# Luke Davidson
# ME 5659
# HW1

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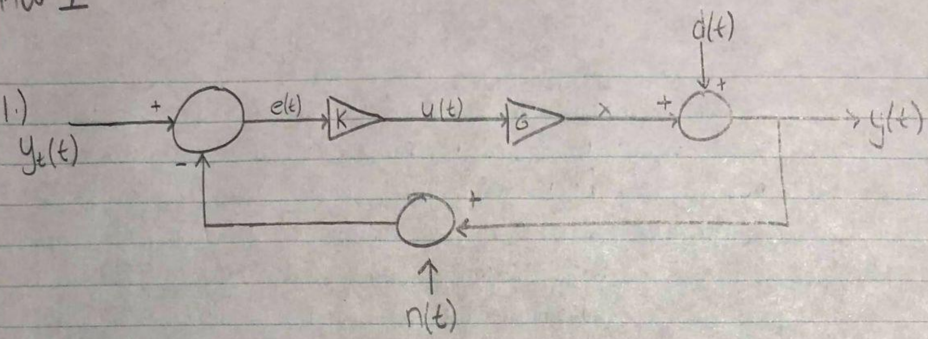
Luke Davidson
ME 5659
HW 1

1.)




a.) $n(t) = 0$ $\quad$ $y(t) = d(t) + x$

$y_t(t) = 0$

$$e(t) = y_t(t) - (n(t) + y(t))$$

$$u(t) = k(e(t))$$

$$x = u(t)(G)$$

$$y(t) = d(t) + KG\left[\cancel{y_t(t)}^{0} - (\cancel{n(t)}^{0} + y(t))\right]$$

$$y(t) = d(t) - KG(y(t))$$

$$y(t) = \frac{d(t)}{1+KG}$$

for $y(t) \to 0$, we would want large values of $K$ because it's in the denominator

b) $d(t) = 0$ $\quad$ $y(t) = \cancel{d(t)}^{0} + KG\left[\cancel{y_t(t)}^{0} - n(t) - y(t)\right]$

$y_t(t) = 0$

$$y(t) = KG(-n(t) - y(t))$$

$$y(t) = -KGn(t) - KGy(t)$$

$$y(t) = \frac{-KG\,n(t)}{1+KG}$$

we will now want small values of $K$ that will lead to $-KGn(t)$ to be close to 0.

c.) The disturbances can be limited by applying a controller to that aspect of the system, however noise is inevitable. Since noise is often associated with higher frequencies and theoretically averages out to zero, a higher emphasis should be placed on controlling the disturbance factor with an appropriate controller (PID, etc.)

2.) $J_e = (N^2 J_m + J)$

$$u = t \begin{bmatrix} & t & \\ & & \\ & & \end{bmatrix}^2$$

$\dot{\theta} = \omega$

$\dot{\omega} = \frac{NK_m}{J_e} i - \frac{1}{J_e} T_L$

$\dot{i} = -\frac{NK_m}{L}\omega - \frac{R}{L} i + v\left(\frac{1}{L}\right)$

$y_1 = \theta \qquad \dot{y}_1 = \dot{\theta}$

$y_2 = \omega \qquad \dot{y}_2 = \dot{\omega}$

$y_3 = i \qquad \dot{y}_3 = \dot{i}$

$$\left.\begin{aligned} \dot{y}_1 &= y_2 \\ \dot{y}_2 &= \frac{NK_m}{J_e} y_3 - \frac{T_L}{J_e} \\ \dot{y}_3 &= \frac{-NK_m}{L} y_2 - \frac{R}{L} y_3 + \frac{v}{L} \end{aligned}\right\} \quad \begin{aligned} \dot{y}_1 &= y_2 \\ \dot{y}_2 &= 4.438\, y_3 \\ \dot{y}_3 &= -12 y_2 - 24 y_3 + \boxed{\frac{v}{L}\ ?} \end{aligned}$$

c) equiv inertia seen at motor?

