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Current characteristics of mesoscopic rings in quantum Smoluchowski regime

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Abstract. In normal mesoscopic metals of a ring topology persistent currents can be induced by threading the center of the ring with a magnetic flux. This phenomenon is an example of the famous Aharonov-Bohm effect. In the paper we study the current *vs* the external constant magnetic flux characteristics of the system driven by both the classical and the quantum thermal fluctuations. The problem is formulated in terms of Langevin equations in classical and quantum Smoluchowski regimes. We analyze the impact of the quantum thermal fluctuations on the current-flux characteristics. We demonstrate that the current response can be changed from paramagnetic to diamagnetic when the quantum nature of the thermal fluctuations increases.

1 Introduction

Mesoscopic systems lay at the border between macroscopic and microscopic worlds. Mesoscopic systems, consisting of a large number of atoms, are too big to study their properties by the quantum methods pertinent to individual atoms, and are too small to apply physical laws of the macro-world. For their description and modeling one should combine both methods appreciating and recognizing the role of quantum and classical processes. The appearance of a new length scale - the phase coherence length l_c of electronic wave functions – introduces various regimes for transport phenomena influenced by quantum interference effects. The mesoscopic regime is characterized by small length scales and low temperatures. When the temperature is lowered, the phase coherence length increases and the mesoscopic regime is extended to larger length scales. At sub-Kelvin temperatures, the length scales are of the order of micrometers. The most prominent mesoscopic effects are: the Aharonov-Bohm oscillations in the conductance of mesoscopic structures, the quantum Hall effects, the universal conductance fluctuations and the persistent currents in mesoscopic normal metal rings threaded by a magnetic flux. Persistent currents have been known to exist in superconductors in which they are related to the existence of a state with zero resistance and the fact that a superconductor is a perfect diamagnet. The existence of persistent currents in normal (i.e. non-superconducting) metals is, in fact, a manifestation of the famous Aharonov-Bohm effect. Persistent currents have been predicted by Hund in 1938 [1] and re-discovered later by others [2,3]. Inspired by these findings, the first experiment was performed in the early 1990s by measuring the magnetization of an array of about ten million not connected micron-sized copper rings [4]. Other experiments also supported existence of persistent currents [5–10]. We should also recall recent definitive measurements of persistent currents in nanoscale gold and aluminium rings [11,12]. The team [11] has developed a new technique for detecting persistent currents that allows to measure the persistent current over a wide range of temperatures, ring sizes, and magnetic fields. They have used nanoscale

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cantilevers, an entirely novel approach to indirectly measure the current through changes in the magnetic force it produces as it flows through the ring. The second team [12] has studied thirty three individual rings, in which they have employed a scanning technique (a SQUID microscope). The rings are very small, each only between one and two micrometers in diameter and 140 nanometers thick. They are made of high-purity gold. Each was scanned individually, unlike past experiments on persistent currents conducted by other groups. In total they were scanned approximately 10 million times. Both works mark the first time that the theory has been *experimentally proven* to a high degree.

In our previous papers [13–15] we have proposed a two-fluid model for the dynamics of the magnetic flux that passes through a mesoscopic ring. It is described in terms of an ordinary differential equation with an additional random force. It is analogous to the well known model of a capacitively and a resistively shunted Josephson junction [16]. The classical part consists of 'normal' electrons carrying dissipative current. The quantum part is formed by those electrons which maintain their phase coherence around the circumference of the ring (it is a counterpart of the Cooper pairs of the electrons in the superconducting systems). The effective dynamics is then determined by a classical Langevin equation [17,18] with a Johnson noise describing classical thermal equilibrium fluctuations. For low temperatures, quantum nature of thermal fluctuations should be taken into account. To this aim we apply the approach based on the so called quantum Smoluchowski equation as introduced in Ref. [19] and in other versions in the following Refs. [20–27].

The paper is organized as follows. First, in the Sec. 2, we present a model of capacitively and resistively shunted Josephson junction in order to demonstrate readers the analogy between both models. Next, in the Sec. 3, we briefly present our model for the flux dynamics in the normal metal rings. In the Sec. 4, we define the quantum Smoluchowski regime following by the Sec. 5, presenting the dimensionless variables and parameters. In the Sec. 6, we study the current characteristics in the stationary states for both classical and quantum Smoluchowski domain. We end this work with the summary and conclusions.

