

Side 1

Section 1: Position and orientation kinematics

This section contains 2 questions: Q1.1 and Q1.2

Q1.1

A frame is defined with respect to the world frame by means of the following sequence of operations:

- 10° rotation around the global x-axis
- 20° rotation about the global z-axis
- 30mm translation along the local x-axis
- 40° rotation about the local y-axis

The frame is then described by the homogeneous matrix

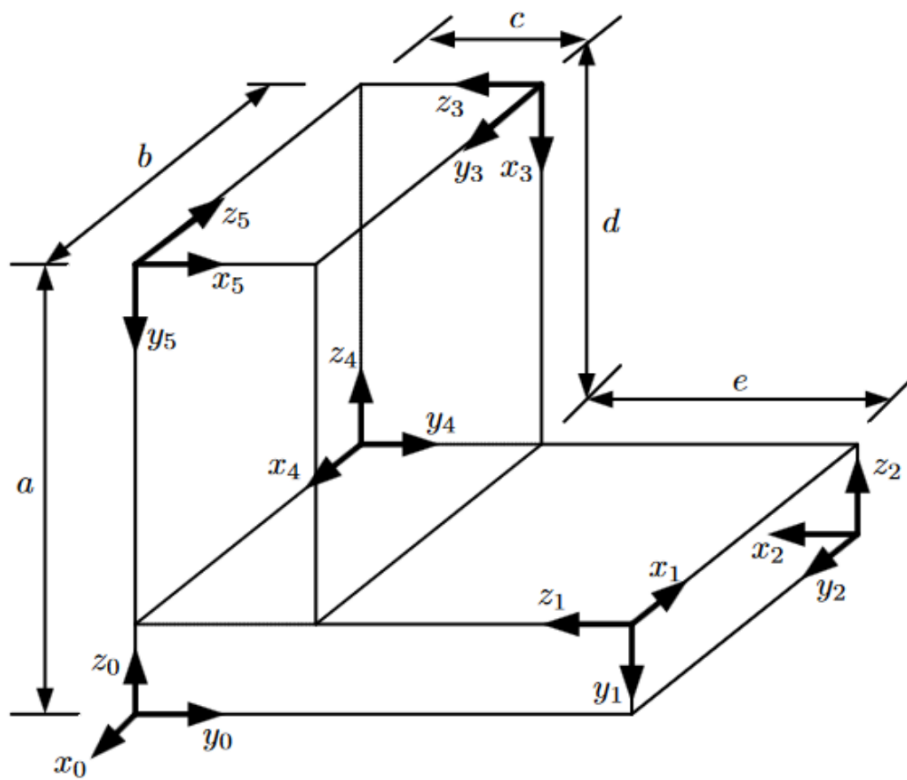
$$T = \begin{bmatrix} \cdot & \cdot & \cdot & \cdot \\ \text{col}_1 & \text{col}_2 & \text{col}_3 & \text{col}_4 \\ \cdot & \cdot & \cdot & \cdot \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

Choose which of the following options (rounded to few significant decimals) corresponds to each of the 4 columns.

