T_task_wrt_C:invert(T_C_wrt_task);

$$\begin{pmatrix} 0.0 & -1.0 & 0.0 & 0.1 \\ -0.9999 & -1.85\;10^{-16} & -0.0 & 0.2 \\ 0.0 & -0.0 & -1.0 & 1.0 \\ -0.0 & 0.0 & -0.0 & 1.0 \end{pmatrix}$$

4 Question 4

(%i68)
```
zO:z;
rO:340;
cO:360;
xO:float((o_r−rO)·zO/f_x);
yO:float((o_c−cO)·zO/f_y);
```

1
340
360
-0.04
-0.24

Hence the object location in the camera frame (with one extra 1) is

(%i69)
```
POc:[xO,yO,zO,1];
```

$[-0.04, -0.24, 1, 1]$

Knowing the transformations from camera to task, and from task to base, it is easy to convert the object position to base coordinates:

(%i70)
```
POB:T_task_wrt_B.T_C_wrt_task.POc;
```

$$\begin{pmatrix} 0.5122 \\ 0.6101 \\ 0.1 \\ 1.0 \end{pmatrix}$$

Final result found after deleting row 4:

(%i71)
```
POB:submatrix(4,POB);
```

$$\begin{pmatrix} 0.5122 \\ 0.6101 \\ 0.1 \end{pmatrix}$$

Results for problem 4 and 5 exam 31383 2017

Problem 4.1

Tbt =

0.7071	-0.7071	0	0.3000
0.7071	0.7071	0	0.2000
0	0	1.0000	0.1000
0	0	0	1.0000

Problem 4.2

p0c =

-0.1000
-0.2000
1.0000

pxc =

-0.1000
0.3000
1.0000

pyc =

0.2000
-0.2000
1.0000

Problem 4.3

Tct =

0	1.0000	0	-0.1000
1.0000	0	0	-0.2000
0	0	-1.0000	1.0000
0	0	0	1.0000

Problem 4.4

Pobj =

0.5122 0.6101 0.1000

Problem 5.1

```
q_gr =

        0.15 s + 0.015
  ------------------------------
  s^3 + 0.5 s^2 + 0.15 s + 0.015

Continuous-time transfer function.


q_D =

            0.5 s
  ------------------------------
  s^3 + 0.5 s^2 + 0.15 s + 0.015

Continuous-time transfer function.
```

Problem 5.2

integrator in controller -> ess=0

TECHNICAL UNIVERSITY OF DENMARK

Written test, date **May 26, 2020**

Course name: **Robotics** Course number: **31383**

Aids allowed: **All according to DTU regulations.**

Exam duration: **4 Hours**

Weighting: **Problem 1 counts for 10 %, Problem 2 counts for 20 %, Problem 3 counts for 20 %, Problem 4 counts for 25 % and Problem 5 counts for 25%**

Problem 1

Part of a homogeneous transformation defined by D-H parameters is given by

$$\begin{bmatrix} ? & ? & -\frac{3}{4} & 1 \\ ? & \frac{1}{4} & ? & \sqrt{3} \\ 0 & ? & ? & \frac{1}{3} \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

Question 1
Find the D-H parameters (assume that all the angles are between 0 and 2π).

Question 2
Find the remaining parameters in the homogeneous transformation.

Problem 2

A rotation matrix, $[R_1^0]$ is given by:

$$[R_1^0] = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 1 & 0 & 0 \end{bmatrix}$$

A rotation matrix, $[R_2^1]$, for frame-2 relative to the frame-1 is defined by three rotations

1. First rotate θ_1 around x-axis of fixed frame.
2. Second rotate θ_2 around current y-axis.
3. Last rotate θ_3 around current z-axis.

Question 1
Find the rotation matrix $[R_2^0]$.

Question 2
Find the angular velocity of frame-2 relative to frame-0 ($\vec{\omega}^0_{0,2}$), as a function of $\dot{\theta}_1$, $\dot{\theta}_2$ and $\dot{\theta}_3$ (the time derivative of the three rotation variables).

Problem 3

A trajectory is specified by the dimensionless time variable $\bar{t} = \frac{t}{\mathrm{s}}$. The initial conditions at time $\bar{t} = 0$ are:

Position: $s(0) = 0\mathrm{m}$
Velocity: $v(0) = 0\mathrm{m/s}$
Acceleration $a(0) = 0\mathrm{m/s^2}$

Furthermore the acceleration, $a(\bar{t})$, is given by

$$\frac{a(\bar{t})}{\mathrm{m/s^2}} = \begin{cases} 2\bar{t} & , \quad \bar{t} \in [0,1] \\ 3 - \bar{t} & , \quad \bar{t} \in [1,3] \\ 0 & , \quad \bar{t} \in [3,T] \\ T - \bar{t} & , \quad \bar{t} \in [T, T+2] \\ 2\bar{t} - 6 - 2T & , \quad \bar{t} \in [T+2, T+3] \end{cases}$$

Find:

1. The maximum speed.
2. The position at $\bar{t} = 3$.
3. T such that the position at $\bar{t} = T + 3$ is $s = 40\mathrm{m}$.
4. The maximum jerk.

Problem 4

A UR-5 robot is mounted on a table. A task coordinate system (Task1) is defined by three points on the table $P0$, Px and Py. $P0$ is the origo of the system and Px is a point on the x-axis and Py is a point on the y-axis. The coordinate system is right handed with the z-axis going up from the table. The coordinates of the points in the robot basesystem are found by moving the TCP to each of the points and read the coordinates from the controller. The coordinates are:
$P0 = [0.2500, 0.2500, 0.1]$ meter
$Px = [0.4526, 0.3357, 0.1]$ meter
$Py = [0.0942, 0.6184, 0.1]$ meter

Question 1
Find the homogeneous transformation from taskspace to basespace T^B_{Task1}.

The robot has a camera mounted at its end-effector. The camera configuration w.r.t the robot base is defined by the homogeneous transformation:

$$T^B_C = \begin{bmatrix} 1 & 0 & 0 & 0.3 \\ 0 & -1 & 0 & 0.1 \\ 0 & 0 & -1 & 0.9 \\ 0 & 0 & 0 & 1 \end{bmatrix} \tag{1}$$

An object is located at the following coordinates w.r.t. the base coordinate system of the robot: $P_o = [0.1; 0.1; 0.1]$

Question 2
Define the 3D position of the point P_o in the camera frame.

Given that the intrinsic parameters of the camera are:

principal point: [320,240] pixel
focal length: fx=fy=500 pixel

Question 3
Find the pixel coordinates of P_o

Problem 5

An electric motor is connected to a mechanical load through a gearbox with transmission ration $n = 1/50$, meaning that given a motor rotational speed $\dot{\theta}_m$, the load rotates with speed $\dot{\theta}_l = n\dot{\theta}_m$. Assuming that the motor electrical dynamics is much faster then the mechanical dynamics, let's consider the electrical dynamic governed by the equation:

$$i_a = (V_{in} - K_b\dot{\theta}_m)/R$$

The dynamic equations governing the motor dynamics can therefore be simplified as:

$$J_m\ddot{\theta}_m + D_m\dot{\theta}_m = \tau_m - n\tau_l$$

where τ_l is an external disturbance applied at the load side of the transmission and

$$\tau_m = K_m i_a$$

Question 1
Derive the transfer function $\dot{\theta}_m/V_{in}$, given that:

$$K_m = 0.7\text{kg}\frac{\text{m}^2}{\text{s}^2\text{A}} \quad K_b = 0.7\frac{\text{Vs}}{\text{rad}} \quad R = 100\frac{\text{V}}{\text{A}} \quad J_m = 0.01\text{kg}\frac{\text{m}^2}{\text{rad}} \quad D_m = 0.003\text{kg}\frac{\text{m}^2}{\text{rad}\cdot\text{s}}$$

Close a position control loop, using a proportional controller that defines the control input V_{in}

$$V_{in} = K_P e_{\theta_m}$$

where the error is defined by $e_{\theta_m} = \theta_m^\star - \theta_m$ and where $\theta_m^\star$ and θ_m represent the reference signal and the actual motor position, respectively, and given that:

$$K_P = 1.2\frac{\text{V}}{\text{rad}}$$

Question 2
For a step reference input $\theta_m^\star = 1.0\text{rad}$ and constant disturbance $\tau_l = 0.0\text{Nm}$ determine the steady-state error e_{ss}.