2 Superconducting ring

For clarity of modeling the current characteristics in non-superconducting rings, we present the well-known approach to describe a quasi-classical regime of superconducting rings. To this aim, let us consider a superconducting loop (ring, cylinder, torus) interrupted by a Josephson junction. This element is a basic unit of various SQUID devices. The phase difference ψ of the Cooper pair wave function across the junction is related to the magnetic flux ϕ threading the ring via the relation [16]

$$\psi = 2\pi(n - \phi/\phi_0), \tag{1}$$

where $2\pi n$ is the phase change per cycle around the ring and $\phi_0 = h/2e$ is the flux quantum. When the external magnetic field is applied, the total flux is

$$\phi = \phi_e + LI, \tag{2}$$

where ϕ_e is the flux generated by an applied external magnetic field, L is the self-inductance of the ring and I is the total current flowing in the ring. We model the Josephson element in terms of the resistively and capacitively shunted junction for which the current consists of three components [28], namely,

$$I = I_C + I_R + I_J = \frac{\phi - \phi_e}{L}, \tag{3}$$

where I_C is a displacement current accompanied with the junction capacitance C, I_R is a normal (Ohmic) current characterized by the normal state resistance R and I_J is the Josephson supercurrent. In the right-hand side, the relation (2) has beed used.

Combining Eqs. (1)–(3) with the second Josephson relation $d\psi/dt = 2eU/\hbar$, where U is the voltage drop across the junction, we get the Langevin-type equation in the form [16]

$$C\frac{d^2\phi}{dt^2} + \frac{1}{R}\frac{d\phi}{dt} + I_0 \sin\phi = -\frac{\phi - \phi_e}{L} + \sqrt{\frac{2k_B T}{R}}\Gamma(t). \tag{4}$$

This equation has a mechanical interpretation: it can describe the 'position' $\phi(t)$ of the Brownian particle moving in the washboard potential

$$W(\phi) = \frac{(\phi - \phi_e)^2}{2L} - I_0 \cos \phi. \tag{5}$$

The first term originates from the external bias and the self–inductive interaction of the magnetic flux whereas the second term is the supercurrent modified by the quantum flux. The ubiquitous thermal equilibrium noise $\Gamma(t)$ consists of Johnson noise associated with the resistance R. The parameter k_B denotes the Boltzmann constant and T is temperature of the system. The Johnson noise is modeled by δ-correlated Gaussian white noise of zero mean, $\langle \Gamma(t) \rangle = 0$, and unit intensity, i.e., $\langle \Gamma(t)\Gamma(u) \rangle = \delta(t - u)$.

3 Normal metal ring

Now, let us consider a non-superconducting ring. When the external magnetic field is applied, the actual flux is given by

$$\phi = \phi_e + LI, \tag{6}$$

where ϕ_e is the flux generated by an applied external magnetic field, L is the self-inductance of the ring and I is the total current flowing in the ring. At zero temperature, the ring displays persistent and non-dissipative currents I_P run by phase-coherent electrons. It is analogue of the Josephson supercurrent I_J. At non-zero temperature, a part of electrons becomes 'normal' (non-coherent) and the amplitude of the persistent current decreases. Moreover, resistance of the ring and thermal fluctuations should be taken into account. Therefore for temperatures $T > 0$, the total current consists of three parts, namely,

$$I = I_C + I_R + I_P = \frac{\phi - \phi_e}{L}. \tag{7}$$

What we need is the expression for the persistent current I_P. It is s function of the magnetic flux ϕ and depends on the parity of the number of coherent electrons. Let p denotes the probability of an even number of coherent electrons and $1 - p$ is the probability of an odd number of coherent electrons. Then the persistent current can be expressed in the form [29]

