# MATH 323 - Homework 3

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September 20, 2018

**Problem 3.1.1d.**

$$\begin{vmatrix} \cos\theta & \sin\theta \\ -\sin\theta & \cos\theta \end{vmatrix} = \cos^2\theta - (-\sin^2\theta) = \boxed{1}$$

**Problem 3.1.1h.**

$$\begin{vmatrix} -6 & 0 & 0 \\ 0 & 2 & 0 \\ 0 & 0 & 5 \end{vmatrix} = -6(2)(5) = \boxed{-60}$$

**Problem 3.1.5b.**

$$\begin{aligned} \begin{vmatrix} 0 & 5 & 4 & 0 \\ 4 & 1 & -2 & 7 \\ -1 & 0 & 3 & 0 \\ 0 & 2 & 1 & 5 \end{vmatrix} &= -5\begin{vmatrix} 4 & -2 & 7 \\ -1 & 3 & 0 \\ 0 & 1 & 5 \end{vmatrix} + 4\begin{vmatrix} 4 & 1 & 7 \\ -1 & 0 & 0 \\ 0 & 2 & 5 \end{vmatrix} \\ &= -5(4(15) + 2(-5) + 7(-1)) + 4(-1(-5) + 7(-2)) \\ &= -5(43) + 4(-9) \\ &= \boxed{-251} \end{aligned}$$

**Problem 3.1.5c.**

$$\begin{vmatrix} 2 & 1 & 9 & 7 \\ 0 & -1 & 3 & 8 \\ 0 & 0 & 5 & 2 \\ 0 & 0 & 0 & 6 \end{vmatrix} = 6\begin{vmatrix} 2 & 1 & 9 \\ 0 & -1 & 3 \\ 0 & 0 & 5 \end{vmatrix} = 6(2(-1)(5)) = \boxed{-60}$$

**Problem 3.1.11b.**

$$\text{Volume} = |\det[\mathbf{x}|\mathbf{y}|\mathbf{z}]| = \left|\det\begin{bmatrix} 4 & -2 & 7 \\ -1 & 3 & 0 \\ 0 & 1 & 5 \end{bmatrix}\right| = |(-5) + 12| = \boxed{7}$$

**Problem 3.1.11d.**

$$\text{Volume} = |\det[\mathbf{x}|\mathbf{y}|\mathbf{z}]| = \left|\det\begin{bmatrix} 1 & 2 & 0 \\ 3 & 2 & -1 \\ 5 & -2 & -1 \end{bmatrix}\right| = |(-2-2) - 2(-3+5)| = \boxed{8}$$

**Problem 3.13.**

a.) *Proof.* Let $A \in \mathbf{R}^{n\times n}$. Consider the determinant of $A$ multiplied by a scalar $c$.

$$\det(cA) = \begin{vmatrix} ca_{11} & ca_{12} & \dots & ca_{1n} \\ ca_{21} & ca_{22} & \dots & ca_{2n} \\ \vdots & \vdots & \ddots & \vdots \\ ca_{n1} & ca_{n2} & \dots & ca_{nn} \end{vmatrix}$$

Recall that if we take a matrix $A$ and multiply any row or column by a scalar $c$ the new determinant of that matrix will be $c$-times the original since cofactor expansion along that row would yield a determinant $c$-times greater. In this case we have $n$ rows. So,

$$\det(cA) = \underbrace{c \cdot c \cdot c \cdots c}_{\text{n times}} \det(A) = c^n \det(A)$$

$\square$

b.) Let $\mathbf{x} = [x_1, x_2, x_3]$, $\mathbf{y} = [y_1, y_2, y_3]$, $\mathbf{z} = [z_1, z_2, z_3]$. The volume of the parallelepiped is given by

$$\det([\mathbf{x}|\mathbf{y}|\mathbf{z}]) = \begin{vmatrix} x_1 & x_2 & x_3 \\ y_1 & y_2 & y_3 \\ z_1 & z_2 & z_3 \end{vmatrix}$$

If each side is doubled by 2, then the sides become $[2x_1, 2x_2, 2x_3]$, $[2y_1, 2y_2, 2y_3]$, $[2z_1, 2z_2, 2z_3]$. So every entry in our matrix is multiplied by 2. We know that this is a $3 \times 3$ matrix, so $n = 3$. By part(a), the volume is then $\det(2[\mathbf{x}|\mathbf{y}|\mathbf{z}]) = 2^3 \det([\mathbf{x}|\mathbf{y}|\mathbf{z}]) = 8 \det([\mathbf{x}|\mathbf{y}|\mathbf{z}])$.

**Problem 3.1.16.**

a.) $\begin{vmatrix} 1 & 1 & 1 \\ a & b & c \\ a^2 & b^2 & c^2 \end{vmatrix} = \begin{vmatrix} 1 & a & a^2 \\ 1 & b & b^2 \\ 1 & c & c^2 \end{vmatrix} = \begin{vmatrix} 1 & a & a^2 \\ 0 & b-a & b^2-a^2 \\ 0 & c-a & c^2-a^2 \end{vmatrix} = (b-a)(c-a) \begin{vmatrix} 1 & a & a \\ 0 & 1 & b+a \\ 0 & 1 & c+a \end{vmatrix} = \boxed{(b-a)(c-a)(c-b)}$

b.) $\begin{vmatrix} 1 & 1 & 1 \\ 2 & 3 & -2 \\ 4 & 9 & 4 \end{vmatrix} = (2-3)(3+2)(-2-2) = \boxed{20}$