Question 3
For a step reference input $\theta_m^\star = 1.5\text{rad}$ and constant disturbance $\tau_l = 0.1\text{Nm}$, determine the steady-state error e_{ss}.

```
> restart; with(linalg);
```

[BlockDiagonal, GramSchmidt, JordanBlock, LUdecomp, QRdecomp, Wronskian, addcol, addrow, adj, adjoint, angle, augment, backsub, band, basis, bezout, blockmatrix, charmat, charpoly, cholesky, col, coldim, colspace, colspan, companion, concat, cond, copyinto, crossprod, curl, definite, delcols, delrows, det, diag, diverge, dotprod, eigenvals, eigenvalues, eigenvectors, eigenvects, entermatrix, equal, exponential, extend, ffgausselim, fibonacci, forwardsub, frobenius, gausselim, gaussjord, geneqns, genmatrix, grad, hadamard, hermite, hessian, hilbert, htranspose, ihermite, indexfunc, innerprod, intbasis, inverse, ismith, issimilar, iszero, jacobian, jordan, kernel, laplacian, leastsqrs, linsolve, matadd, matrix, minor, minpoly, mulcol, mulrow, multiply, norm, normalize, nullspace, orthog, permanent, pivot, potential, randmatrix, randvector, rank, ratform, row, rowdim, rowspace, rowspan, rref, scalarmul, singularvals, smith, stackmatrix, submatrix, subvector, sumbasis, swapcol, swaprow, sylvester, toeplitz, trace, transpose, vandermonde, vecpotent, vectdim, vector, wronskian] **(1)**

```
> #31383 Ex2020 Problem 1;
> # Question 1;
```

$> \; d\_1 := \frac{1}{3};$

$$d\_1 := \frac{1}{3} \quad \textbf{(2)}$$

$> \; the\_1 := \arctan\left(\sqrt{3}, 1\right); \; \frac{\% \cdot 180}{\mathrm{Pi}};$

$$the\_1 := \frac{\pi}{3}$$

$$60 \quad \textbf{(3)}$$

```
>
```

$> \; alp\_1 := \arctan\left(-\frac{3}{4 \cdot \sin(the\_1)}, \frac{1}{4 \cdot \cos(the\_1)}\right); \; \frac{\% \cdot 180}{\mathrm{Pi}};$

$$alp\_1 := -\frac{\pi}{3}$$

$$-60 \quad \textbf{(4)}$$

$> \; a\_1 := \frac{1}{\cos(the\_1)};$

$$a\_1 := 2 \quad \textbf{(5)}$$

```
> # Question 2;
```

$> \; Rot\_z\_theta := Matrix([[\cos(the), -\sin(the), 0, 0], [\sin(the), \cos(the), 0, 0], [0, 0, 1, 0], [0, 0, 0, 1]])$

$$Rot\_z\_theta := \begin{bmatrix} \cos(the) & -\sin(the) & 0 & 0 \\ \sin(the) & \cos(the) & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \quad \textbf{(6)}$$

$> \; Trans\_z\_d := Matrix([[1, 0, 0, 0], [0, 1, 0, 0], [0, 0, 1, d], [0, 0, 0, 1]])$

$$Trans\_z\_d := \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & d \\ 0 & 0 & 0 & 1 \end{bmatrix} \tag{7}$$

> $Trans\_x\_a := Matrix([[1, 0, 0, a], [0, 1, 0, 0], [0, 0, 1, 0], [0, 0, 0, 1]])$

$$Trans\_x\_a := \begin{bmatrix} 1 & 0 & 0 & a \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \tag{8}$$

> $Rot\_x\_alpha := Matrix([[1, 0, 0 , 0], [0, \cos(alp), -\sin(alp), 0], [0, \sin(alp), \cos(alp), 0,],$
> $[0, 0, 0, 1]])$

$$Rot\_x\_alpha := \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & \cos(alp) & -\sin(alp) & 0 \\ 0 & \sin(alp) & \cos(alp) & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \tag{9}$$

> $DH := (Rot\_z\_theta.Trans\_z\_d.Trans\_x\_a.Rot\_x\_alpha);$

$$DH := \begin{bmatrix} \cos(the) & -\sin(the)\cos(alp) & \sin(the)\sin(alp) & \cos(the)\, a \\ \sin(the) & \cos(the)\cos(alp) & -\cos(the)\sin(alp) & \sin(the)\, a \\ 0 & \sin(alp) & \cos(alp) & d \\ 0 & 0 & 0 & 1 \end{bmatrix} \tag{10}$$

> $eval(DH, \{d = d\_1, the = the\_1, a = a\_1, alp = alp\_1\});$

$$\begin{bmatrix} \frac{1}{2} & -\frac{\sqrt{3}}{4} & -\frac{3}{4} & 1 \\ \frac{\sqrt{3}}{2} & \frac{1}{4} & \frac{\sqrt{3}}{4} & \sqrt{3} \\ 0 & -\frac{\sqrt{3}}{2} & \frac{1}{2} & \frac{1}{3} \\ 0 & 0 & 0 & 1 \end{bmatrix} \tag{11}$$

> *#31383 Ex2020 problem 2*

> *# Question 1;*

>

> $R10 := Matrix([[0, 1, 0], [0, 0, 1], [1, 0, 0]]);$

$$R10 := \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 1 & 0 & 0 \end{bmatrix} \tag{12}$$

> $R1 := Matrix([[1, 0, 0], [0, \cos(t1(t)), -\sin(t1(t))], [0, \sin(t1(t)), \cos(t1(t))]]); R2 := Matrix([[\cos(t2(t)), 0, \sin(t2(t))], [0, 1, 0], [-\sin(t2(t)), 0, \cos(t2(t))]]); R3 := Matrix([[\cos(t3(t)), -\sin(t3(t)), 0], [\sin(t3(t)), \cos(t3(t)), 0], [0, 0, 1]]);$

$$R1 := \begin{bmatrix} 1 & 0 & 0 \\ 0 & \cos(t1(t)) & -\sin(t1(t)) \\ 0 & \sin(t1(t)) & \cos(t1(t)) \end{bmatrix}$$

$$R2 := \begin{bmatrix} \cos(t2(t)) & 0 & \sin(t2(t)) \\ 0 & 1 & 0 \\ -\sin(t2(t)) & 0 & \cos(t2(t)) \end{bmatrix}$$

$$R3 := \begin{bmatrix} \cos(t3(t)) & -\sin(t3(t)) & 0 \\ \sin(t3(t)) & \cos(t3(t)) & 0 \\ 0 & 0 & 1 \end{bmatrix} \tag{13}$$

> $R21 := eval(R1 \cdot R2 \cdot R3);$

$$R21 := [[\cos(t2(t)) \cos(t3(t)), -\cos(t2(t)) \sin(t3(t)), \sin(t2(t))], [\sin(t1(t)) \sin(t2(t)) \cos(t3(t)) + \cos(t1(t)) \sin(t3(t)), -\sin(t1(t)) \sin(t2(t)) \sin(t3(t)) + \cos(t1(t)) \cos(t3(t)), -\sin(t1(t)) \cos(t2(t))], [-\cos(t1(t)) \sin(t2(t)) \cos(t3(t)) + \sin(t1(t)) \sin(t3(t)), \cos(t1(t)) \sin(t2(t)) \sin(t3(t)) + \sin(t1(t)) \cos(t3(t)), \cos(t1(t)) \cos(t2(t))]] \tag{14}$$

> $R20 := eval(R10 \cdot R21);$

$$R20 := [[\sin(t1(t)) \sin(t2(t)) \cos(t3(t)) + \cos(t1(t)) \sin(t3(t)), -\sin(t1(t)) \sin(t2(t)) \sin(t3(t)) + \cos(t1(t)) \cos(t3(t)), -\sin(t1(t)) \cos(t2(t))], [-\cos(t1(t)) \sin(t2(t)) \cos(t3(t)) + \sin(t1(t)) \sin(t3(t)), \cos(t1(t)) \sin(t2(t)) \sin(t3(t)) + \sin(t1(t)) \cos(t3(t)), \cos(t1(t)) \cos(t2(t))], [\cos(t2(t)) \cos(t3(t)), -\cos(t2(t)) \sin(t3(t)), \sin(t2(t))]] \tag{15}$$

> # Question 2;

