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Wind Energy: Energy Yield

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Power available from the wind

To extract wind energy we need to decelerate the moving air to extract its kinetic energy.

If we decelerated the air to standstill we could extract a power equal to all the kinetic energy in the volume of air being processed per unit time.

Here we consider a volume of air arriving at a location in a period of time.

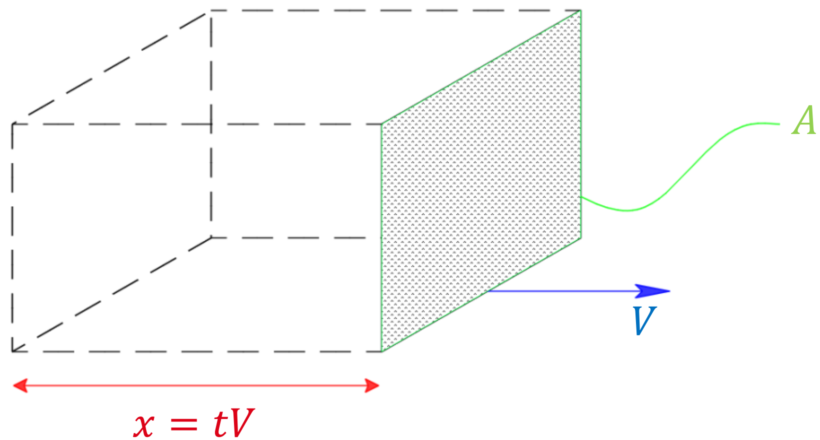
$$E_W = \tfrac{1}{2} m V^2 = \tfrac{1}{2} \rho A x \, V^2$$

$$m = \rho A x = \rho A V t$$

$$P_W = \frac{d}{dt} E_W$$

$$= \frac{d}{dt}\left(\tfrac{1}{2}\rho A V t V^2\right)$$

$$= \tfrac{1}{2}\rho A V^3$$



V is the wind velocity which is also $^{dx}/_{dt}$.

A is the cross-sectional area of volume being captured (swept area of blades for instance).

x is the distance in the direction of the wind of the volume of wind processed per unit time.

ρ is the density of air.

Power extracted from a wind turbine

However, the power extracted by a turbine, P_T , is a product of the force and velocity so we can't completely decelerate the air. We are limited to taking a fraction of the wind's power and we call that fraction the power coefficient of the turbine, C_P.

$$P_T = C_P \frac{1}{2} \rho A V^3$$

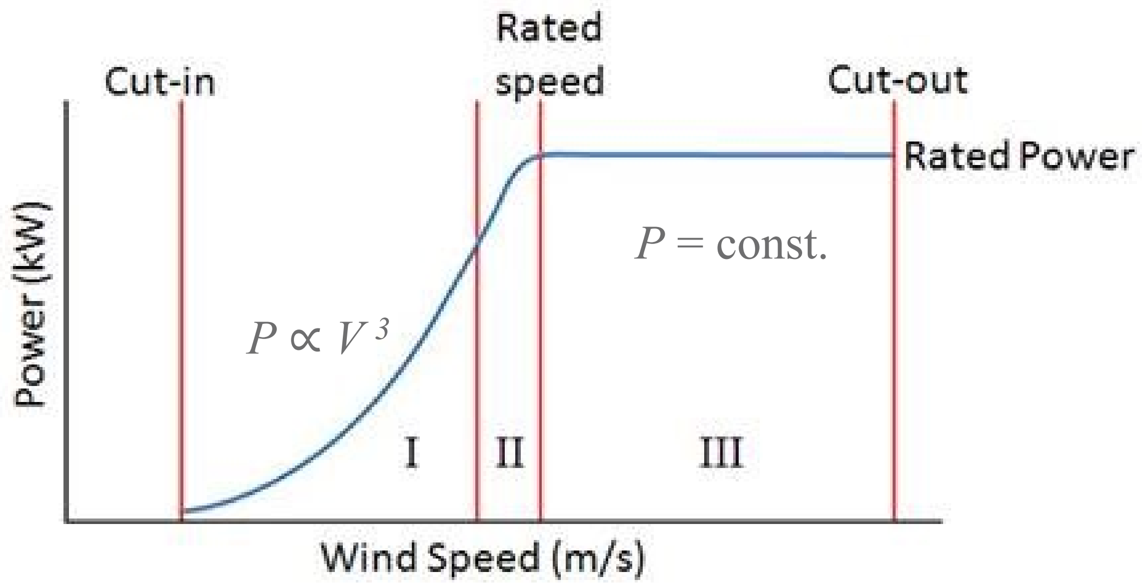
In principle, the power coefficient, C_P , can not exceed the "Betz Limit" of $^{16}/_{27} \simeq 59\%$.

A good turbine has $C_P > 50\%$.

We will cover the derivation of the Betz Limit later.

Wind Turbine Operating Regions (or modes)

Every wind turbine has a characteristic **power curve** which tells us how much power it can produce for each possible wind speed:



Wind turbines operate in different modes depending on the wind speed:

- At low wind speeds (region I), the goal is to extract as much power as possible by maximising the power coefficient.
- At high wind speeds (region III), the goal is to limit the rotational speed of the turbine and the forces on the structure to safe levels, in keeping with the design, and this means extracting less power than is technically available and normally means holding power constant.
- There is often a transition mode (region II) between the two main modes.

