Exam 2023

Problem 1 Homogeneous Transformation Matrices and Rotations

Key Observations:

- Definitions of rotation matrices Rx, Ry, Rz and their conversion to 4x4 homogeneous transformation matrices by appending translations and a bottom row [0,0,0,1].
- The computation of a composite transformation matrix H, combining multiple rotations and a translation.
- Use of symbolic variables for constructing transformation matrices.

Likely Questions:

1. **Q1.1**: Define or construct a homogeneous transformation matrix for a combination of rotations about x, y, and z-axes and a translation along x-axis. Compute the resulting matrix H.
2. **Q1.3**: Given symbolic vectors x,y,z,t, form a homogeneous transformation matrix H24 with these components representing orientation and translation. Identify the significance of each term in H24.

```
% Q1.1
syms theta
R_x(theta) = [1 0 0;
              0 cos(theta) -sin(theta);
              0 sin(theta) cos(theta)];
R_y(theta) = [cos(theta) 0 sin(theta);
              0 1 0;
              -sin(theta) 0 cos(theta)];
R_z(theta) = [cos(theta) -sin(theta) 0;
              sin(theta) cos(theta) 0;
              0 0 1];
R_x = [R_x,[0;0;0];0 0 0 1];
R_y = [R_y,[0;0;0];0 0 0 1];
R_z = [R_z,[0;0;0];0 0 0 1];

angle1 = deg2rad(10);
angle2 = deg2rad(20);
trans3 = 30;
angle4 = deg2rad(40);

T_x = [eye(3),[trans3;0;0];0 0 0 1]
```

```
T_x = 4×4
     1     0     0    30
     0     1     0     0
     0     0     1     0
     0     0     0     1
```

```
H = R_y(angle4)*R_y(angle1)*R_z(angle2)*T_x;
vpa(H,4)
```

```
ans =
```

$$\begin{pmatrix} 0.604 & -0.2198 & 0.766 & 18.12 \\ 0.342 & 0.9397 & 0 & 10.26 \\ -0.7198 & 0.262 & 0.6428 & -21.6 \\ 0 & 0 & 0 & 1.0 \end{pmatrix}$$

```
% Q1.3
syms a b c d e
x = [0; 1; 0];
y = [-1; 0; 0];
z = [0; 0; 1];
t = [c+e; 0; a-d];
H_24 = [x,y,z,t; 0 0 0 1]
```

H_24 =

$$\begin{pmatrix} 0 & -1 & 0 & c+e \\ 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & a-d \\ 0 & 0 & 0 & 1 \end{pmatrix}$$

Problem 2 Denavit-Hartenberg Parameters

Key Observations:

- Use of the DH convention to define transformation matrices between joints.
- Joint parameters provided for a robotic manipulator.
- Inverse kinematics: solving for joint angles and link displacements using transformation matrix equations and a target point o2.

Likely Questions:

1. **Q2.2**: Using the provided DH parameters for each joint, derive the transformation matrices for a 4-DOF robotic manipulator.
2. **Q2.3**: Solve an inverse kinematics problem to determine joint variables θ1 and d2 based on a given target position o2.

```
clear;

syms DH(th,d,a,alpha)
DH(th,d,a,alpha) = [cos(th) -sin(th)*cos(alpha) sin(th)*sin(alpha) a*cos(th);
                    sin(th) cos(th)*cos(alpha) -cos(th)*sin(alpha) a*sin(th);
                    0 sin(alpha) cos(alpha) d;
                    0 0 0 1];

% Q2.2
% Joint i: [theta d a alpha]
% Joint 1: [0 180 80 90]
% Joint 2: [80 0 180 0]
```

```
% Joint 3: [30 0 0 90]
% Joint 4: [180 180 0 90]

% Q2.3
syms theta_1 theta_2 d_2
T_02 = [cos(theta_1) 0 sin(theta_1) d_2*sin(theta_1)+90*cos(theta_1);
        sin(theta_1) 0 -cos(theta_1) -d_2*cos(theta_1)+90*sin(theta_1);
        0 1 0 0;
        0 0 0 1];
o2 = [76.1; 87.4; 0];

eqns = [T_02(1,4) == o2(1);
        T_02(2,4) == o2(2)];
S = solve(eqns,[theta_1 d_2])
```

```
S = struct with fields:
    theta_1: [2×1 sym]
        d_2: [2×1 sym]
```

```
% theta_1 = 1.54rad, d_2 = 73.0 mm
disp(vpa([S.theta_1 S.d_2],5))
```