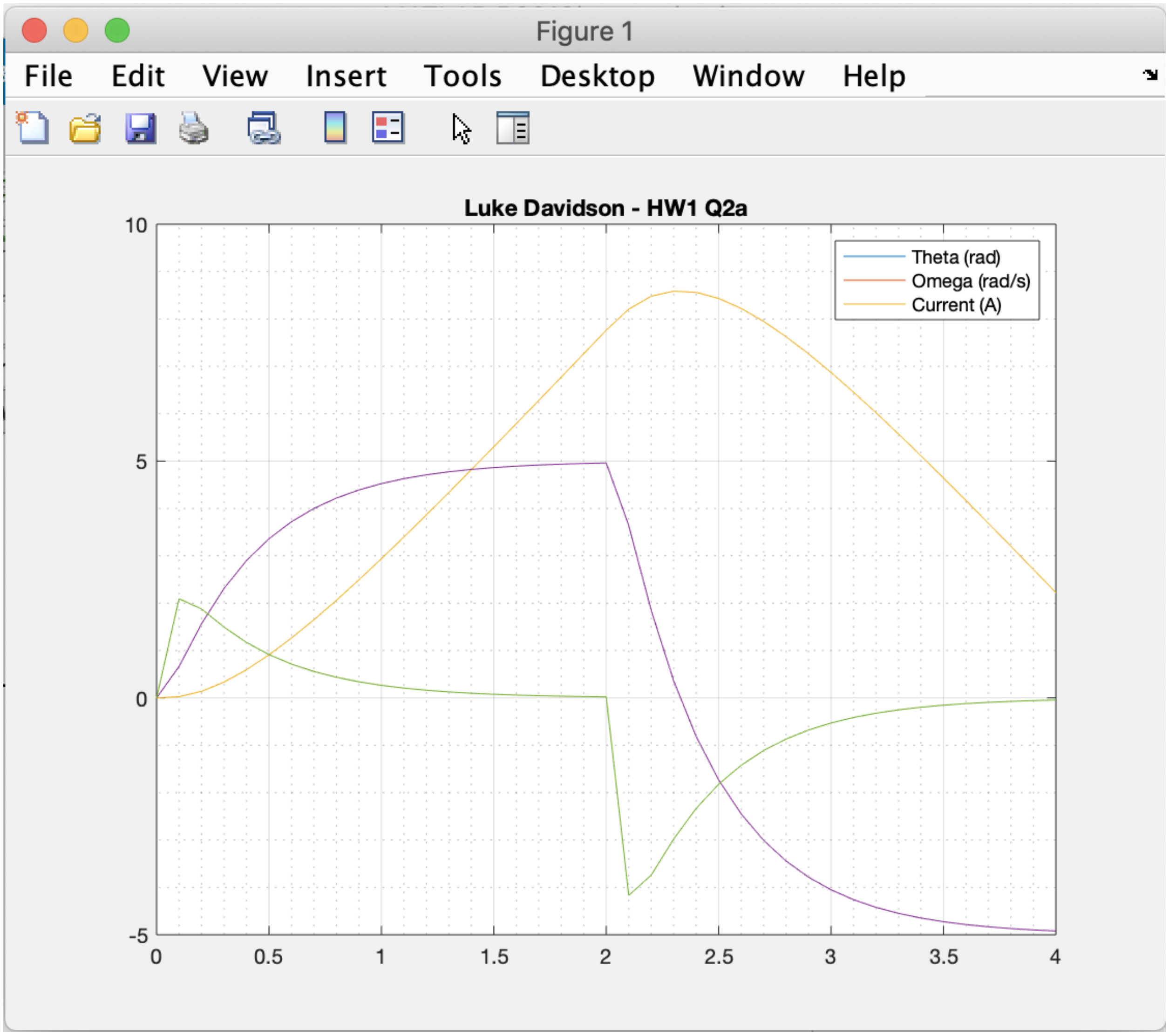
$J_{e_L} = J + N^2 J_m$

$J_{e_m} = J_m + \frac{J}{N^2}$

These will be different due to the gear ratio acting in opposite directions.