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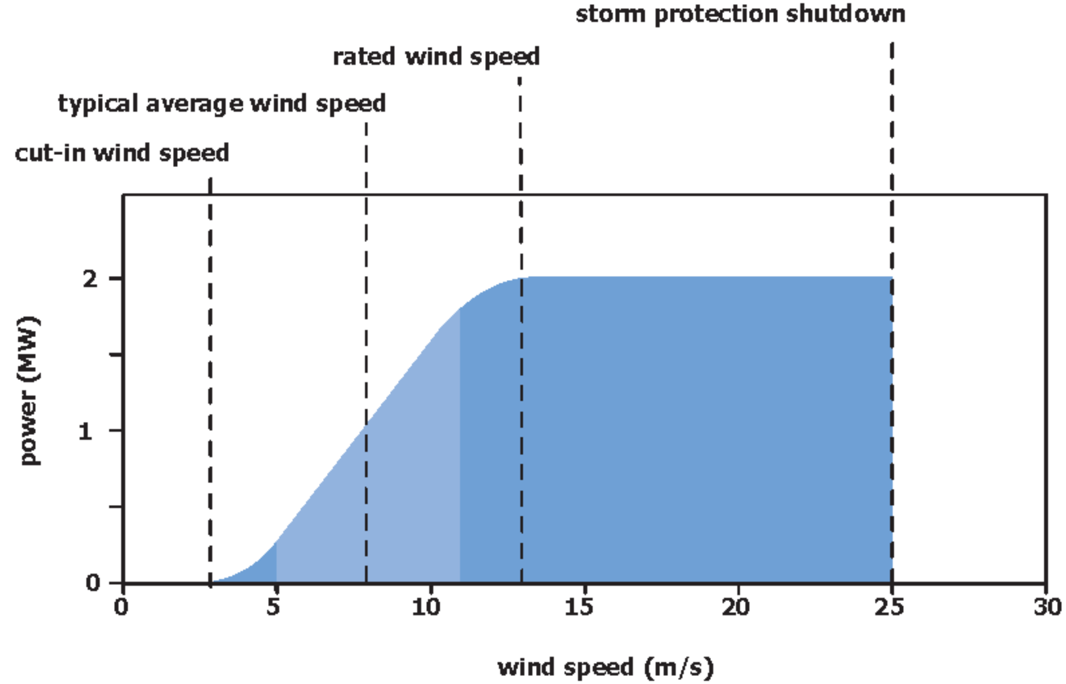
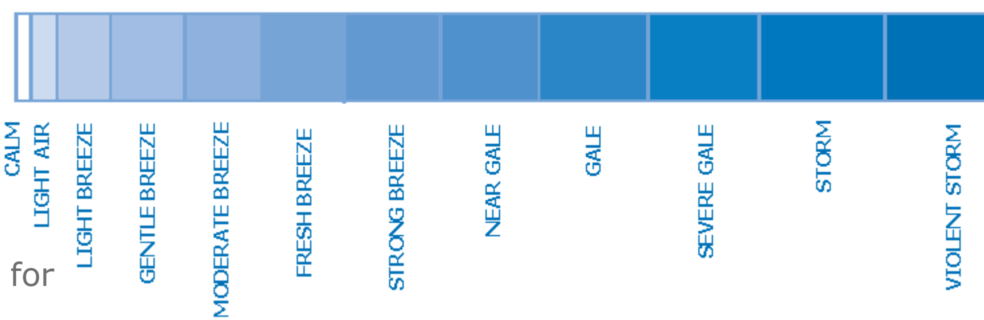
Nomenclature and Typical Values

- **Cut-in speed**: minimum wind speed at which useful power can be generated (~3 m/s).
- **Rated** (or nominal) speed: lowest wind speed at which rated power is produced (~10-16 m/s).
- **Cut-out speed**: maximum wind speed for safe operation (~20-30m/s). Also known as storm protection shut down.
- Survival speed: maximum wind speed the turbine can handle without being damaged (~50 m/s).



