Quiz 2

Question 1 (1 point)

What is the size of a homogeneous transformation matrix in 3D?

- 3
- 4
- 6
- 9

the size is 4 since we have a 3x3 rotation matrix and 3x1 position vector

Question 2 (1 point)

What is the size of a homogeneous transformation matrix in 2D?

- 2
- 3
- 4

the size is 3 since we have a 2x2 rotation matrix and 2x1 position vector

Question 3 (1 point)

A homogenization transformation matrix can be used

- For rotations and translations with respect to the **current** frame axes.
- For rotations and translations with respect to the **world** frame axes.
- Both

both

Question 4 (2 points)

Which of the following statements are correct

- [ ] $$Rot_{z,30}Trans_{x,0.5} = Trans_{x,0.5}Rot_{z,30}$$
- [ ] $$Rot_{x,30}Trans_{x,1.5}Rot_{z,45} = Trans_{x,1.5}Rot_{x,30}Rot_{z,45}$$
- [ ] $$Rot_{x,30}Trans_{x,1.5}Rot_{z,45} = Rot_{z,45}Rot_{x,30}Trans_{x,1.5}$$
- [ ] $$Rot_{y,30}Rot_{y,15} = Rot_{y,45}$$
- [ ] $$Trans_{x,1.5}Trans_{x,-1.5} = I_{4\times 4}$$
- [ ] $$Trans_{x,1.5}Rot_{z,15}Trans_{x,-1.5} = Rot_{z,15}$$

RotX,30 TransX,1.5 RotZ,45 = TransX,1.5 RotX,30 RotZ,45 % the changes happen to the same axis

RotY,30 RotY,15 = RotY,45 % adding to each other

TransX,1.5 TransX,-1.5 = I4x4 % going back and fourth same position

Question 5 (2 points)

Which of the following inputs are relevant for robot kinematics

- [ ] The mass of the links
- [ ] The speed of the motors at the joints
- [ ] The number and type of joints
- [ ] The torque of the motors at the joints
- [ ] The moment of inertia of the links
- [ ] The orientation and position of joint axes
- [ ] Gravity

Speed, Number, Orientation

Quiz 3

Question 1 (1 point)

A serial link manipulator with n=5 joints, has in total

[ ] links, and

[ ] frames.

6 links between the joints and each end

6 frames including the base

Question 2 (1 point)

With the usual numbering of joints and frames, when joint i is activated, then

- ○ frame i moves with respect to frame i-1
- ○ frame i+1 moves with respect to frame i

frame i-1 moves since joint i relates to frame i-1

Question 3 (1 point)

There are [ ] Denavit-Hartenberg parameters. [ ] of them are constants.

4 parameters 3 constant, only 1 parameter can be variable per frame

Question 4 (1 point)

The joint angle θ_4 is the angle between frame axis

[ ] and frame axis

[ ] about frame axis

[ ]

x3,x4,z3 You are turning about the z axis so you get the angle between the X axis relative to the z axis

Question 5 (1 point)

When the frame axes z_5 an z_6 are intersecting, then the link length a_6 is equal to

[ ] .

0 since otherwise they wouldnt intersect

Question 6 (2 points) ✓ *Saved*

If H is the homogeneous matrix of the end-effector for a serial link robot, select which of the following options (one or more) constitute a valid (sufficient) input for solving the inverse kinematics problem:

- [ ] The values H_{11}, H_{21}, H_{31}, H_{12}, H_{22}, H_{32}, H_{13}, H_{23}, H_{33} are known
- [ ] The values H_{14}, H_{24}, H_{34} are known
- [x] The values H_{11}, H_{12}, H_{13}, H_{21}, H_{22}, H_{23}, as well as H_{14}, H_{24}, H_{34} are known
- [x] The values H_{12}, H_{22}, H_{32}, H_{13}, H_{23}, H_{33}, as well as H_{14}, H_{24}, H_{34} are known
- [ ] The values H_{11}, H_{21}, H_{31}, H_{12}, H_{22}, H_{32}, H_{13}, H_{23}, H_{33}, as well as H_{14} and H_{24} are known
- [ ] The values H_{13}, H_{23}, H_{33}, as well as H_{14}, H_{24}, H_{34} are known
- [x] The values H_{11}, H_{21}, H_{31}, H_{12}, H_{22}, H_{32}, as well as H_{14}, H_{24}, H_{34} are known

No clue

Question 7 (2 points)