$$I_P = I_P(\phi) = p\, I_E(\phi) + (1 - p)\, I_O(\phi), \tag{8}$$

where

$$I_E(\phi) = I_O(\phi + \phi_0/2) = I_0 \sum_{n=1}^{\infty} A_n(T/T^*) \cos(nk_F l) \sin(2n\pi\phi/\phi_0), \tag{9}$$

where I_0 is the maximal current at zero temperature. The temperature dependent amplitudes are determined by the relation [29]

$$A_n(T/T^*) = \frac{4T}{\pi T^*} \frac{\exp(-nT/T^*)}{1 - \exp(-2nT/T^*)}, \tag{10}$$

where the characteristic temperature T^* is proportional to the energy gap Δ_F at the Fermi surface, k_F is the Fermi momentum and l is the circumference of the ring. If the number N of electrons is fixed then $k_F = \pi N/l$ and the persistent current takes the form

$$I_P(\phi) = I_0 \sum_{n=1}^{\infty} A_n(T/T^*) \sin(2n\pi\phi/\phi_0)[p + (-1)^n(1 - p)]. \tag{11}$$

As a result, from Eq. (7) we obtain the equation of motion in the form [13–15]

$$C\frac{d^2\phi}{dt^2} + \frac{1}{R}\frac{d\phi}{dt} = -\frac{1}{L}(\phi - \phi_e) + I_P(\phi) + \sqrt{\frac{2k_BT}{R}}\,\Gamma(t)$$
$$= -\frac{dV(\phi)}{d\phi} + \sqrt{\frac{2k_BT}{R}}\,\Gamma(t), \qquad (12)$$

where the "potential" $V(\phi)$ reads

$$V(\phi) = \frac{1}{2L}(\phi - \phi_e)^2 + \phi_0 I_0 \sum_{n=1}^{\infty} \frac{A_n(T/T^*)}{2n\pi} \cos\left(2n\pi\frac{\phi}{\phi_0}\right)\,[p + (-1)^n(1-p)]. \qquad (13)$$

In the above Langevin equation, C and L are, respectively, the capacitance and inductance of the ring. It was shown [30], that the energy associated with long-wavelength and low-energy charge fluctuations is determined by classical charging energies and therefore the ring behaves as it were a classical capacitor. The flux dependence of these energies yields the contribution to the persistent current [31]. The capacitance becomes essential if the ring accommodates a stationary impurity or a quantum dot [32,33]. Moreover, in the mesoscopic domain the standard description of a capacitor in terms of the geometric capacitance (that relates the charge on the plate to the voltage across the capacitor) gives way to a more complex notion of capacitance which depends on the properties of conductors [34].

Note that Eqs. (4) and (12) have a similar structure. The difference is not only in the form of the potential but also in temperature dependence of the potential in the case the normal metal rings.

4 Quantum Smoluchowski regime

In both models (for superconducting and non-superconducting rings), thermal equilibrium fluctuations are modeled as classical fluctuations of zero correlation time. When temperature is lowered, quantum nature of fluctuations starts to play a role, fluctuations become correlated and leading quantum corrections should be taken into account. It is not a simple task and a general method how to incorporate quantum corrections in a case described by Eq. (12) is not known. However, in the quantum Smoluchowki regimes [19], where the charging effects (related to the capacitance C) can be neglected, the system can be described by the "overdamped" Langevin equation – the so-named quantum Smoluchowski equation [19,20]. For a Brownian particle it corresponds to neglecting inertial effects related to the mass of a particle. The quantum Smoluchowski equation has the same structure as a classical Smoluchowski equation, in which the diffusion coefficient $D_0 = k_BT/R$ is modified due to quantum effects like tunnelling, quantum reflections and purely quantum fluctuations. In terms of the Langevin equation (12), it assumes the form

$$\frac{1}{R}\frac{d\phi}{dt} = -\frac{dV(\phi)}{d\phi} + \sqrt{2D_\Lambda(\phi)}\,\Gamma(t). \qquad (14)$$

