Robotics Project
Kalman Filter

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Abstract

Kalman filter.

1 Introduction

In this project we will use Kalman filter to estimate the motion path of an aircraft.

1.1 Physics model

The following model defines the state estimation:

$$\hat{\mathbf{x}}_t = \mathbf{A}\hat{\mathbf{x}}_{t-1} + \mathbf{B}\mathbf{u}_t + \mathbf{w}_{t-1} \tag{1}$$

Which is

$$\begin{bmatrix} p_x^{(t)} \\ p_y^{(t)} \\ v_x^{(t)} \\ v_y^{(t)} \end{bmatrix} = \underbrace{\begin{bmatrix} 1 & 0 & t & 0 \\ 0 & 1 & 0 & t \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}}_{\text{state transition}} \begin{bmatrix} p_x^{(t-1)} \\ p_y^{(t-1)} \\ v_x^{(t-1)} \\ v_y^{(t-1)} \end{bmatrix} + \underbrace{\begin{bmatrix} t^2/2 & 0 \\ 0 & t^2/2 \\ t & 0 \\ 0 & t \end{bmatrix}}_{\text{control matrix}} \begin{bmatrix} a_x^{(t-1)} \\ a_y^{(t-1)} \end{bmatrix} + \text{noise} \tag{2}$$

whose noise $\sim \mathcal{N}(0, \Sigma_x)$. And the following defines the observation from the state

$$\mathbf{z}_k = \mathbf{H}\hat{\mathbf{x}}_k + \text{noise} \tag{3}$$

which is

$$\begin{bmatrix} p_x^{(t)} \\ p_y^{(t)} \end{bmatrix} = \underbrace{\begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \end{bmatrix}}_{\text{estimation model}} \begin{bmatrix} p_x \\ p_y \\ v_x \\ v_y \end{bmatrix} + \text{noise} \tag{4}$$

whose noise $\sim \mathcal{N}(0, \Sigma_z)$.

1.2 Prediction

$$p_t = \mathbf{A} p_{t-1} \mathbf{A}^T + Q \tag{5}$$

where Q is the convarience matrix of E_x.

1.3 Measure

The Kalman gain would be

$$k_k = p_k \mathbf{H}^T \left(\mathbf{H} p_k \mathbf{H}^T + R \right)^{-1} \tag{6}$$

where r is the convarience matrix of E_z.

$$E_x = \begin{bmatrix} \sigma_1^2 & & \sigma_1\sigma_3 & \\ & \sigma_2^2 & & \sigma_2\sigma 4 \\ \sigma_3\sigma_1 & & \sigma_3^2 & \\ & \sigma_4\sigma_2 & & \sigma_4^2 \end{bmatrix} \tag{7}$$

2 Conclusion