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# Exercise sheet 4

Rocq

**Exercise 1** (Proving with tactics)**.**

Prove the following formulas using basic tactics for logical connectives. Do not use automation (`auto`, `tauto` or `firstorder`).

1. $(P \to Q) \to \neg Q \to \neg P$.
2. $P \to (P \to Q) \to Q$.
3. $P \to \neg\neg P$.
4. $P \vee \neg P \to \neg\neg P \to P$.
5. $\neg\neg(P \to Q) \to \neg\neg P \to \neg\neg Q$.
6. $(\neg\neg P \to \neg\neg Q) \to \neg\neg(P \to Q)$.
7. $\neg(P \wedge \neg P)$.
8. $(P \vee Q \to R) \to (P \to R) \wedge (Q \to R)$.
9. $(\exists x.P \wedge Rx) \leftrightarrow P \wedge \exists x.Rx$.
10. $P \vee (\forall x.Rx) \to \forall x.P \vee Rx$.
11. $(\exists x.Rx) \to \neg(\forall x.\neg Rx)$.
12. $\neg(P \vee Q) \leftrightarrow \neg P \wedge \neg Q$.

**Exercise 2** (Induction on natural numbers)**.**

Prove by induction the following facts about addition. Do not use `lia` or `omega`.

*Hint.* Use the `auto` tactic. It knows some basic facts about natural numbers.

1. Commutativity: `forall x y, x + y = y + x.`
2. Associativity: `forall x y z, (x + y) + z = x + (y + z).`
3. Distributivity of multiplication: `forall x y z, x * (y + z) = x * y + x * z.`

   *Hint.* Use the previous two points and the tactic `replace t with t'` which replaces `t` with `t'` in the goal.

**Exercise 3** (Induction on lists)**.**

Consider the following function which reverses a list.

```
Fixpoint reverse {A} (l : list A) :=
  match l with
  | [] => []
  | h :: t => reverse t ++ [h]
  end.
```

This is *not* an efficient implementation of list reversal (quadratic complexity), but it is easier to reason with than the linear implementation from the first lecture.

Prove by induction the following facts about `reverse`.

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1. `forall l1 l2 : list A, reverse (l1 ++ l2) = reverse l2 ++ reverse l1.`

   *Hint.* Use `SearchRewrite` to find helper lemmas in Coq's standard library.

2. `forall l : list A, reverse (reverse l) = l.`

3. `forall l : list A, reverse l = List.rev l.`

   Is it possible to prove `reverse = List.rev`?