> $DR20 := diff(R20, t);$

$$DR20 := \Bigg[\Bigg[\left(\frac{d}{dt} t1(t)\right) \cos(t1(t)) \sin(t2(t)) \cos(t3(t)) + \sin(t1(t)) \left(\frac{d}{dt} t2(t)\right) \cos(t2(t)) \cos(t3(t)) - \sin(t1(t)) \sin(t2(t)) \left(\frac{d}{dt} t3(t)\right) \sin(t3(t)) - \left(\frac{d}{dt} t1(t)\right) \sin(t1(t)) \sin(t3(t)) + \cos(t1(t)) \left(\frac{d}{dt} t3(t)\right) \cos(t3(t)), -\left(\frac{d}{dt} t1(t)\right) \cos(t1(t)) \sin(t2(t)) \sin(t3(t)) - \sin(t1(t)) \left(\frac{d}{dt} t2(t)\right) \cos(t2(t)) \sin(t3(t)) - \sin(t1(t)) \sin(t2(t)) \left(\frac{d}{dt} t3(t)\right) \cos(t3(t)) - \left(\frac{d}{dt} t1(t)\right) \sin(t1(t)) \cos(t3(t)) - \cos(t1(t)) \left(\frac{d}{dt} t3(t)\right) \sin(t3(t)), -\left(\frac{d}{dt} t1(t)\right) \cos(t1(t)) \cos(t2(t)) + \sin(t1(t)) \left(\frac{d}{dt} t2(t)\right) \sin(t2(t))\Bigg], \tag{16}$$

$$\left[\left(\frac{d}{dt}\,t1(t)\right)\sin(t1(t))\sin(t2(t))\cos(t3(t)) - \cos(t1(t))\left(\frac{d}{dt}\,t2(t)\right)\cos(t2(t))\cos(t3(t)) + \cos(t1(t))\sin(t2(t))\left(\frac{d}{dt}\,t3(t)\right)\sin(t3(t)) + \left(\frac{d}{dt}\,t1(t)\right)\cos(t1(t))\sin(t3(t)) + \sin(t1(t))\left(\frac{d}{dt}\,t3(t)\right)\cos(t3(t)), -\left(\frac{d}{dt}\,t1(t)\right)\sin(t1(t))\sin(t2(t))\sin(t3(t)) + \cos(t1(t))\left(\frac{d}{dt}\,t2(t)\right)\cos(t2(t))\sin(t3(t)) + \cos(t1(t))\sin(t2(t))\left(\frac{d}{dt}\,t3(t)\right)\cos(t3(t)) + \left(\frac{d}{dt}\,t1(t)\right)\cos(t1(t))\cos(t3(t)) - \sin(t1(t))\left(\frac{d}{dt}\,t3(t)\right)\sin(t3(t)), -\left(\frac{d}{dt}\,t1(t)\right)\sin(t1(t))\cos(t2(t)) - \cos(t1(t))\left(\frac{d}{dt}\,t2(t)\right)\sin(t2(t))\right],$$

$$\left[-\left(\frac{d}{dt}\,t2(t)\right)\sin(t2(t))\cos(t3(t)) - \cos(t2(t))\left(\frac{d}{dt}\,t3(t)\right)\sin(t3(t)), \left(\frac{d}{dt}\,t2(t)\right)\sin(t2(t))\sin(t3(t)) - \cos(t2(t))\left(\frac{d}{dt}\,t3(t)\right)\cos(t3(t)), \left(\frac{d}{dt}\,t2(t)\right)\cos(t2(t))\right]\Big]$$

> $E := simplify(DR20 \cdot transpose(R20));$

$$E := \left[\left[0, -\sin(t2(t))\left(\frac{d}{dt}\,t3(t)\right) - \frac{d}{dt}\,t1(t), \cos(t1(t))\cos(t2(t))\left(\frac{d}{dt}\,t3(t)\right) + \sin(t1(t))\left(\frac{d}{dt}\,t2(t)\right)\right],\right.$$
$$\left[\sin(t2(t))\left(\frac{d}{dt}\,t3(t)\right) + \frac{d}{dt}\,t1(t), 0, \sin(t1(t))\cos(t2(t))\left(\frac{d}{dt}\,t3(t)\right) - \cos(t1(t))\left(\frac{d}{dt}\,t2(t)\right)\right],$$
$$\left.\left[-\cos(t1(t))\cos(t2(t))\left(\frac{d}{dt}\,t3(t)\right) - \sin(t1(t))\left(\frac{d}{dt}\,t2(t)\right), -\sin(t1(t))\cos(t2(t))\left(\frac{d}{dt}\,t3(t)\right) + \cos(t1(t))\left(\frac{d}{dt}\,t2(t)\right), 0\right]\right] \tag{17}$$

> $omega\_x := E[3, 2]; omega\_y := E[1, 3]; omega\_z := E[2, 1];$

$$omega\_x := -\sin(t1(t))\cos(t2(t))\left(\frac{d}{dt}\,t3(t)\right) + \cos(t1(t))\left(\frac{d}{dt}\,t2(t)\right)$$
$$omega\_y := \cos(t1(t))\cos(t2(t))\left(\frac{d}{dt}\,t3(t)\right) + \sin(t1(t))\left(\frac{d}{dt}\,t2(t)\right)$$
$$omega\_z := \sin(t2(t))\left(\frac{d}{dt}\,t3(t)\right) + \frac{d}{dt}\,t1(t) \tag{18}$$

>

> *#31383 Ex2020 Problem 3;*

> *# Question 1 and 2;*

> $a1 := 2 \cdot t;$

$$a1 := 2\,t \tag{19}$$

> $v1 := integrate(a1, t) + c0;$

$$v1 := t^2 + c0 \tag{20}$$

> $s1 := integrate(v1, t) + c1;$

$$s1 := \frac{1}{3}\,t^3 + c0\,t + c1 \tag{21}$$

> $solve(\{t=0, v1=0, s1=0\}, \{t, c0, c1\});$

$$\{c0=0, c1=0, t=0\} \tag{22}$$

> $v11 := eval(v1, \{c0=0, c1=0, t=1\}); s11 := eval(s1, \{c0=0, c1=0, t=1\})$

$$v11 := 1$$

$$s11 := \frac{1}{3} \tag{23}$$

> $a2 := 3 - t;$

$$a2 := 3 - t \tag{24}$$

> $v2 := integrate(a2, t) + c0;$

$$v2 := 3\,t - \frac{1}{2}\,t^2 + c0 \tag{25}$$

> $s2 := integrate(v2, t) + c1;$

$$s2 := \frac{3}{2}\,t^2 - \frac{1}{6}\,t^3 + c0\,t + c1 \tag{26}$$

> $solve(\{t=1, v2=v11, s2=s11\}, \{t, c0, c1\})$

$$\left\{c0 = -\frac{3}{2}, c1 = \frac{1}{2}, t=1\right\} \tag{27}$$

> $v21 := eval\left(v2, \left\{c0 = -\frac{3}{2}, c1 = \frac{1}{2}, t=3\right\}\right); s21 := eval\left(s2, \left\{c0 = -\frac{3}{2}, c1 = \frac{1}{2}, t=3\right\}\right)$

$$v21 := 3$$

$$s21 := 5 \tag{28}$$

> #position at t=3 is therefore 5 meters, and the maximum speed is 3 $\frac{meters}{sek}$

> #Question 3;

>

> $a3 := 0;$

$$a3 := 0 \tag{29}$$

> $v3 := integrate(a3, t) + c0;$

$$v3 := c0 \tag{30}$$

> $s3 := integrate(v3, t) + c1;$

$$s3 := c0\,t + c1 \tag{31}$$

> $solve(\{t=3, v3=v21, s3=s21\}, \{t, c0, c1\})$

$$\{c0=3, c1=-4, t=3\} \tag{32}$$

> $v31 := eval(v3, \{c0=3, c1=-4, t=T\}); s31 := eval(s3, \{c0=3, c1=-4, t=T\})$

$$v31 := 3$$

$$s31 := 3\,T - 4 \tag{33}$$

> $a4 := T - t;$

$$a4 := T - t \tag{34}$$

> $v4 := integrate(a4, t) + c0;$

$$v4 := T\,t - \frac{1}{2}\,t^2 + c0 \tag{35}$$

> $s4 := integrate(v4, t) + c1;$

$$s4 := \frac{1}{2}\,T\,t^2 - \frac{1}{6}\,t^3 + c0\,t + c1 \tag{36}$$

> $solve(\{t = T, v4 = v31, s4 = s31\}, \{t, c0, c1\})$

$$\left\{c0 = -\frac{T^2}{2} + 3,\ c1 = \frac{T^3}{6} - 4,\ t = T\right\} \tag{37}$$

> $v41 := eval\left(v4, \left\{c0 = -\frac{T^2}{2} + 3, c1 = \frac{T^3}{6} - 4, t = T + 2\right\}\right); s41 := eval\left(s4, \left\{c0 = -\frac{T^2}{2} + 3, c1 = \frac{T^3}{6} - 4, t = T + 2\right\}\right)$

$$v41 := T\,(T + 2) - \frac{(T + 2)^2}{2} - \frac{T^2}{2} + 3$$

$$s41 := \frac{T\,(T + 2)^2}{2} - \frac{(T + 2)^3}{6} + \left(-\frac{T^2}{2} + 3\right)(T + 2) + \frac{T^3}{6} - 4 \tag{38}$$