$$\begin{pmatrix} 0.17288 & -73.007 \\ 1.5359 & 73.007 \end{pmatrix}$$

```
rad2deg(1.5359)
```

```
ans = 88.0006
```

```
% X1 = 8, Y1 = 8
% X2 = 7, Y2 = 3
```

Problem 3 Jacobians and Trajectory Planning

Key Observations:

- **Q3.1**: Definition and use of the Jacobian matrix J to map joint velocities (q1′,q2′) to end-effector velocities ζ.
- **Q3.2**: Derivation of a quintic polynomial for trajectory generation and calculation of acceleration at a specific time ttt.

Likely Questions:

1. **Q3.1**: Given a Jacobian matrix and joint velocities, calculate the end-effector velocities for different scenarios and plot the resultant velocity vectors.
2. **Q3.2**: Compute the coefficients of a quintic polynomial for a specified trajectory. Derive and evaluate the acceleration of the trajectory at a specific time.

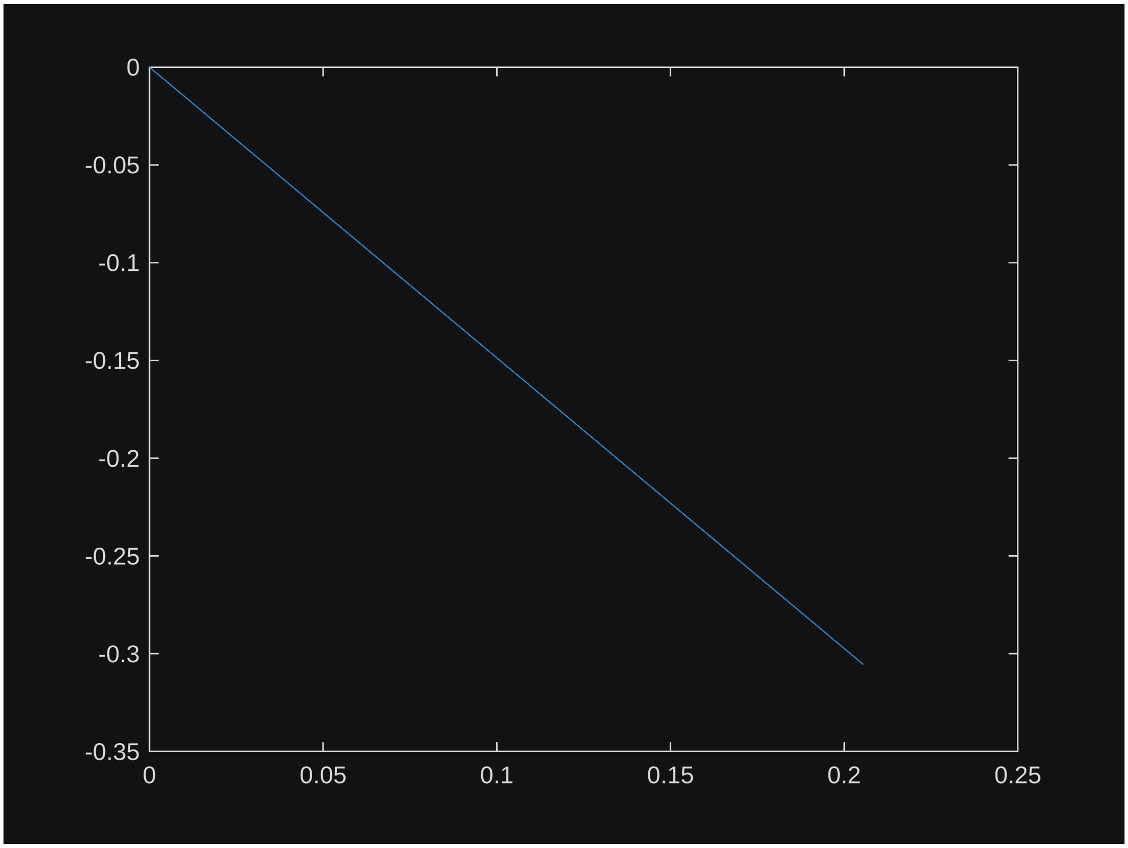
```
% Q3.1
function plot_resultant(vect)
```

```
    % for Q3.1
    vx = [vect(1);0];
    vy = [0; vect(2)];
    wz_magnitude = abs(vect(6));
    wz_direction = sign(vect(6));
    wz = wz_direction*[-sqrt(wz_magnitude);sqrt(wz_magnitude)];
    resultant = vx+vy+wz;
    plot([0 resultant(1)],[0 resultant(2)])
end

J = [-0.3 0.96;
     0.4 -0.28;
     0 0;
     0 0;
     0 0;
     1 0];
syms q1_dot q2_dot
q_dot = [q1_dot; q2_dot];
zeta = J*q_dot;

qd_1 = vpa(subs(zeta,[q1_dot q2_dot],[-0.0933 -0.1333]),5)
```