	$\begin{bmatrix} 28.19 \\ 10.26 \\ 0 \end{bmatrix}$	$\begin{bmatrix} 0.758 \\ 0.337 \\ -0.559 \end{bmatrix}$	$\begin{bmatrix} -0.337 \\ 0.925 \\ 0.174 \end{bmatrix}$	$\begin{bmatrix} 0.682 \\ 0.367 \\ -0.633 \end{bmatrix}$	$\begin{bmatrix} 22.98 \\ 0 \\ -19.28 \end{bmatrix}$	$\begin{bmatrix} 0.72 \\ 0.37 \\ -0.588 \end{bmatrix}$	$\begin{bmatrix} 28.19 \\ 10.1 \\ 1.78 \end{bmatrix}$	$\begin{bmatrix} 0.65 \\ 0.095 \\ 0.754 \end{bmatrix}$	$\begin{bmatrix} 0.604 \\ 0.083 \\ 0.793 \end{bmatrix}$
Vælg en svarmulighed på hver linje									
Column 1	<input type="radio"/>	<input type="radio"/>	<input type="radio"/>	<input type="radio"/>	<input type="radio"/>	<input type="radio"/>	<input type="radio"/>	<input type="radio"/>	<input type="radio"/>
Column 2	<input type="radio"/>	<input type="radio"/>	<input type="radio"/>	<input type="radio"/>	<input type="radio"/>	<input type="radio"/>	<input type="radio"/>	<input type="radio"/>	<input type="radio"/>
Column 3	<input type="radio"/>	<input type="radio"/>	<input type="radio"/>	<input type="radio"/>	<input type="radio"/>	<input type="radio"/>	<input type="radio"/>	<input type="radio"/>	<input type="radio"/>
Column 4	<input type="radio"/>	<input type="radio"/>	<input type="radio"/>	<input type="radio"/>	<input type="radio"/>	<input type="radio"/>	<input type="radio"/>	<input type="radio"/>	<input type="radio"/>

Q1.2

Consider the frames shown in the figure



Choose the columns of the homogeneous matrix  $H_1^4$ , which describes frame {1} with respect to frame {4}, among the following options

Vælg en svarmulighed på hver linje	$\begin{bmatrix} 1 \\ 0 \\ 0 \\ 0 \end{bmatrix}$	$\begin{bmatrix} 0 \\ 1 \\ 0 \\ 0 \end{bmatrix}$	$\begin{bmatrix} 0 \\ 0 \\ 1 \\ 0 \end{bmatrix}$	$\begin{bmatrix} -1 \\ 0 \\ 0 \\ 0 \end{bmatrix}$	$\begin{bmatrix} 0 \\ -1 \\ 0 \\ 0 \end{bmatrix}$	$\begin{bmatrix} 0 \\ 0 \\ -1 \\ 0 \end{bmatrix}$	$\begin{bmatrix} b \\ 0 \\ c+e \\ 1 \end{bmatrix}$	$\begin{bmatrix} b \\ c+e \\ 0 \\ 1 \end{bmatrix}$	$\begin{bmatrix} c+e \\ a \\ b \\ 1 \end{bmatrix}$	$\begin{bmatrix} c+e \\ b \\ a \\ 1 \end{bmatrix}$
Column 1	<input type="radio"/>	<input type="radio"/>	<input type="radio"/>	<input type="radio"/>	<input type="radio"/>	<input type="radio"/>	<input type="radio"/>	<input type="radio"/>	<input type="radio"/>	<input type="radio"/>
Column 2	<input type="radio"/>	<input type="radio"/>	<input type="radio"/>	<input type="radio"/>	<input type="radio"/>	<input type="radio"/>	<input type="radio"/>	<input type="radio"/>	<input type="radio"/>	<input type="radio"/>
Column 3	<input type="radio"/>	<input type="radio"/>	<input type="radio"/>	<input type="radio"/>	<input type="radio"/>	<input type="radio"/>	<input type="radio"/>	<input type="radio"/>	<input type="radio"/>	<input type="radio"/>
Column 4	<input type="radio"/>	<input type="radio"/>	<input type="radio"/>	<input type="radio"/>	<input type="radio"/>	<input type="radio"/>	<input type="radio"/>	<input type="radio"/>	<input type="radio"/>	<input type="radio"/>

