Topic 5 - Joint Distributions

Car example

X/Y	0	1	2	3	4
0	1/2	1/16	1/32	1/32	1/32
1	1/16	1/32	1/32	1/32	1/32
2	1/32	1/32	1/32	1/32	1/32

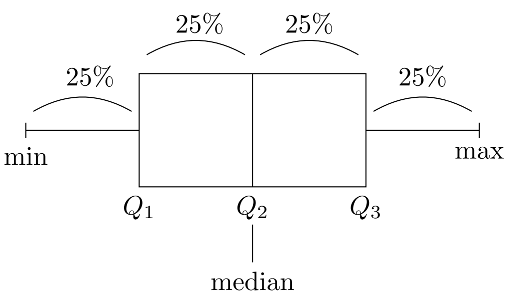
$P(X \geq 1, Y \geq 1) = 8/32$
$P(X \geq 1) = 6/32 + 5/32 = 11/32$
$E(X + Y) =$

sample mean: $\bar{x} = \dfrac{\sum_{i=1}^{n} x_i}{n}$
sample median: middle value if arranged in increasing order

1^{st} quartile (Q_1) - 25^{th} percentile
2^{nd} quartile (Q_2) - 50^{th} percentile
3^{rd} quartile (Q_3) - 75^{th} percentile

5 number summary: Min, Q_1, Q_2, Q_3, Max

Box Plot



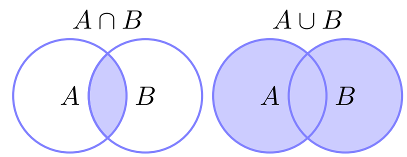
Measures of spread: IQR ($Q_3 - Q_1$) and sample variance (s^2 or σ^2)

$\sigma^2 = \dfrac{\sum_{i=1}^{n} (x_i - \bar{x})^2}{n-1}$,where $n - 1$ is the degrees of freedom

sample standard deviation: $\sqrt{s^2}$

Trimmed mean: take off from both ends and compute new mean. *Example*: Sample of 20. 10% trimmed mean → take 2 values off from beginning and end so mean of 16 values. Usually to get rid of outliers.

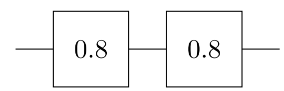
Topic 2



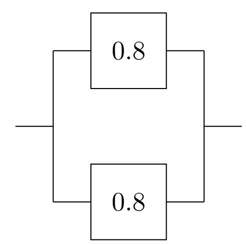
Properties of probability

- $P(A \cup B) = P(A) + P(B) - P(A \cap B)$
- $P(A \cap B) = P(A) + P(B) - P(A \cup B)$
- $P(A^c) = 1 - P(A)$
- Conditional probability of A given B: $P(A|B) = \dfrac{P(A \cap B)}{P(B)}$
- A and B are independent if $P(A \cap B) = P(A)P(B)$

The reliability of a system is the overall probability of success.
$P(\text{failure}) = 1 - P(\text{success})$



$$\begin{aligned} P(S) &= P(A \cap B) = P(A)P(B) \\ &= 0.8 \cdot 0.8 \\ &= 0.64 \end{aligned}$$



$$\begin{aligned} P(S) &= P(A \cup B) = P(A) + P(B) - P(A \cap B) \\ &= 0.8 + 0.8 - 0.64 \\ &= 0.96 \end{aligned}$$

Adding more components to a series system results in loss of reliability, vice versa for parallel systems.

Topic 3

discrete random variable takes on a countable number of values
probability mass function (pmf) is a distribution of a discrete random variable x

$$f(x) = P(X = x)$$

requirements for pmf: $f(x) > 0$ for all x, and $\sum_{\text{all } x} f(x) = 1$

cumulative distribution function (cdf): $F(x) = P(X \leq x) = \sum_{\text{all } t \leq x} f(t)$

Bernoulli random variable: Success $= p$, failure $= 1 - p$
Example:

$$SSS = p^3 \qquad SFS = p^2(1-p) \qquad FFS = p(1-p)^2 \qquad FFF = (1-p)^3$$

Mean of discrete random variable: $\mu_x = E(x) = \sum x \cdot p(x)$
$E(h(x)) = \sum h(x) \cdot f(x)$ is the expected value of $h(x)$
$V(x) = \sigma_x^2 = E(x^2) - (E(x))^2$

Binomial Distribution: Bernoulli trials: each trial can result in one or two outcomes (success or fail), trials are independent
Binomial pmf: probability of any outcome sequence from n Bernoulli trials with x successes and $n - x$ fails

$$f(x) = P(X = x) = \binom{n}{x} p^x (1-p)^{n-x}$$

Binomial properties: $E(x) = \mu_x = np$, $\sigma_x^2 = np(1-p)$

Poisson Distribution: number of events occurring in an interval where the expected number of events is proportional to the length of the interval

$$f(x) = P(X = x) = \frac{e^{-\lambda}\lambda^x}{x!},\ \lambda : \text{rate}$$

Poisson properties: $E(x) = \lambda$, $\sigma_x^2 = \lambda$

Topic 4

Probability Density function (pdf): A function $f(x)$ such that for $a \leq x \leq b$

$$P(a \leq x \leq b) = \int_a^b f(x)\, dx$$

cdf of X is $F(X) = P(X = x) = \int_{-\infty}^{x} f(t)\, dt$

$f(x) = F'(x)$
$P(a \leq x \leq b) = F(b) - F(a)$

Properties of pdf:
$E(x) = \int_a^b x \cdot f(x)\, dx$
$\sigma_x^2 = E(x^2) - (E(x))^2 = \int_a^b x^2 \cdot f(x)\, dx - \mu_x^2$
$E(h(x)) = \int_a^b h(x) \cdot f(x)\, dx$

If it is a *uniform* distribution, then