This equation has to be interpreted in the Ito sense [35]. The modified diffusion coefficient $D_\Lambda(\phi)$ takes the form [20,21]

$$D_\Lambda(\phi) = \frac{D_0}{1 - \Lambda V''(\phi)/k_BT}, \qquad (15)$$

where the prime denotes differentiation with respect to the argument of the function. The quantum correction is characterized by the parameter Λ. It measures a deviation of the quantal flux fluctuations from its classical counterpart, namely,

$$\Lambda = \langle\phi^2\rangle_Q - \langle\phi^2\rangle_C, \qquad (16)$$

where $\langle \cdots \rangle$ denotes equilibrium average, the subscripts Q and C refer to quantal and classical cases, respectively. Let us determine the range of applicability of the quantum Smoluchowski regime. The classical Smoluchowski limit corresponds to the case when charging effects can be neglected. Formally, one can put $C = 0$ in the inertial term of Eq. (12), which is related to the strong damping limit of the Brownian particle. In the case studied here it means that [19]

$$\omega_0 CR \ll 1, \tag{17}$$

where the frequency ω_0 is a typical frequency of the bare system and its inverse corresponds to a characteristic time of the system. In such a case, Eq. (16) takes the form [13]

$$\Lambda = \frac{\hbar R}{\pi} \left[\gamma + \Psi \left(1 + \frac{\hbar}{2\pi CR k_B T} \right) \right], \tag{18}$$

where the psi function $\Psi(z)$ is the logarithmic derivative of the Gamma function and $\gamma \simeq 0.5772$ is the Euler constant.

The separation of time scales, on which the flux relaxes and the conjugate observable (a charge) is already equilibrated, requires the second condition, namely,

$$\omega_0 CR \ll k_B T / \hbar \omega_0. \tag{19}$$

In the deep quantum regime, i.e. when

$$k_B T \ll \frac{\hbar}{2\pi CR}, \tag{20}$$

the correction parameter (18) simplifies to the form

$$\Lambda = \frac{\hbar R}{\pi} \left[\gamma + \ln \left(\frac{\hbar}{2\pi CR k_B T} \right) \right]. \tag{21}$$

In order to identify precisely the quantum Smoluchowski regime, we have to determine a typical frequency ω_0 or the corresponding characteristic time $\tau_0 \propto 1/\omega_0$. There are many characteristic times in the system, which can be explicitly extracted from the evolution equation (12), e.g. CR, $\hbar/k_B T$, $\phi_0/(RI_0)$. The characteristic time $\tau_0 = L/R$ is the inductive time of the ring and for a typical mesoscopic ring, L/R is in the picosecond range. Therefore, in the quantum Smoluchowski regime, all the above inequalities (17), (19) and (20) should be fulfilled for $\omega_0 = 2\pi/\tau_0$. Because the diffusion coefficient cannot be negative, the parameter Λ should be chosen small enough to satisfy the condition $D_\Lambda(\phi) \geq 0$ for all values of ϕ.

The "overdamped" Langevin equation (14) describes a classical Markov stochastic process and its probability density $P(\phi, t)$ obeys the Fokker-Planck equation [35], namely,

$$\frac{1}{R} \frac{\partial}{\partial t} P(\phi, t) = \frac{\partial}{\partial \phi} \left[\frac{dV(\phi)}{d\phi} P(\phi, t) \right] + \frac{\partial^2}{\partial \phi^2} \left[D_\Lambda(\phi) P(\phi, t) \right]. \tag{22}$$

We wish to analyze an averaged stationary current $\langle I \rangle$ flowing in the ring which can be obtained from Eq. (7):

$$\langle I \rangle = \frac{1}{L} \left[\langle \phi \rangle - \phi_e \right], \tag{23}$$

where the averaged stationary magnetic flux $\langle \phi \rangle$ is calculated from the equation

$$\langle \phi \rangle = \int_{-\infty}^{\infty} \phi \, P(\phi) d\phi, \quad P(\phi) = \lim_{t \to \infty} P(\phi, t), \tag{24}$$

where $P(\phi)$ is a stationary probability density,

$$P(\phi) = C_0 D_\Lambda^{-1}(\phi) \exp \left[-U(\phi) \right], \tag{25}$$

where C_0 is the normalization constant and the generalized thermodynamic potential $U(\phi)$ reads

$$U(\phi) = \int \frac{dV(\phi)}{d\phi} D_\Lambda^{-1}(\phi)\, d\phi. \tag{26}$$

Because the potential $V(\phi)$ depends on the external flux ϕ_e, the averaged stationary current (23) is a non-linear function of ϕ_e. Equations (23)–(26) form a closed set of equations from which the current characteristics $\langle I \rangle = f(\phi_e)$ as a certain function f of the external magnetic flux ϕ_e can be obtained. It is an analogous of the current-voltage characteristics for electrical circuits.