> $a5 := 2 \cdot t - 6 - 2 \cdot T;$

$$a5 := 2\,t - 6 - 2\,T \tag{39}$$

> $v5 := integrate(a5, t) + c0;$

$$v5 := -2\,T\,t + t^2 + c0 - 6\,t \tag{40}$$

> $s5 := integrate(v5, t) + c1;$

$$s5 := -T\,t^2 + \frac{1}{3}\,t^3 + c0\,t - 3\,t^2 + c1 \tag{41}$$

> $solve(\{t = T + 2, v5 = v41, s5 = s41\}, \{t, c0, c1\});$

$$\left\{c0 = T^2 + 6\,T + 9,\ c1 = -\frac{1}{3}\,T^3 - 3\,T^2 - 6\,T - 8,\ t = T + 2\right\} \tag{42}$$

> $eval\left(s5, \left\{c0 = T^2 + 6\,T + 9, c1 = -\frac{1}{3}\,T^3 - 3\,T^2 - 6\,T - 8, t = T + 3\right\}\right)$

$$-T\,(T + 3)^2 + \frac{(T + 3)^3}{3} + (T^2 + 6\,T + 9)\,(T + 3) - 3\,(T + 3)^2 - \frac{T^3}{3} - 3\,T^2 - 6\,T - 8 \tag{43}$$

> $evalf(-2\,T\,(T + 3) + (T + 3)^2 + T^2 - 9);$

$$-2.\,T\,(T + 3.) + (T + 3.)^2 + T^2 - 9. \tag{44}$$

> $solve\left(-T\,(T + 3)^2 + \frac{(T + 3)^3}{3} + (T^2 + 6\,T + 9)\,(T + 3) - 3\,(T + 3)^2 - \frac{T^3}{3} - 3\,T^2 - 6\,T - 8 = 40, T\right);$

$$13 \tag{45}$$

> # T=13;

> #Question 4;

> # Maximum jerk is $2\,\frac{meters}{sek^3}$

>
>
>
>

problem 4.1

```
0.9210   -0.3895        0    0.2500
0.3896    0.9210        0    0.2500
     0         0   1.0000    0.1000
     0         0        0    1.0000
```

problem 4.2:

POC =

```
 -0.2000
       0
  0.8000
```

problem 4.3:

pOC =

```
  445   240
```

pOC =

```
  195   240
```

----

problem 5.1

joint =

```
      0.007
  ---------------
  0.01 s + 0.0079
```

Continuous-time transfer function.

problem 5.1-2

joint2 =

```
    0.7
  --------
  s + 0.79
```

Continuous-time transfer function.

problem 5.2

-4.4409e-16

problem 5.3

0.2381

TECHNICAL UNIVERSITY OF DENMARK

Written test, date **Dec 16, 2020**

Course name: **Robotics**

Course number: **31383**

Aids allowed: **All according to DTU regulations.**

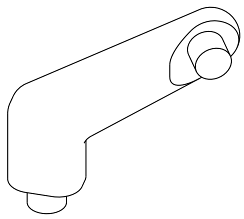
Exam duration: **4 Hours**

Weighting:

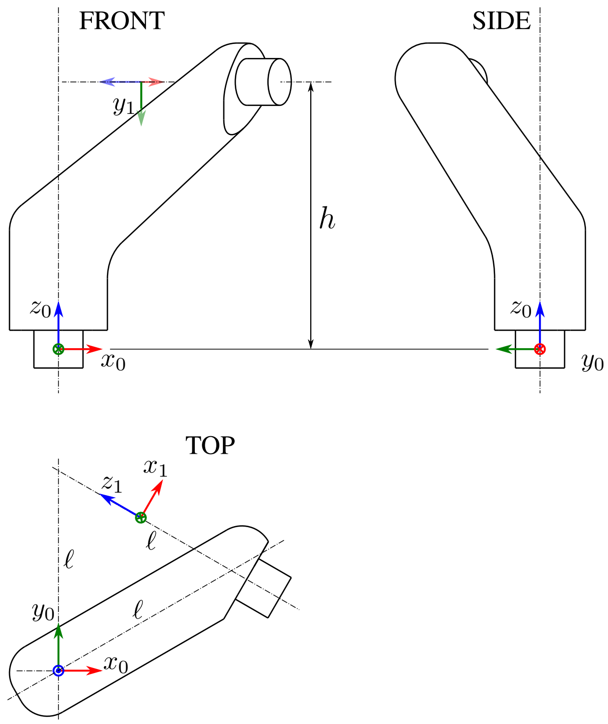
Problem 1 counts for 10%
Problem 2 counts for 20%
Problem 3 counts for 20%
Problem 4 counts for 25%
Problem 5 counts for 25%

Problem 1

The figure below shows a bracket used as the first link in a robotic arm manipulator.



Frames 0 and 1 are assigned, according to the Denavit-Hartenberg convention, as shown in the front, side and top views below.



The essential dimensions of the bracket are defined by means of the height $h = 400$ mm, shown in the front view, and the side length $\ell = 200$ mm of the equilateral triangle, shown in the top view.

Question 1
Determine the Denavit-Hartenberg parameters θ, d, a and α for joint 1, in the configuration shown in the figure.

Question 2
Calculate the homogeneous transformation matrix T_1^0 for the configuration shown in the figure.

Problem 2

A four-joint robotic arm manipulator is defined by the following Denavit-Hartenberg parameters

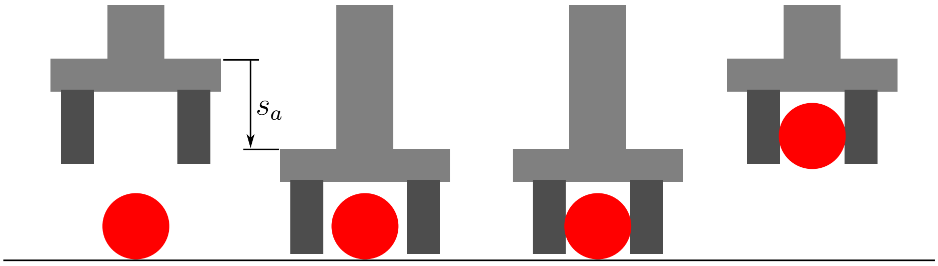
Joint	Type	θ	d [mm]	a [mm]	α
1	Rev	θ_1	100	0	90°
2	Prism	90°	d_2	0	90°
3	Rev	θ_3	0	60	90°
4	Rev	θ_4	0	0	90°

Question 1
Find the homogeneous transformation matrix T_4^0 that describes the placement of the end-effector for the configuration with $\theta_1 = 135°$, $d_2 = 200\,\text{mm}$, $\theta_3 = 45°$ and $\theta_4 = -60°$.

Question 2
In the same configuration, find the linear velocity of the end-effector v_4^0 for $\dot{\theta}_1 = 0.0025\pi\,\text{rad}/\text{sec}$ and all other joints locked, i.e. $\dot{d}_2 = 0$ and $\dot{\theta}_3 = \dot{\theta}_4 = 0$.

Problem 3

A prismatic joint performs an approach motion of a robot end-effector against a running production band. There is a need for minimizing the time t_a it takes for the grip to cover the distance s_a, shown in the figure below.



Assuming an acceleration function in the form

$$a(t) = \ddot{s}(t) = a_{\max} \sin(2\pi t/t_a)$$

a position function $s(t)$ is sought that fulfills the following terminal conditions:

$$s(0) = 0\,\mathrm{m}$$
$$\dot{s}(0) = 0\,\mathrm{m/s}$$
$$s(t_a) = s_a$$

Answer the following questions.

Question 1
Determine the position function $s(t)$, the velocity function $v(t) = \dot{s}(t)$, and the jerk function $j(t) = \dot{a}(t)$, assuming that $a_{\max}$ and t_a are known.

Question 2
For $s_a = 0.03\,\mathrm{m}$, find the minimum approach time t_a in seconds in order for the acceleration not to exceed the limit of $100g$, with $g = 9.82\,\mathrm{m/s^2}$.

Question 3
What is the maximum velocity and maximum jerk, in $\mathrm{m/s}$ and $\mathrm{m/s^3}$, respectively?