Næste

1

2

3

4

5



? 0/39

Aflever

Slet alle svar

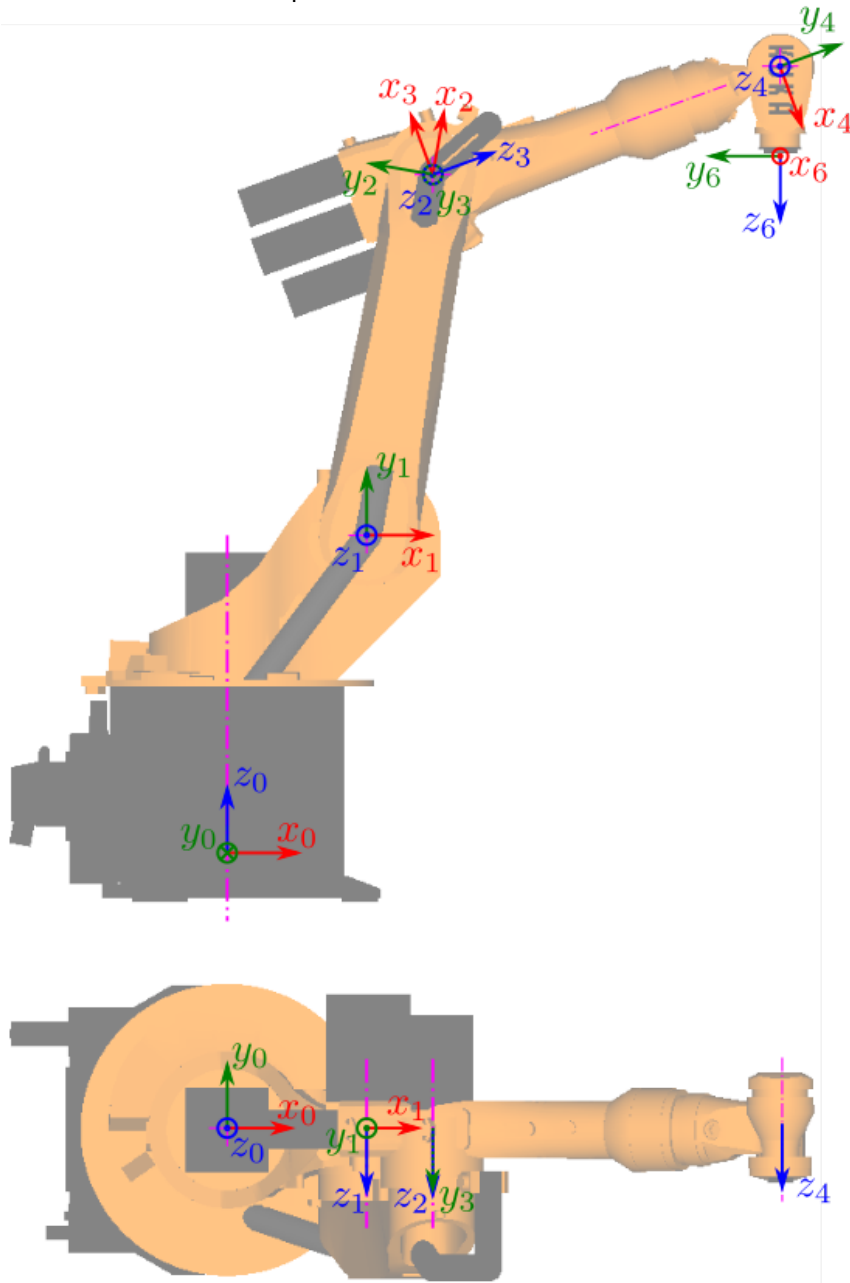
## Side 2

## Section 2: Assignment of Denavit-Hartenberg coordinate frames

This section contains 3 questions: Q2.1, Q2.2 and Q2.3

## Q2.1

The following illustration shows a side and top view of serial link robot with 6 degrees of freedom. It also shows frames  $\{0\}$ ,  $\{1\}$ ,  $\{2\}$ ,  $\{3\}$ ,  $\{4\}$ ,  $\{6\}$ , defined according to the (original) Denavit-Hartenberg convention. Frame  $\{5\}$  is skipped from the illustration due to limited space.

