MATH 489 — Intro to Topological Data Analysis

Notes taken by Lukas Zamora

Spring 2019

The Nature of Data and Data Analysis
Data - form and function, big data, Data Analysis - Graphical, Statistical, Topological. [1]

Review of Linear Algebra
Vector spaces and subspaces, basis and dimension, linear transformations and matrix representations, kernal and cokernal of a linear transformation, quotient vector spaces, inner product spaces, normed spaces. [2]

Metric Spaces and Point Clouds
Definition of a metric space, examples of metric spaces, open and closed sets, continuous maps between metric spaces. [3]

Data Clustering
Clustering Theory vs. practice, desiriable properties and Kleinberg's Impossibility Theorem, functional clustering, single linkage clustering, Kruskal's algorithms, mixture models and generalized PCA. [4]

Topology
Basic topological properties of metric spaces: connectedness, compactness, equivalence relations, Homeomorphism, introduction to the ideas of π_0, π_1. [12]

Homology of Simplicial Complexes
Simple examples and calculations, chain complexes of vector spaces and boundary maps, Euler characteristic, rigorous definition of homology of a simplicial complex, computing homology, introduction to functionality: inclusion maps on homology. [10]

Greens Functions for Time-Independent Problems
Green's functions for boundary value problems for ODEs, method of eigenvalue expansion, nonhomogeneous boundary conditions. [9]

Contents

1 The Heat Equation