Problem 4

A UR-5 robot is mounted on a table. A task coordinate system (Task1) is defined by three points on the table $P0$, Px and Py. $P0$ is the origo of the system and Px is a point on the x-axis and Py is a point on the y-axis. The coordinate system is right handed with the z-axis going up from the table. The coordinates of the points in the robot basesystem are found by moving the TCP to each of the points and read the coordinates from the controller. The coordinates are:

$P0$ = [0.2500, 0.2500, 0.1500] m
Px = [0.4526, 0.3357, 0.1500] m
Py = [0.0942, 0.6184, 0.1500] m

Question 1
Find the homogeneous transformation from taskspace to basespace T^B_{Task1}.

A second taskspace Task2 is defined by 3 points in the table plane $P0$, $P1$ and $P2$. $P0$ is the origo of the system and $P1$ is a point on the x-axis and $P2$ is a point in the x-y plane. The coordinate system is right handed with the z-axis defined by the cross-product $P0\vec{P}1 \times P0\vec{P}2$ The coordinates are:

$P0$ = [0.4000; 0.2000; 0.1500] m
$P1$ = [0.6388; 0.2240; 0.1500] m
$P2$ = [0.5321; 0.2903; 0.1500] m

Question 2
Find the transformation from taskspace Task2 to basespace T^B_{Task2}

The robot has a stereo camera mounted on its end-effector. The stereo camera system consists of two cameras that share a common field of view (see Figure 1). The two cameras are placed at

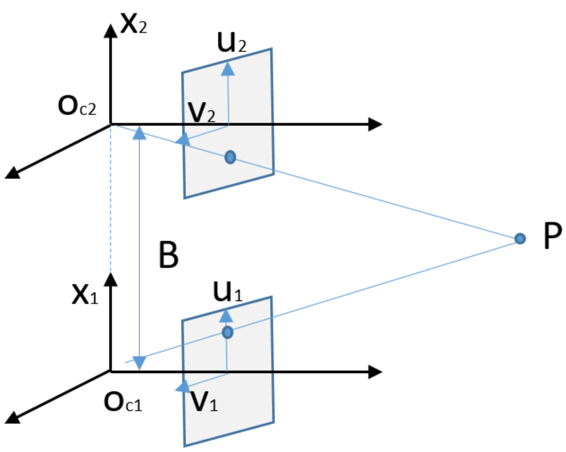


Figure 1: stereo camera system.

coordinate frames $o_{c1}x_{c1}y_{c1}z_{c1}$ and $o_{c2}x_{c2}y_{c2}z_{c2}$, both located in the center of the camera plane and with focal axis aligned with the z-direction of the two coordinate frames, such that

$$T_{c2}^{c1} = \begin{bmatrix} 1 & 0 & 0 & B \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

being $B = 0.1\,\mathrm{m}$ the baseline between the two cameras. The focal length for both the cameras is $\lambda = 1\,\mathrm{m}$. A 3D point P projects onto the two images with image plane coordinates $(u_1, v_1) = (0.05, 0)$ for the first camera, and $(u_2, v_2) = (-0.05, 0)$ for the second one.

Question 3

3.1) Determine the depth (z) of point P from the two cameras.
Note: as the translation along the z-axis between the cameras is zero, $z_1 = z_2$

3.2) Determine the distance vector from point P to $o_{c1}x_{c1}y_{c1}z_{c1}$

3.3) Considering that the configuration of camera 1 with respect to the robot base is defined by the homogeneous transformation:

$$T_{c1}^{B} = \begin{bmatrix} 1 & 0 & 0 & 0.4 \\ 0 & -1 & 0 & 0.2 \\ 0 & 0 & -1 & 1.0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

Define the 3D position of the point P in the base frame.

Problem 5

With reference to Figure 2, consider a robot manipulator's joint consisting of a direct drive electric motor, whose dynamics described by the transfer function $G(s)$. The motor can be actuated by a control input τ_m, while the load can be described by an external torque τ_l. The joint is position controlled, meaning that the input torque τ_m is defined to track the position reference $q^\star$, using a PI (Proportional-Integral) feedback control loop that is defined by transfer function $K(s)$.

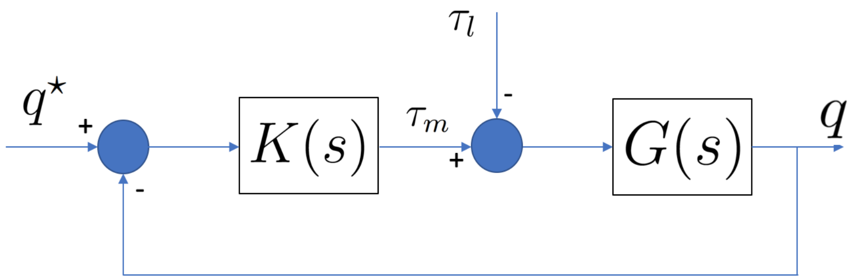


Figure 2: Block scheme for direct drive robot joint.

Problem data:

- $G(s) = \dfrac{1}{0.2s^2 + 0.8s}$
- $K(s) = k_p + \dfrac{k_i}{s}$
- $k_p = 0.5$, $k_i = 0.1$

Question 1
Derive the transfer function in the form

$$q(s) = \frac{F(s)q^\star(s) - H(s)\tau_l(s)}{N(s)}$$

Question 2
For a step reference input with step value $q^\star = 1.0\,\mathrm{rad}$, and constant disturbance $\tau_l = 0.1\mathrm{Nm}$, determine the steady-state error e_{ss}.

```
fpprintprec : 4$
```

Problem 1

1 D-H parameters

```
h:400;
l:200;
```

$$400$$
$$200$$

```
theta: %pi/3;
d: h;
a: l·sin(%pi/3);
alpha:−%pi/2;
```

$$\frac{\pi}{3}$$
$$400$$
$$100\sqrt{3}$$
$$-\frac{\pi}{2}$$

2 Homogeneous transformation

```
T_01: matrix(
 [cos(theta),−sin(theta)·cos(alpha),sin(theta)·sin(alpha),a·cos(theta)],
 [sin(theta),cos(theta)·cos(alpha),−cos(theta)·sin(alpha),a·sin(theta)],
 [0,sin(alpha),cos(alpha),d],
 [0,0,0,1]
);
```

$$\begin{pmatrix} \frac{1}{2} & 0 & -\frac{\sqrt{3}}{2} & 50\sqrt{3} \\ \frac{\sqrt{3}}{2} & 0 & \frac{1}{2} & 150 \\ 0 & -1 & 0 & 400 \\ 0 & 0 & 0 & 1 \end{pmatrix}$$

```
float(T_01);
```

$$\begin{pmatrix} 0.5 & 0.0 & -0.866 & 86.6 \\ 0.866 & 0.0 & 0.5 & 150.0 \\ 0.0 & -1.0 & 0.0 & 400.0 \\ 0.0 & 0.0 & 0.0 & 1.0 \end{pmatrix}$$

Problem 2

1 Homogeneous transformation

```
theta_2:90·%pi/180$
theta_3:45·%pi/180$
theta_4:−60·%pi/180$
d_1:100$
d_2:200$
d_3:0$
d_4:0$
a_1:0$
a_2:0$
a_3:60$
a_4:0$
alpha_1:90·%pi/180$
alpha_2:90·%pi/180$
alpha_3:90·%pi/180$
alpha_4:90·%pi/180$

T(theta,d,a,alpha):= matrix(
 [cos(theta),−sin(theta)·cos(alpha),sin(theta)·sin(alpha),a·cos(theta)],
 [sin(theta),cos(theta)·cos(alpha),−cos(theta)·sin(alpha),a·sin(theta)],
 [0,sin(alpha),cos(alpha),d],
 [0,0,0,1]
);
```

$$T(theta,d,a,alpha):=\begin{pmatrix} \cos(theta) & (-\sin(theta))\cos(alpha) & \sin(theta)\sin(alpha) & a\cos(theta) \\ \sin(theta) & \cos(theta)\cos(alpha) & (-\cos(theta))\sin(alpha) & a\sin(theta) \\ 0 & \sin(alpha) & \cos(alpha) & d \\ 0 & 0 & 0 & 1 \end{pmatrix}$$

```
T01:float(T(theta_1,d_1,a_1,alpha_1));
T12:float(T(theta_2,d_2,a_2,alpha_2));
T23:float(T(theta_3,d_3,a_3,alpha_3));
T34:float(T(theta_4,d_4,a_4,alpha_4));
```

$$\begin{pmatrix} \cos(theta_1) & 0.0 & \sin(theta_1) & 0.0 \\ \sin(theta_1) & 0.0 & -1.0\cos(theta_1) & 0.0 \\ 0.0 & 1.0 & 0.0 & 100.0 \\ 0.0 & 0.0 & 0.0 & 1.0 \end{pmatrix}$$

