MATH 407 - Homework 5

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Problem 4.32c.

From $(0,1)$ to $(0,5)$, $x=0$, $dx=0$. We then have

$$\int_1^5 2y\,dy = y^2\Big|_1^5 = 24$$

From $(0,5)$ to $(2,5)$, $y=0$, $dy=0$. We then have

$$\int_0^2 3x+5\,dx = \frac{3}{2}x^2+5x\Big|_0^2 = 16$$

Thus the total integral is $24+16=\boxed{40}$.

Problem 4.33a.

$$\begin{aligned}
\oint_C (x+2y)\,dx+(y-2x)\,dy &= \int_0^{2\pi}(4\cos\theta+6\sin\theta)(-4\sin\theta)+(3\sin\theta-8\cos\theta)(3\cos\theta)\,d\theta \\
&= \int_0^{2\pi} -16\sin\theta\cos\theta-24\sin^2\theta+9\sin\theta\cos\theta-24\cos^2\theta\,d\theta \\
&= \int_0^{2\pi} -5\sin\theta\cos\theta-24\,d\theta \\
&= \frac{5}{2}\cos^2\theta\Big|_0^{2\pi}-48\pi \\
&= \boxed{-48\pi}
\end{aligned}$$

Problem 4.34a.

Using the parameterization $x=t$, $y=2t^2$, $dx=dt$, $dy=4t\,dt$, $dz=(1+i4t)\,dt$, the integral becomes

$$\begin{aligned}
\int_C (x^2-iy^2)\,dz &= \int_1^2 (t^2-4it^4)(1+4it)\,dt \\
&= \int_1^2 t^2+16t^5\,dt + i\int_1^2 4t^3-4t^4\,dt \\
&= \frac{1}{3}t^3\Big|_1^2+\frac{16}{6}t^6\Big|_1^2+i\left(t^4\Big|_1^2-\frac{4}{5}t^5\Big|_1^2\right) \\
&= \frac{8}{3}-\frac{1}{3}+\frac{32}{3}-\frac{16}{6}+i\left(15-\frac{128}{5}-\frac{4}{5}\right) \\
&= \boxed{\frac{511}{3}-\frac{49}{5}i}
\end{aligned}$$

Problem 4.36a.

Using the parameterization $h(\theta) = 2e^{i\theta}$, $h'(\theta) = 2ie^{i\theta}$, $0 < \theta < \frac{\pi}{2}$, the integral becomes

$$\begin{aligned}\int_C z^2 + 3z\,dz &= \int_0^{\pi/2} \left(4e^{2i\theta} + 6e^{i\theta}\right) 2ie^{i\theta}\,d\theta \\ &= \int_0^{\pi/2} 8ie^{3i\theta} + 12ie^{2i\theta}\,d\theta \\ &= \frac{8}{3}e^{3i\theta}\Big|_0^{\pi/2} + 6e^{2i\theta}\Big|_0^{\pi/2} \\ &= -\frac{8}{3}i - \frac{8}{3} - 6 - 6 \\ &= \boxed{-\frac{44}{3} - \frac{8}{3}i}\end{aligned}$$

Problem 4.36b.

Using the parameterization $h(t) = 2 + t(-2+2i)$, $h'(t) = -2+2i$, $0 < t < 1$, the integral becomes

$$\begin{aligned}\int_C z^2 + 3z\,dz &= \int_0^1 [(2 + t(-2+2i))^2 + 3(2 + t(-2+ti))](-2+2i)\,dt \\ &= \int_0^1 (-20+20i) - 56it + (16+16i)t^2\,dt \\ &= -20 + 20i - 56i\left(\frac{1}{2}t^2\right)\Big|_0^1 + (16+16i)\left(\frac{1}{3}t^3\right)\Big|_0^1 \\ &= \boxed{-\frac{44}{3} - \frac{8}{3}i}\end{aligned}$$

Problem 4.39a.

Using the parameterization $h(\theta) = e^{i\theta}$, $h'(\theta) = ie^{i\theta}$, $0 < \theta < 2\pi$, the integral becomes

$$\begin{aligned}\oint_C \bar{z}^2\,dz &= \int_0^{2\pi} \left(e^{-i\theta}\right)^2 \left(ie^{i\theta}\right)\,d\theta \\ &= i\int_0^{2\pi} e^{-i\theta}\,d\theta \\ &= -e^{-i\theta}\Big|_0^{2\pi} = -\left(e^{-2\pi i} - 1\right) = \boxed{0}\end{aligned}$$

Problem 4.39b.

Using the parameterization $h(\theta) = 1 + e^{i\theta}$, $h'(\theta) = ie^{i\theta}$, $0 < \theta < 2\pi$, the integral becomes

$$\begin{aligned}\oint_C \bar{z}^2\,dz &= \int_0^{2\pi} \left(1 + e^{-i\theta}\right)^2 \left(ie^{i\theta}\right)\,d\theta \\ &= \int_0^{2\pi} \left(1 + 2e^{-i\theta} + e^{-2i\theta}\right)\left(ie^{i\theta}\right)\,d\theta \\ &= \int_0^{2\pi} 2i + ie^{-i\theta} + ie^{i\theta}\,d\theta \\ &= 4\pi i - e^{i\theta}\Big|_0^{2\pi} + e^{i\theta}\Big|_0^{2\pi} \\ &= 4\pi i - \left(e^{-2\pi i} - 1\right) + \left(e^{2\pi i} - 1\right) \\ &= \boxed{4\pi i}\end{aligned}$$