Let the endpoints of interval 1 be a, b and the endpoints of interval 2 be c, d. Let m be the center of interval 1 and n be the center of interval 2. Then,

$$m = \frac{a+b}{2} \text{ and } n = \frac{c+d}{2}.$$

Now let p be the percentage tolerance of interval 1 and q be the percentage tolerance of interval 2.

$$p = 100\left(\frac{b - \frac{a+b}{2}}{\frac{a+b}{2}}\right) \text{ and } q = 100\left(\frac{d - \frac{c+d}{2}}{\frac{c+d}{2}}\right).$$

This simplifies to:

$$p = 100\left(\frac{b-a}{a+b}\right) \text{ and } q = 100\left(\frac{d-c}{c+d}\right).$$

When $a, b, c, d \in Z^+$, then the lower bound of the new interval formed by the multiplication of interval 1 and 2 is ac and the upper bound is bd. This means that the new center is $\frac{ac+bd}{2}$ and the new percentage tolerance, r is:

$$r = 100\left(\frac{bd - \frac{ac+bd}{2}}{\frac{ac+bd}{2}}\right) = 100\left(\frac{bd-ac}{ac+bd}\right).$$

The following reveals how r can be approximated in terms of p and q given small percentage tolerances. The percentage conversion is omitted for brevity and clarity.

$$\begin{aligned}
p + q &= \left(\frac{b-a}{a+b}\right) + \left(\frac{d-c}{c+d}\right) \\
&= \frac{(b-a)(c+d) + (d-c)(a+b)}{(a+b)(c+d)} \\
&= \frac{bc + bd - ac - ad + ad + bd - ac - bc}{ac + ad + bc + bd} \\
&= \frac{2bd - 2ac}{ac + ad + bc + bd} \\
&= \frac{(bd - ac) + bd - ac}{(ac + bd) + bc + bd}
\end{aligned}$$

If $\frac{bd-ac}{bc+bd} \approx \frac{bd-ac}{ac+bd} \iff bc \approx ac$, then $p + q$ is a good approximation for r. This means that $p + q$ is a good approximation when the values of a and b are similar, which is the case when the percentage tolerance is small. Therefore, under the assumption of small percentage tolerances the percentage tolerance of the product of two intervals can be approximated by the sum of the tolerances of the factors.