MATH 407 - Homework 6

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Problem 4.61a.

Using the parameterization $h(t) = e^{it}$, $0 \leq t \leq 2\pi$, we have

$$\begin{aligned}
\int_C f(z)\,dz = \int_C f(h(t))h'(t)\,dt &= \int_0^{2\pi} \left(e^{3it} - ie^{2it} - 5e^{it} + 2i\right) ie^{it}\,dt \\
&= \int_0^{2\pi} ie^{4it} + e^{3it} - 5ie^{2it} - 2e^{it}\,dt \\
&= \frac{1}{4}e^{4it}\Big|_0^{2\pi} + \frac{1}{3i}e^{3it}\Big|_0^{2\pi} - \frac{5}{2}e^{2it}\Big|_0^{2\pi} - \frac{2}{i}e^{it}\Big|_0^{2\pi} \\
&= \frac{1}{4}e^{8\pi i} - \frac{1}{4} + \frac{1}{3i}e^{6\pi i} - \frac{1}{3i} - \frac{5}{2}e^{4\pi i} + \frac{5}{2} - \frac{2}{i}e^{2\pi i} + \frac{2}{i} \\
&= 0
\end{aligned}$$

Thus Cauchy's Theorem is satisfied.

Problem 4.62.

a.) Since the pole $z = 3$ is inside C, we can evaluate this integral. Comparing to Cauchy's integral formula for $f(z_0)$, we see that $f(z) = 1$ and $z_0 = 3$. So we have

$$1 = \frac{1}{2\pi i}\oint_C \frac{dz}{z-3}$$

or

$$\oint_C \frac{dz}{z-3} = 2\pi i$$

b.) No, since $f(z) = \frac{1}{z-3}$ is not analytic at $z = 3$.

Problem 4.72ac.

Solving directly,

$$\begin{aligned}
\int_{3+4i}^{4-3i} 6z^2 + 8iz\,dz &= 2z^3\Big|_{3+4i}^{4-3i} + 4iz^2\Big|_{3+4i}^{4-3i} \\
&= 2(4-3i)^3 - 2(3+4i)^3 + 4i(4-3i)^2 - 4i(3+4i)^2 \\
&= 338 - 266i
\end{aligned}$$

a.) Using the parameterization $h(t) = 3 + 4i + t(1-7i)$, $0 \leq t \leq 1$, the integral becomes

$$\begin{aligned}
\int_{3+4i}^{4-3i} 6z^2 + 8iz\,dz &= \int_0^1 \Big(6(3+4i+t(1-7i))^2 + 8i(3+4i+t(1-7i))\Big)(1-7i)\,dt \\
&= \int_0^1 -876t^2 - 944t + 1102\,dt + i\int_0^1 1932t^2 - 3192t + 686\,dt \\
&= 338 - 266i
\end{aligned}$$

c.) Using the parameterization $h(t) = 5e^{it}, 0 \leq t \leq 2\pi$, the integral becomes

$$\begin{aligned}\int_{3+4i}^{4-3i} 6z^2 + 8iz\,dz &= \int_0^{2\pi} 6\left(25e^{2it}\right) + 8ie^{it}\,dt \\ &= 338 - 266i\end{aligned}$$

Problem 5.30ab.

a.) Since the pole $z = 2$ is inside C, we can evaluate this integral. Comparing to Cauchy's integral formula for $f(z_0)$, we see that $f(z) = e^z$ and $z_0 = 2$. So we have

$$e^z\Big|_{z=2} = \oint_C \frac{e^z}{z-2}\,dz$$

or

$$\oint_C \frac{e^z}{z-2}\,dz = e^2$$

b.) Since C does not enclose the pole $z = 2$, the integral is 0.

Problem 5.32.

a.) Since the pole $z = \pi i$ is inside C, we can evaluate this integral. Comparing to Cauchy's integral formula for $f(z_0)$, we see that $f(z) = e^{3z}$ and $z_0 = \pi i$. So we have

$$e^{3z}\Big|_{z=\pi i} = \frac{1}{2\pi i}\oint_C \frac{e^{3z}}{z-\pi i}\,dz$$

or

$$\begin{aligned}\oint_C \frac{e^{3z}}{z-\pi i}\,dz &= 2\pi i e^{3\pi i} \\ &= -2\pi i\end{aligned}$$

b.) Since the pole $z = \pi i$ is outside the curve C, integral is 0.