Asteroid and Meteor Impact Modeling: Formulas, Assumptions, and Models with Justifications

Contents

1 Simulation Assumptions

To simplify and standardize the simulation, the following assumptions are made:

1. The position vector ($\mathbf{r}$) is assumed the same as Earth's.
2. Planet and asteroid are considered perfect spheres.
3. Crater geometry is simplified: the hole depth is considered to be 0.2 times the diameter D (*final rim-to-floor depth 0.2 D*), based on classic lunar morphology. In reality, Earth exhibits more complex craters, and complex craters generally have smaller depth-to-diameter ratios [6].
4. Last pericenter passage is at $t = 0$.
5. Mass loss during atmospheric entry: 95% for water-based, 75% for sedimentary, 50% for crystalline meteors [3].
6. Due to lack of trajectory data, a custom trajectory identical to Earth's but on a different plane is used for the asteroid [5].
7. For the estimation of de-orbiting requirements, it is assumed that a hydrogen-based propulsion system intercepts the asteroid approximately 50 days before its perigee or apogee passage to initiate the de-orbiting operation. The hydrogen system provides the necessary ΔV impulse to gradually alter the trajectory before Earth encounter.

2 Impact and Trajectory Models

2.1 Forward Trajectory

$$dx = \frac{\cos(\mathrm{az}) \cdot H}{\tan(\mathrm{at})},$$
$$dy = \frac{\sin(\mathrm{az}) \cdot H}{\tan(\mathrm{at})}$$

Source: Vallado (2013) [5].

Justification: The straight-line trajectory approximation assumes small deflections over the atmospheric entry distance, providing a reasonable estimate of the ground-track displacement before impact.

2.2 Backward Trajectory (Orbital Position)

$$r(\theta) = \frac{1.4956 \times 10^{11}}{1 + 0.0167\cos(\theta)},$$
$$x = r(\theta)\cos(\theta),$$
$$y = r(\theta)\sin(\theta)\cos(\mathrm{at}),$$
$$z = r(\theta)\sin(\theta)\sin(\mathrm{at})$$

Source: Vallado (2013) [5].

Justification: A simplified two-body orbit is assumed. The eccentricity and scaling factors are based on Earth's orbit to provide a reasonable orbital framework for the asteroid.

2.3 De-orbit Energy Requirement

To alter the trajectory of an asteroid from an orbiting path to an Earth-impacting trajectory, a finite velocity change (ΔV) must be applied. The total energy required for this maneuver is given by:

$$E_{\text{deorbit}} = \frac{1}{2}m(\Delta V)^2 + v_0\Delta V$$

where:

- E_{deorbit} = energy required to alter trajectory (J),
- ΔV = required change in velocity (assumed 1.476 m/s^2),
- v_0 = initial orbital velocity of the asteroid (m/s),
- m = mass of the asteroid (kg).

Source: Vallado (2013) [5].

Justification: This equation derives from the expansion of the kinetic energy difference between the new and old orbital states:

$$E = \frac{1}{2}m(v_0 + \Delta V)^2 - \frac{1}{2}mv_0^2 = \frac{1}{2}m(\Delta V)^2 + mv_0\Delta V$$

It estimates the additional kinetic energy needed to change the asteroids velocity enough to drop it from its near-orbital trajectory toward an Earth-intersecting path. The assumed $\Delta V = 1.476$ m/s^2 provides a reasonable order of magnitude for minimal deflection thresholds for kilometer-scale bodies in heliocentric orbits.

2.4 Impact Energy

$$E = \frac{1}{2}mv^2,$$

$$E_{\text{TNT}} = \frac{\frac{1}{2}mv^2}{4.184 \times 10^{15}}$$

Source: Melosh (1989/1996) [3].

Justification: Standard kinetic energy formula; converting to TNT equivalent is conventional in impact studies for comparison purposes.

2.5 Crater Diameter

$$D = 0.0162 \left(E \cdot s_f \cdot 4.184 \times 10^{15}\right)^{0.29}$$

$s_f = 0.05$ water, 0.30 sedimentary, 0.50 crystalline.

Source: Holsapple (1993) [1], Housen et al. (1983) [2].

Justification: The exponent 0.29 is within the expected range (0.25–0.33) for energy scaling in π-scaling laws. The s_f values represent the fraction of mass remaining after atmospheric entry, chosen based on typical material behavior: water loses most mass, sedimentary loses less, crystalline loses the least.

2.6 Blast Diameter

$$D_{\text{blast}} = 1.25 \times D$$

Source: Pierazzo et al. (2005) [4].

Justification: The factor 1.25 is empirically determined from crater-to-blast scaling in planetary impact studies; it accounts for the additional area affected beyond the crater rim.

2.7 Atmospheric Entry Mass Disintegration

$$\Delta m = \frac{v^2 \cdot Q_{\text{air}}^{1/3}}{Q_{\text{meteor}}^{1/3}} \cdot t$$

Source: Melosh (1989/1996) [3].

Justification: This model expresses the mass loss of a meteor due to aerodynamic heating and ablation during atmospheric entry. It is derived from the proportionality of drag-related energy loss to velocity squared, scaled by the cube-root dependence on material density. The constant factors are absorbed for simplicity, and air density is taken as an average representative value through the atmosphere.

2.8 Blast Effect Radius (Assumed Model)

Assuming energy E is inversely proportional to the cube of the affected radius r:

$$E = \frac{C}{r^3}.$$

Applying initial conditions $r = R_0$, $E = E_0$, we get $C = E_0 R_0^3$, giving

$$r = \left(\frac{E_0}{E}\right)^{1/3} R_0$$

Justification: This is an assumed model, based on the idea that energy dissipates with volume ($\sim r^3$). Setting initial conditions allows calculation of the proportionality constant. The formula provides a simple way to scale the blast radius as energy decreases. While heuristic, it is reasonable for preliminary estimation of the affected area.

3 References

References

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