

DEPARTMENT OF INFORMATICS

TECHNICAL UNIVERSITY OF MUNICH

Bachelor's Thesis in Informatics

**Formalisation of a Congruence Closure
Algorithm in Isabelle/HOL**

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**Formalisierung eines
Kongruenzhüllen-Algorithmus in
Isabelle/HOL**

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I confirm that this bachelor's thesis in informatics is my own work and I have documented all sources and material used.

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Rebecca Ghidini

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Abstract

Contents

Acknowledgments	iii
Abstract	iv
1 Introduction	1
1.1 Outline	1
2 Preliminaries	2
2.1 Union Find with Explain Operation	2
2.2 Congruence Closure with Explain Operation	2
2.3 Isabelle/HOL	2
2.3.1 Union Find in Isabelle	2
3 Explain Operation for Union Find	3
3.1 The Union Find Data Structure	3
3.2 Implementation	3
3.2.1 Union	3
3.2.2 Helper Functions for Explain	3
3.2.3 Explain	3
3.3 Proofs	3
3.3.1 Invariant and Induction Rule	3
3.3.2 Termination Proof	3
3.3.3 Correctness Proof	3
4 Congruence Closure with Explain Operation	4
4.1 Implementation	4
4.1.1 Modified Union Find Algorithm	4
4.1.2 Congruence Closure Data Structure	4
4.1.3 Congruence Closure Algorithm	4
4.2 Abstract Formalisation of Congruence Closure	4
4.3 Correctness Proof	4
4.4 Implementation of the Explain Operation	4

Contents

5 Conclusion	5
5.1 Future work	5
List of Figures	6
Bibliography	7

1 Introduction

1.1 Outline

Citation test [Lam94].

```
apply(simp)
apply(auto)
done
```

Figure 1.1: An example for a source code listing.

2 Preliminaries

2.1 Union Find with Explain Operation

2.2 Congruence Closure with Explain Operation

2.3 Isabelle/HOL

2.3.1 Union Find in Isabelle

3 Explain Operation for Union Find

3.1 The Union Find Data Structure

3.2 Implementation

3.2.1 Union

3.2.2 Helper Functions for Explain

3.2.3 Explain

3.3 Proofs

3.3.1 Invariant and Induction Rule

3.3.2 Termination Proof

3.3.3 Correctness Proof

4 Congruence Closure with Explain Operation

4.1 Implementation

4.1.1 Modified Union Find Algorithm

`add_edge`

`add_label`

4.1.2 Congruence Closure Data Structure

4.1.3 Congruence Closure Algorithm

4.2 Abstract Formalisation of Congruence Closure

4.3 Correctness Proof

4.4 Implementation of the Explain Operation

5 Conclusion

5.1 Future work

List of Figures

1.1	Example listing	1
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Bibliography

- [Lam94] L. Lamport. *LaTeX : A Documentation Preparation System User's Guide and Reference Manual*. Addison-Wesley Professional, 1994.