

## Results from UVA

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### 0.3 Problem E (page 9)

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17306383	1714 Keyboarding	Accepted	JAVA	13.780	2016-05-04 01:50:35
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# 1 Problem A: Amalgamated Artichokes

## Results:

17246735	1709 Amalgamated Artichokes	Accepted	JAVA	4.410	2016-04-22 19:22:25
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## Background:

Fatima Cynara is an analyst at Amalgamated Artichokes (AA). As with any company, AA has had some very good times as well as some bad ones. Fatima does trending analysis of the stock prices for AA, and she wants to determine the largest decline in stock prices over various time spans. For example, if over a span of time the stock prices were 19, 12, 13, 11, 20 and 14, then the largest decline would be 8 between the first and fourth price. If the last price had been 10 instead of 14, then the largest decline would have been 10 between the last two prices.

Fatima has done some previous analyses and has found that the stock price over any period of time can be modelled reasonably accurately with the following equation:

$$price(x) = p \cdot (\sin(a \cdot x + b) + \cos(c \cdot x + d) + 2)$$

where  $p, a, b, c$ , and  $d$  are constants. Fatima would like you to write a program to determine the largest price decline over a given sequence of prices. You have to consider the prices only for integer values of  $x$ .

## Input:

The input file contains several test cases. Each test case is on a single line containing 6 integers,  $p$  ( $1 \leq p \leq 1000$ ),  $a$ ,  $b$ ,  $c$ ,  $d$  ( $0 \leq a, b, c, d \leq 1000$ ), and  $n$  ( $1 \leq n \leq 10^6$ ). The first 5 integers are described above. The sequence of stock prices to consider are  $price(1), price(2), \dots, price(n)$ .

## Output:

For each test case, display the maximum decline in stock prices. If there is no decline, display the number '0'. Your output should have an absolute or relative error of at most  $10^{-6}$ .

## Sample Input:

```
42 1 23 4 8 10
100 7 615 998 801 3
100 432 406 867 60 1000
```

## Sample Output:

```
104.855110477
0.00
399.303813
```

## 1.1 Mathematical Formulation

Given an input of integers  $p, a, b, c, d$ , and  $n$ , the formula  $f(x) = p \cdot (\sin(a \cdot x + b) + \cos(c \cdot x + d) + 2)$  where  $x \in [1, n]$ , determine the largest decrease between the integer values  $x_i, x_j$  where  $i < j$  and  $x_i \geq x_j$  and there does not exist another pair  $x_k, x_l$  where  $k < l$  and  $x_k \geq x_l$  but  $x_k - x_l > x_i - x_j$ .

## 1.2 Solution

The main functionality of this algorithm is to plug in each point keeping track of the highest seen point,  $h$ , the lowest seen point occurring after  $l$ , and the largest difference,  $d = h - l$ . It should be noted that since we are always taking the difference between the two values, we can factor out the  $\cdot p$  as well as neglect the  $+2$  portions of the formula. Also, to cut down on run time, it works in the java system if you `% pi` each of the entries before putting them into the sine and cosine functions. For whatever reason the larger the input, the more costly the operation is.

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### Algorithm 1 Main

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```

procedure F(x)
   $ab \leftarrow (a \cdot x + b) \% \pi$ ,
   $cd \leftarrow (c \cdot x + d) \% \pi$ ;
  return (Math.sin(ab) + Math.cos(cd))