$$\begin{pmatrix} 0.0 & 0.0 & 1.0 & 0.0 \\ 1.0 & 0.0 & 0.0 & 0.0 \\ 0.0 & 1.0 & 0.0 & 200.0 \\ 0.0 & 0.0 & 0.0 & 1.0 \end{pmatrix}$$

$$\begin{pmatrix} 0.7071 & 0.0 & 0.7071 & 42.42 \\ 0.7071 & 0.0 & -0.7071 & 42.42 \\ 0.0 & 1.0 & 0.0 & 0.0 \\ 0.0 & 0.0 & 0.0 & 1.0 \end{pmatrix}$$

$$\begin{pmatrix} 0.5 & 0.0 & -0.866 & 0.0 \\ -0.866 & 0.0 & -0.5 & 0.0 \\ 0.0 & 1.0 & 0.0 & 0.0 \\ 0.0 & 0.0 & 0.0 & 1.0 \end{pmatrix}$$

```
TT(theta_1):=''(float(T01.T12.T23.T34));
```

$$TT(theta_1) := \begin{pmatrix} 0.3535\ \sin(theta_1) - 0.866\ \cos(theta_1) & -0.7071\ \sin(theta_1) & -0.6123\ \sin(theta_1) - 0.5\ \cos(theta_1) & 242.4\ \sin(theta_1) \\ -0.866\ \sin(theta_1) - 0.3535\ \cos(theta_1) & 0.7071\ \cos(theta_1) & 0.6123\ \cos(theta_1) - 0.5\ \sin(theta_1) & -242.4\ \cos(theta_1) \\ 0.3535 & 0.7071 & -0.6123 & 142.4 \\ 0.0 & 0.0 & 0.0 & 1.0 \end{pmatrix}$$

```
float(TT(135·%pi/180));
```

$$\begin{pmatrix} 0.8623 & -0.4999 & -0.07945 & 171.4 \\ -0.3623 & -0.4999 & -0.7865 & 171.4 \\ 0.3535 & 0.7071 & -0.6123 & 142.4 \\ 0.0 & 0.0 & 0.0 & 1.0 \end{pmatrix}$$

2 Linear velocity

```
dthetadt:0.0025·%pi;
```

$$0.0025\ \pi$$

```
dTTdtheta(theta_1):=''(diff(TT(theta_1),theta_1,1));
```

$$dTTdtheta(theta_1) := \begin{pmatrix} 0.866\ \sin(theta_1) + 0.3535\ \cos(theta_1) & -0.7071\ \cos(theta_1) & 0.5\ \sin(theta_1) - 0.6123\ \cos(theta_1) & 242.4\ \cos(theta_1) \\ 0.3535\ \sin(theta_1) - 0.866\ \cos(theta_1) & -0.7071\ \sin(theta_1) & -0.6123\ \sin(theta_1) - 0.5\ \cos(theta_1) & 242.4\ \sin(theta_1) \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix}$$

```
dTTdt:float(dTTdtheta(135·%pi/180)·dthetadt);
```

$$\begin{pmatrix} 0.002846 & 0.003926 & 0.006177 & -1.346 \\ 0.006773 & -0.003926 & -6.24\ 10^{-4} & 1.346 \\ 0.0 & 0.0 & 0.0 & 0.0 \\ 0.0 & 0.0 & 0.0 & 0.0 \end{pmatrix}$$

```
v_40:submatrix(4,dTTdt,1,2,3);
```

$$\begin{pmatrix} -1.346 \\ 1.346 \\ 0.0 \end{pmatrix}$$

Problem 3

1 Position, velocity and jerk functions

```
assume(t>0);assume(t_a>0);
```

$$[t > 0]$$
$$[t_a > 0]$$

```
a(t):=a_max·sin(2·%pi·t/t_a);
```

$$a(t) := a_{max} \sin\left(\frac{2\pi t}{t_a}\right)$$

```
v(t):=''(integrate(a(t), t,0,t));
```

$$v(t) := a_{max}\left(\frac{t_a}{2\pi} - \frac{\cos\left(\frac{2\pi t}{t_a}\right) t_a}{2\pi}\right)$$

```
s(t):=''(integrate(v(t), t,0,t));
```

$$s(t) := -\frac{a_{max}\left(\sin\left(\frac{2\pi t}{t_a}\right) t_a^{2} - 2\pi t\, t_a\right)}{4\pi^{2}}$$

```
j(t):=''(diff(a(t), t));
```

$$j(t) := \frac{2\pi a_{max} \cos\left(\frac{2\pi t}{t_a}\right)}{t_a}$$

```
j(0);j(t_a);
```

$$\frac{2\pi a_{max}}{t_a}$$

$$\frac{2\pi a_{max}}{t_a}$$

```
a(0);a(t_a);
```

$$0$$

$$0$$

```
v(0);v(t_a);
```

$$0$$

$$0$$

```
s(0);s(t_a);
```

$$0$$

$$\frac{a_{max}\, t_a^{2}}{2\pi}$$

2 Minimum approach time

```
s_a:0.03;
```

0.03

```
g:9.82;
```

9.82

```
a_max:100·g;
```

982.0

```
sol:solve(s(t_a)=s_a,t_a);
```

rat: replaced −0.03 by −3/100 = −0.03
rat: replaced 491.0 by 491/1 = 491.0

$$[t_a = -\frac{\sqrt{3}\sqrt{\pi}}{10\sqrt{491}}, t_a = \frac{\sqrt{3}\sqrt{\pi}}{10\sqrt{491}}]$$

```
t_a:float(rhs(sol[2]));
```

0.01385

3 Maximum velocity and maximum jerk

```
float(v(0));float(v(t_a/2));float(v(t_a));
```

0.0
4.33
0.0

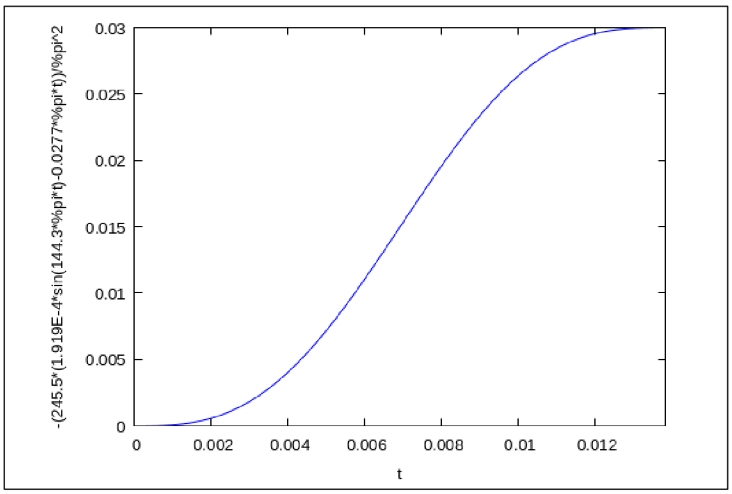
```
float(j(0));float(j(t_a/2));float(j(t_a));
```