qd_1 =

$$\begin{pmatrix} -0.099978 \\ 4.0\text{e-}6 \\ 0 \\ 0 \\ 0 \\ -0.0933 \end{pmatrix}$$

```
plot_resultant(qd_1)    % SE
```

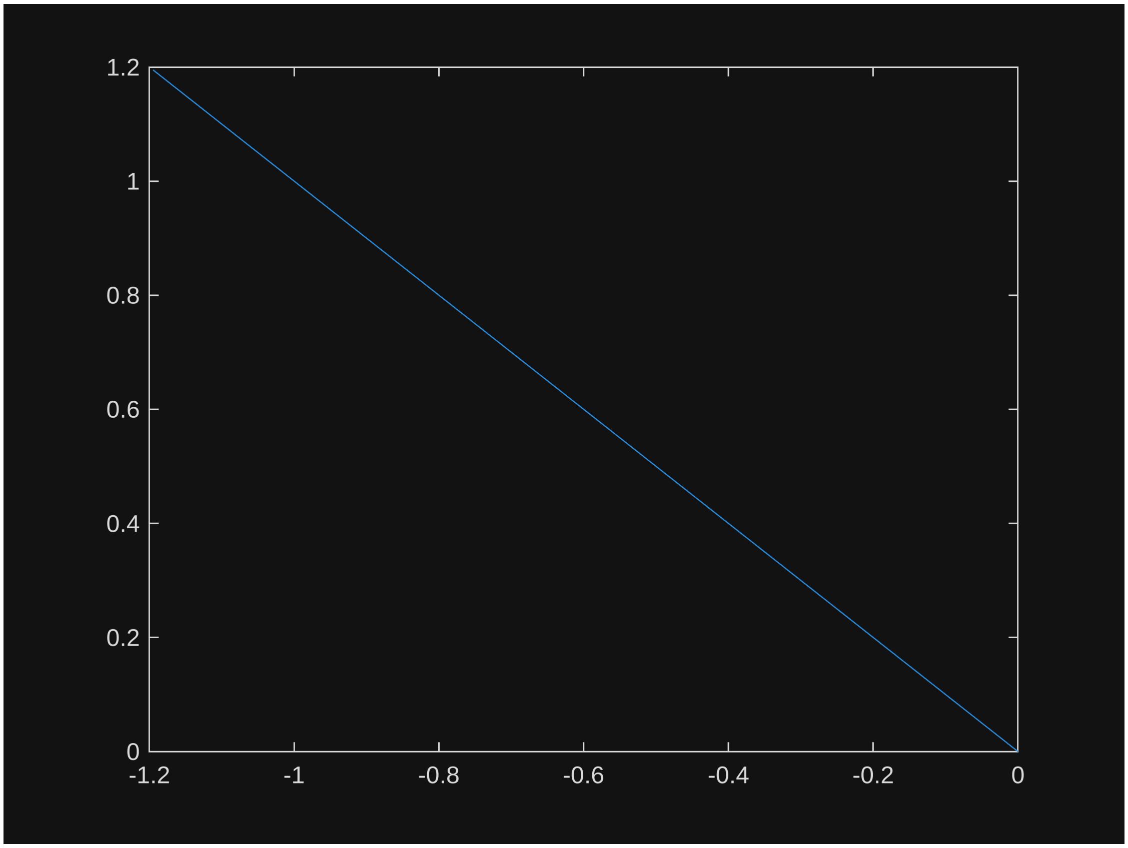



```
qd_2 = vpa(subs(zeta,[q1_dot q2_dot],[0.748 -0.11]),5)
```

