Let us consider the set of m points

$$\mathcal{P} = \{p_1, \dots, p_m\} \subset \mathbb{R}^d \ .$$

The problem is to find a center of ball such that the maximum distance between this center and points in $\mathcal{P}$ is minimal, i.e.

$$\bar{p} := \arg_p \min_{p \in \mathbb{R}^d} \{ \max_{p_i \in \mathcal{P}} \|p_i - p\| \} \ , \tag{1}$$

and the radius of this ball is given by the value of maximum distance between the center and points from $\mathcal{P}$, i.e.

$$r := \max_{p_i \in \mathcal{P}} \|p_i - p\| \ .$$

Similarly to *polytope example (test 03)*, we will search for a coeficient vector of the convex linear combination

$$p = \sum_{i=1}^{m} \alpha_i p_i, \text{ where } \sum_{i=1}^{m} \alpha_i = 1 \text{ and } 0 \leq \alpha_i \leq 1 \ \forall i = 1, \dots, m \ .$$

The reason is that the center of enclosing ball lies in the convex hull of $\mathcal{P}$ (see Schönherr). Afterwards, we denote

$$\begin{array}{rcl} y & := & [\alpha_1, \dots, \alpha_m]^T \in \mathbb{R}^m \ , \\ C & := & [p_1, \dots, p_m] \in \mathbb{R}^{d,m} \ , \\ b & := & [p_1^T p_1, \dots, p_m^T p_m] \in \mathbb{R}^m \ . \end{array}$$

The problem (1) can be reformulated to QP. See next lemma.

Lemma 1. *The solution of optimization problem*

$$\bar{p} := C\bar{y}, \qquad \bar{y} := \arg \min_{y \in \Omega_E \cap \Omega_I} y^T C^T C y - b^T C y,$$

where

$$\begin{array}{rcl} \Omega_E & := & \{y \in \mathbb{R}^m : Bx = 1\} \ , \\ \Omega_I & := & \{y \in \mathbb{R}^m : x \geq 0\} \ , \\ B & := & [1, \dots, 1] \in \mathbb{R}^{1,m} \end{array}$$

is equivalent to the solution of the problem (1).

Proof. See Schönherr . The proof is based on KKT optimality conditions. □

Moreover, the problem can be homogenized and solved using the same methodology as in *polytope example (test 03)*. We solve the problem to obtain the center of ball $\bar{p}$. Afterwards, the radius can be computed by

$$r = \sqrt{-\bar{p}^T A \bar{p} + 2b^T \bar{p}} \ .$$

Numerical example We work with random data in our benchmark. We generate $m = 100$ random points from circle

$$\{p \in \mathbb{R}^2 : \|p - [1, 1]^T\|^2 \leq 1\}.$$

Afterwards, we discard the information about the circle and try to find the enclosing ball using the process described above.

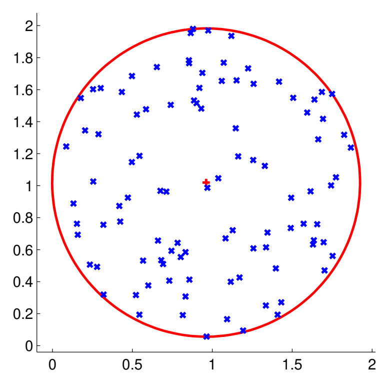

Figure 1: Enclosing ball: the solution of the benchmark.