procedure SOLVE(p, a, b, c, d, n)
   $val, h, l \leftarrow f(1)$ ;  $diff \leftarrow 0$ 
  for  $x \in [2, n]$  do // if  $n = 1$ , do not execute
     $val \leftarrow f(x)$ 
    if  $val > h$  then // higher than current highest
       $h, l \leftarrow val$ ;
    else if  $val < l$  then // lower than current lowest
       $l \leftarrow val$ ;  $curDiff \leftarrow h - l$ ;
      if  $curDiff > diff$  then  $diff \leftarrow curDiff$ ;
  PRINT( $p \cdot diff$ )

```

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## 1.3 Correctness

### Proposition 1.

*We will determine the value of largest price decline over the interval  $[1, n]$ , only considering  $f(1), f(2), \dots, f(n)$ .*

*Proof.*

We do this by keeping track of the largest price decline seen thus far,  $diff$ , the current highest point seen,  $h = f(x_i)$ , and the current lowest point seen,  $l = f(x_j)$ , such that  $x_i \leq x_j$ , and  $f(x_i) \geq f(x_j)$ . Therefore whenever we see a higher point,

$f(x_k) > f(x_i), x_k > x_i$ , we update our  $h = f(x_k)$  and reset our lowest point to be  $l = h$  since we are searching for the largest decline  $\implies l$  must occur after  $h$ . Now every time that we see a number  $f(x_m) \leq l$ , we update  $l$  and check to see if our  $h - l \geq \text{diff}$ , if so we update  $\text{diff}$ , else we continue to the next point. If we see a higher point than  $h$  we will repeat this process. Therefore we will be looking at each subsequent highest corresponding following lowest points  $\implies$  we will see this largest price decline.  $\square$

## 1.4 Analysis

**Proposition 2.** The space complexity of this algorithm is  $O(1)$

*Proof.*

This is due to the fact that we will only store the values  $p, a, b, c, d, n$ , and  $\text{diff}$  as integer variables  $O(1)$ :

Giving us a space complexity of  $O(1)$

$\square$

**Proposition 3.** The time complexity of this algorithm is  $O(N)$

*Proof.* This is the case because our algorithm goes through the points  $1, 2, \dots, n$  once and only calculates each value one time.

Giving us a time complexity of  $O(N)$

$\square$

## 1.5 An Example

Given the input of: 42 1 23 4 8 10, we will read this in as  $p = 42, a = 1, b = 23, c = 4, d = 8$ , and  $n = 10$ . Then we will initialize our  $h = l = f(1)$  and  $\text{diff} = 0$ . Then starting with the second point until the 10th we will read through and record the values of what  $h, l, \text{curDiff}$ , and  $\text{diff}$  are:

$x =$	1	2	3	4	5	6	7	8	9	10
$f(x)$	-0.061724	-1.090011	1.170641	1.380555	-0.691700	0.170589	-1.115995	-1.070976	1.551270	0.359768
$h$	-0.061724	-0.061724	1.170641	1.380555	1.380555	1.380555	1.380555	1.380555	1.551270	1.551270
$l$	-0.061724	-1.090011	1.170641	1.380555	-0.691700	-0.691700	-1.115995	-1.115995	1.551270	0.359768
$\text{curDiff}$	—	1.028286	—	—	2.072255	—	2.496550	—	—	1.191502
$\text{diff}$	0	1.028286	1.028286	1.028286	2.072255	2.072255	2.496550	2.496550	2.496550	2.496550

Now multiplying our  $\text{diff}$  by  $p \implies 2.496550 \cdot 42 = 104.855110$  which is our solution.

## 2 Problem D: Cutting Cheese

### Results:

17274629	1712 Cutting Cheese	Accepted	JAVA	1.700	2016-04-27 22:19:28
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### Background:

Of course you have all heard of the International Cheese Processing Company. Their machine for cutting a piece of cheese into slices of exactly the same thickness is a classic. Recently they produced a machine able to cut a spherical cheese (such as Edam) into slices – no, not all of the same thickness, but all of the same weight! But new challenges lie ahead: cutting Swiss cheese.

Swiss cheese such as Emmentaler has holes in it, and the holes may have different sizes. A slice with holes contains less cheese and has a lower weight than a slice without holes. So here is the challenge: cut a cheese with holes in it into slices of equal weight.

By smart sonar techniques (the same techniques used to scan unborn babies and oil elds), it is possible to locate the holes in the cheese up to micrometer precision. For the present problem you may assume that the holes are perfect spheres.

Each uncut block has size  $100 \times 100 \times 100$  where each dimension is measured in millimeters. Your task is to cut it into  $s$  slices of equal weight. The slices will be  $100\text{mm}$  wide and  $100\text{mm}$  high, and your job is to determine the thickness of each slice.

### Input:

The input file contains several test cases, each of them as described below:

The first line of the input contains two integers  $n$  and  $s$ , where  $0 \leq n \leq 10000$  is the number of holes in the cheese, and  $1 \leq s \leq 100$  is the number of slices to cut. The next  $n$  lines each contain four positive integers,  $r, x, y$ , and  $z$  are the coordinates of the center, all in micrometers. The cheese block occupies the points  $(x, y, z)$  where  $0 \leq x, y, z \leq 100000$ , except for the points that are part of some hole. The cuts are made perpendicular to the  $z$ -axis.

You may assume that holes do not overlap but may touch, and that the holes are fully contained in the cheese but may touch its boundary.

### Output:

For each test case, display the  $s$  slice thicknesses in millimeters, starting from the end of the cheese with  $z = 0$ . Your output should have an absolute or relative error of at most  $10^{-6}$ .