qd_2 =

$$\begin{pmatrix} -0.33 \\ 0.33 \\ 0 \\ 0 \\ 0 \\ 0.748 \end{pmatrix}$$

```
plot_resultant(qd_2)    % NW
```

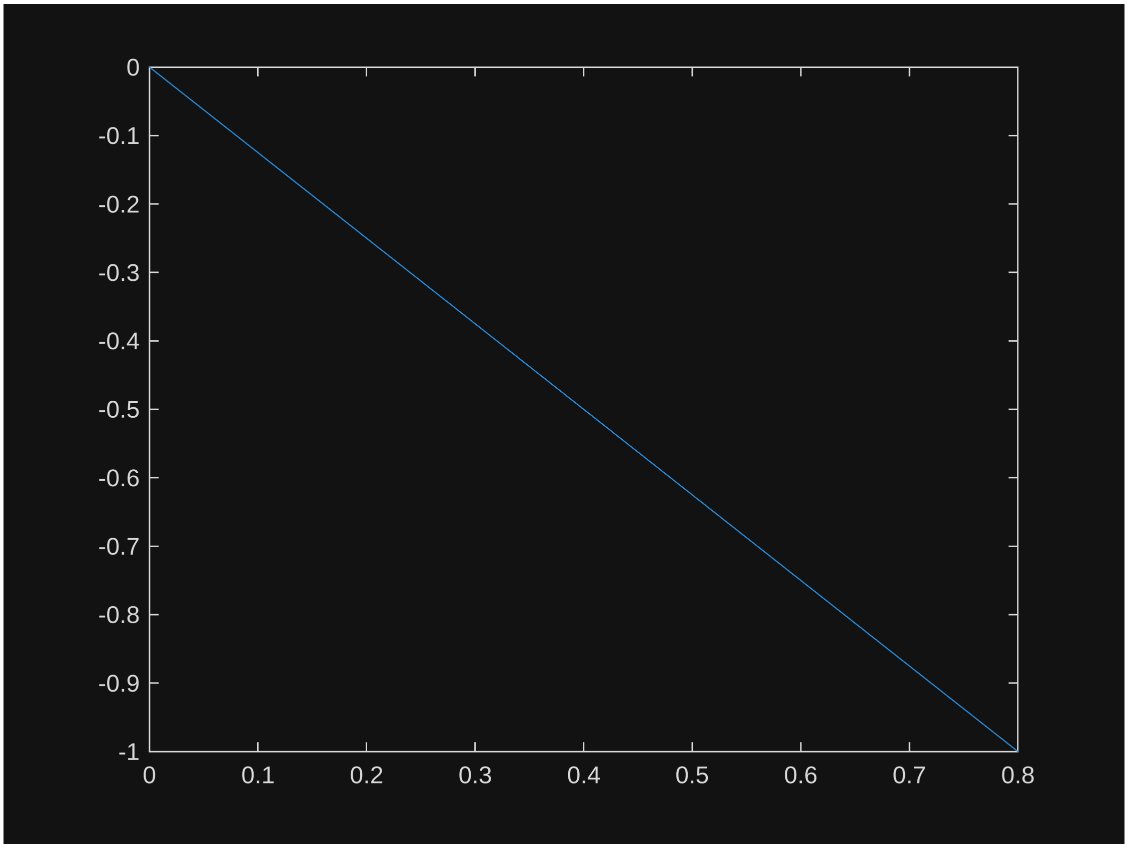



```
qd_3 = vpa(subs(zeta,[q1_dot q2_dot],[-0.64 -0.2]),5)
```

