Factoring Formulas

$$
\begin{aligned}
\mathbf{a^2 - b^2} &= (a-b)(a+b)\\
\mathbf{(a+b)^2} &= a^2 + 2ab + b^2\\
\mathbf{(a-b)^2} &= a^2 - 2ab + b^2\\
\mathbf{a^3 - b^3} &= (a-b)(a^2+ab+b^2)\\
\mathbf{a^3 + b^3} &= (a+b)(a^2-ab+b^2)\\
\mathbf{(a+b)^3} &= a^3 + 3a^2b + 3ab^2 + b^3\\
\mathbf{(a-b)^3} &= a^3 - 3a^2b + 3ab^2 - b^3
\end{aligned}
$$

Radicals

$$
\begin{array}{lcl@{\qquad}lcl}
\sqrt[n]{a} &=& a^{\frac{1}{n}} & \sqrt[n]{a^m} &=& a^{\frac{m}{n}}\\
\sqrt[n]{\frac{a}{b}} &=& \frac{\sqrt[n]{a}}{\sqrt[n]{b}} & \sqrt[n]{a\cdot b} &=& \sqrt[n]{a}\cdot\sqrt[n]{b}\\
\sqrt[n]{a^n} &=& |a| & \sqrt[m]{\sqrt[n]{a}} &=& \sqrt[m\cdot n]{a}
\end{array}
$$

Exponents

$$
\begin{array}{lcl@{\qquad}lcl}
a^n \cdot a^m &=& a^{n+m} & \frac{a^n}{a^m} &=& a^{n-m}\\
(a\cdot b)^n &=& a^n \cdot b^n & \left(\frac{a}{b}\right)^n &=& \frac{a^n}{b^n}\\
(a^n)^m &=& a^{n\cdot m} & \left(\frac{a}{b}\right)^{-n} &=& \left(\frac{b}{a}\right)^n
\end{array}
$$

Quadratic Formula

$$\mathbf{a}x^2 + \mathbf{b}x + \mathbf{c} \qquad \text{for} \quad a \neq 0$$

$$\Delta = b^2 - 4ac$$

$$x_{1,2} = \frac{-b \pm \sqrt{\Delta}}{2a}$$

$$p = \frac{-b}{2a} \quad , \quad q = \frac{-\Delta}{4a}$$

$$a(x-p)^2 + q \qquad \text{(vertex form)}$$

$$a(x-x_1)(x-x_2) \qquad \text{(factored form)}$$

Logarithms

$$a^c = x \Leftrightarrow \log_a x = c \qquad for \qquad a \in R^+_{\setminus\{0\}}, x \in R^+$$

$$
\begin{array}{ll@{\qquad}ll}
\log_a a & = 1 & \log_a 1 & = 0\\
\ln x & = \log_e x & \log x & = \log_{10} x\\
\log_a b \cdot \log_b c & = \log_a c & \log_a b & = \frac{1}{\log_b a}
\end{array}
$$

$$\log_a (x \cdot y) = \log_a x + \log_a y$$

$$\log_a \left(\frac{x}{y}\right) = \log_a x - \log_a y$$

Binomial Coefficient

$$\binom{n}{k} = \frac{n!}{k!(n-k)!}$$

Matrices

$$A_{m\times n} = \begin{bmatrix} a_{1,1} & a_{1,2} & \cdots & a_{1,n}\\ a_{2,1} & a_{2,2} & \cdots & a_{2,n}\\ \vdots & \vdots & \ddots & \vdots\\ a_{m,1} & a_{m,2} & \cdots & a_{m,n} \end{bmatrix}$$

$$A_{ij} = (-1)^{i+j} M_{ij} \qquad \text{(Cofactor expansion of element } a_{ij}\text{)}$$

Minor M_{ij} is the determinant of submatrix (of rank $n-1$), cut down from A by removing i row and j column.

$$A^{-1} = \frac{1}{\det A} \cdot [A_{ij}]^T \qquad \text{where} \qquad A^{-1} \cdot A = A \cdot A^{-1} = \mathbf{I}$$

$$\det A = \sum_{j=1}^{n} a_{ij} A_{ij} \qquad \text{(Laplace expansion)}$$

$$
\begin{array}{ccc:cc}
+ & + & + & & \\
a_{11} & a_{12} & a_{13} & a_{11} & a_{12}\\
a_{21} & a_{22} & a_{23} & a_{21} & a_{22}\\
a_{31} & a_{32} & a_{33} & a_{31} & a_{32}\\
- & - & - & &
\end{array}
$$

$$\det A = \det A^T$$

$$\det(A \cdot B) = \det A \cdot \det B$$

$$\det(A^{-1}) = (\det A)^{-1}$$

$$\det(k \cdot A) = k^n \cdot \det A \qquad k \in R, n \text{ order of } A$$

Limits

$$
\begin{array}{lcl@{\qquad}lcl}
\lim\limits_{n\to\infty} \frac{1}{n} &=& 0 & \lim\limits_{n\to\infty} \frac{1}{\sqrt{n}} &=& 0\\
\lim\limits_{n\to\infty} 2^n &=& \infty & \lim\limits_{n\to\infty} \left(\frac{1}{2}\right)^n &=& 0\\
\lim\limits_{n\to\infty} n &=& \infty & \lim\limits_{n\to\infty} \left(1+\frac{1}{n}\right)^n &=& e\\
\lim\limits_{n\to\infty} \sqrt[n]{a} &=& 1 & \lim\limits_{n\to\infty} \frac{n}{0} &=& \infty
\end{array}
$$

$$
\lim_{x\to 0} \frac{\sin x}{x} = 1 \qquad \lim_{x\to 0} \left(1+\frac{1}{x}\right)^x = 1
$$

Indeterminate forms

$$\frac{0}{0}, \quad \frac{\infty}{\infty}, \quad \infty - \infty, \quad 0\cdot\infty, \quad \infty^0, \quad 1^\infty, \quad 0^0$$

Derivatives

$$y = f\Big(g(x)\Big) \qquad y' = f'(g) \cdot g'$$

$$y = f(x) \cdot g(x) \qquad y' = f' \cdot g + f \cdot g'$$

$$y = \frac{f(x)}{g(x)} \qquad y' = \frac{f' \cdot g - f \cdot g'}{g^2}$$

$y = x^n$	$y' = n \cdot x^{n-1}$	$y = ax$	$y' = a$
$y = a^x$	$y' = a^x \cdot \ln a$	$y = e^x$	$y' = e^x$
$y = \log_a x$	$y' = \frac{1}{x \cdot \ln a}$	$y = \ln x$	$y' = \frac{1}{x}$
$y = \sin x$	$y' = \cos x$	$y = \cos x$	$y' = -\sin x$

Integrals

$$\int 0 \ dx = 0 + c \qquad \int 1 \ dx = x + c$$

$$\int x^n \ dx = \frac{1}{n} x^{n+1} + c \qquad \int a^x \ dx = \frac{a^x}{\ln a} + c$$

$$\int x \ dx = \frac{1}{2} x^2 + c \qquad \int \sqrt{x} \ dx = \frac{2}{3} x \sqrt{x} + c$$

$$\int \frac{1}{x} \ dx = \ln|x| + c \qquad \int \ln x \ dx = x \ln x - x + c$$

$$\int e^x \ dx = e^x + c \qquad \int e^{ax} \ dx = \frac{1}{a} e^{ax} + c$$