## 2.1 Mathematical Formulation

Given an input of  $n$  totally encapsulated, non-overlapping spheres, each with a  $x$  position and  $r$ , radius we can determine where to make cuts in a block of cheese  $100000 \times 100000 \times 100000$

## 2.2 Solution

The main functionality of this algorithm is to compute the total volume of the block of cheese, then determine what the weight should be of each equally sliced piece and perform a binary search of segments (from the left side,  $z = 0$ ) to find the segments which are within  $10^{-6}$  of this target weight. We calculate the volumes of each block by using spherical segment calculations of all spheres within the range in question.

The only data structure we used was a two dimensional array **holes** $[n][2]$  such that each entry *holes* $[i][0]$  corresponds to the center  $z$  coordinate and *holes* $[i][1]$  is the radius of hole  $i$ . Asside from this we use the instance variable  $v$ , *goal*, *numHoles*, *numSlices* to keep track of the total volume, the goal weight for evenly cut slices of cheese, the number of holes and the number of slices.

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### Algorithm 2 Main

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#### **procedure** MAIN

$v, numHoles, numSlices, holes[numHoles][2] \leftarrow$  initialized

**for** hole  $i \in [1..numHoles]$  **do**

    store  $i_z, i_r$  in index  $i$

    update  $v$

$goal \leftarrow v / numSlices$

Sort(*holes*) based off of left-most point on sphere on z-axis

BINARYSEARCH( )

#### **procedure** BINARYSEARCH

(double)  $low, high, last, cut \leftarrow 0, 100000, 0.0$

(int)  $slicePerformed \leftarrow 1$

**while** true **do**

$cut \leftarrow (h + l) / 2$

    (double)  $diff \leftarrow goal - ALLVOL(last, cut)$

**if**  $diff == 0$  **then**

        PRINT((cut-last)/1000)

$last \leftarrow cut; l \leftarrow last; h \leftarrow 100000;$

**if**  $slicePerformed++ == numSlices-1$  **then** break;

**else if**  $goal - val > 0$  **then**  $l \leftarrow cut$

**else**  $h \leftarrow cut$

    PRINT((100000 - last)/1000);

---

Since we have sorted by left most part of each sphere, as soon as the leftmost point of the  $i^{\text{th}}$  hole is past the mark  $b$ , then the points  $j \geq i$  are not contained in the bounds so we can break out of the loop and not calculate any more spheres. Otherwise we calculate the spherical segments of each sphere for which there is some portion of it in the band  $[a, b]$ . The link for equations used can be found here: <http://mathworld.wolfram.com/SphericalSegment.html>.

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**Algorithm 3** Computations
 

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**procedure** ALLVOL(double a, double b)

$vol \leftarrow DIM \cdot DIM \cdot (b - a)$

**for**  $i \in 1..numHoles$  **do**

**if**  $i_z - i_r > b$  **then** break;

**if**  $i_z + i_r < a$  **then** continue;

$val \leftarrow val + \text{VOLINRANGE}(a, b, i);$

  return  $val$ ;

**procedure** VOLINRANGE(double a, double b, int i)

  Computes the spherical segment based off of the formulas in the link above.

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## 2.3 Correctness

### Corollary 4.

*It is sufficient for us to simply view the spheres on a 1-dimensional plane because we are ensured that each sphere is fully encapsulated within the block of cheese and that no two spheres are overlapping, therefore if we take the band of  $[a, b]$ , if some portion of sphere  $i$  is within this we can calculate the volume displaced using only the portion  $(i_a, i_b)$  that overlaps with sphere  $i$  and its radius,  $i_r$ .*

### Proposition 5.

*Given the correct process for determining the volume of portions of spheres, a binary search for cuts in the cheese will give us the correct cuts to make.*

*Proof.*

We know that binary search is a viable option for when we know the stopping criteria and can calculate or lookup each intermediate stage. Therefore if we perform  $s$  binary searches, decreasing our range appropriately each time we find a cut, and we know our stopping criteria as *goal*; we can calculate the volume displaced by each portion of a sphere in intermediate ranges  $[a, b]$ , ultimately giving us the appropriate cut coordinates along the z-axis.  $\square$

## 2.4 Analysis

**Proposition 6.** *The space complexity of this algorithm is  $O(N)$*

*Proof.*

This is due to the fact that all we store are the z-coordinate and the radius of each hole.

Giving us a space complexity of  $O(N)$

□

**Proposition 7.** *The time complexity of this algorithm is  $O(N \cdot \log(N) + N \cdot S \cdot \log(S))$*

*Proof.* This is the case because our initial sorting of the holes takes  $N \cdot \log(N)$ , then within the binary search ( $S \cdot \log(S)$ ) we do at most (worstcase if every interval contains every hole)  $N$  volume calculations.

Giving us a time complexity of  $O(N \cdot \log(N) + N \cdot S \cdot \log(S))$

□

## 2.5 An Example

Given the input of:

1 2

10000 10000 10000 50000

Which is asking for 2 slices with one hole positioned at  $(x, y, z) = (10000, 10000, 50000)$  and the radius is 10000. As we read in we learn:

$$vol = 9.958112 \cdot 10^{14} \implies goal = 4.979056 \cdot 10^{14}.$$

For this case we record everything and make our initial cut at 50000, for which we calculate the sphere takes up  $2.094395 \cdot 10^{12} \text{ micrometers}^3$  which gives us our goal of  $4.979056 \cdot 10^{14}$ . Therefore we print out:

50.000000

50.000000



