Mesoscopic rings: multi-states induced by quantum thermal fluctuations

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When temperature of a conducting mesoscopic ring is decreased, the magnetic flux states can change drastically, e.g. from bistable to multistable states. Possibility of the application of the multistability of the magnetic flux is emphasised in the information-theoretical context. The proposal of the quantum kinetics description is qualitatively discussed.

Key words: Persistent currents, quantum Smoluchowski limit, qutrit

Mesoscopic multiply connected samples such as conducting rings, cylinders or tori exhibit quantum size phenomena originated from Aharonov-Bohm effects. One of the most spectacular phenomena is a persistent current [1] which is a direct manifestation of quantum phase coherence of the current carriers over a mesoscopic length scale. At non-zero temperature $T > 0$, the phase coherence of carriers is weakened and the samples exhibit the dissipative Ohmic contribution to the total current. There is a regime of temperatures in which both phase-coherent and dissipative Ohmic currents coexist. Such currents generate a magnetic flux. Its properties can be studied via modeling based on kinetic equations like Langevin equations [2]. The long-time, steady states of the magnetic flux are characterized by properties of the stationary probability density obtained from the corresponding Langevin equation. E.g., if the stationary probability density is bimodal (it has two maxima) then it reflects the possibility of occurrence of non-zero magnetic fluxes and in consequence the currents. One can notice an obvious analogy to the Josephson-based flux qubits [3]. There are serious advantages for using non-superconducting samples like rings as a building blocks for qubit storage because due to a small size they seem to be more stable with respect to decoherence and dephasing [4]. For higher temperatures, when the Ohmic current dominates over the persistent current, thermal fluctuations can be modeled as classical white noise with state-independent intensity determined by the fluctuation-dissipation relation. For moderate and low temperatures, when persistent current starts to dominate over the Ohmic current, this modeling is insufficient and leading quantum corrections to thermal fluctuations should be incorporated. This regime can be described by the so-called quantum Smoluchowski equation [5] in which quantum corrections modify both diffusion and drift coefficients [6, 7]. The resulting steady states of the system can change drastically. E.g. the bistable states evolve to the tristable states when

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the temperature decreases. It opens the door for using non-superconducting rings in qutrit architecture with potential applications, e.g. in the quantum cryptography [8].

The magnetic flux ϕ threaded a mesoscopic system of the ring topology, operating in the quantum Smoluchowski regime, can be described as a noisy overdamped *RLC* circuit driven by the non-linear 'force' created by the persistent current [9]. In this regime, the ϕ-dynamics is described by a Langevin equation in the dimensionless form [9, 10]

$$\frac{dx}{ds} = -\frac{dV_{eff}(x)}{dx} + \sqrt{2D(x)}\xi(s) \qquad (1)$$

where the rescaled flux $x=\phi/\phi_0$ and rescaled time $s=t/\tau_0$. The flux quantum $\phi_0=h/e$ is the ratio of the Planck constant h and the charge of the electron. The characteristic time $\tau_0=L/R$, where L is the self-inductance and R is the resistance of the ring, respectively.

The effective 'potential' $V_{eff}(x)$ takes the form

$$V_{eff}(x) = V(x) + \frac{1}{2}\lambda B''(x), \ \ V(x) = \frac{1}{2}(x - x_e)^2 + B(x), \qquad (2)$$

where

$$B(x) = \alpha \sum_{n=1}^{\infty} \frac{A_n(T_0)}{2n\pi} \{ p\cos(2n\pi x) + (1-p)\cos[2n\pi(x+1/2)] \} \qquad (3)$$

The dimensionless amplitude of the coherent current $\alpha=LI_0/\phi_0$ with I_0 being the maximal amplitude of the coherent part of the current at zero temperature. The temperature is rescaled with respect to T^* which correspondes to the size induced energy gap at the Fermi surface, i.e. $T_0 = T/T^*$. The detailed form of $A_n(T_0)$ given e.g. in [2] is not reproduced here.

The modified diffusion coefficient $D(x)$ with the Maxwell demon successfully exorcised [6] ensures the equilibrium character of quantum thermal fluctuations:

$$D(x) = \beta^{-1} \{ 1 - \lambda\beta[1 + B''(x)] \}^{-1}. \qquad (4)$$

The dimensionless parameters are: $1/\beta = k_B T/2E_m = k_0 T_0$, where the elementary magnetic flux energy $E_m = \phi_0^2/2L$ and $k_0 = k_B T^*/2E_m$ is the ratio of two characteristic energies. The δ-correlated stochastic force $\xi(s)$ is zero-mean Gaussian white noise of unit intensity. The prominent parameter [10]

$$\lambda = \lambda_0\left[\gamma + \ln\left(1 + \frac{\varepsilon}{T_0}\right)\right], \quad \lambda_0 = \frac{R}{R_0}, \quad \varepsilon = \frac{E_0}{k_B T^*} \tag{5}$$

characterizes quantum fluctuations in the magnetic flux space. Here, γ is the Euler constant, the resistance quantum $R_0 = \pi\phi_0^2/\hbar$ is expressed by the fundamental constants and the energy $E_0 = \hbar/2\pi CR$ characterizes the CR-properties of the ring.