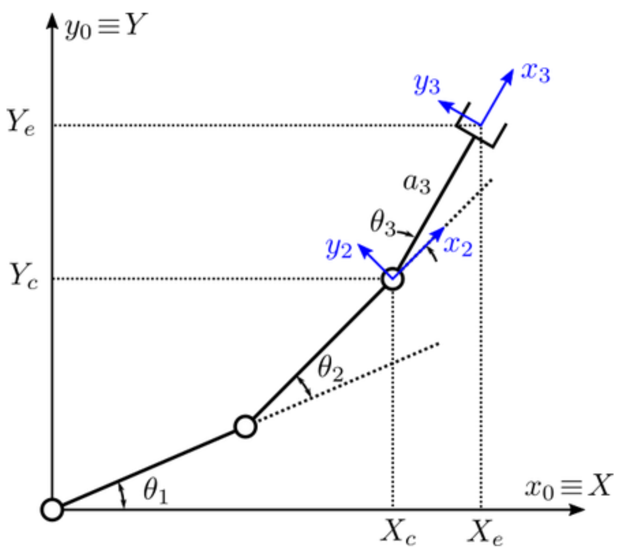
For the planar (2D) robot arm shown in the figure, the quantities

$$X_e,\ Y_e$$

as well as the unit vector

$$x_3$$

are given. Then, the inverse kinematics problem can be solved by the kinematics decoupling method. When you apply this method, you find the following intermediate quantities. Put them in the correct order.



4

$$R_{23}$$

this means finding the relative orientation of

$$x_3$$

with respect to

$$x_2$$

1

$$X_c,\ Y_c,\ R_{03}$$

5

$$\theta_3$$

2

$$\theta_1,\ \theta_2$$

3

$$R_{02}$$

5

This means finding the direction of

dont understand the order

Quiz 4

Question 1 (1 point)

A rigid body has an angular velocity given by the vector

$$\omega = [2, -1, 3]^T \text{ rad}/\text{s}$$

Select all correct alternative formats that can express the same quantity.

- [x] Angular velocity

$$\dot{\theta} = 3.7417 \text{ rad}/\text{s}$$

and rotation axis

$$k = [0.5345, -0.2673, 0.8018]^T$$

- [x] The skew symmetric matrix

$$S = \begin{bmatrix} 0 & -3 & -1 \\ 3 & 0 & -2 \\ 1 & 2 & 0 \end{bmatrix}$$

- [ ] Angular velocity

$$\dot{\theta} = 3.7417 \text{ rad}/\text{s}$$

and rotation axis

$$k = [-0.5345, 0.2673, -0.8018]^T$$

- [ ] The skew symmetric matrix

$$S = \begin{bmatrix} 0 & 3 & 1 \\ -3 & 0 & 2 \\ -1 & -2 & 0 \end{bmatrix}$$

- [x] Angular velocity

$$\dot{\theta} = -3.7417 \text{ rad}/\text{s}$$

and rotation axis

$$k = [-0.5345, 0.2673, -0.8018]^T$$

- [ ] The skew symmetric matrix

$$S = \begin{bmatrix} 0 & -3 & 1 \\ 3 & 0 & -2 \\ -1 & 2 & 0 \end{bmatrix}$$

$$v = \omega \times r = \begin{bmatrix} 0 & -\omega_z & \omega_y \\ \omega_z & 0 & -\omega_x \\ -\omega_y & \omega_x & 0 \end{bmatrix} \begin{bmatrix} r_x \\ r_y \\ r_z \end{bmatrix}$$

We look at the w vector and match with the matrix from the slides

Question 2 (1 point) ✓ *Saved*

Select which matrix or matrices are plausible time derivatives for a time dependent rotation matrix which at the current time is equal to

$$R = \begin{bmatrix} 0.328 & -0.737 & -0.591 \\ 0.591 & -0.328 & 0.737 \\ 0.737 & 0.591 & -0.328 \end{bmatrix}$$

(up to a certain precision)

- [ ] $$\dot{R} = \begin{bmatrix} 4.130 & 1.708 & 0.162 \\ 2.130 & -0.292 & -1.838 \\ 2.494 & -3.604 & -0.890 \end{bmatrix}$$

- [ ] $$\dot{R} = \begin{bmatrix} -2.474 & -2.182 & -0.344 \\ 0.838 & -3.130 & -0.708 \\ 4.150 & -4.078 & -1.072 \end{bmatrix}$$

- [x] $$\dot{R} = \begin{bmatrix} -4.130 & -1.708 & -0.162 \\ -0.818 & -2.656 & -0.526 \\ 2.494 & -3.604 & -0.890 \end{bmatrix}$$

$$\frac{dR}{d\theta} R^T + R \frac{dR^T}{d\theta} = 0$$

```
R = [0.328 -0.737 -0.591; 0.591 -0.328 0.737; 0.737 0.591 -0.328];
R1 = [4.130 1.708 0.162; 2.130 -0.292 -1.838; 2.494 -3.604 -0.890];
R2 = [-2.474 -2.182 -0.344; 0.838 -3.130 -0.708; 4.150 -4.078 -1.072];
R3 = [-4.130 -1.708 -0.162; -0.818 -2.656 -0.526; 2.494 -3.604 -0.890];

R1*R.'+R*R1.'
R2*R.'+R*R2.'
R3*R.'+R*R3.' % closest to 0
```