### 3 Problem E: Evolution

#### Results:

17311732	1713 Evolution in Parallel	Accepted	JAVA	0.620	2016-05-04 17:06:22
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#### Background:

It is 2178, and alien life has been discovered on a distant planet. There seems to be only one species on the planet and they do not reproduce as animals on Earth do. Even more amazing, the genetic makeup of every single organism is identical!

The genetic makeup of each organism is a single sequence of nucleotides. The nucleotides come in three types, denoted by 'A' (Adenine), 'C' (Cytosine), and 'M' (Muamine). According to one hypothesis, evolution on this planet occurs when a new nucleotide is inserted somewhere into the genetic sequence of an existing organism. If this change is evolutionarily advantageous, then organisms with the new sequence quickly replace ones with the old sequence.

It was originally thought that the current species evolved this way from a single, very simple organism with a single-nucleotide genetic sequence, by way of mutations as described above. However, fossil evidence suggests that this might not have been the case. Right now, the research team you are working with is trying to validate the concept of "parallel evolution" – that there might actually have been two evolutionary paths evolving in the fashion described above, and eventually both paths evolved to the single species present on the planet today. Your task is to verify whether the parallel evolution hypothesis is consistent with the genetic material found in the fossil samples gathered by your team.

#### Input:

The input file contains several test cases, each of them as described below.

The input begins with a number  $n$ , ( $1 \leq n \leq 4000$ ) denoting the number of nucleotide sequences found in the fossils. The second line describes the nucleotide sequence of the species currently living on the planet. Each of the next  $n$  lines describes one nucleotide sequence found in the fossils.

Each nucleotide sequence consists of a string of at least one but no more than 4 000 letters. The strings contain only upper-case letters 'A', 'C', and 'M'. All the nucleotide sequences, including that of the currently live species, are distinct.

#### Output:

For each test case, display an example of how the nucleotide sequences in the fossil record participate in two evolutionary paths. The example should begin with one line containing two integers  $s_1$  and  $s_2$ , the number of nucleotide sequences in the fossil record that participate in the first path and second path, respectively. This should be followed by  $s_1$  lines containing the sequences attributed to the first path,

in chronological order (from the earliest), and then  $s_2$  lines containing the sequences attributed to the second path, also in chronological order. If there are multiple examples, display any one of them. If it is possible that a sequence could appear in the genetic history of both species, your example should assign it to exactly one of the evolutionary paths.

If it is impossible for all the fossil material to come from two evolutionary paths, display the word 'impossible'.

### 3.1 Mathematical Formulation

Given a currently present string  $s_g$  and  $N$  strings of average length  $S$ , determine if the  $N$  strings can be grouped into 2 sequences of evolutionary paths  $sub_1$  and  $sub_2$ . A valid evolutionary path constitutes of having each string to be a sub-sequence [defined below] of the proceeding string. Both  $sub_1$  and  $sub_2$  should be sub-sequences of  $s_g$  and if there is a string which does not belong to either  $sub_1$  or  $sub_2$  or they are not a sub-sequence of  $s_g$ , then the algorithm will return "impossible".

### 3.2 Solution

The main functionality of this algorithm is to first order the given strings in ascending order in terms of length of string. Then we will build  $sub_1$  and  $sub_2$  top down, first appending  $s_g$  then ensuring that every subsequent string added to either  $sub_1$  or  $sub_2$  must be a sub-sequence [see helpers section for this] of the currently smallest length element on them.

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#### Algorithm 4 Main

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```

procedure MAIN
  for each test case do
    INITIALIZE // see below
    boolean failed, shared  $\leftarrow$  false; String lastS1, lastS2  $\leftarrow$  initialized
    sub1.PUSH(sg); sub2.PUSH(sg);
    for each string i from N..1 do
      token  $\leftarrow$  sequence[i]
      if shared then
        if isSubSequence(token, sharedList.peekFirst()) then
          sharedList.addFirst(token);
        else if isSubSequence(token, lastS1) then
          sub1.PUSH(token)
          sub2.PUSH(sharedList)
          shared  $\leftarrow$  true;
        else if isSubSequence(token, lastS2) then
          sub2.PUSH(token)
          sub1.PUSH(sharedList)
          shared  $\leftarrow$  true;
        else failed  $\leftarrow$  true
      else
        (boolean) inS1, inS2  $\leftarrow$  isSubSequence(token, sub1,2.peek())
        if inS1 && inS2 then
          shared  $\leftarrow$  true; lastS1, lastS2  $\leftarrow$  sub1,2.peek();
          sharedList.ADDFIRST(token)
        else if inS1 then sub1.PUSH(token)
        else if inS2 then sub2.PUSH(token)
        else failed  $\leftarrow$  true
    if failed then print("impossible");
    else
      if shared then sub1.PUSH(sharedList)
      print(sub1.size() sub2.size()); print(sub1); print(sub2);