The stationary probability density $P(x)$ can be obtained from the Fokker-Planck equation corresponding to the Langevin equation (1). It takes the form

$$P(x) \propto D^{-1}(x)\exp[-\Phi(x)], \tag{6}$$

where the generalized thermodynamic potential

$$\Phi(x) = \int\frac{V'_{eff}(x)}{D(x)}dx. \tag{7}$$

The stationary state (6) is a thermal equilibrium state. However, due to both the x-dependence of the modified diffusion coefficient $D(x)$ and the temperature dependence of the effective potential $V_{eff}(x)$, it is not a Gibbs state (which is characterized by the distribution $P_G(x) \propto \exp[-\beta V(x)]$).

There are parameters regimes when in the formal absence of the quantum fluctuations, i.e. in the $\lambda \to 0$ limit, the system is bistable reflecting the possibility of occurrence of self-sustaining fluxes. There is one-to-one correspondence between probability density and the exponentiated deterministic potential $V(x)$. Passing into the quantum Smoluchowski limit characterized by non-zero λ with the effective potentials this correspondence is lost: the steady state at sufficiently low temperatures becomes mulitistable with three central peaks dominated the other. The result is plotted in Fig.1.

The peaks in the probability distribution correspond to stationary states of the system. The natural measure of stability of the formally meta-stable state $x = x_m$ is its lifetime $\tau(x_m)$ calculated as an expected time of the 'particle' starting at $x = x_m$ to leave the well of the effective potential [10]. The plot presenting numerical results is given in Fig.2. The lifetime of two fundamental peaks at $x_m = 0, \pm 1/2$ exceeds the lifetime of $x_m = 1$ by almost one order of magnitude.

Bistable systems are natural candidates for qubits. The famous examples are Josephson-junction based devices which can be generally divided into two classes: charge and flux qubits [3]. It seems that a qubit can also be based on non-superconducting materials [4]. Because within tailored parameter regimes in the quantum Smoluchowski domain there are symmetric peaks in the multistable state, such a system is a good candidate for a *qutrit*. The problem of the qutrit implementation is of a central importance for quantum cryptography [8].

The discusion below is limited to purely qualitative aspects of the qutrit kinetics. We assume for simplicity that there are only three significant (in the statistical sense) peaks in the probability distribution, as e.g. in Fig. 2. Replicating Feynman's discussion of the ammonia molecule [11] one can propose the 'Hamiltonian' of the system as a 3×3 real symmetric matrix with diagonal elements proportional to the energy of the system calculated at magnetic flux extremal value.

An attempt of direct quantization of the classical system which is essentially dissipative (overdamped) suffers serious difficulties. The choice of the quantum Hamiltonian is not obvious. Moreover, this quantum dissipative system is strongly damped. As there is no a generic master equation describing reduced dynamics of such a system [12], a phenomenological modeling shall be used. In the regime where only three fundamental peaks are relevant, the presented system is a perfect example of the celebrated *V*-system [13], i.e. it consists of three levels: $|x = 0\rangle$ and $|x = \pm 1/2\rangle$. The rates of spontaneous transitions can be directly related to the *classical* inter-peaks mean passage times $\tau(0 \to \pm 1/2)$ [10]. The transition $|x = \mp 1/2\rangle \to |x = \pm 1/2\rangle$ can clearly be neglected. It is also natural to control the system either by incoherent or coherent driving.

In the following the simplest case of the dynamics of the incoherently driven system is introduced. We limit our consideration to the dynamics populations i.e. we consider only diagonal elements of the full density matrix of the *V*-system. It reads [13]:

$$\frac{d\rho_{11}}{ds} = -\left(S_1 + I_1\right)\rho_{11} + I_1\rho_{00} \tag{8}$$

$$\frac{d\rho_{22}}{ds} = -\left(S_2 + I_2\right)\rho_{22} + I_2\rho_{00} \tag{9}$$

$$\frac{d\rho_{00}}{ds} = -\left(I_1 + I_2\right)\rho_{00} + \left(S_1 + I_1\right)\rho_{11} + \left(S_2 + I_2\right)\rho_{22} \tag{10}$$

with the normalization condition $\sum_i \rho_{ii} = 1$. The parameters I_i and S_i are the rates of induced and spontaneous transitions respectively. The stationary state [13]:

$$\rho_{11}(s \to \infty) = \frac{I_1(S_2 + I_2)}{S_1(S_2 + 2I_2) + I_2(2S_2 + 3I_2)} \tag{11}$$

$$\rho_{22}(s \to \infty) = \frac{I_2(S_1 + I_1)}{S_1(S_2 + 2I_2) + I_2(2S_2 + 3I_2)} \tag{12}$$

$$\rho_{00}(s \to \infty) = 1 - \sum_{i \geq 1} \rho_{ii}(s \to \infty) \tag{13}$$