Question 3 (1 point)

The current orientation of a drone is given by the rotation matrix

$$R_1^0 = \begin{bmatrix} 0.983 & 0.017 & 0.183 \\ 0.017 & 0.983 & -0.183 \\ -0.183 & 0.183 & 0.966 \end{bmatrix}$$

representing a frame {1} fixed on the drone, with respect to the world frame {0}. The angular velocity of the drone is:

$$\omega_{0,1}^0 = \begin{bmatrix} 2 \\ -2 \\ 0 \end{bmatrix} \mathrm{rad}\,/\mathrm{s}$$

The angular velocity of one of the drone propellers is

$$\omega_{1,2}^1 = \begin{bmatrix} 0 \\ 0 \\ -200 \end{bmatrix} \mathrm{rad}\,/\mathrm{s}$$

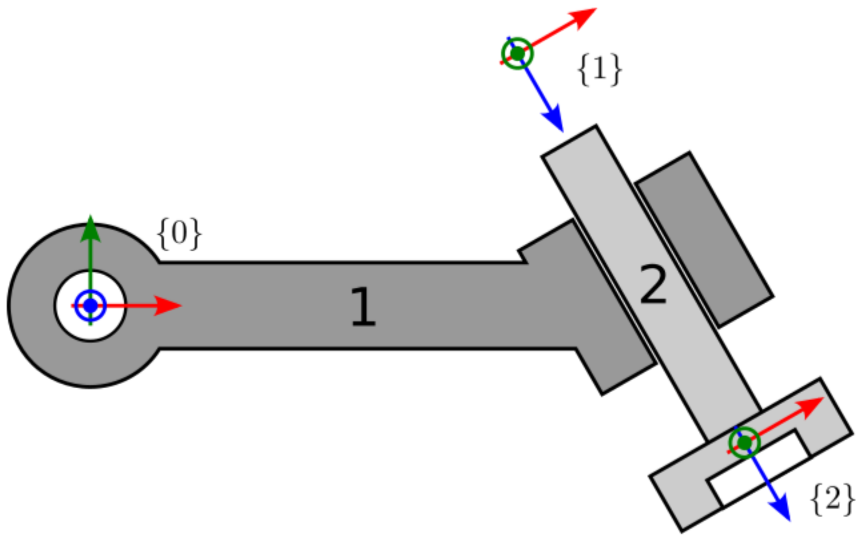
with respect to the drone (frame {2} is attached to the propeller).

Select the correct answers for each of the three components of the angular velocity of the drone propeller, with respect to the world frame.

```
R = [0.983 0.017 0.183; 0.017 0.983 -0.183; -0.183 0.183 0.966];
w1 = [2;-2;0];
w2 = [0;0;-200];
R*w2+w1
```

Question 4 (1 point)

The figure shows a planar robotic arm consisting of one revolute and one prismatic joint along with Denavit-Hartenberg frames assigned to it.



Match the following quantities with their correct values.

$$A_i = \begin{bmatrix} c_{\theta_i} & -s_{\theta_i}c_{\alpha_i} & s_{\theta_i}s_{\alpha_i} & a_i c_{\theta_i} \\ s_{\theta_i} & c_{\theta_i}c_{\alpha_i} & -c_{\theta_i}s_{\alpha_i} & a_i s_{\theta_i} \\ 0 & s_{\alpha_i} & c_{\alpha_i} & d_i \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

1. z_0
2. z_1
3. o_0
4. o_1
5. o_2
6. Jacobian for origin of frame {2}
7. Wrong Jacobian
8. Jacobian for origin of frame {1}

3 $\begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$

5 $\begin{bmatrix} a_1\cos\theta_1 + d_2\sin\theta_1 \\ a_1\sin\theta_1 - d_2\cos\theta_1 \\ 0 \end{bmatrix}$

7 $\begin{bmatrix} -a_1\sin\theta_1 - a_2\sin(\theta_1+\theta_2) & -a_2\sin(\theta_1+\theta_2) \\ a_1\cos\theta_1 + a_2\cos(\theta_1+\theta_2) & a_2\cos(\theta_1+\theta_2) \\ 0 & 0 \\ 0 & 0 \\ 0 & 0 \\ 1 & 1 \end{bmatrix}$

6 $\begin{bmatrix} -a_1\sin\theta_1 + d_2\cos\theta_1 & \sin\theta_1 \\ a_1\cos\theta_1 + d_2\sin\theta_1 & -\cos\theta_1 \\ 0 & 0 \\ 0 & 0 \\ 0 & 0 \\ 1 & 0 \end{bmatrix}$

8 $\begin{bmatrix} -a_1\sin\theta_1 & 0 \\ a_1\cos\theta_1 & 0 \\ 0 & 0 \\ 0 & 0 \\ 0 & 0 \\ 1 & 0 \end{bmatrix}$