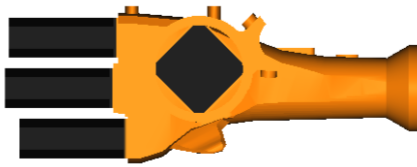


Choose which frame is attached to each of the three robot links shown below:

Vælg en svarmulighed på hver linje

Frame  $\{0\}$    Frame  $\{1\}$    Frame  $\{2\}$    Frame  $\{3\}$    Frame  $\{4\}$    Frame  $\{5\}$    Frame  $\{6\}$


☐
☐
☐
☐
☐
☐
☐



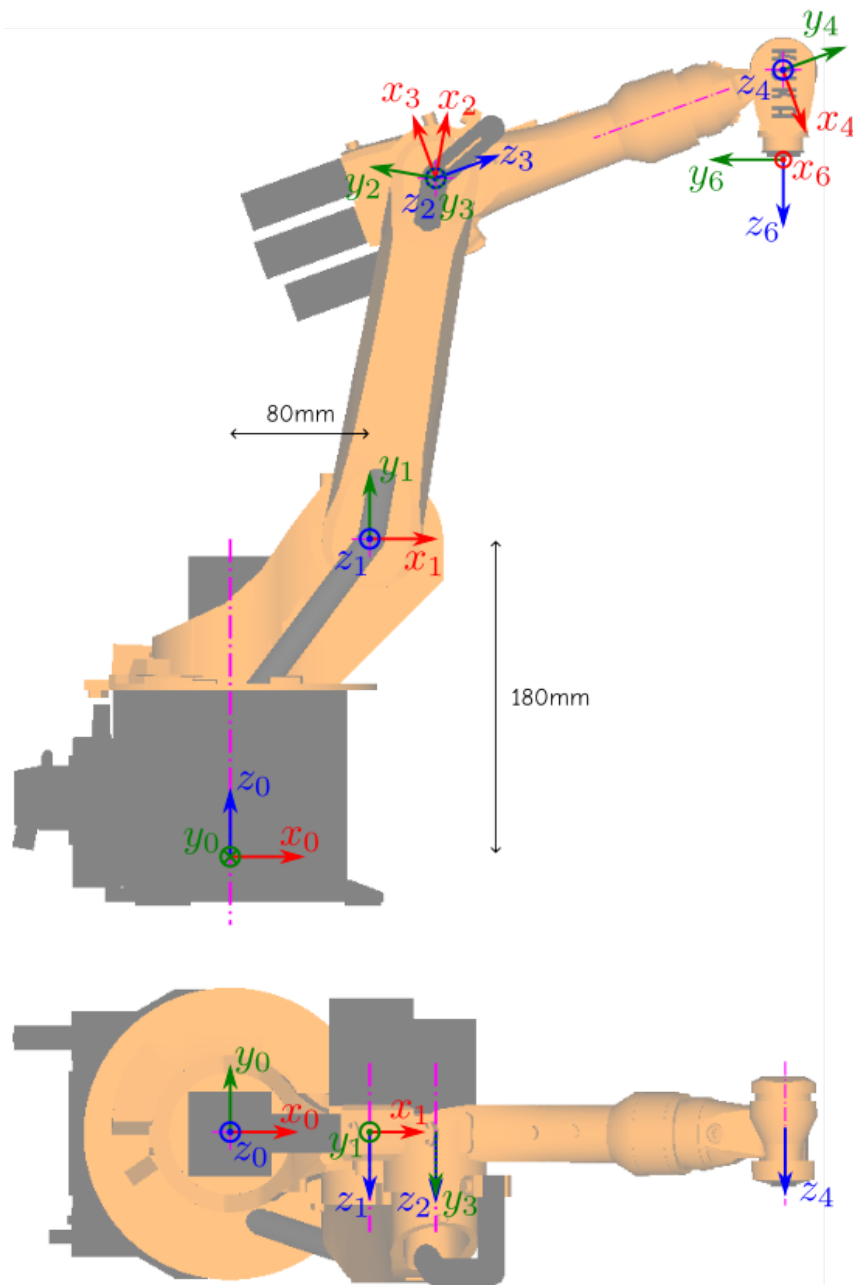
☐ ☐ ☐ ☐ ☐ ☐ ☐



☐ ☐ ☐ ☐ ☐ ☐ ☐

Q2.2

The following illustration shows a side and top view of the same serial link robot with 6 degrees of freedom as in the previous question. In addition, two dimensions are provided in the side view of the robot.