qd_3 =

$$\begin{pmatrix} 0 \\ -0.2 \\ 0 \\ 0 \\ 0 \\ -0.64 \end{pmatrix}$$

```
plot_resultant(qd_3)    % SE
```

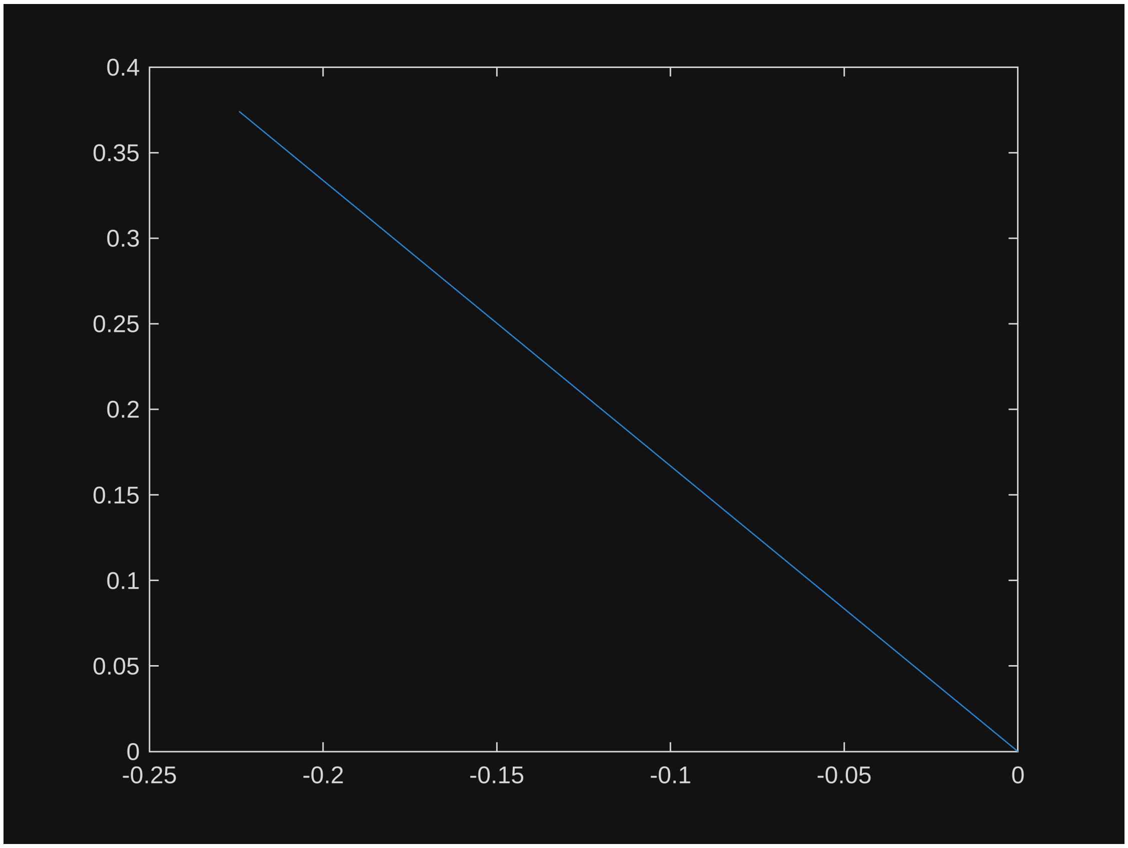



```
qd_4 = vpa(subs(zeta,[q1_dot q2_dot],[0.14 0.2]),5)
```

qd_4 =

$$\begin{pmatrix} 0.15 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0.14 \end{pmatrix}$$

```
plot_resultant(qd_4)     % NW
```

