

Exercise 1 (Binary search trees: membership).

Define the membership operation for binary search trees:

```
member {A} {dto : DecTotalOrder A} (x : A) (t : tree A) : bool
```

Prove: `forall x t, BST t -> List.In x (elements t) -> member x t`.

Exercise 2 (Binary search trees: minimum).

Define a function which returns the smallest element in a binary search tree and a function which deletes the smallest element:

```
minimum {A} {dto : DecTotalOrder A} (t : tree A) : t <> empty -> A
deletemin {A} {dto : DecTotalOrder A} (t : tree A) : tree A
```

Prove:

1. `forall t p, BST t -> All (leb (minimum t p)) t`
2. `forall t p, List.In (minimum t p) (elements t)`
3. `forall t p, All P t -> P (minimum t p)`
4. `forall t p, Permutation (minimum t p :: elements (deletemin t)) (elements t)`
5. `forall t, All P t -> All P (deletemin t)`
6. `forall t, BST t -> BST (deletemin t)`