$4.453\ 10^{5}$

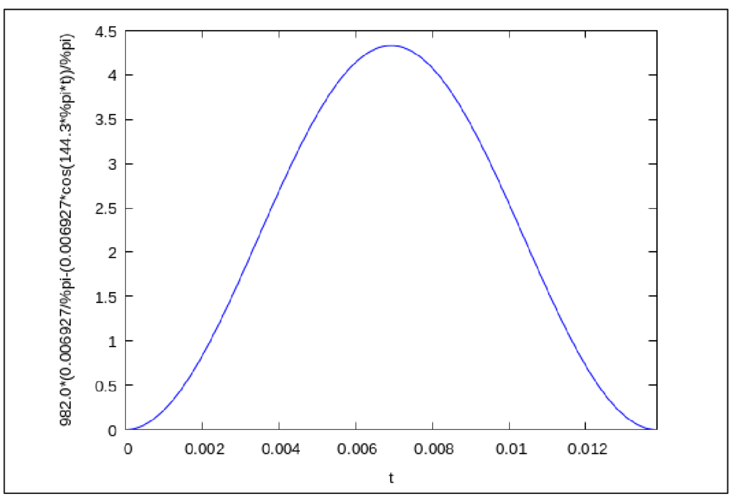
$-4.453\ 10^{5}$

$4.453\ 10^{5}$

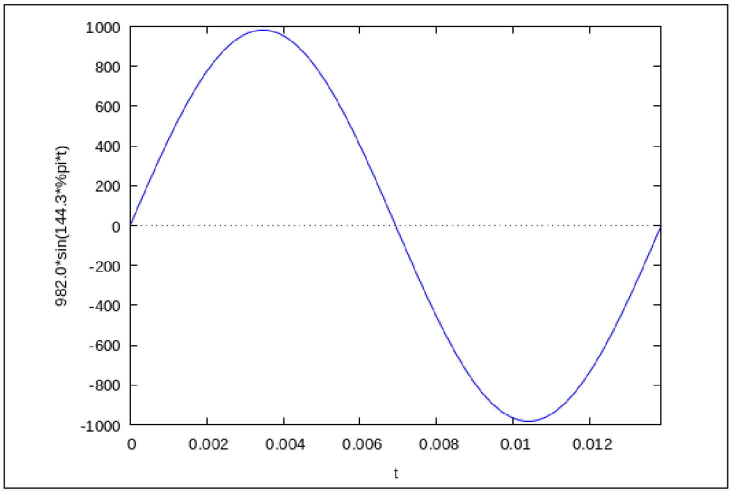
```
wxplot2d([s(t)], [t,0,t_a])$
```



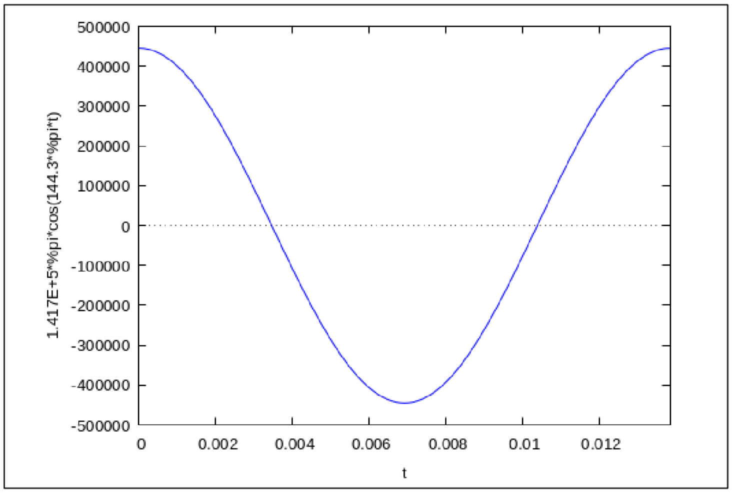

```
wxplot2d([v(t)], [t,0,t_a])$
```



```
wxplot2d([a(t)], [t,0,t_a])$
```




```
wxplot2d([j(t)], [t,0,t_a])$
```




```
T(theta0,d0,0,0);
```

$$\begin{pmatrix} \cos(theta0) & -\sin(theta0) & 0 & 0 \\ \sin(theta0) & \cos(theta0) & 0 & 0 \\ 0 & 0 & 1 & d0 \\ 0 & 0 & 0 & 1 \end{pmatrix}$$

```
T(theta0,d0,0,0).T(0,0,a0,alpha0);
```

$$\begin{pmatrix} \cos(theta0) & -\cos(alpha0)\sin(theta0) & \sin(alpha0)\sin(theta0) & a0\cos(theta0) \\ \sin(theta0) & \cos(alpha0)\cos(theta0) & -\sin(alpha0)\cos(theta0) & a0\sin(theta0) \\ 0 & \sin(alpha0) & \cos(alpha0) & d0 \\ 0 & 0 & 0 & 1 \end{pmatrix}$$

```
T(theta0,0,0,0).T(0,d0,0,0).T(0,0,a0,0).T(0,0,0,alpha0);
```

$$\begin{pmatrix} \cos(theta0) & -\cos(alpha0)\sin(theta0) & \sin(alpha0)\sin(theta0) & a0\cos(theta0) \\ \sin(theta0) & \cos(alpha0)\cos(theta0) & -\sin(alpha0)\cos(theta0) & a0\sin(theta0) \\ 0 & \sin(alpha0) & \cos(alpha0) & d0 \\ 0 & 0 & 0 & 1 \end{pmatrix}$$

```
%Problem 4.1

P0a=[0.25, 0.25, 0.15];
Pxa=[0.4526, 0.3357, 0.15];
Pya=[0.0942, 0.6184, 0.15];

vx=(Pxa-P0a);
vy=(Pya-P0a);
vx=vx/norm(vx);
vy=vy/norm(vy);
vz=cross(vx,vy);

Ttask0_B=[[vx' vy' vz' P0a'];0 0 0 1];
disp('problem 4.1')
```

```
problem 4.1
```

```
disp(Ttask0_B)
```

```
    0.9210   -0.3895         0    0.2500
    0.3896    0.9210         0    0.2500
         0         0    1.0000    0.1500
         0         0         0    1.0000
```

```
%Problem 4.2

P0 = [0.4000, 0.2000, 0.1500];
P1 = [0.6388    0.2240    0.1500];
P2 = [0.5321    0.2903    0.1500];

vx=(P1-P0);
vy=(P2-P0);
vx=vx/norm(vx);
vz=cross(vx,vy);
vz=vz/norm(vz);
vy=cross(vz,vx);
Ttask2_B=[[vx' vy' vz' P0'];0 0 0 1];
disp('problem 4.2')
```

```
problem 4.2
```

```
disp(Ttask2_B)
```

```
    0.9950   -0.1000         0    0.4000
    0.1000    0.9950         0    0.2000
         0         0    1.0000    0.1500
         0         0         0    1.0000
```

```
% Question 4.3
```

```
z1=1
```

```
z1 = 1
```

```
B=0.1
```

```
B = 0.1000
```

```
l1=1
```

```
l1 = 1
```

```
l2=1
```

```
l2 = 1
```

```
u1=0.05
```

```
u1 = 0.0500
```

```
v1=0
```

```
v1 = 0
```

```
u2=-0.05
```

```
u2 = -0.0500
```

```
v2=0
```

```
v2 = 0
```

```
% Question 4.3.1
%x for camera 1 (x1) is equal to x camera 2 (x2) minus B x1=x2-B
%x=z/l*u
%x2-x1=B -> I can compute z
z=B/(u1/l1-u2/l2)
```

```
z = 1
```

```
% Question 4.3.2
x1=u1*z/l1
```

```
x1 = 0.0500
```

```
y1=v1*z/l1
```

```
y1 = 0
```

```
z1=z
```

```
z1 = 1
```

```
PC1=[x1;y1;z1;1]
```

```
PC1 = 4×1
    0.0500
         0
    1.0000
    1.0000
```

```
%Question 4.3.3
TC1B=[1 0 0 0.4; 0 -1 0 0.2; 0 0 -1 1; 0 0 0 1]
```

```
TC1B = 4×4
    1.0000         0         0    0.4000
         0   -1.0000         0    0.2000
         0         0   -1.0000    1.0000
         0         0         0    1.0000
```

```
PB1=TC1B*PC1
```

```
PB1 = 4×1
    0.4500
    0.2000
         0
    1.0000
```

```
% Ex 5

% Question 5.1\\

kp=0.5;
ki=0.1;

G1=tf(1,[0.2 0.8 0])
```

```
G1 =

         1
  ---------------
  0.2 s^2 + 0.8 s

Continuous-time transfer function.
Model Properties
```

```
K=tf([kp ki],[1 0])
```

```
K =

  0.5 s + 0.1
  -----------
       s

Continuous-time transfer function.
Model Properties
```