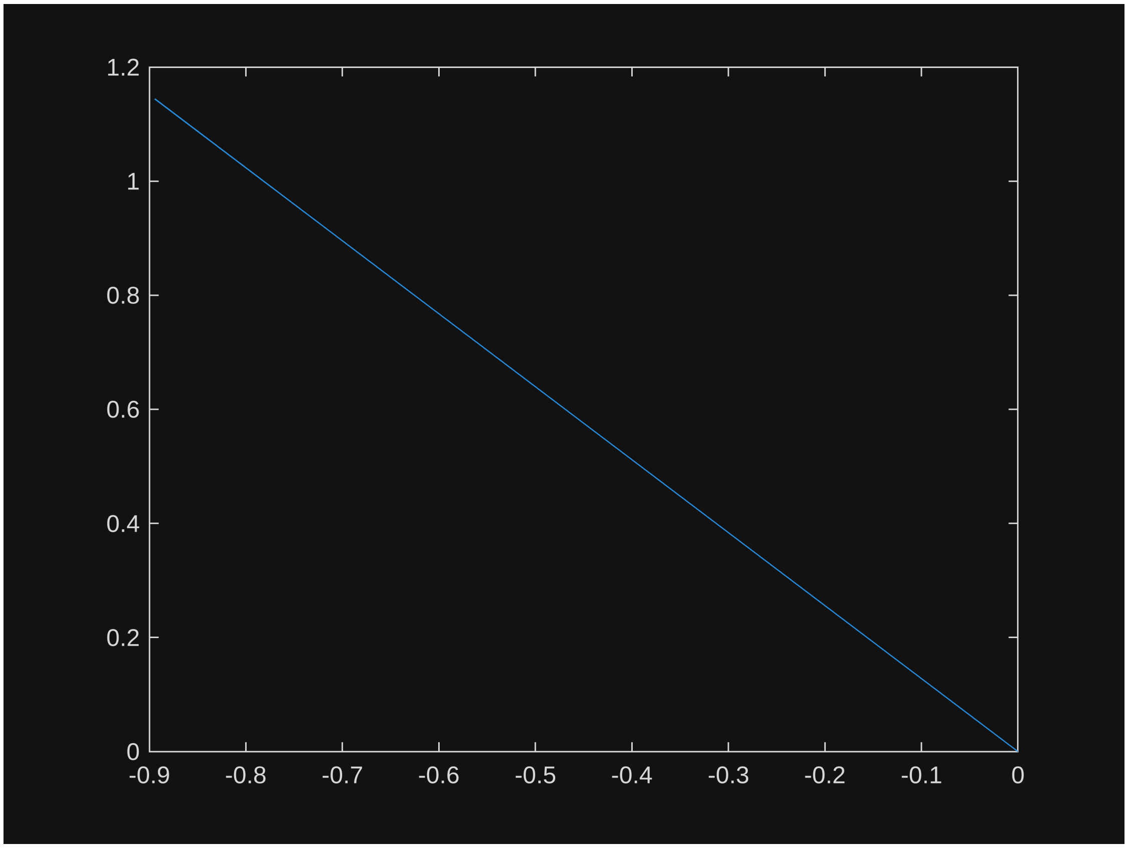



```
qd_5 = vpa(subs(zeta,[q1_dot q2_dot],[0.8 0.25]),5)
```