For the configuration shown in the figure, select realistic values for each of the Denavit-Hartenberg parameters listed

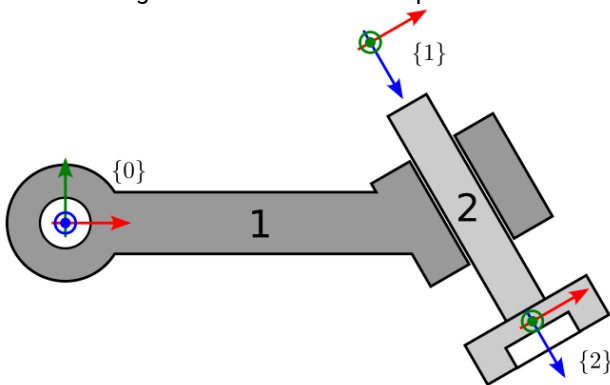
below:

Vælg en svarmulighed  
på hver linje

	-90	-80	-30	0	30	80	90	120	180
$\theta_1$ in °	<input type="radio"/>	<input type="radio"/>	<input type="radio"/>	<input type="radio"/>	<input type="radio"/>	<input type="radio"/>	<input type="radio"/>	<input type="radio"/>	<input type="radio"/>
$\theta_2$ in °	<input type="radio"/>	<input type="radio"/>	<input type="radio"/>	<input type="radio"/>	<input type="radio"/>	<input type="radio"/>	<input type="radio"/>	<input type="radio"/>	<input type="radio"/>
$\theta_3$ in °	<input type="radio"/>	<input type="radio"/>	<input type="radio"/>	<input type="radio"/>	<input type="radio"/>	<input type="radio"/>	<input type="radio"/>	<input type="radio"/>	<input type="radio"/>
$\theta_4$ in °	<input type="radio"/>	<input type="radio"/>	<input type="radio"/>	<input type="radio"/>	<input type="radio"/>	<input type="radio"/>	<input type="radio"/>	<input type="radio"/>	<input type="radio"/>
$d_1$ in mm	<input type="radio"/>	<input type="radio"/>	<input type="radio"/>	<input type="radio"/>	<input type="radio"/>	<input type="radio"/>	<input type="radio"/>	<input type="radio"/>	<input type="radio"/>
$a_1$ in mm	<input type="radio"/>	<input type="radio"/>	<input type="radio"/>	<input type="radio"/>	<input type="radio"/>	<input type="radio"/>	<input type="radio"/>	<input type="radio"/>	<input type="radio"/>
$\alpha_1$ in °	<input type="radio"/>	<input type="radio"/>	<input type="radio"/>	<input type="radio"/>	<input type="radio"/>	<input type="radio"/>	<input type="radio"/>	<input type="radio"/>	<input type="radio"/>
$\alpha_2$ in °	<input type="radio"/>	<input type="radio"/>	<input type="radio"/>	<input type="radio"/>	<input type="radio"/>	<input type="radio"/>	<input type="radio"/>	<input type="radio"/>	<input type="radio"/>
$\alpha_3$ in °	<input type="radio"/>	<input type="radio"/>	<input type="radio"/>	<input type="radio"/>	<input type="radio"/>	<input type="radio"/>	<input type="radio"/>	<input type="radio"/>	<input type="radio"/>

Q2.3

The following illustration shows a simple robot arm with only 2 joints (revolute+prismatic):