```

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A string,  $s_1$  is called a sub-sequence of  $s_2$  if every letter in  $s_1$  is present in  $s_2$  and they occur in the same order (though there can be different letters in between them).

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**Algorithm 5** Helpers
 

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```

procedure ISSUBSEQUENCE(String  $s_1$ , String  $s_2$ )
  (int)  $i \leftarrow 0$ 
  for  $j \in [0..s_2.length]$  do
    if  $s_1.charAt(i) == s_2.charAt(j)$  then
      if  $++i == s_2.length()$  then return true;
  return false;

procedure INITIALIZE( )
   $N \leftarrow$  number of strings;  $sequence[N] \leftarrow$  initialized and filled;
   $s_g \leftarrow$  goal string (currently present string)
  SORT(sequence) by ascending order
  Stack<String>  $sub_1, sub_2 \leftarrow$  initialized;
  LinkedList<String>  $sharedList \leftarrow$  initialized;
  
```

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### 3.3 Correctness

**Proposition 8.**

*Given*

*Proof.*

We know that

□

### 3.4 Analysis

Here we will refer to  $N$  being number of strings inputed and  $S$  as the average length of all the strings.

**Proposition 9.** The space complexity of this algorithm is  $O(N \cdot S)$

*Proof.*

This is due to the fact that we store every string in our  $sequence[N]$ , our two stacks, and the queue. Worst case the sum of the sizes of the two stacks and the queue will be  $N$  because each string is only present at one at a time. Therefore, since each of these contain every string, it becomes  $S \cdot (N + N)$ .

Giving us a space complexity of  $O(N \cdot S)$

□

**Proposition 10.** The time complexity of this algorithm is  $O(N \cdot \log(N) + N \cdot S)$

*Proof.* This is the case because we first sort all of the strings based off of length ( $N \cdot \log(N)$ ). Then we will perform the *isSubSequence* method on each string a maximum of 4 times. Twice to determine sub for the top of  $s_1$  and  $s_2$  then possibly to the string that may be place on top of it and then possibly again if they are the end of a split as described in the proof above therefore giving us  $N \cdot 4 \cdot S$  as worst case.

Giving us a time complexity of  $O(N \cdot \log(N) + N \cdot S)$

□

### 3.5 An Example

Given the input of: