

$T_{pq}$  transforms the pair  $(a, b)$  such that  $a \leftarrow bq + aq + ap$  and  $b \leftarrow bp + aq$ .

After two transformations we get:

$$\begin{aligned}a &\leftarrow (bp + aq)q + (bq + aq + ap)q + (bq + aq + ap)p \\&= pqb + q^2a + q^2b + q^2a + pqa + pqb + pqa + p^2a \\&= p^2a + 2pqa + 2pqb + 2q^2a + q^2b \\&= a(p^2 + 2pq + 2q^2) + b(2pq + q^2) \\b &\leftarrow (bp + aq)p + (bq + aq + ap)q \\&= p^2b + pqa + q^2b + q^2a + pqa \\&= p^2b + 2pqa + q^2a + q^2b \\&= b(p^2 + q^2) + a(2pq + q^2).\end{aligned}$$

From above,  $a \leftarrow a(p^2 + 2pq + 2q^2) + b(2pq + q^2)$

$$= b(2pq + q^2) + a(2pq + q^2) + a(p^2 + q^2).$$

Therefore, if we let  $p' = p^2 + q^2$  and  $q' = 2pq + q^2$ , then we can represent applying  $T_{pq}$  twice with the single transformation  $T_{p'q'}$ .