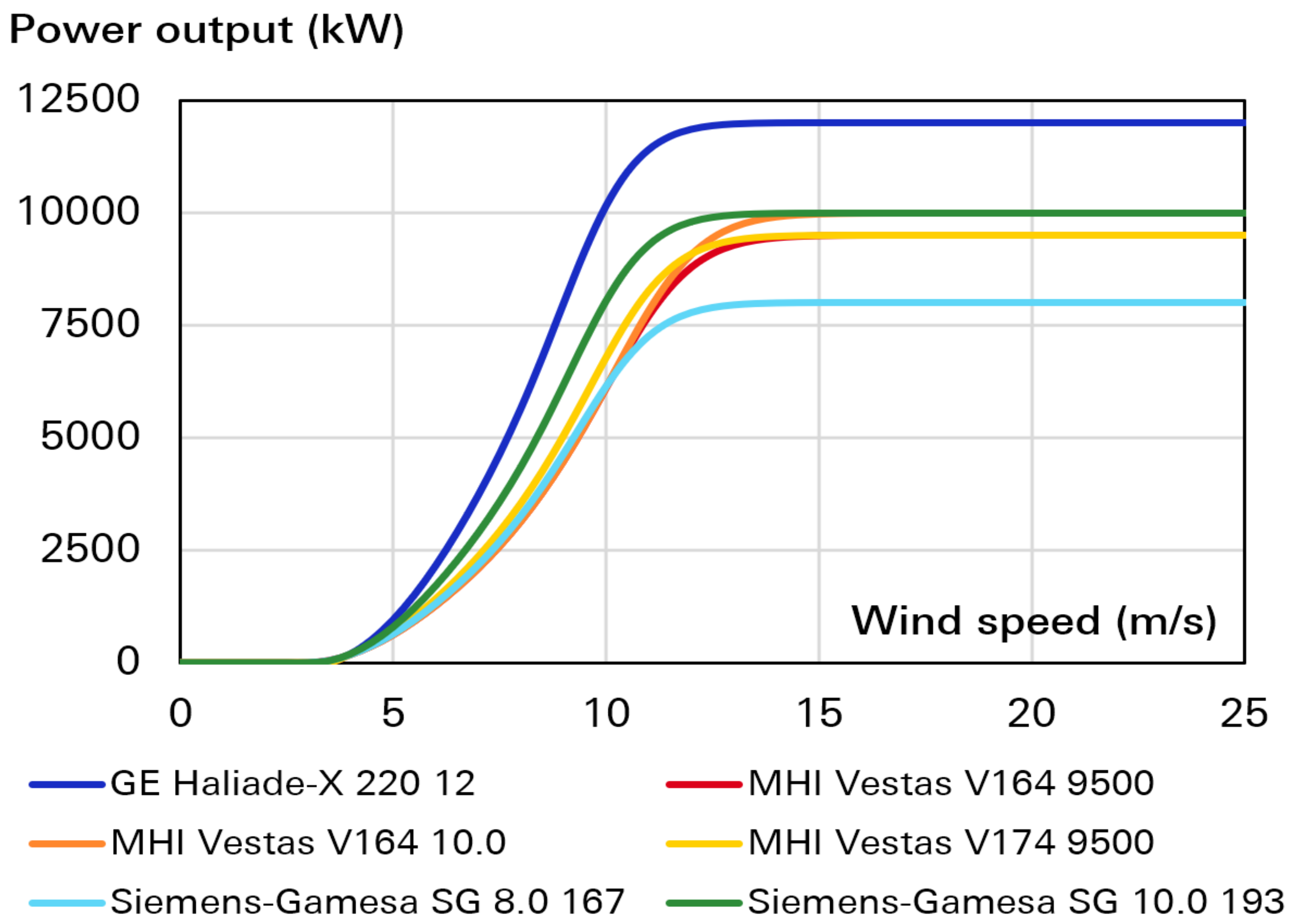


Note that the unit of speed when describing wind speed for turbines is metres per second, m/s.
There are several other units in use for wind speed such as

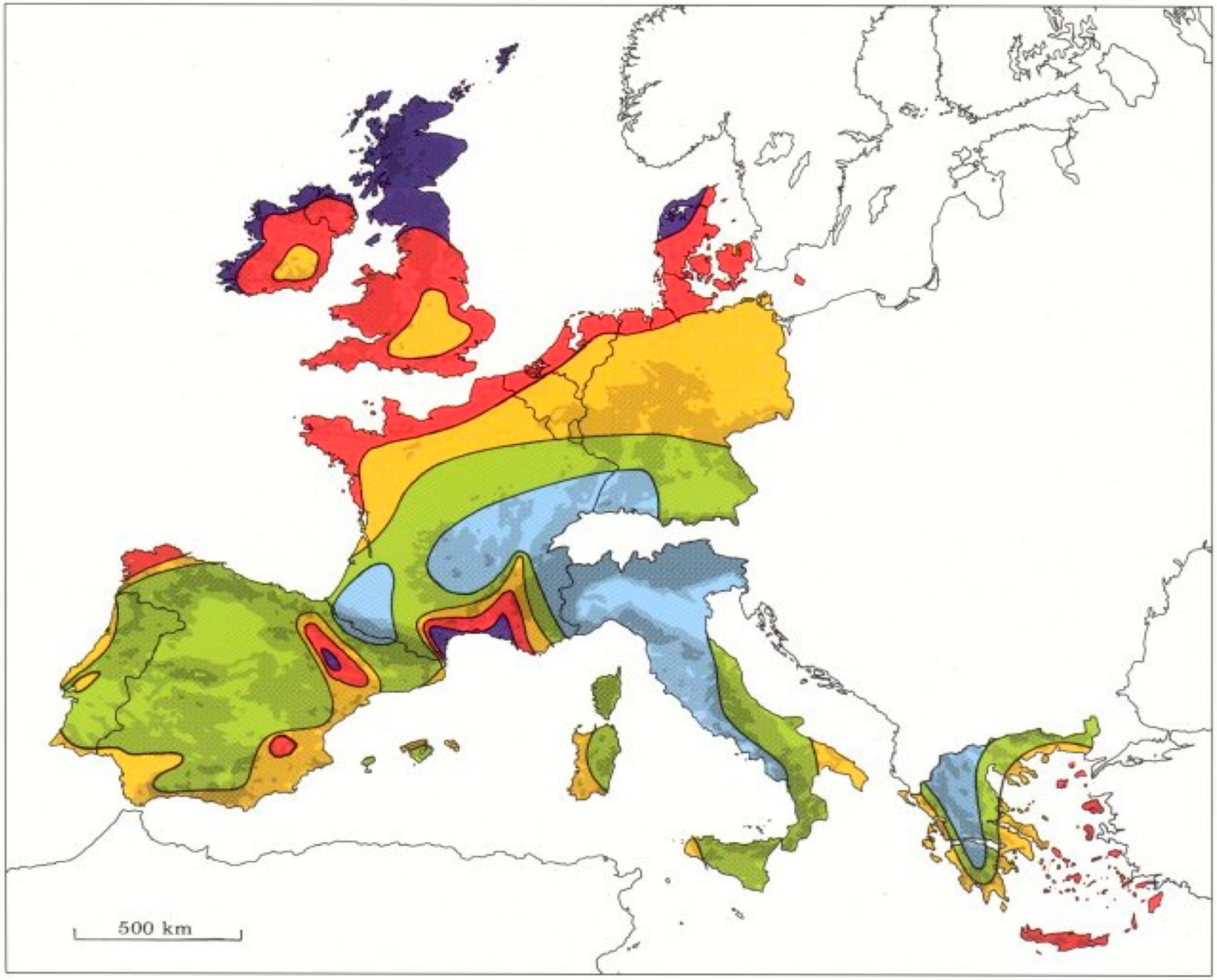
- kilometres per hour, kph or km/h: 1 kph = 0.277 m/s
- miles per hour, mph: 1 mph = 0.447 m/s
- knots (nautical mile per hour), kn: 1 kn = 0.514 m/s

