

$$\begin{array}{c}
\overline{\Gamma, X : \varphi \vdash X : \varphi} \text{ (Ax)} \\[10pt]
\frac{\Gamma, X : \varphi_1 \vdash M : \varphi_2}{\Gamma \vdash (\lambda X : \varphi_1.M) : \varphi_1 \rightarrow \varphi_2} \text{ (}\rightarrow\text{I)} \quad \frac{\Gamma \vdash M_1 : \varphi \rightarrow \psi \quad \Gamma \vdash M_2 : \varphi}{\Gamma \vdash M_1 M_2 : \psi} \text{ (}\rightarrow\text{E)} \\[10pt]
\frac{\Gamma \vdash M_1 : \varphi_1 \quad \Gamma \vdash M_2 : \varphi_2}{\Gamma \vdash (M_1, M_2) : \varphi_1 \wedge \varphi_2} \text{ (}\wedge\text{I)} \quad \frac{\Gamma \vdash M : \varphi_1 \wedge \varphi_2 \quad \Gamma, X_1 : \varphi_1, X_2 : \varphi_2 \vdash N : \psi}{\Gamma \vdash (\text{case } M \text{ of } (X_1, X_2) \Rightarrow N) : \psi} \text{ (}\wedge\text{E)} \\[10pt]
\frac{\Gamma \vdash M : \varphi_1}{\Gamma \vdash \text{inl } M : \varphi_1 \vee \varphi_2} \text{ (}\vee\text{I}_1\text{)} \quad \frac{\Gamma \vdash M : \varphi_2}{\Gamma \vdash \text{inr } M : \varphi_1 \vee \varphi_2} \text{ (}\vee\text{I}_2\text{)} \\[10pt]
\frac{\Gamma \vdash M : \varphi_1 \vee \varphi_2 \quad \Gamma, X_1 : \varphi_1 \vdash N_1 : \psi \quad \Gamma, X_2 : \varphi_2 \vdash N_2 : \psi}{\Gamma \vdash (\text{case } M \text{ of inl } X_1 \Rightarrow N_1 \mid \text{inr } X_2 \Rightarrow N_2) : \psi} \text{ (}\vee\text{E)} \\[10pt]
\frac{\Gamma \vdash M : \perp}{\Gamma \vdash (\text{case}_\psi M) : \psi} \text{ (}\perp\text{E)} \\[10pt]
\frac{\Gamma \vdash M : \varphi \quad x : A \quad x \notin \text{FV}(\Gamma)}{\Gamma \vdash (\lambda x : A.M) : \forall x : A.\varphi} \text{ (}\forall\text{I)} \quad \frac{\Gamma \vdash M : \forall x : A.\varphi \quad t : A}{\Gamma \vdash Mt : \varphi[t/x]} \text{ (}\forall\text{E)} \\[10pt]
\frac{\Gamma \vdash M : \varphi[t/x] \quad t : A \quad x : A}{\Gamma \vdash [t, M] : \exists x : A.\varphi} \text{ (}\exists\text{I)} \\[10pt]
\frac{\Gamma \vdash M : \exists x : A.\varphi \quad \Gamma, X : \varphi \vdash N : \psi \quad x \notin \text{FV}(\Gamma, \psi)}{\Gamma \vdash (\text{case } M \text{ of } [x, X] \Rightarrow N) : \psi} \text{ (}\exists\text{E)}
\end{array}$$

Figure 1: The rules of intuitionistic multisorted first-order predicate logic