Despite its simplicity the classical treatment described above allows for an effective control of populations via suitable external driving, e.g. to find 'equidistribution' conditions $\rho_{ii} = 1/3$, satisfied e.g. for $I_i \gg S_i$. The potential application for quantum computing also requires an effective way of controlling coherences. It can be done either phenomenologically via suitable driving inducing inter-peak transitions or by consistent calculations beyond the quantum Smoluchowski expansion resulting in the master equation with non-diagonal elements included. Since the system is mesoscopic we expect the driving to operate at *microwave* [14] rather than optical frequencies.

The approach including kinetics of coherences can be formulated in terms of the phenomenological quantum jump formalism [13]. This problem is postponed to further studies.

References

[1] F. HUND, *Ann. Phys*. (Leipzig) **32**, 102 (1938); I. O. KULIK, *JETP Lett.* **11**, 275 (1970); M. BÜTTIKER, Y. IMRY, R. LANDAUER, *Phys. Lett. A* **96**, 365 (1993).

[2] J. DAJKA, J. ŁUCZKA, M. SZOPA, E. ZIPPER, *Phys. Rev B* **67**, 073305 (2003); J. DAJKA, M. KOSTUR, J. ŁUCZKA, M. SZOPA AND E. ZIPPER *Acta Phys. Pol. B* **34** 3793 (2003).

[3] Y. MAKHLIN, G. SCHÖN, A. SCHNIRMAN, *Rev. Mod. Phys.* **73**, 357 (2001).

[4] E. ZIPPER, M. KURPAS, M. SZELĄG, J. DAJKA, M. SZOPA, *Phys. Rev B* **74**, 125462 (2006).

[5] J. ANKERHOLD, P. PECHUKAS, H. GRABERT, *Phys. Rev. Lett*. **87**, 086801 (2001).

[6] Ł. MACHURA, M. KOSTUR, P. HÄNGGI, P. TALKNER, J. ŁUCZKA *Phys. Rev. E* **70**, 031107 (2004); J. ŁUCZKA, R. RUDNICKI, P. HÄNGGI , *Physica A* **351**, 60 (2005).

[7] J. ANKERHOLD, *Europhys. Lett.* **67**, 280 (2004); J. ANKERHOLD, P. PECHUKAS, H. GRABERT *Chaos* **15**, 026106 (2005).

[8] G. MOLINA-TERRIZA, A. VAZIRI, R. URSIN, A. ZEILINGER, *Phys. Rev. Lett*. **94**, 040501 (2005).

[9] J. DAJKA, S. ROGOZIŃSKI Ł. MACHURA, J. ŁUCZKA, *Acta Physica Polonica B* **17** (2007) accepted, cond-mat/0611773.

[10] J. DAJKA, Ł. MACHURA, S. ROGOZIŃSKI, J. ŁUCZKA, *Phys. Rev. B* **76**, 045337 (2007)

[11] R. P. FEYNMAN, LECTURES ON PHYSICS, Addison-Wesley, Reading (1964), Vol 3.

[12] U. WEISS, QUANTUM DISSIPATIVE SYSTEMS, Second Edition, World Scientific, Singapore (1999).

[13] M.B. PLENIO, P.L. KNIGHT, *Rev. Mod. Phys.* **70**, 101 (1998).

[14] J. DAJKA, M. SZOPA, A. VOURDAS, E. ZIPPER, *Phys. Rev. B* **69**, 045305 (2005); J. DAJKA, E. ZIPPER, *Material Science Poland*, **24**, 769 (2006).

Figure captions

1. The quantum Smoluchowski regime is compared to its classical counterpart. The potential $V(x)$ and the modified potential $V_{eff}(x)$ in (2) are shown in the upper panel. In the inset, the modified diffusion function $D(x)$ defined in (4) is depicted. The lower panel shows the stationary probability density $P(x)$ in the classical Smoluchowski ($\lambda_0 = 0$) and quantum Smoluchowski ($\lambda_0 = 0.01$) regimes. Other parameters are set as following: $x_e = 0$, $T_0 = 0.5$, $\varepsilon = 2$, $\alpha = 2$, p=0.5, $k_0 = 1.0$ and $k_F l = 0.001$.

2. The lifetime of peaks (log-scale) vs. temperature T_0 for the following set of parameters: $\alpha = 0.1$, $\varepsilon = 10$, $k_0 = 0.5$, $\lambda_0 = 0.002$, and $k_F l = 0.001$