4 $\begin{bmatrix} a_1\cos\theta_1 \\ a_1\sin\theta_1 \\ 0 \end{bmatrix}$

1 $\begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix}$

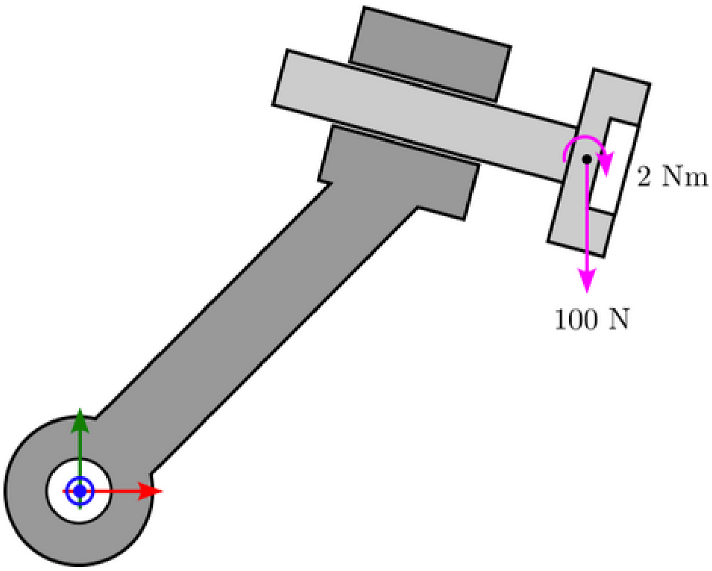
2 $\begin{bmatrix} \sin\theta_1 \\ -\cos\theta_1 \\ 0 \end{bmatrix}$

```
syms DH(th,d,a,alpha)
DH(th,d,a,alpha) = [cos(th) -sin(th)*cos(alpha) sin(th)*sin(alpha) a*cos(th);
                    sin(th) cos(th)*cos(alpha) -cos(th)*sin(alpha) a*sin(th);
                    0 sin(alpha) cos(alpha) d;
                    0 0 0 1];

syms theta_1 theta_2 a_1 d_2
T_01 = DH(theta_1, 0, a_1, pi/2);
T_12 = DH(theta_2, d_2, 0, 0);
T_02 = T_01*T_12;

z0 = [0;0;1];
z1 = T_01(1:3,3)
z2 = T_02(1:3,3)
o0 = [0;0;0];
o1 = T_01(1:3,4)
o2 = T_02(1:3,4)
% Jacobian for o1
J1 = [cross(z0,o1-o0);z0];
J2 = [0;0;0;0;0;0];
J = [J1 J2]
% Jacobian for o2
```

```
J1 = [cross(z0,o2-o0);z0];
J2 = [z1; [0;0;0]];
J = [J1 J2]
```

Question 5 (1 point)

The planar arm of the following figure has one revolute and one prismatic joint.



In the show configuration, the Jacobian for the end-effector is

$$J = \begin{bmatrix} -0.1673 & 0.9659 \\ 0.1484 & -0.2588 \\ 0 & 0 \\ 0 & 0 \\ 0 & 0 \\ 1 & 0 \end{bmatrix}$$

and the end-effector is loaded with the force and moment shown in the figure. Select the correct answers for the units of the Jacobian matrix components and for the required joint moment for joint 1 and the required force for joint 2.

- [x] J_{11} in m/rad
- [ ] J_{11} in rad/m
- [ ] J_{11} dimensionless
- [ ] J_{12} in m/rad
- [ ] J_{12} in rad/m
- [x] J_{12} dimensionless
- [ ] J_{61} in m/rad
- [ ] J_{61} in rad/m
- [x] J_{61} dimensionless
- [x] Moment for joint 1 $|\tau_1|$=2 Nm
- [ ] Moment for joint 1 $|\tau_1|$=14.836 Nm
- [ ] Moment for joint 1 $|\tau_1|$=16.836 Nm
- [ ] Force for joint 2 $|\tau_2|$=100 N
- [x] Force for joint 2 $|\tau_2|$=25.88 N
- [ ] Force for joint 2 $|\tau_2|$=16.73 N

```
J = [-0.1673 0.9659; 0.1484 -0.2588; 0 0; 0 0; 0 0; 1 0];
F = [0,100,0,0,0,2]';
tau = J'*F % see that the moment should be 16.8 not 2
```

Quiz 7 part 1

Question 1 (1 point) ✓ *Saved*

Which of the following statements is not correct?

- (●) An open loop system uses a measurement signal of the output, to define the control signal
- ( ) An open-loop control system utilizes an actuating device to control the process directly without using feedback.
- ( ) A closed-loop control system uses a measurement of the output and feedback of this signal to compare it with the desired output (reference or command).