qd_5 =

$$\begin{pmatrix} 0 \\ 0.25 \\ 0 \\ 0 \\ 0 \\ 0.8 \end{pmatrix}$$

```
plot_resultant(qd_5)    % NW
```




```
% Q3.2
clear;

syms t_a t_b

T = [1 t_a t_a^2 t_a^3 t_a^4 t_a^5;
    0 1 2*t_a 3*t_a^2 4*t_a^3 5*t_a^4;
    0 0 2 6*t_a 12*t_a^2 20*t_a^3;
    1 t_b t_b^2 t_b^3 t_b^4 t_b^5;
    0 1 2*t_b 3*t_b^2 4*t_b^3 5*t_b^4;
    0 0 2 6*t_b 12*t_b^2 20*t_b^3]
```

T =

$$\begin{pmatrix} 1 & t_a & {t_a}^2 & {t_a}^3 & {t_a}^4 & {t_a}^5 \\ 0 & 1 & 2\,t_a & 3\,{t_a}^2 & 4\,{t_a}^3 & 5\,{t_a}^4 \\ 0 & 0 & 2 & 6\,t_a & 12\,{t_a}^2 & 20\,{t_a}^3 \\ 1 & t_b & {t_b}^2 & {t_b}^3 & {t_b}^4 & {t_b}^5 \\ 0 & 1 & 2\,t_b & 3\,{t_b}^2 & 4\,{t_b}^3 & 5\,{t_b}^4 \\ 0 & 0 & 2 & 6\,t_b & 12\,{t_b}^2 & 20\,{t_b}^3 \end{pmatrix}$$

```
T = subs(T,[t_a t_b],[0 2])
```

```
T =
```

$$\begin{pmatrix} 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 2 & 0 & 0 & 0 \\ 1 & 2 & 4 & 8 & 16 & 32 \\ 0 & 1 & 4 & 12 & 32 & 80 \\ 0 & 0 & 2 & 12 & 48 & 160 \end{pmatrix}$$

```
q_a = pi/5;
q_d_a = -pi/10;
q_dd_a = 0;

q_b = pi/2;
q_d_b = 3*pi/20;
q_dd_b = 0;
c = inv(T)*[q_a; q_d_a; q_dd_a; q_b; q_d_b; q_dd_b]
```

```
c =
```

$$\begin{pmatrix} \frac{\pi}{5} \\ -\frac{\pi}{10} \\ 0 \\ \frac{3\,\pi}{8} \\ -\frac{\pi}{4} \\ \frac{3\,\pi}{64} \end{pmatrix}$$

```
vpa(c,5)          % coefficients for quintic polynomial
```

```
ans =
```

$$\begin{pmatrix} 0.62832 \\ -0.31416 \\ 0 \\ 1.1781 \\ -0.7854 \\ 0.14726 \end{pmatrix}$$

```
syms t
qdd = 2*c(3) + 6*c(4)*t + 12*c(5)*t^2 + 20*c(6)*t^3
```

```
qdd =
```

$$\frac{15\,\pi\,t^3}{16} - 3\,\pi\,t^2 + \frac{9\,\pi\,t}{4}$$

```
vpa(subs(qdd,t,1),5)        % acceleration at t=1
```

ans = 0.58905

Problem 4 Camera Calibration and Motion Estimation

Key Observations:

- Camera model with pixel-to-world coordinate conversion.
- Calculation of object dimensions and motion parameters using depth and calibration data.
- Determination of whether a couch can fit through a door based on their relative dimensions in the camera frame.
- Use of pixel differences to calculate the velocity of a camera.

