

The order of growth of the number of steps needed to encode the most frequent symbol in the alphabet described in Exercise 2.71, where the relative frequencies of symbols are  $2^{-n}$  where  $n$  starts at 0, is  $\Theta(1)$ . This is because the most frequent symbol is always a leaf node that is a child of the root node, which means the number of steps to traverse the tree from root to leaf will be the same no matter the size of  $n$  (number of total leaf nodes in tree).

The order of growth of the number of steps to encode the least frequent symbol is  $\Theta(n^2)$ . The number of steps to check whether the symbol is in each node (choose-branch, which calls element-of-set?) is roughly  $n$  where  $n$  is the number of symbols in said node. Starting from the root node and traversing the tree to reach the least frequent symbol, the element-of-set? procedure is called once on a branch with  $n - k$  symbols where  $k$  is the level of the tree, which requires  $n - k$  many steps, and once on a leaf node which always requires only 1 step. Incrementing  $n$  by one yields another level with  $n$  many steps. The total number of steps involved with choose-branch thus grows as  $n^2$  because there will be  $n$  many levels where element-of-set? invokes up to  $n$  many steps.