5 Dimensionless variables and parameters

To analyze the current-flux characteristics in the stationary state, we first introduce dimensionless variables and parameters. The rescaled flux $x = \phi/\phi_0$. Then Eq. (23) can be rewritten in the dimensionless form

$$i = \langle x \rangle - x_e, \quad i = \langle I \rangle L/\phi_0, \quad x_e = \phi_e/\phi_0, \tag{27}$$

where $i, \langle x \rangle$ and x_e are dimensionless averaged current, averaged flux and external flux, respectively.

The stationary probability density $p(x)$ takes the form

$$p(x) = N_0 D^{-1}(x) \exp\left[-\Psi_\lambda(x)\right], \tag{28}$$

where N_0 is the normalization constant and the generalized thermodynamic potential reads

$$\Psi_\lambda(x) = \int \frac{dV(x)}{dx} D_\lambda^{-1}(x)\, dx. \tag{29}$$

The rescaled potential reads

$$V(x) = \frac{1}{2}(x - x_e)^2 + B(x), \tag{30}$$

where

$$B(x) = \alpha \sum_{n=1}^{\infty} \frac{A_n(T_0)}{2n\pi} \cos(2n\pi x)[p + (-1)^n(1-p)] \tag{31}$$

with the dimensionless temperature $T_0 = T/T^*$ and $\alpha = LI_0/\phi_0$. The rescaled modified diffusion function $D_\lambda(x)$ assumes the form

$$D_\lambda(x) = \frac{\beta^{-1}}{1 - \lambda\beta V''(x)} \tag{32}$$

with $\beta^{-1} = k_B T/2E_m = k_0 T_0$, the elementary magnetic flux energy $E_m = \phi_0^2/2L$ and $k_0 = k_B T^*/2E_m$ is the ratio of two characteristic energies. The dimensionless quantum correction parameter

$$\lambda = \lambda_0\left[\gamma + \Psi\left(1 + \frac{\epsilon}{T_0}\right)\right], \quad \lambda_0 = \frac{\hbar R}{\pi\phi_0^2}, \quad \epsilon = \frac{\hbar/2\pi CR}{k_B T^*}. \tag{33}$$

Remember that the Smoluchowski regime corresponds to the strong coupling limit. For classical systems, i.e. when the quantum correction parameter $\lambda = 0$, the stationary state is a Gibbs state, i.e. $p(x) \propto \exp[-\beta V(x)]$. For quantum systems, due to the x-dependence of the modified diffusion coefficient $D_\lambda(x)$, the stationary state (28) is not a Gibbs state. However, it is a thermal equilibrium state.
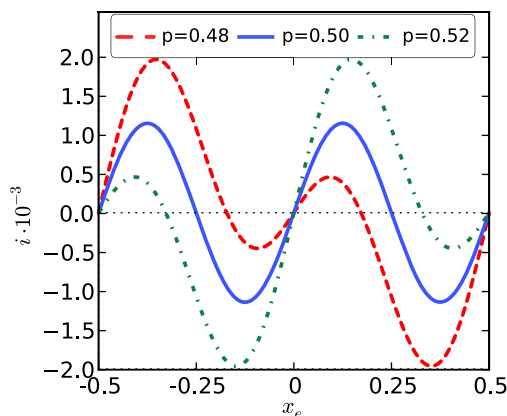
Fig. 1. (Color online) The dimensionless current i *vs* the external magnetic flux x_e for three values of the probability p of an even number of coherent electrons in the ring. Please note that one can obtain the persistent current for slightly asymmetric case $p = 0.48$ just by shifting the presented characteristic for $p = 0.52$ by $x_e = 0.5$. It is the case of "classical" thermal fluctuations, i.e. when the quantum correction parameter $\lambda_0 = 0$. Other dimensionless parameters are: $\alpha = 0.1$, $T_0 = 0.5$, $k_0 = 0.08$.