Example Power Curves of Commercial Turbines

Curves from Saint-Drenan et al. http://arxiv.org/abs/1909.13780



Importance of Location



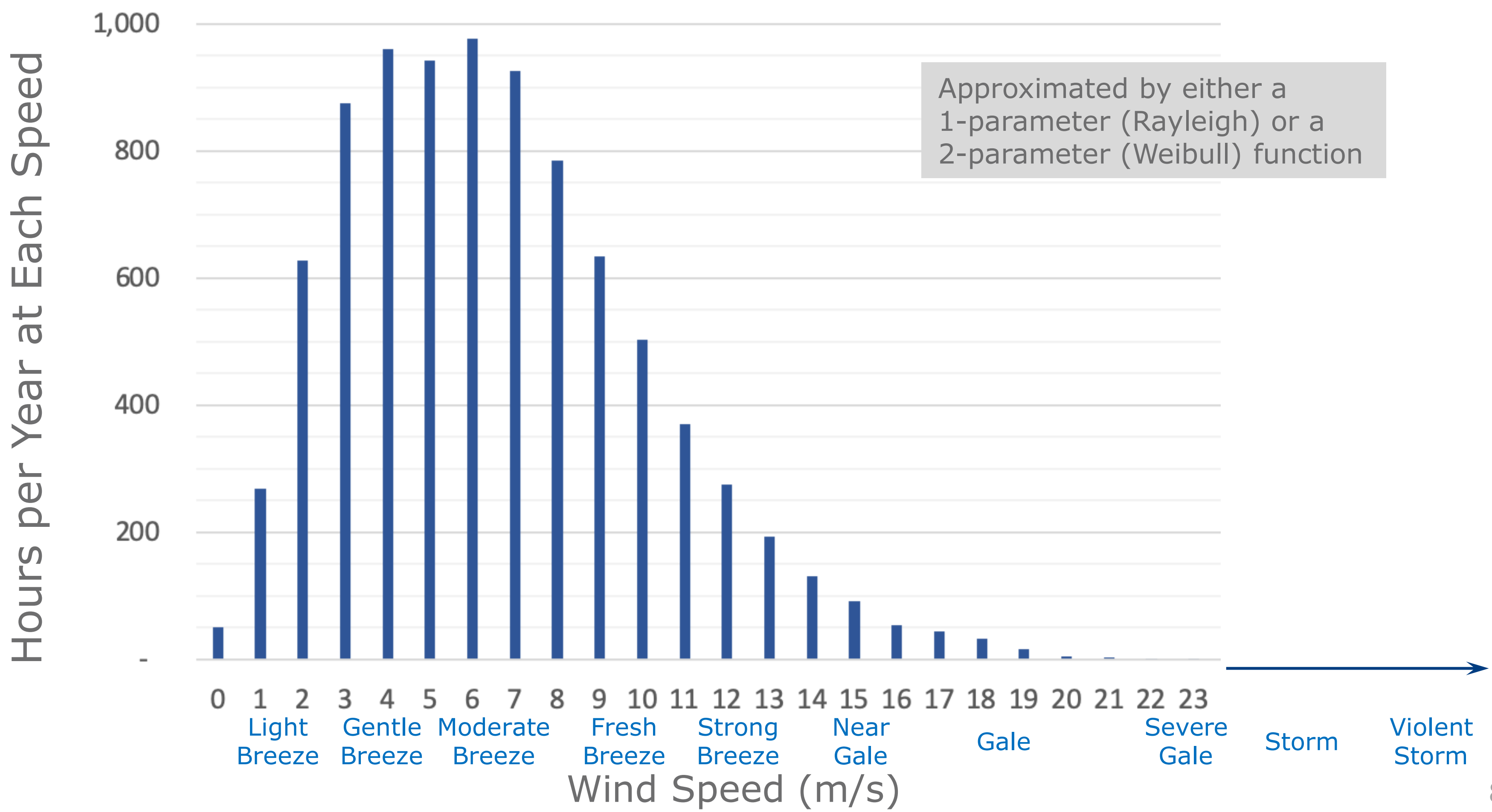
Wind resources[1] at 50 metres above ground level for five different topographic conditions										
	Sheltered terrain[2]		Open plain[3]		At a sea coast[4]		Open sea[5]		Hills and ridges[6]	
	$m\,s^{-1}$	Wm^{-2}	$m\,s^{-1}$	Wm^{-2}	$m\,s^{-1}$	Wm^{-2}	$m\,s^{-1}$	Wm^{-2}	$m\,s^{-1}$	Wm^{-2}
	> 6.0	> 250	> 7.5	> 500	> 8.5	> 700	> 9.0	> 800	> 11.5	> 1800
	5.0-6.0	150-250	6.5-7.5	300-500	7.0-8.5	400-700	8.0-9.0	600-800	10.0-11.5	1200-1800
	4.5-5.0	100-150	5.5-6.5	200-300	6.0-7.0	250-400	7.0-8.0	400-600	8.5-10.0	700-1200
	3.5-4.5	50-100	4.5-5.5	100-200	5.0-6.0	150-250	5.5-7.0	200-400	7.0- 8.5	400- 700
	< 3.5	< 50	< 4.5	< 100	< 5.0	< 150	< 5.5	< 200	< 7.0	< 400

Importance of Wind Speed Distribution

Distribution of wind speeds is skewed

Speeds below the average (mean) are common but speeds above the average extend out to a long tail of low occurrences of gales and storms.