Question 2 (1 point) ✓ *Saved*

Only closed loop systems can be unstable, if not properly designed

- ( ) True
- (●) False

Question 3 (1 point) ✓ *Saved*

Indicate which of the following statements is not correct: the Laplace transform has the following properties

- ( ) it has the property of linearity
- (●) none of the above
- ( ) it is possible to compute the derivative of a function, just by knowing the function and its initial condition
- ( ) it is a function in the complex space

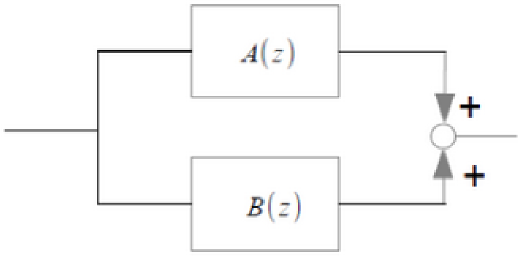
Question 4 (1 point) ✓ *Saved*

The type of a transfer function is the number of real and not null poles

- ( ) True
- (●) False

Question 5 (1 point) ✓ *Saved*

Given the following block diagram:



the input/output relation D(s) can be computed a

- ◯ D(s)=A(s)B(s)
- ◯ D(s)=A(s)/(1-A(s)B(s))
- ◉ D(s)=A(s)+B(s)
- ◯ none of the above

Quiz 7 part 2

Question 1 (1 point) ✓ *Saved*

The current passing through the coils of a DC motor is, at any moment, directly proportional to the applied voltage

- ◯ True
- ◉ False

Question 2 (1 point) ✓ *Saved*

The transfer function of a DC motor

$$\frac{\Theta_m(s)}{V_{in}(s)}$$

which is coupled to its load by means of an ideal rigid gear train:

- ○ has more than 3 poles
- ◉ has 3 poles
- ○ has two poles
- ○ has one pole

Question 3 (1 point) ✓ *Saved*

The transfer function of a DC motor

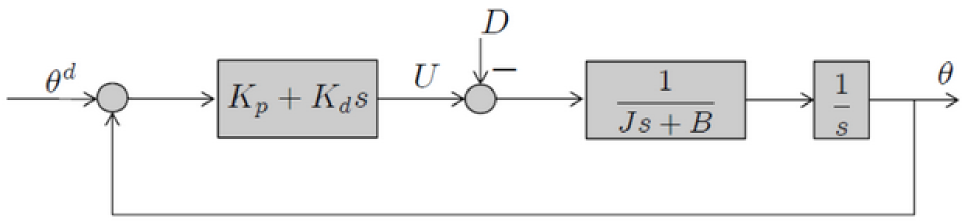
$$\frac{\Theta_m(s)}{V_{in}(s)}$$

which is coupled to its load by means of an ideal rigid gear train, under the assumption the electrical dynamics is much faster than the mechanical dynamics:

- ◉ can be approximated to have 2 poles, one of which in the origin
- ○ cannot be approximated
- ○ can be approximated to have 3 poles, two of which in the origin
- ○ has the same numbers of poles and zeroes

Question 4 (1 point) Saved

The controlled motor scheme in figure

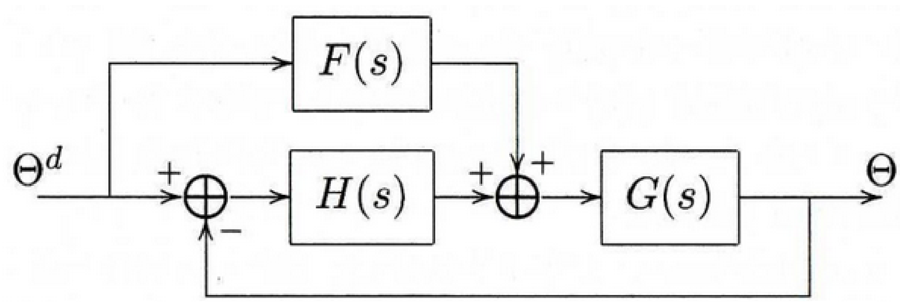


is:

- (selected) stable under certain conditions of Kp and Kd
- always unstable
- stable for all values of Kp and Kd
- stable for all Kp>0

Question 5 (1 point) Saved

A feedforward control scheme for tracking time varying trajectories, such as



is stable if the denominators of both the closed loop and the open loop functions are Hurwitz

- (selected) True
- False

Quiz 6

Question 1 (1 point) ✓ *Saved*

A robot end-effector needs to move from point A to point B. Between the two points there is an obstacle C that needs to be avoided. This problem is addressed in:

- (●) Path planning
- ( ) Trajectory planning

- **Path planning**:
  - –Geometric path, e.g. point-wise in configuration space or in workspace
  - –Avoidance of obstacles, shortest path …

Question 2 (1 point) ✓ *Saved*

A robot end-effector needs to move from point A to point B in less than 2 seconds. This problem is addressed in:

- ( ) Path planning
- (●) Trajectory planning

Question 3 (1 point) ✓ *Saved*

A robot arm is used to relocate a sensitive payload. The payload should not be subjected to an acceleration larger than $3\times9.82\ m/s^2$. This requirement is addressed in:

- ( ) Path planning
- (●) Trajectory planning

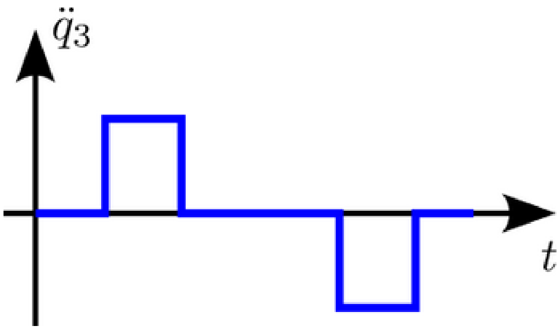
Question 4 (1 point) ✓ *Saved*

A robot end-effector is used to relocate a payload from point A to point B. The payload needs to remain in a certain fixed orientation during the operation. This requirement is mainly addressed in:

- (●) Path planning
- ( ) Trajectory planning

Question 5 (1 point) ✓ *Saved*

The 3rd joint of a serial link manipulator is driven following the acceleration history over time, shown in the figure



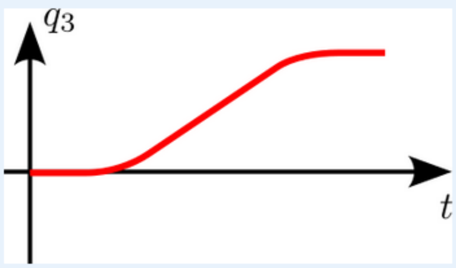
Select all correct statements among the options below.

(You get points only if you find all correct options)

- [x] The history of the joint variable

$$q_3$$

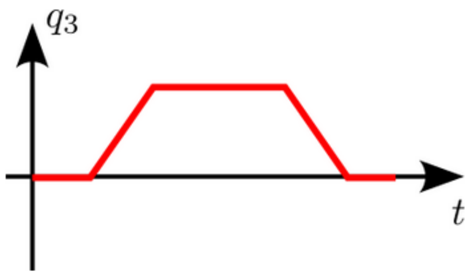
can look like this



- [ ] The history of the joint variable

$$q_3$$

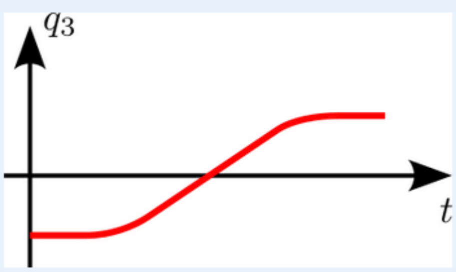
can look like this



- [x] The history of the joint variable

$$q_3$$

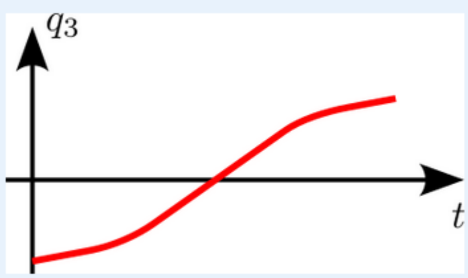
can look like this



- [x] The history of the joint variable

$$q_3$$

can look like this



Question 6 (1 point)

The motion of a prismatic robot joint is described by the cubic polynomial

$$q(t) = c_0 + c_1 t + c_2 t^2 + c_3 t^3$$

At t =1 sec: q =15mm, dq/dt = 0 mm/s

At t =3 sec: q =55mm, dq/dt = 0 mm/s

Then, c_0 must be equal to

55

mm

c_1 must be equal to

-90

mm/s

c_2 must be equal to

60

mm/s^2

and c_3 must be equal to

-10

mm/s^3

$$\begin{bmatrix} 1 & t_{in} & t_{in}^2 & t_{in}^3 \\ 0 & 1 & 2t_{in} & 3t_{in}^2 \\ 1 & t_A & t_A^2 & t_A^3 \\ 0 & 1 & 2t_A & 3t_A^2 \end{bmatrix} \begin{bmatrix} c_0 \\ c_1 \\ c_2 \\ c_3 \end{bmatrix} = \begin{bmatrix} q_{in} \\ v_{in} \\ q_A \\ v_A \end{bmatrix}$$

```
t_a = 1;
t_b = 3;
q_a = 15;
qd_a = 0;
q_b = 55;
qd_b = 0;
T = [1 t_a t_a^2 t_a^3;
    0 1 2*t_a 3*t_a^2;
    1 t_b t_b^2 t_b^3;
    0 1 2*t_b 3*t_b^2];

c = inv(T)*[q_a; qd_a; q_b; qd_b]
```

```
c = 4×1
    55
   -90
    60
   -10
```

Question 7 (1 point)