6 Current − flux characteristics in stationary states

A persistent current is a periodic function of the magnetic flux with a period given by a single-electron flux unit $\phi_0 = h/e$. To what extent the current is highly sensitive to a variety of subtle effects such as an electron–electron interaction, defects, disorder, coupling to an environment and other degrees of freedom, it is still a topic of controversy and persistent discussion. Fortunately, novel techniques developed recently such as the microtorsional magnetometer [11] and scanning SQUID [12] allow to measure the persistent current in metal rings over a wide range of magnetic fields, temperatures and ring sizes. Like many mesoscopic effects, the persistent current in real systems depends on the particular realization of disorder and thus varies between nominally identical rings, cf. the term $\cos(nk_F l)$ in Eq. (9) which in practice is random. In Fig. 1 we depict the well-known dependence of current upon the external magnetic flux x_e for three selected values of the probability p of an even number of coherent electrons in the ring. The choice of values of p is arbitrary but shapes of the current are similar to those observed in experiments.

We now focus on the influence of quantum thermal fluctuations on persistent currents. The deviation from the "classicality" is measured by the dimensionless parameter λ which depends on λ_0 (the material constant) and temperature, see Eq. (33). It is instructive to compare basic quantities characterizing the system. In Fig. 2 we show the generalized thermodynamic potential $\Psi_\lambda(x)$, the modified diffusion function $D_\lambda(x)$ and the stationary probability density $p(x)$ for two values of the quantum correction strength λ_0. Three panels (a), (b) and (c) are presented for the case $x_e = 0$ (the vanishing external flux). The case $\lambda_0 = 0$ corresponds to classical thermal fluctuations and $\Psi_0(x)$ is a bare potential $V(x)/k_0 T_0$. We note that the generalized thermodynamic potential $\Psi_\lambda(x)$ for various λ changes only slightly. On the contrary, the state-dependent diffusion function is a periodic function of the magnetic flux and possess maxima and minima. It is a radical difference to the classical case $\lambda = 0$ for which $D_0(x) = D_0 = k_0 T_0$ is a constant function (thin solid blue line and thin dashed red line in panel (c)). The maxima of $D_\lambda(x)$ can be interpreted as a higher effective local temperature. They are located at $x_e = 1/4 \mod(1/2)$. The impact of quantum corrections on the stationary probability density $p(x)$ seems to be rather insignificant. One can observe a small deformation around the peak of the density: for lower temperature and non-zero λ_0 the peak becomes slightly higher and narrower and the tails do not diverge in the quantum case as fast as in the classical one.

