

**Exercise 1** (Insertion sort).

Prove `forall l, Permutation (isort l) l` where `isort` is the insertion sort function from the lecture.

**\*Exercise 2** (Sortedness).

Prove

```
forall l,
  (forall i j, i < j < List.length l -> List.nth i l 0 <= List.nth j l 0) ->
  Sorted l.
```

where `Sorted` is the inductive predicate from the lecture expressing that a list of natural numbers is sorted.

**\*Exercise 3** (Fibonacci numbers).

Recall the `fib` and `fib'` functions from the first lecture:

```
Fixpoint fib (n : nat) : nat :=
  match n with
  | 0 => 1
  | 1 => 1
  | S m as m' => fib m + fib m'
  end.
```

```
Fixpoint itfib a b k :=
  match k with
  | 0 => a
  | S m => itfib b (a + b) m
  end.
```

**Definition** `fib' n := itfib 1 1 n`.

The functions compute the  $n$ th Fibonacci number, in time exponential in  $n$  and linear<sup>1</sup> in  $n$ , respectively.

Prove: `forall n, fib n = fib' n`.

---

<sup>1</sup>Assuming addition on `nat` is a basic operation, i.e., counting it as constant time.