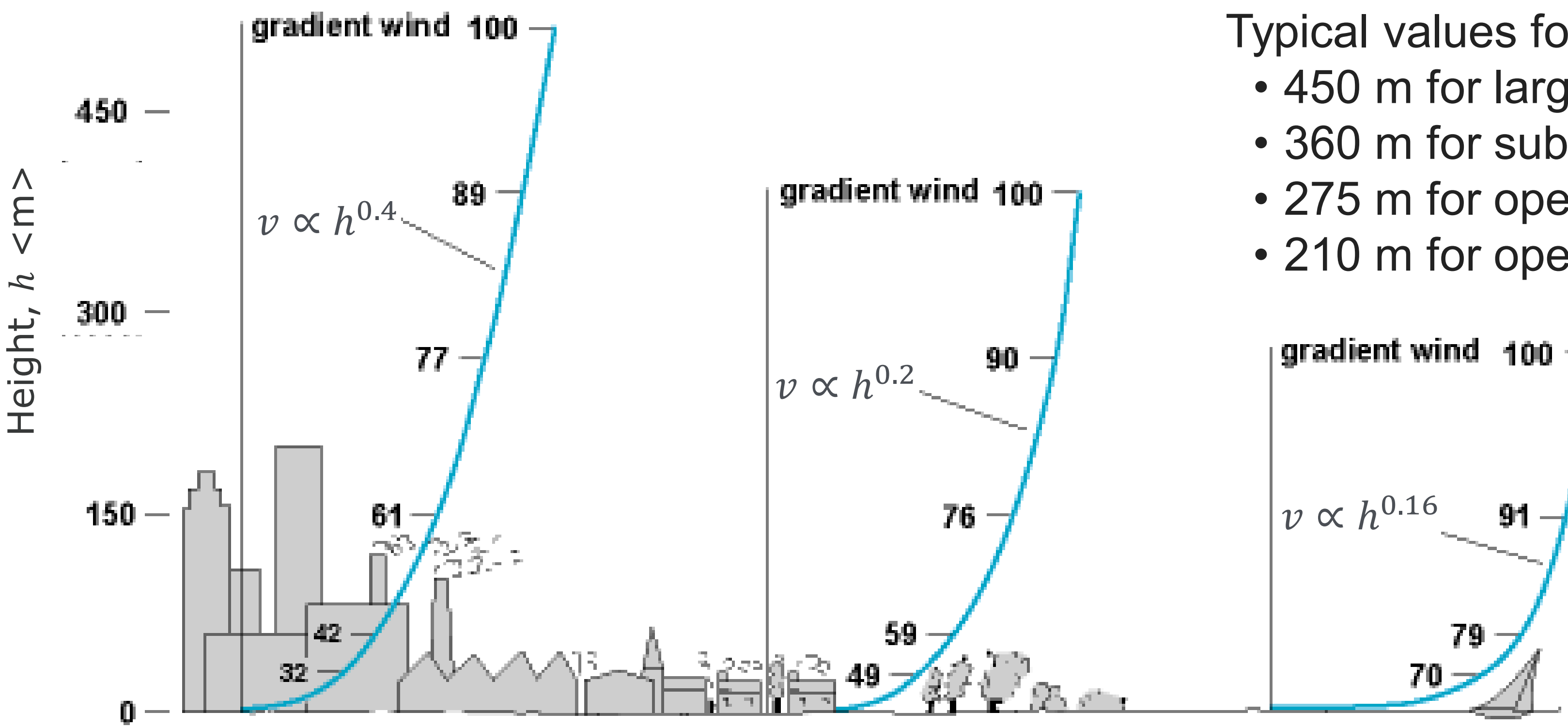
This example is for a site on the west coast of Scotland.



Importance of Hub Height

The friction created by the ground surface create a boundary layer of stationary air at the surface. Wind speed increases as one moves up from the surface. The wind speed reaches the speed of the main flow at a height known as the "gradient height", h_G.

This figure shows the "gradient wind" as a percentage of the wind speed in the main air flow.



Typical values for h_G are

- 450 m for large cities,
- 360 m for suburbs,
- 275 m for open terrain
- 210 m for open sea

Importance of Ground Roughness

The way in which the wind speed changes between zero at the surface and its value in the main flow is determined by the Hellmann exponent, a. The following equation identifies wind velocity at a height of interest from a measurement of velocity at a reference height (10 m in this case) and is valid for heights, $h < h_G$. Height h is measure with respect to local ground level not elevation above sea level.

$$v(h) = v_{10} \left(\frac{h}{h_{10}}\right)^a$$

$v(h) =$ velocity of the wind at height, h

$v_{10} =$ velocity at height of 10 m

$a =$ Hellmann exponent

The Hellman exponent depends on the nature of the surface and particularly its roughness.

Surface Description	a
Unstable (turbulent) air above open water	0.06
Neutral (laminar) air above open water	0.10
Unstable air above flat open coast	0.11
Neutral air above flat open coast	0.16
Stable air above open water surface	0.27
Unstable air above human inhabited areas	0.27
Neutral air above human inhabited areas	0.34
Stable air above flat open coast	0.40
Stable air above human inhabited areas	0.60

Physical Limits on Speed and Torque of a Turbine / Generator

Storm-force winds occur occasionally, and the turbine must be strong enough to at least withstand those wind speeds.