Finally, we analyze the influence of quantum thermal fluctuations on the current-flux characteristics $i = i(x_e)$. It is illustrated in panel (d) of Fig. 2 and in Fig. 3. Two solid blue lines in panel (d) of Fig. 2 are qualitatively similar to the experimental curve shown in figure
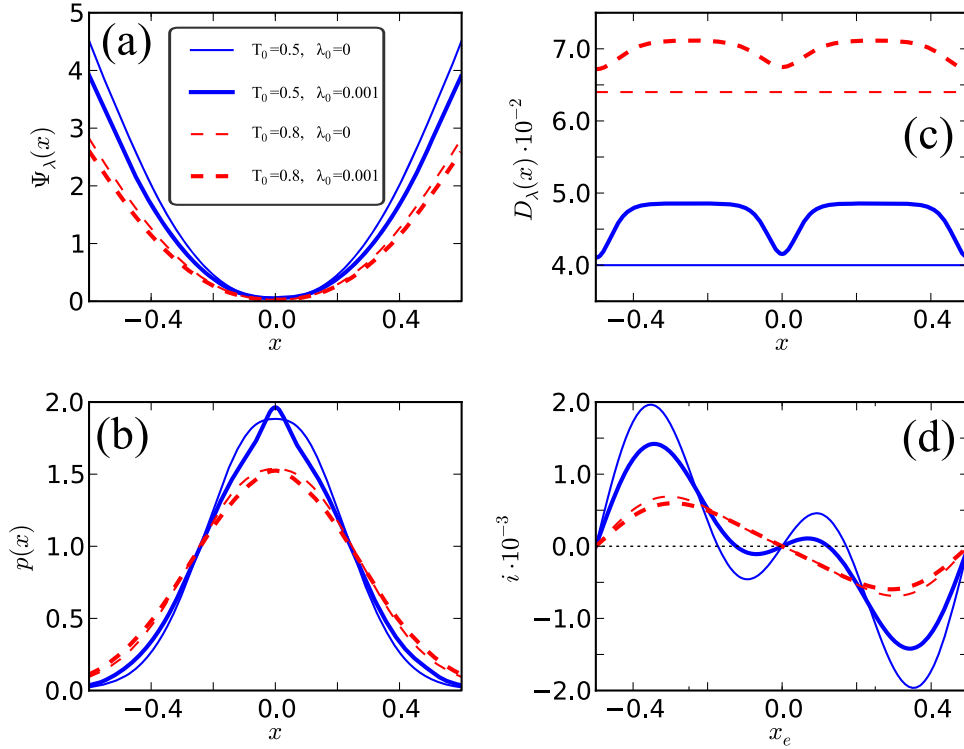
Fig. 2. (Color online) Four characteristics of the normal metallic ring in the absence (panels a–c) and presence (panel d) of the external flux are demonstrated for two temperatures: $T_0 = 0.5, 0.8$ and for two different quantum fluctuations strengths: $\lambda_0 = 0, 0.001$. The key for reading this plot is as the following: blue solid lines denote curves corresponding to the lower temperature $T_0 = 0.5$ and thickness mimic the quantum fluctuation strength: the thin line corresponds to $\lambda_0 = 0$ and thick line to $\lambda_0 = 0.001$. The red dashed lines correspond to the higher temperature $T_0 = 0.8$ with the same rule for thickness as in the previous case. In panel (a) we present the thermodynamic potential defined in (29). In the classical case ($\lambda_0 = 0$) it reduces to $V(x)/k_0T_0$. One can notice only small deviations from the classical case when the rescaled parameter λ_0 is increased to 0.001. In the panel (b) we illustrate the corresponding stationary probability density function $p(x)$. The bell–shaped curve in the classical limit is slightly deformed for non-zero λ_0. The peaks tend to be narrower and reach higher values and the tails decay slower as we increase λ_0. This effect is stimulated collectively by the thermodynamic potential and the effective diffusion presented on panels (a) and (c), respectively. Please note that for the classical case flux dependence of $D_\lambda(x)$ disappears and constantly equals $k_0T_0 = 0.04$ and 0.064 for $T_0 = 0.5$ and 0.8, respectively. The most significant influence of the temperature is found in panel (d), where the current–flux characteristics are displayed. For small external load, around flux $x_e = 0$, the system responses in a completely different way for the two selected temperatures. For $T_0 = 0.5$ in both classical and quantum cases the persistent current is paramagnetic. If we, however, increase the temperature to $T_0 = 0.8$, the situation changes drastically and the susceptibility for this higher temperature becomes diamagnetic. Other rescaled parameters are set as the following $\alpha = 0.1$, $k_0 = 0.08$, $p = 0.48$, $\epsilon = 100$.