The forward kinematics solution for its end-effector frame is given as:

$$T_2^0 = \begin{bmatrix} \cos \theta_1 & 0 & \sin \theta_1 & d_2 \sin \theta_1 + 90 \cos \theta_1 \\ \sin \theta_1 & 0 & -\cos \theta_1 & -d_2 \cos \theta_1 + 90 \sin \theta_1 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

Find the inverse kinematics solution  $(\theta_1, d_2)$  so that the origin of the end-effector frame is located at

$$o_2 = \begin{bmatrix} 94.822 \\ 28.102 \\ 0 \end{bmatrix} \text{ mm}$$

with respect to frame {0}. Among all possible solutions, look only for a solution with  $d_2 > 0$ .

Choose the correct results for the joint angle  $\theta_1$  in degrees °, and the joint distance  $d_2$  in mm, rounded to an integer value, among the following options.

Vælg en  
svarmulighed  
på hver linje

23      29      31      37      41      43      47      53      73      79

$\theta_1$  in  $^\circ$

☐☐☐☐☐☐☐☐☐☐

$d_2$  in  
mm

☐☐☐☐☐☐☐☐☐☐

Forrige

Næste



1

2

3

4

5



0/39

.....

Aflever

Slet alle svar



$$\dot{d}_2 = -0.05 \text{ m/s}$$

Q3.2

At time

$$t_A = 0 \text{ s}$$

a robot revolute joint is at an angle

$$q_A = 0 \text{ rad}$$

moving at a speed

$$\dot{q}_A = 0.2 \text{ rad/s}$$

with zero acceleration

$$\ddot{q}_A = 0 \text{ rad/s}^2$$

At time

$$t_B = 2.5 \text{ s}$$

the joint needs to be at an angle

$$q_B = \pi/4$$

moving at a speed

$$\dot{q}_B = 0 \text{ rad/s}$$

with zero acceleration

$$\ddot{q}_B = 0 \text{ rad/s}^2$$

You are asked to find the coefficients of the quintic interpolation polynomial

$$q(t) = c_0 + c_1 t + c_2 t^2 + c_3 t^3 + c_4 t^4 + c_5 t^5$$

which performs this transition, and fill in the following table correctly.

Vælg en

svarmulighed  
på hver linje

	-1.5708	-0.31066	-0.19919	-0.03289	0	0.03289	0.19919	0.2	0.31066	1.5708
$c_0$	<input type="radio"/>	<input type="radio"/>	<input type="radio"/>	<input type="radio"/>	<input type="radio"/>	<input type="radio"/>	<input type="radio"/>	<input type="radio"/>	<input type="radio"/>	<input type="radio"/>
$c_1$ in $\text{s}^{-1}$	<input type="radio"/>	<input type="radio"/>	<input type="radio"/>	<input type="radio"/>	<input type="radio"/>	<input type="radio"/>	<input type="radio"/>	<input type="radio"/>	<input type="radio"/>	<input type="radio"/>
$c_2$ in $\text{s}^{-2}$	<input type="radio"/>	<input type="radio"/>	<input type="radio"/>	<input type="radio"/>	<input type="radio"/>	<input type="radio"/>	<input type="radio"/>	<input type="radio"/>	<input type="radio"/>	<input type="radio"/>
$c_3$ in $\text{s}^{-3}$	<input type="radio"/>	<input type="radio"/>	<input type="radio"/>	<input type="radio"/>	<input type="radio"/>	<input type="radio"/>	<input type="radio"/>	<input type="radio"/>	<input type="radio"/>	<input type="radio"/>
$c_4$ in $\text{s}^{-4}$	<input type="radio"/>	<input type="radio"/>	<input type="radio"/>	<input type="radio"/>	<input type="radio"/>	<input type="radio"/>	<input type="radio"/>	<input type="radio"/>	<input type="radio"/>	<input type="radio"/>
$c_5$ in $\text{s}^{-5}$	<input type="radio"/>	<input type="radio"/>	<input type="radio"/>	<input type="radio"/>	<input type="radio"/>	<input type="radio"/>	<input type="radio"/>	<input type="radio"/>	<input type="radio"/>	<input type="radio"/>

Forrige

Næste



1

2

3

4

5



0/39

Aflever

Slet alle svar



## Side 4

## Section 4: Robot Vision

This section contains 1 problem with 5 questions

## Q4.1

A police drone has taken off from a take-off area H, and it is flying monitoring the traffic. The drone is stationary and it is monitoring a road using a camera pointing downwards, so that its principal axis is perpendicular to the ground.