A robot arm consists of a prismatic joint #1 and a revolute joint #2. Its Lagrangian function is in the form

$$L = \frac{1}{2}A\dot{q}_1^2 + \frac{1}{2}B\dot{q}_2^2 + C\sin(q_2)\dot{q}_1\dot{q}_2 - Dq_1 - E\cos(q_2) - F\sin(q_2)$$

(A,B,C,D,E,F are just symbols for some constants, do not try to find them in the textbook)

Select all correct statements:

- [ ] Constant A has units in [kg.m]
- [x] Constant A has units in [kg]
- [x] Constant B has units in [kg.m²]
- [ ] Constant B has units in [kg]
- [ ] Constant C has units in [kg.m²]
- [x] Constant C has units in [kg.m]
- [ ] Constant C has units in [kg]
- [ ] Constant D has units in [kg]
- [x] Constant D has units in [N]
- [ ] Constant D has units in [N.m]
- [ ] Constant E has units in [kg]
- [ ] Constants E and F have units in [N]
- [x] Constants E and F have units in [N.m]
- [ ] Constants E and F have units in [kg]

q1 = m, q1 dot=m/s, q2=rad=enhedsløst, q2 dot = 1/s, q1 dot ^2 = (m/s)^2, q2dot^2 =1/s^2, L=kg*(m/s)^2

Question 8 (2 points)

A robot arm consists of a prismatic joint #1 and a revolute joint #2. Its Lagrangian function is in the form

$$L = \frac{1}{2}0.8\dot{q}_1^2 + \frac{1}{2}0.003\dot{q}_2^2 - 0.03\sin(q_2)\dot{q}_1\dot{q}_2 - 7q_1 -$$

$$0.2\cos(q_2) - 0.2\sin(q_2)$$

Select the correct dynamic equations for joint #1 and joint #2 (expecting 2 selections in total):

Select the correct dynamic equations for joint #1 and joint #2 (expecting 2 selections in total):

- [ ] For joint #1

$$0.8\dot{q}_1 - 0.03\sin(q_2)\dot{q}_2 + 7 = \tau_1$$

- [ ] For joint #1

$$0.8\ddot{q}_1 + 0.03\sin(q_2)\ddot{q}_1 - 7 = \tau_1$$

- [x] For joint #1

$$0.8\ddot{q}_1 - 0.03\sin(q_2)\ddot{q}_2 - 0.03\cos(q_2)\dot{q}_2^2 + 7 = \tau_1$$

- [x] For joint #2

$$0.003\ddot{q}_2 - 0.03\sin(q_2)\ddot{q}_1 + 0.2\cos(q_2) - 0.2\sin(q_2) = \tau_2$$

- [ ] For joint #2

$$0.003\ddot{q}_2 - 0.03\sin(q_2)\ddot{q}_1 - 0.06\cos(q_2)\dot{q}_1\dot{q}_2 + 0.2\sin(q_2) - 0.2\cos$$

- [ ] For joint #2

$$0.003\ddot{q}_2 - 0.03\sin(q_2)\ddot{q}_1 - 0.03\cos(q_2)\dot{q}_1\dot{q}_2 + 0.2\sin(q_2) - 0.2\cos$$

- [ ] For joint #2

$$0.003\dot{q}_2 - 0.03\sin(q_2)\dot{q}_1 - 0.2\sin(q_2) + 0.2\cos(q_2) = \tau_2$$

$$\frac{d}{dt}\frac{\partial \mathcal{L}}{\partial \dot{q}_k} - \frac{\partial \mathcal{L}}{\partial q_k} = \tau_k$$

```
syms q1(t) q2(t) t
q1d(t) = diff(q1(t),t);
q2d(t) = diff(q2(t),t);
L =
1/2*0.8*q1d(t)^2+1/2*0.003*q2d(t)^2-0.03*sin(q2(t))*q1d(t)*q2d(t)-7*q1(t)-0.2
*cos(q2(t))-0.2*sin(q2(t));
```

```
diff(diff(L,q1d(t)),t)-diff(L,q1(t))
```

ans =

$$\frac{4\frac{\partial^2}{\partial t^2}q_1(t)}{5}-\frac{3\sin(q_2(t))\frac{\partial^2}{\partial t^2}q_2(t)}{100}-\frac{3\cos(q_2(t))\left(\frac{\partial}{\partial t}q_2(t)\right)^2}{100}+7$$

```
diff(diff(L,q2d(t)),t)-diff(L,q2(t))
```

ans =

$$\frac{\cos(q_2(t))}{5}-\frac{\sin(q_2(t))}{5}-\frac{3\sin(q_2(t))\frac{\partial^2}{\partial t^2}q_1(t)}{100}+\frac{3\frac{\partial^2}{\partial t^2}q_2(t)}{1000}$$

Question 9 (3 points)