S6(A) in the Supporting Online Material [36] of the paper [11]. We observe that in all cases of quantum thermal fluctuations the amplitude of persistent currents is reduced in comparison to classical thermal fluctuations case ($\lambda_0 = 0$) both in the paramagnetic regime ($T_0 = 0.5$) and diamagnetic regime ($T_0 = 0.8$). The parameter regime depicted in Fig. 3 is much more interesting. For $T_0 = 0.6$, in the "classical" case, the persistent current is *paramagnetic*, i.e. $i = \eta x_e$ with the positive slope $\eta > 0$ in the vicinity of $x_e = 0$. It is a linear response regime where the transport coefficient (susceptibility) $\eta = \lim_{x_e \to 0}[i(x_e)/x_e]$. If temperature is a litlle bit higher ($T_0 = 0.642$), the susceptibility $\eta = 0$ in the classical case. If quantum corrections are taken into account, the susceptibility $\eta < 0$, the slope of the $i - x_e$ curve is negative and the
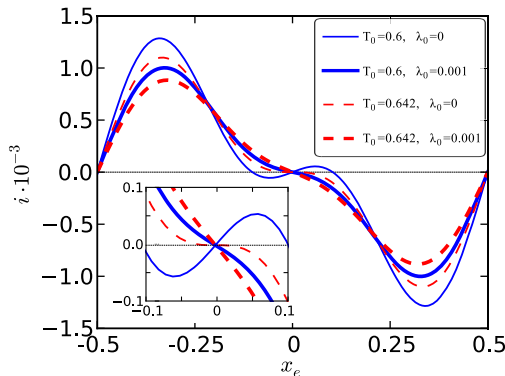
Fig. 3. (Color online) The dimensionless current i *vs* the external magnetic flux x_e for two values of the temperature $T_0 = 0.6$, 0.642 and two values of the parameter $\lambda_0 = 0$ (classical) and 0.001. Again, the key for reading this plot is as for the previous figure: blue solid lines denote curves corresponding to the lower temperature $T_0 = 0.6$ and thickness mimic the quantum fluctuation strength: the thin line corresponds to $\lambda_0 = 0$ and thick line to $\lambda_0 = 0.001$. The red dashed lines correspond to the higher temperature $T_0 = 0.642$ with the same rule for thickness as in the previous case. The most significant influence of the 'quantum parameter' is found for the rescaled temperature $T_0 = 0.6$, where the current i changes its behavior from paramagnetic to diamagnetic one just by adjusting λ_0 from 0 to 0.001. Moreover, for the presented set of the system parameters we can find characteristic cross-temperature at $T_0 = 0.642$ where the magnetic susceptibility is zero in the classical case and diamagnetic in quantum (see red dashed line for details). Rescaled parameters are set as the following $\alpha = 0.1$, $k_0 = 0.08$, $p = 0.48$, $\epsilon = 100$.

current becomes diamagnetic. The most interesting observation is that the persistent current can change its character from the paramagnetic to diamagnetic phase and the sign of the low-field magnetic response depends on the level of the quantum corrections. Our detailed numerical analysis shows that the sign of magnetic susceptibility can easily be affected by system para-meters and therefore is not robust against small perturbations. This is what has been observed in many experiments regarding the paramagnetic or/and diamagnetic persistent currents. The best illustration of what we state here is the response of 15 nominally identical ring presented in Fig. 2 in Ref. [12]: e.g. for the ring 1 the current is paramagnetic while for the ring 2 it is diamagnetic.

7 Conclusions

In many cases and for various systems at the "intermediate" temperatures, the semi-classical theory is insufficient and the quantum corrections should be involved. It has been shown in the literature that in the strong friction limit, the quantum effects are not restricted to low temperatures only and should be considered for the higher temperatures as well. This is so because the quantum fluctuations, even if reduced for one variable, are enlarged for the conju-gate variable. The dynamics as well as the stationary states in this regime can be modeled by the quantum Smoluchowski equation. In other words, the quantum non-Markovian stochastic process is approximated by the classical Markovian process with the modified, state-dependent diffusion function.

The role of the quantum corrections on the current-flux characteristics is addressed in this work. A general conclusion is that the quantum thermal fluctuations reduce the amplitude of the persistent currents: the current amplitude is always smaller than the corresponding "classical" one. In the quantum case, the diffusion constant becomes a periodic function of the magnetic flux. Maxima and minima of the diffusion function can be interpreted in terms of the higher and lower local temperature. There are parameters regimes where the system response changes the character from the paramagnetic to diamagnetic, when the quantum effects of thermal

fluctuations increase. It would be interesting to extend the current study by including the time-dependent drivings modeled by the time-periodic magnetic fields. One could expect novel transport phenomena like a negative susceptibility which for Brownian particles corresponds to negative mobility [37] or negative conductances [28].

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