All the elements of the drive train of a turbine will have some physical limitations. For instance,

- the blades must support the centripetal forces and respect the stress limitation of the material; this will place a limit on the maximum speed of rotation.
- the gearbox will have been designed for some maximum torque determine by gear tooth dimensions etc.

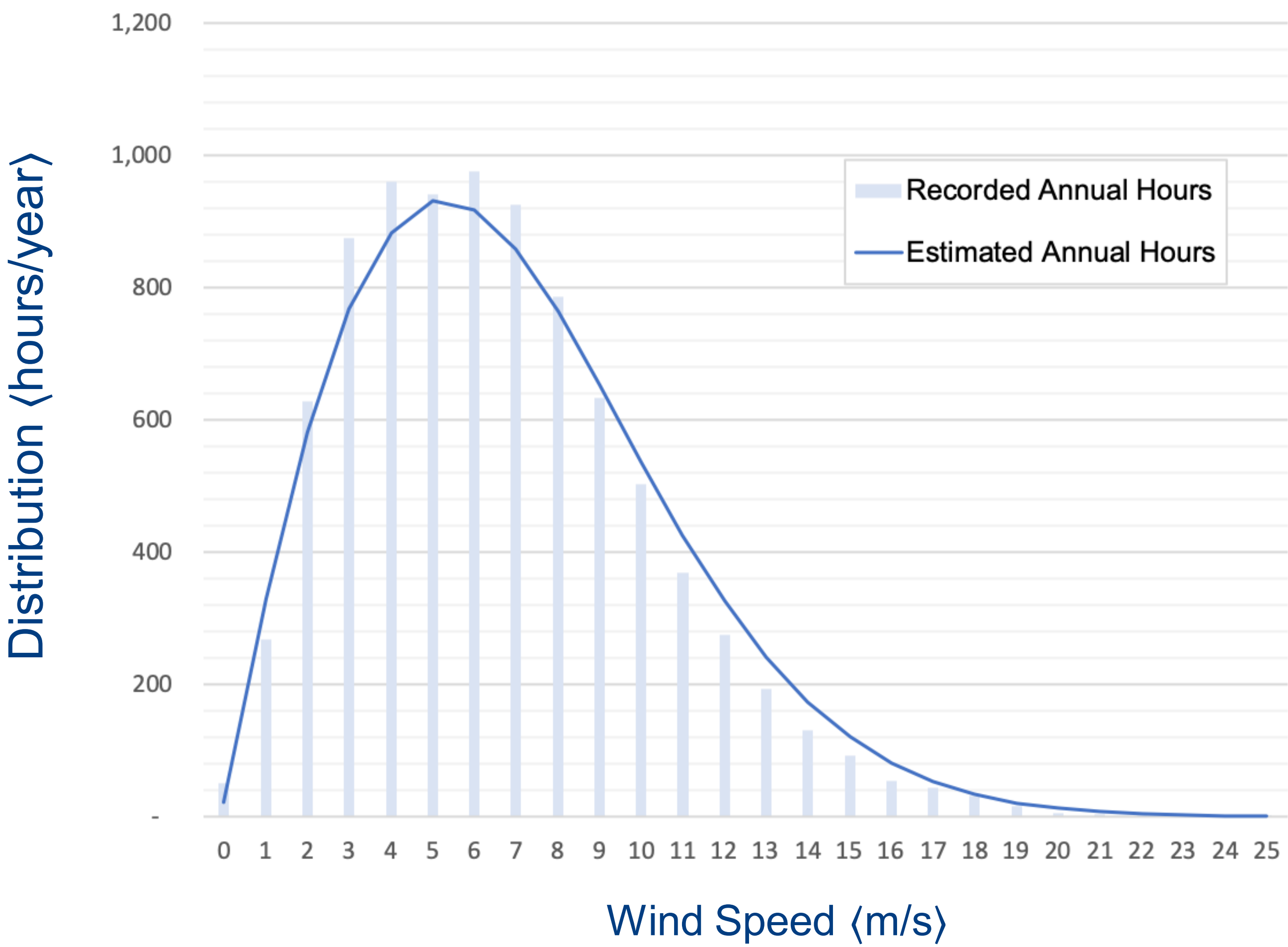
The control regime must respect these physical limitations of the plant and ensure operation remains in a safe range.

Note

- physical limits on speed and torque give rise to a maximum power capability.
- for a turbine, torque and power are functions of wind speed and rotor speed.
- we normally just talk about trying to observe a power limit for the turbine but underlying this are individual limits on speed and torque.

Why place the power limit at ~12 m/s ?

The **skewed** distribution of wind speeds means that winds above 12 m/s only occur for a small fraction of the year (a few hundred hours) and the energy they can produce is low.



Here, the distribution has also been estimated by a **Weibull** distribution with:

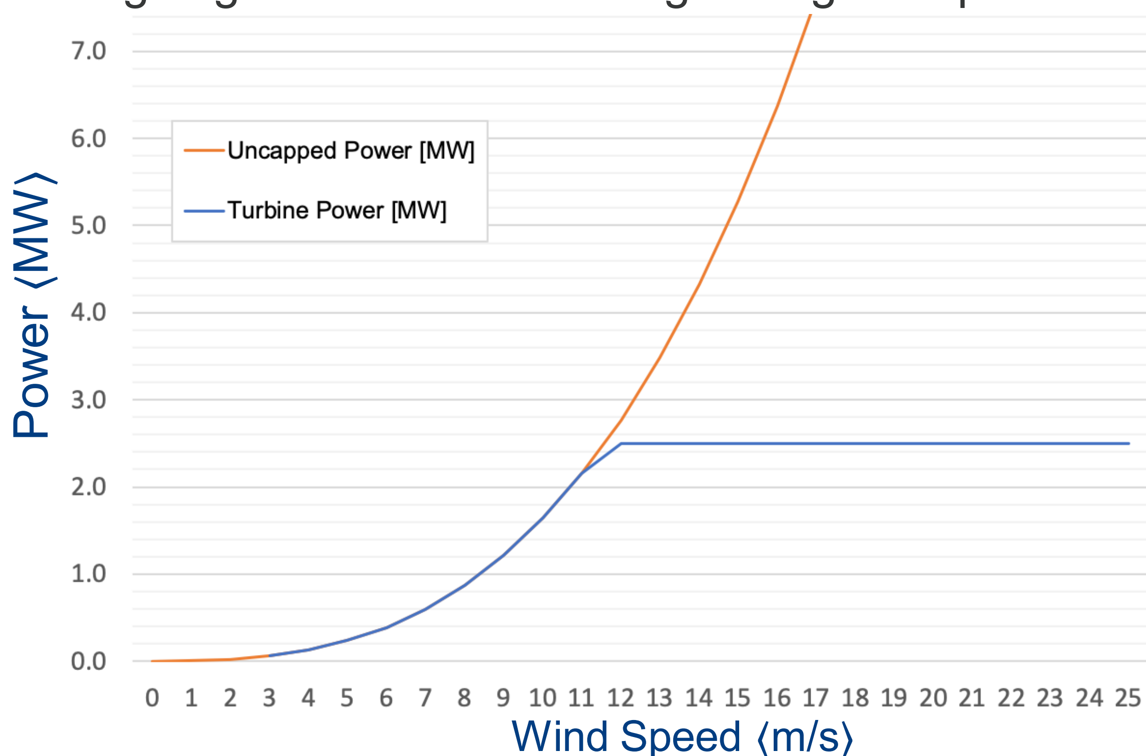
$k = 1.9$

$C = 7.8$

$$p(V) = \frac{k}{C}\left(\frac{V}{C}\right)^{k-1} e^{-\left(\frac{V}{C}\right)^{k}}$$

Wind Power rather than Wind Speed

- Expected wind energy is determined by the likelihood of a wind speed existing and the cube law for power
- Below 3 m/s there is little power so little energy
- Above 25 m/s there is very high power but for few hours so little energy
- Turbines would need to be very strong and contain a generator with a high power rating to operate above 25 m/s. There is no economic value in designing turbines to be strong enough to operate for these few hours.



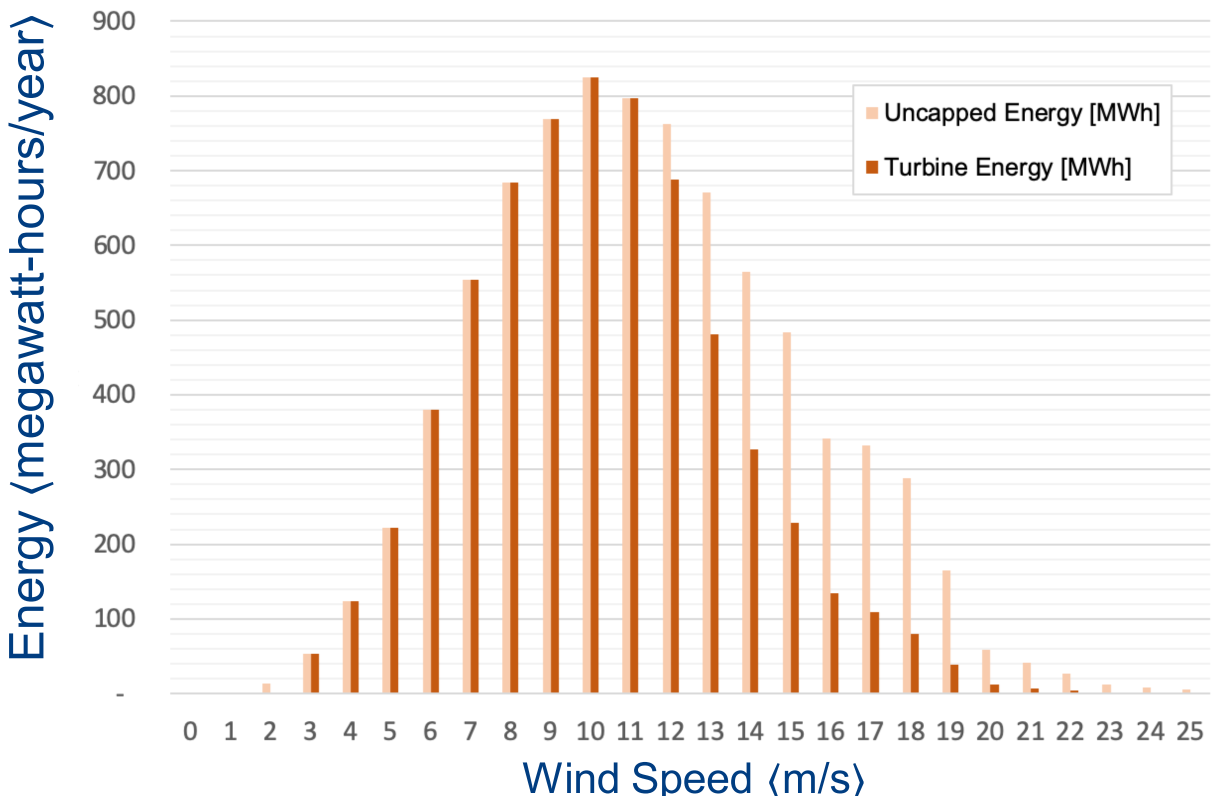
The turbine power shown here is based on a turbine of:

R = 38 m (A = 4,536 m²)
C_p = 0.5

with ρ = 1.25 kg/m^3

How Much Energy is Foregone?