At time  $t_1=15s$ , the drone is hovering at a certain altitude Z, when it takes the picture (picture 1) of a car identified as point  $P_1$ , located at pixel

$$p_1=[476; 380];$$

At time  $t_2=18s$ , the drone takes a second picture (picture 2) of the same car identified as point  $P_2$ , located at pixel

$$p_2=[147; 117];$$

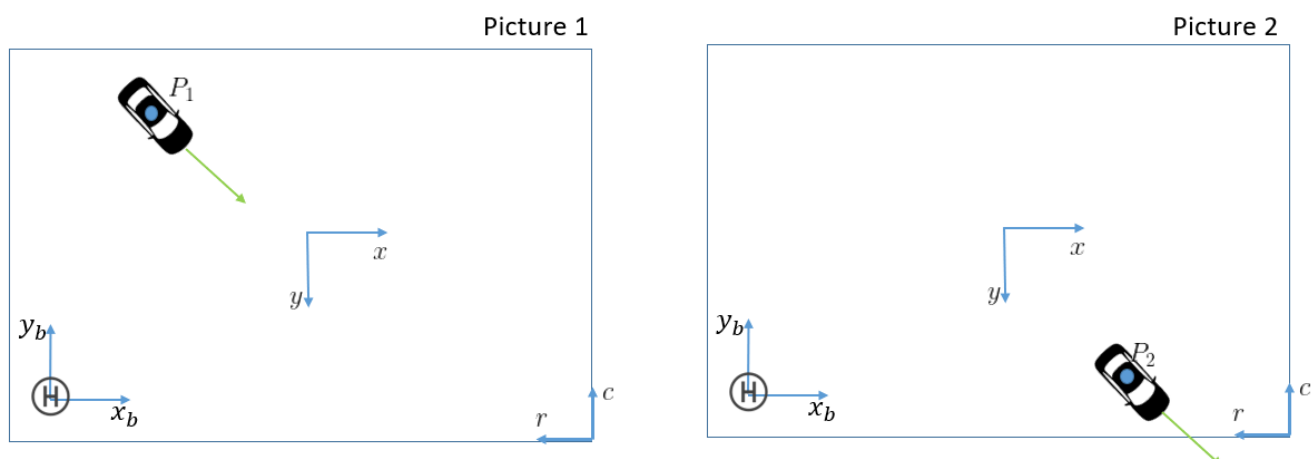
The camera parameters are:

$$\lambda = 8 \text{ mm}$$

$$\alpha_x = \frac{1}{s_x} = 79.2 \text{ pix/mm}$$

$$\alpha_y = \frac{1}{s_y} = 120.5 \text{ pix/mm}$$

The origin of the camera CCD is located in the positive x-y quadrant with respect to frame  $\{C\}$ , as seen in the figure, so that the principal point is at pixel coordinates  $o_r = 250 \text{ pix}$  and  $o_c = 250 \text{ pix}$ .



What is the distance traveled by the car, in CCD units, within the two pictures?

Vælg én svarmulighed

- ☐ 378 pixels
- ☐ 421 pixels
- ☐ 423 pixels
- ☐ 451 pixels

#### Q4.2

What is the distance travelled in the image plane?

