

DEPARTMENT OF INFORMATICS

TECHNICAL UNIVERSITY OF MUNICH

Bachelor's Thesis in Informatics

**Formalisation of a Congruence Closure  
Algorithm in Isabelle/HOL**

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**Formalisierung eines  
Kongruenzhüllen-Algorithmus in  
Isabelle/HOL**

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I confirm that this bachelor's thesis in informatics is my own work and I have documented all sources and material used.

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Rebecca Ghidini

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# Abstract

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# 1 Introduction

## 1.1 Outline

Citation test [Lam94].

```
apply(simp)
apply(auto)
done
```

Figure 1.1: An example for a source code listing.



## **2 Preliminaries**

### **2.1 Union Find with Explain Operation**

### **2.2 Congruence Closure with Explain Operation**

### **2.3 Isabelle/HOL**

#### **2.3.1 Union Find in Isabelle**

## **3 Explain Operation for Union Find**

### **3.1 The Union Find Data Structure**

### **3.2 Implementation**

#### **3.2.1 Union**

#### **3.2.2 Helper Functions for Explain**

#### **3.2.3 Explain**

### **3.3 Proofs**

#### **3.3.1 Invariant and Induction Rule**

#### **3.3.2 Termination Proof**

#### **3.3.3 Correctness Proof**

## **4 Congruence Closure with Explain Operation**

### **4.1 Implementation of the Congruence Closure Algorithm**

### **4.2 Formalisation of Congruence Closure**

### **4.3 Correctness Proof**

### **4.4 Implementation of the Explain Operation**

## **5 Conclusion**

### **5.1 Future work**

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# Bibliography

- [Lam94] L. Lamport. *LaTeX : A Documentation Preparation System User's Guide and Reference Manual*. Addison-Wesley Professional, 1994.