- Cut-out below ~3 m/s because the power is so low it won't cover the internal power consumption of the turbine (auxiliaries such as yaw and pitch control).
- Cut-out above ~25 m/s because there is very little energy and the cost of a structure to generated in this wind speed is prohibitive.
- Above ~12 m/s the energy falls rapidly with wind speed: a good compromise is to use a generator of limited power and forego some of the available energy but save on capital cost of high power rating components.



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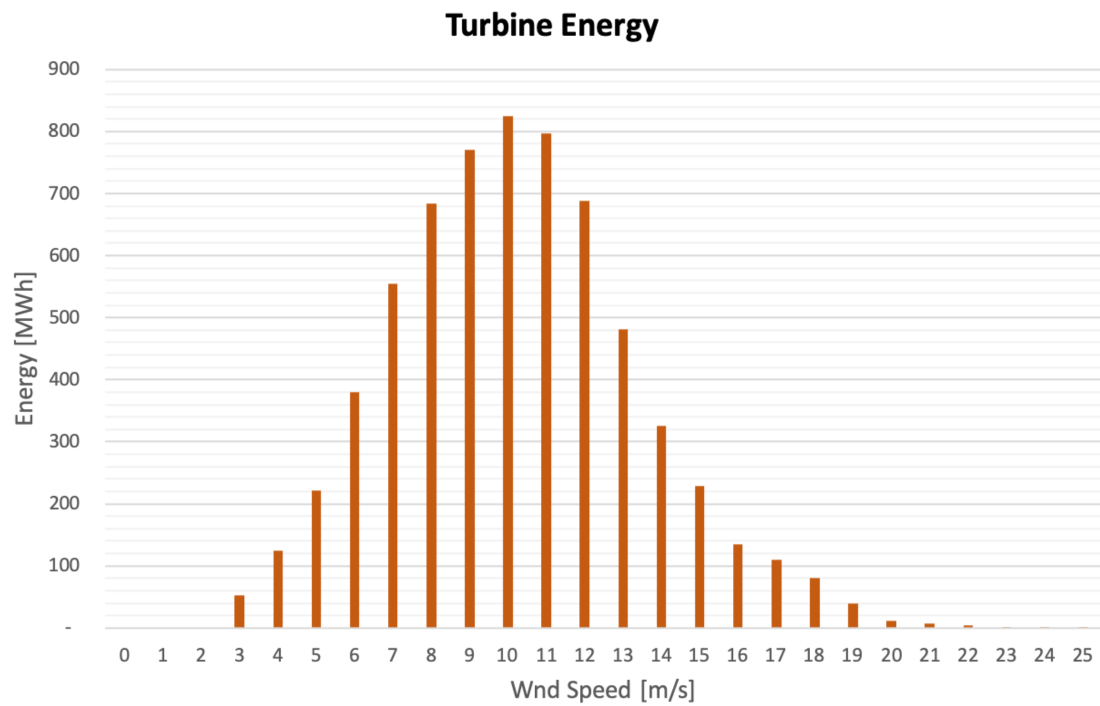
Load Factor

- The sum of the energy at each wind speed gives the total annual energy yield for the combination of site and turbine.
- In this example, the sum is 6,523 MWh
- We can compare this with how much energy the turbine would produce if it were to run at its maximum rating all year which is:

  $2.5 \times 24 \times 365 = 21{,}900$ MWh



- The **ratio of energy yield to rated maximum** is termed the load factor:

$$LF = \frac{Total\ Energy\ Yieid}{Rating\ \times\ 24 \times 365} = \frac{6{,}523}{21{,}900} = 29.6\%$$

- Load factor is a useful figure-of-merit for a turbine-and-site combination. The UK has relatively high wind speeds so load factors of 20-30% can be expected at good sites onshore and 40% or more offshore.

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Wind Turbine Classes

- To recognise that different wind farm sites will have different distributions of wind speed and different average windspeeds, there is a need for wind turbines with different base speeds, cut-out speed etc.

- A small number of standard class of wind turbine have been defined in IEC 61400

Wind Class/Turbulence	Annual average wind speed at hub-height	Extreme 50-year gust
Ia High wind - Higher Turbulence 18%	10 metres per second (36 km/h; 22 mph)	70 metres per second (250 km/h; 160 mph)
Ib High wind - Lower Turbulence 16%	10 metres per second (36 km/h; 22 mph)	70 metres per second (250 km/h; 160 mph)
IIa Medium wind - Higher Turbulence 18%	8.5 metres per second (31 km/h; 19 mph)	59.5 metres per second (214 km/h; 133 mph)
IIb Medium wind - Lower Turbulence 16%	8.5 metres per second (31 km/h; 19 mph)	59.5 metres per second (214 km/h; 133 mph)
IIIa Low wind - Higher Turbulence 18%	7.5 metres per second (27 km/h; 17 mph)	52.5 metres per second (189 km/h; 117 mph)
IIIb Low wind - Lower Turbulence 16%	7.5 metres per second (27 km/h; 17 mph)	52.5 metres per second (189 km/h; 117 mph)
IV	6.0 metres per second (22 km/h; 13 mph)	42 metres per second (150 km/h; 94 mph)

https://en.wikipedia.org/wiki/IEC_61400#Wind_Turbine_Generator_(WTG)_classes