

DEPARTMENT OF INFORMATICS

TECHNICAL UNIVERSITY OF MUNICH

Bachelor's Thesis in Informatics

**Formalisation of a Congruence Closure
Algorithm in Isabelle/HOL**

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**Formalisierung eines
Kongruenzhüllen-Algorithmus in
Isabelle/HOL**

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I confirm that this bachelor's thesis in informatics is my own work and I have documented all sources and material used.

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Rebecca Ghidini

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Abstract

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1 Introduction

1.1 Outline

Citation test [Lam94].

```
apply(simp)
apply(auto)
done
```

Figure 1.1: An example for a source code listing.

2 Preliminaries

2.1 Union Find with Explain Operation

2.2 Congruence Closure with Explain Operation

2.3 Isabelle/HOL

2.3.1 Union Find in Isabelle

3 Explain Operation for Union Find

3.1 The Union Find Data Structure

3.2 Implementation

3.2.1 Union

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5.1 Future work

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Bibliography

- [Lam94] L. Lamport. *LaTeX : A Documentation Preparation System User's Guide and Reference Manual*. Addison-Wesley Professional, 1994.