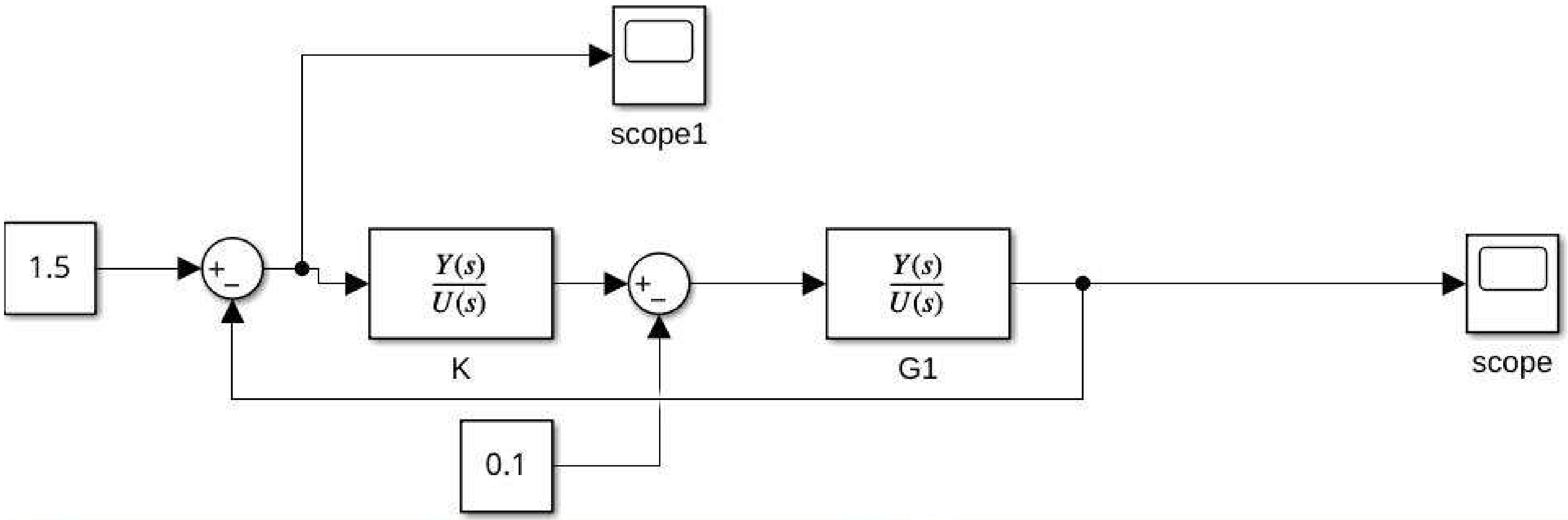
```
L1=feedback(K*G1,1)
```

```
L1 =

           0.5 s + 0.1
  ------------------------------
  0.2 s^3 + 0.8 s^2 + 0.5 s + 0.1

Continuous-time transfer function.
Model Properties
```

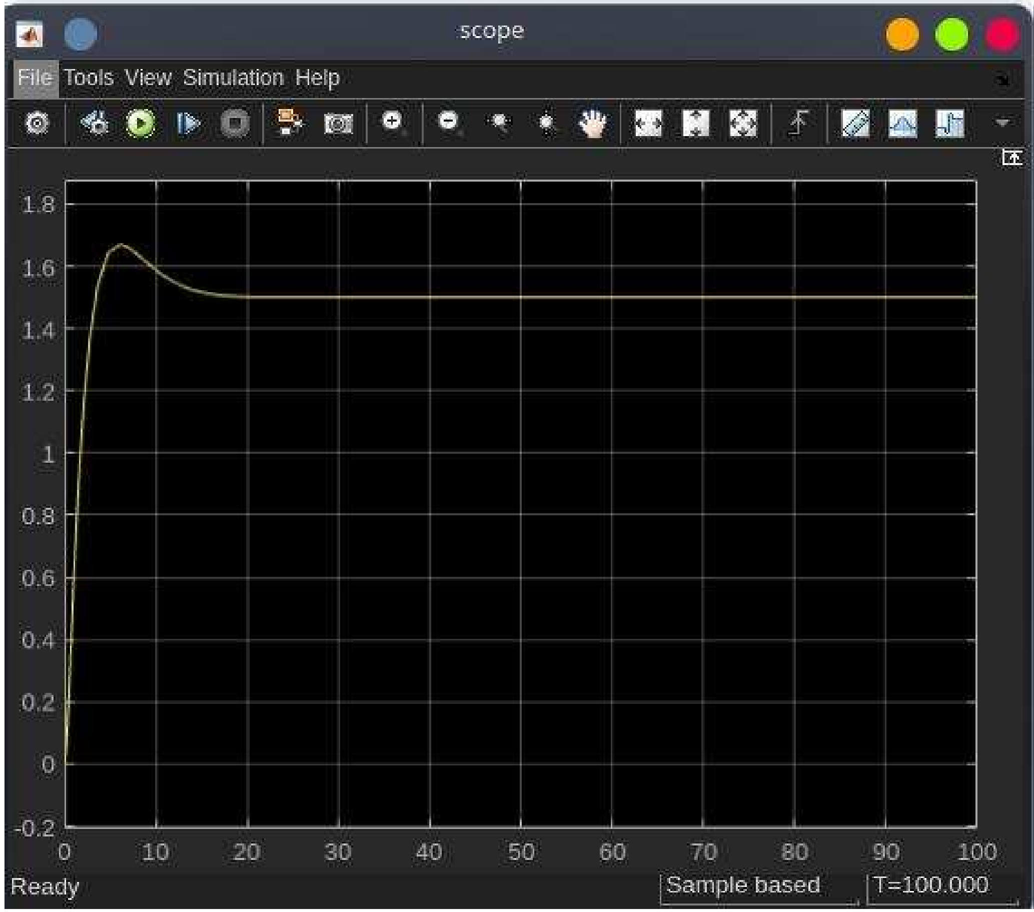
```
L2=feedback(G1,K)
```

```
L2 =

                s
  ------------------------------
  0.2 s^3 + 0.8 s^2 + 0.5 s + 0.1

Continuous-time transfer function.
Model Properties
```

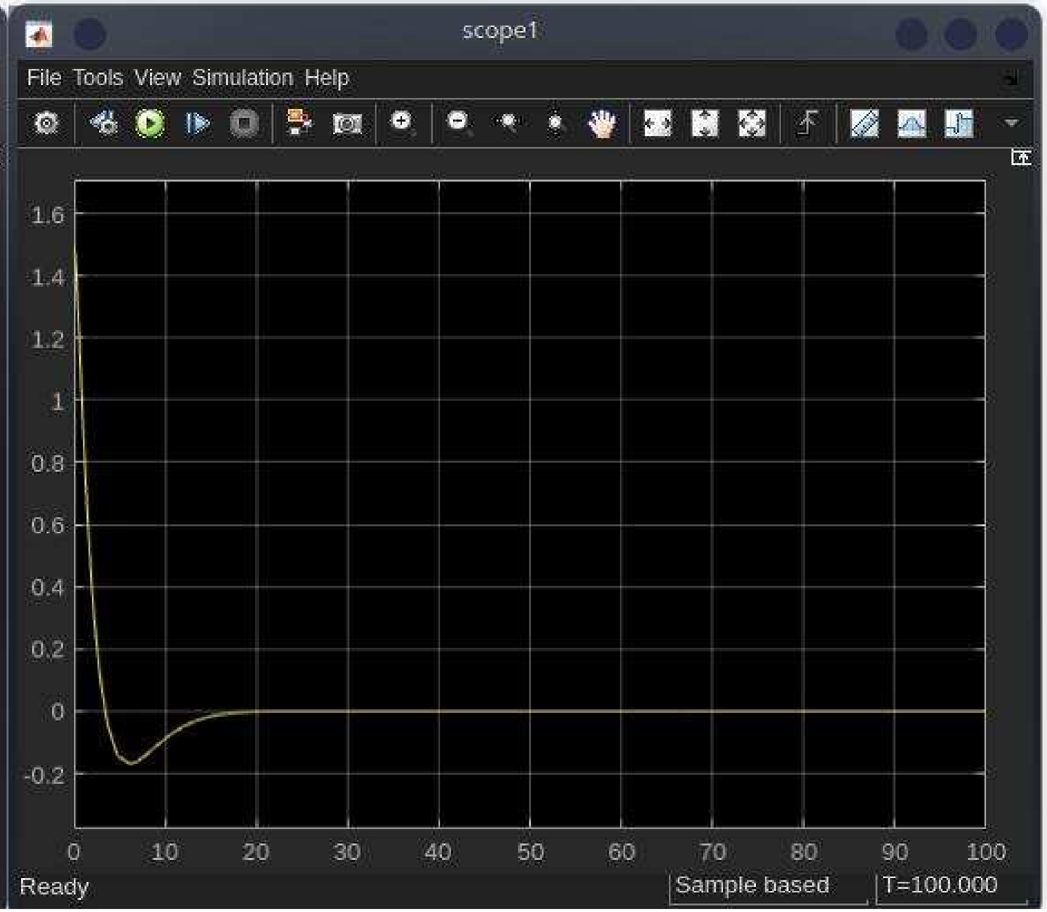
```
% F= numerator of L1
% H= denominator of L2
% provided that they have been normalized to have the same denominator

% Question 5.2
tau=0.1;
qref=1.5;

% use initial value theorem
%integrator in controller -> ess=0
```



scope1
1.5
Y(s)
U(s)
K
Y(s)
U(s)
G1
scope
0.1



scope
File Tools View Simulation Help
1.8
1.6
1.4
1.2
1
0.8
0.6
0.4
0.2
0
-0.2
0 10 20 30 40 50 60 70 80 90 100
Ready
Sample based
T=100.000



scope1
File Tools View Simulation Help
1.6
1.4
1.2
1
0.8
0.6
0.4
0.2
0
-0.2
0 10 20 30 40 50 60 70 80 90 100
Ready
Sample based
T=100.000