Consider the dynamic equation of motion

D(q) q'' + C(q,q') q' + g(q) = τ

of a 2 joint robot arm with a known inertia matrix

$$D=\begin{bmatrix}0.8 & -0.03\sin(q_2)\\ -0.03\sin(q_2) & 0.003\end{bmatrix}$$

Select which of the corresponding Christoffel symbols c_{ijk} are **non-zero**, as well as which of the elements c_{kj} of the robot matrix C(q,q') are **non-zero**.

$$D(q)\ddot{q}+C(q,\dot{q})\dot{q}+g(q)=\tau$$

$$D(q)=\sum_{i=1}^{k}(m_iJ_{v_i}^T(q)J_{v_i}(q)+J_{\omega_i}(q)^TR_i(q)IR_i(q)^TJ_{\omega_i}(q))$$

Element of $C(q,\dot{q})$: $c_{kj}=\sum_{i=1}^{n}c_{ijk}(q)\dot{q}_i=\sum_{i=1}^{n}\frac{1}{2}\left(\frac{\partial d_{kj}}{\partial q_i}+\frac{\partial d_{ki}}{\partial q_j}-\frac{\partial d_{ij}}{\partial q_k}\right)\dot{q}_i$

$$g(q)=[g_1,\ldots,g_n(q)]^T$$

```
syms q1 q2
q = [q1 q2];
D = [0.8 -0.03*sin(q2); -0.03*sin(q2) 0.003];

c = sym(zeros(2,2,2));       % small c
for i=1:2 % sum function
    for j=1:2
        for k=1:2
            c(i,j,k) = 0.5*(diff(D(k,j),q(i)) + diff(D(k,i),q(j)) - 
diff(D(i,j),q(k)));
        end
    end
```

```
end
c
```

c(:,:,1) =

$$\begin{pmatrix} 0 & 0 \\ 0 & -\frac{3\cos(q_2)}{100} \end{pmatrix}$$

c(:,:,2) =

$$\begin{pmatrix} 0 & 0 \\ 0 & 0 \end{pmatrix}$$

```
q = [q1 q2];

% robot matrix C(q,qd) C_kj
C = sym(zeros(2,2));    % big C
for k=1:2
    for j=1:2
        for i=1:2
            C(k,j) = C(k,j) + c(i,j,k);
        end
    end
end
C
```

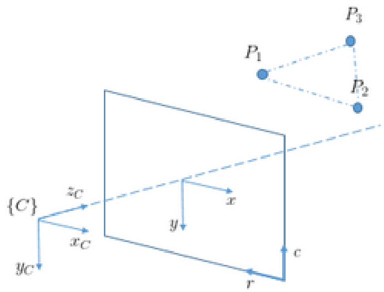
C =

$$\begin{pmatrix} 0 & -\frac{3\cos(q_2)}{100} \\ 0 & 0 \end{pmatrix}$$

Quiz 8

Question 1 (1 point) ✓ *Saved*

A triangular object is placed in front of a camera with frame $\{C\}$, as in figure:



The camera parameters are:

$$f = 10mm$$

$$\alpha_x = \frac{1}{s_x} = 96pix/mm$$

$$\alpha_y = \frac{1}{s_y} = 96pix/mm$$

The frame of reference of the camera is placed in the positive x-y quadrant with respect to frame $\{C\}$, at the corner of the CCD, and the distance between the CCD frame and the focal axis is

$$o_r = 300pix$$

and

$$o_c = 300pix$$

.

The corners of the object are placed in the following positions w.r.t. the camera frame $\{C\}$:

$$P_1 = [-0.05; -0.05; 0.8]$$

m

$$P_2 = [0.05; -0.05; 0.8]$$

m

$$P_3 = [-0.05; -0.05; 1.3]$$

m

Find the projection

$$\hat{P}_i$$

i=1,2,3 in pixels of the three points on the CCD.

NOTE: be careful to the position of the camera frame w.r.t. the CCD frame.

```
% Find pixel coordinates from {camera} points

f = 10*96;  % in pixels
or = 300;       % pix
oc = 300;       % pix

% in the camera frame {C}
P1 = [-0.05;-0.05;0.8];    % m
P2 = [0.05;-0.05;0.8];     % m
P3 = [-0.05;-0.05;1.3];    % m
P1 = P1*1000;
P2 = P2*1000;
P3 = P3*1000;

% negative because x=-r, y=-c
u = -f*P1(1)/P1(3) + or;
v = -f*P1(2)/P1(3) + oc;
P1_p = [u,v]
```

```
P1_p = 1×2
   360   360
```

```
u = -f*P2(1)/P2(3) + or;
v = -f*P2(2)/P2(3) + oc;
P2_p = [u,v]
```

```
P2_p = 1×2
   240   360
```

```
u = -f*P3(1)/P3(3) + or;
v = -f*P3(2)/P3(3) + oc;
P3_p = [u,v]
```

```
P3_p = 1×2
  336.9231  336.9231
```