Vælg én svarmulighed

- ☐ 4.8521 mm
- ☐ 4.6925 mm
- ☐ 5.3182 mm
- ☐ 4.9863 mm
- ☐ 3.4954 mm
- ☐ 5.2956 mm
- ☐ 3.4928 mm

#### Q4.3

The onboard speed radar detects that the car is moving at a constant speed of absolute value 43km/h.

What is the distance travelled by the car (in meters)?

Vælg én svarmulighed

- ☐ 38m
- ☐ 45.7m

- ☐ 23.83m
- ☐ 35.83m
- ☐ 785.48m
- ☐ 34.841m

#### Q4.4

What is the altitude Z from the ground, at which the drone is hovering?

Vælg én svarmulighed

- ☐ 25.438m
- ☐ 35.83m
- ☐ 61.09m
- ☐ approximately 45m
- ☐ 78.32m

#### Q4.5

Because the drone is stationary, the take off area H, identified by the base frame  $[x_b, y_b, z_b]$ , remains at the same pixel position in both pictures, given as

$p_b=[450;50]$

What is the position of the drone's camera origin, with respect to the base frame b located at the landing area H?

Vælg én svarmulighed

- ☐  $P_C^H = \begin{bmatrix} -19.28 \\ 12.67 \\ 61.09 \end{bmatrix} \text{m}$

☐  $P_C^H = \begin{bmatrix} 19.28 \\ -12.67 \\ 61.09 \end{bmatrix} \text{ m}$

☐  $P_C^H = \begin{bmatrix} 19.28 \\ 12.67 \\ 61.09 \end{bmatrix} \text{ m}$

☐  $P_C^H = \begin{bmatrix} 19.28 \\ 12.67 \\ 78.32 \end{bmatrix} \text{ m}$

☐  $P_C^H = \begin{bmatrix} -19.28 \\ 12.67 \\ 78.32 \end{bmatrix} \text{ m}$

☐  $P_C^H = \begin{bmatrix} 19.28 \\ 12.67 \\ 35.83 \end{bmatrix} \text{ m}$

---

Forrige

Næste



1

2

3

4

5



? 0/39

Aflever

Slet alle svar

## Side 5

## Section 5: Motor Control

This section contains 2 problems: Q5.1-Q5.2. Notice that the terms may not use the same symbols as in the slides or book. This is done on purpose, to verify your understanding of the different terms. You can verify that you use the correct values by checking the units (SI units)

## Q5.1

Let us now consider an electric motor characterized by a non-negligible electrical dynamics, with by the following parameters:

$$L_m = 0.001 \text{ H}$$

$$R_m = 0.9 \Omega$$

$$K_m = 0.12 \text{ Vs/rad}$$

$$K_i = 0.12 \text{ Nm/A}$$

The mechanics of the actuator has the following parameters:

$$J_m = 0.08 \text{ kg} \frac{\text{m}^2}{\text{rad}}$$

$$D_m = 0.027 \text{ kg} \frac{\text{m}^2}{\text{rad s}}$$

and load inertia:

$$J_l = 0.18 \text{ kg} \frac{\text{m}^2}{\text{rad}}$$

The load and the motors are connected through a rigid transmission, with gear ratio  $n = 20$ , such that the relation between the motor and load speed is  $\theta_m = n \theta_l$

How many poles characterize the electromechanical dynamics of the motor?

Vælg én svarmulighed

☐ 1

☐ 2

☐ 3

☐ 4

## Q5.2

Implement a proportional position controller with proportional gain  $k_p = 3.78$ . Evaluate the step response to a step reference input  $\theta_m^* = 1.5$  rad, when the load is subject to an external disturbance torque  $\tau_l = 0.05$  Nm. The steady state error is:

Vælg én svarmulighed

- ☐ 0
- ☐ infinite
- ☐ 0.575 rad
- ☐ approximately 0.21 rad
- ☐ -0.0128 rad
- ☐ approximately 0.075 m
- ☐ approximately 0.01rad

Forrige



1

2

3

4

5

? 0/39

Aflevér

Slet alle svar