Likely Questions:

1. **Q4.1**: Use camera calibration parameters to convert pixel coordinates to world coordinates and calculate the dimensions of objects (e.g., couch length, door width). Determine if the couch can fit through the door.
2. **Q4.2**: Compute the height of objects (e.g., P1, P2) based on their pixel positions and camera calibration data.
3. **Q4.3**: Given pixel differences and object displacement, compute the velocity of the camera in the y-direction.

```
% Q4.1
or = 250;
oc = 250;
lambda = 8;      % mm
lambda = lambda/1000;   % m
alpha_x = 79.2;     % pix / mm
alpha_y = 120.5;    % pix / mm
alpha_x = alpha_x*1000;    % pix / m
alpha_y = alpha_y*1000;    % pix / m

p1 = [476;222];
p2 = [295;222];
p3 = [295;99];
p4 = [476;99];
p5 = [448;226];
p6 = [290;226];
p7 = [290;117];
p8 = [210;117];
p9 = [147;117];

% depth of p5, p6, p7, p8, p9 : 8m
```

```
% pixels -> {world}
zc = 8;    % m
p5_xc = (p5(1)-or)*zc/alpha_x;
p5_yc = (p5(2)-oc)*zc/alpha_y;
p5_c = [p5_xc; p5_yc];   % in m

p6_xc = (p6(1)-or)*zc/alpha_x;
p6_yc = (p6(2)-oc)*zc/alpha_y;
p6_c = [p6_xc; p6_yc];

p7_xc = (p7(1)-or)*zc/alpha_x;
p7_yc = (p7(2)-oc)*zc/alpha_y;
p7_c = [p7_xc; p7_yc];

p8_xc = (p8(1)-or)*zc/alpha_x;
p8_yc = (p8(2)-oc)*zc/alpha_y;
p8_c = [p8_xc; p8_yc];

p9_xc = (p9(1)-or)*zc/alpha_x;
p9_yc = (p9(2)-oc)*zc/alpha_y;
p9_c = [p9_xc; p9_yc];

door_width_c = norm(p8_c-p9_c)
```

```
door_width_c = 0.0064
```

```
couch_height_c = norm(p7_c-p6_c)
```

```
couch_height_c = 0.0072
```

```
couch_length_c = norm(p6_c-p5_c)
```

```
couch_length_c = 0.0160
```

```
length = couch_length_c/door_width_c*0.80   % m
```

```
length = 2.0063
```

```
height = couch_height_c/door_width_c*0.80    % m
```

```
height = 0.9097
```

```
% Since door_width in {camera} < couch_height in {camera},
% the couch cannot pass through the door.

% incomplete
P7_cx = (290-250)/alpha_x
```

```
P7_cx = 5.0505e-04
```

```
P7_wx = P7_cx/lambda*8
```

```
P7_wx = 0.5051
```

```
P3_cx = (295-250)/alpha_x
```

```
P3_cx = 5.6818e-04
```

```
P3_wx = P7_wx;
P3_wz = P3_wx*lambda/P3_cx
```

```
P3_wz = 7.1111
```

```
% Q4.2
P1H = [263;485];
P1F = [263;154];
P2H = [234;401];
P2F = [234;190];

z1 = 5;
P1_Hc = (P1H(2)-oc)/alpha_y;
P1_Fc = (P1F(2)-oc)/alpha_y;
P1_height_c = P1_Hc - P1_Fc     % in {camera}, m
```

```
P1_height_c = 0.0027
```

```
P1_height = z1/lambda*P1_height_c
```

```
P1_height = 1.7168
```

```
z2 = 8;     % m
P2_Hc = (P2H(2)-oc)/alpha_y;
P2_Fc = (P2F(2)-oc)/alpha_y;
P2_height_c = P2_Hc - P2_Fc     % in {camera}, m
```

```
P2_height_c = 0.0018
```

```
P2_height = z2/lambda*P2_height_c
```

```
P2_height = 1.7510
```

```
% Q4.3
% c coordinate is increasing, so the camera is moving down
% thus, y_speed is positive
d = 1.5;
pixel_diff = (314-121)/alpha_y;
y_speed = d/lambda*pixel_diff   % m travelled in 1.3s
```

```
y_speed = 0.3003
```

```
time = 1.3;
y_speed = y_speed/time  % final answer
```

```
y_speed = 0.2310
```