Asteroid and Meteor Impact Modeling: Formulas, Assumptions, and Models with Justifications

Contents

1 Simulation Assumptions

To simplify and standardize the simulation, the following assumptions are made:

1. The position vector ($\mathbf{r}$) is assumed the same as Earth's.
2. Planet and asteroid are considered perfect spheres.
3. Crater geometry is simplified: the hole depth is considered to be 0.2 times the diameter D (*final rim-to-floor depth 0.2 D*), based on classic lunar morphology. In reality, Earth exhibits more complex craters, and complex craters generally have smaller depth-to-diameter ratios [6].
4. Last pericenter passage is at $t = 0$.
5. Mass loss during atmospheric entry: 95% for water-based, 75% for sedimentary, 50% for crystalline meteors [3].
6. Due to lack of trajectory data, a custom trajectory identical to Earth's but on a different plane is used for the asteroid [5].

2 Impact and Trajectory Models

2.1 Forward Trajectory

$$dx = \frac{\cos(\mathrm{az}) \cdot H}{\tan(\mathrm{at})},$$
$$dy = \frac{\sin(\mathrm{az}) \cdot H}{\tan(\mathrm{at})},$$

Conversion to latitude and longitude:

$$d_{\mathrm{long}} = \frac{dx}{111\cos(\mathrm{dlati})},$$
$$d_{\mathrm{lati}} = \frac{dy}{111}$$

Source: Vallado (2013) [5].

Justification: The straight-line trajectory approximation assumes small deflections over the atmospheric entry distance, and using Earth's latitude/longitude conversion provides a reasonable estimate for positioning on Earth's surface.

2.2 Backward Trajectory (Orbital Position)

$$\begin{aligned} r(\theta) &= \frac{1.4956 \cdot 10^{11}}{1 + 0.0167\cos(\theta)}, \\ x &= r(\theta)\cos(\theta), \\ y &= r(\theta)\sin(\theta)\cos(\mathrm{at}), \\ z &= r(\theta)\sin(\theta)\sin(\mathrm{at}) \end{aligned}$$

Source: Vallado (2013) [5].

Justification: A simplified two-body orbit is assumed. The eccentricity and scaling factors are based on Earth's orbit to provide a reasonable orbital framework for the asteroid.

2.3 Impact Energy

$$\begin{aligned} E &= \frac{1}{2}mv^2, \\ E_{\mathrm{TNT}} &= \frac{\frac{1}{2}mv^2}{4.184 \cdot 10^{15}} \end{aligned}$$

Source: Melosh (1989/1996) [3].

Justification: Standard kinetic energy formula; converting to TNT equivalent is conventional in impact studies for comparison purposes.

2.4 Crater Diameter

$$D = 0.0162 \left(E \cdot s_f \cdot 4.184 \cdot 10^{15}\right)^{0.29}$$

$s_f = 0.05$ water, 0.30 sedimentary, 0.50 crystalline.

Source: Holsapple (1993) [1], Housen et al. (1983) [2].

Justification: The exponent 0.29 is within the expected range (0.25–0.33) for energy scaling in -scaling laws. The s_f values represent the fraction of mass remaining after atmospheric entry, chosen based on typical material behavior: water loses most mass, sedimentary loses less, crystalline loses the least.

2.5 Blast Diameter

$$D_{\mathrm{blast}} = 1.25 \times D$$

Source: Pierazzo et al. (2005) [4].

Justification: The factor 1.25 is empirically determined from crater-to-blast scaling in planetary impact studies; it accounts for the additional area affected beyond the crater rim.

2.6 Atmospheric Entry Mass Disintegration

$$\Delta m = \frac{v^2 \cdot Q_{\text{air}}^{1/3}}{Q_{\text{meteor}}^{1/3}} \cdot t,$$

$$\Delta m \approx 1.75 \frac{v^2}{Q_{\text{meteor}}^{1/3}} \quad (\text{for 120 km fall, } Q_{\text{air}} = 0.025\ \text{kg/m}^3)$$

Source: Melosh (1989/1996) [3].

Justification: The 1.75 coefficient arises from evaluating the integral of the disintegration formula over the fall from 120 km with average air density. This simplification is practical for coding purposes and approximates mass loss during atmospheric entry.

2.7 Blast Effect Radius (Assumed Model)

Assuming energy E is inversely proportional to the cube of the affected radius r:

$$E = \frac{C}{r^3}.$$

Applying initial conditions $r = R_0$, $E = E_0$, we get $C = E_0 R_0^3$, giving

$$r = \left(\frac{E_0}{E}\right)^{1/3} R_0$$

Justification: This is an assumed model, based on the idea that energy dissipates with volume ($\sim r^3$). Setting initial conditions allows calculation of the proportionality constant. The formula provides a simple way to scale the blast radius as energy decreases. While heuristic, it is reasonable for preliminary estimation of the affected area.

3 References

References

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