## 2a.)

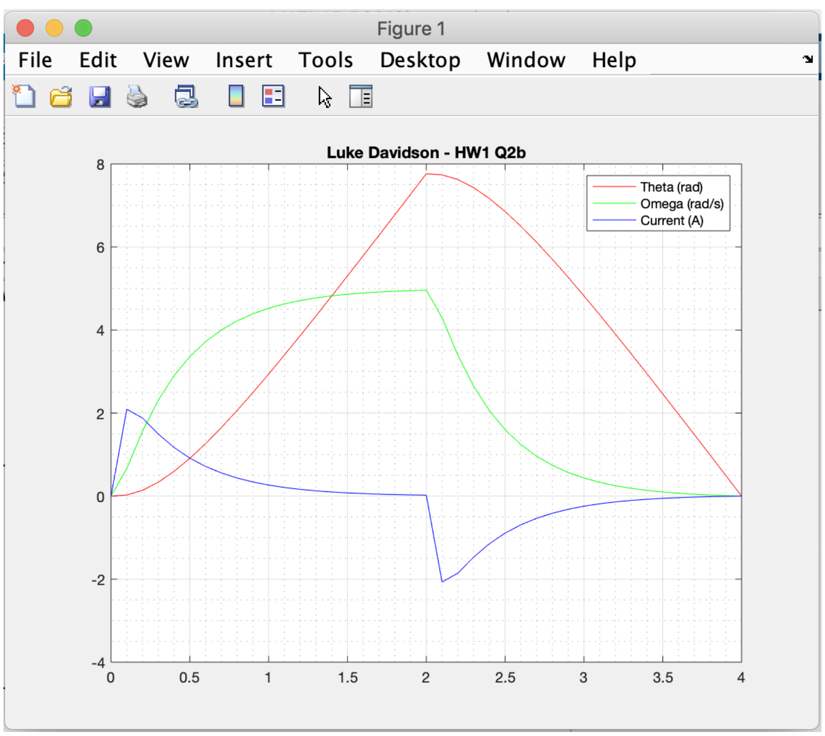
```
%Luke Davidson
%ME 5659
%HW1 Q2a

clc;
clear all;
close all;

%initialize params
N = 12;
Jm = 8e-4;
J = .02;
Je = (N.^2)*Jm + J;
K = 0.05;
R = 1.2;
L = 0.05;

%build state matrices
A = [0 1 0; 0 0 N*K/Je; 0 -N*K/L -R/L];
B = [0 0; 0 -1/Je; 1/L 0];
C = [1 0 0; 0 1 0];
D = [0 0; 0 0];
[By,Bx] = size(B);

%lsim matrices
t = 0:0.1:4;
[ty,tx] = size(t);
U = zeros(Bx,tx);
U(1,1:22) = 3;
U(1,21:end) = -3;

%lsim
sys = ss(A,B,C,D);
[Y_data,T,X_data] = lsim(sys,U,t);
plot(T,Y_data,T,X_data)
title('Luke Davidson - HW1 Q2a');
legend('Theta (rad)','Omega (rad/s)','Current (A)');
grid on;
grid minor;
```




2b.)

```
%Luke Davidson
%ME 5659
%HW1 Q2b

clc;
clear all;
close all;

%ode45
tspan1 = 0:0.1:2;
tspan2 = 2:0.1:4;
[t1,x1] = ode45(@func1,tspan1,[0 0 0]);
[t2,x2] = ode45(@func2,tspan2,[0 0 0]);

%start second plots from end of first
[x1zero,z1one]=size(x1);
```

```
start2 = x1(x1zero,:);

%plot
plot(tspan1,x1(:,1),'-r',tspan1,x1(:,2),'-g',tspan1,x1(:,3),'-b',tspan2,x2(:,1)+start2(1),'-r',tspan2,x2(:,2)+start2(2),'-g',tspan2,x2(:,
3)+start2(3),'-b')
title('Luke Davidson - HW1 Q2b');
legend('Theta (rad)','Omega (rad/s)','Current (A)');
grid on;
grid minor;

%define functions
function ydot=func1(t,y)

ydot(1) = y(2);
ydot(2) = 4.438*y(3);
ydot(3) = -12*y(2)-24*y(3)+60;
ydot = ydot';

end

function ydot=func2(t,y)

ydot(1) = y(2);
ydot(2) = 4.438*y(3);
ydot(3) = -12*y(2)-24*y(3)-60;
ydot = ydot';

end
```




2c.) ON PAGE 2

d.) motor torque = $T_0$ N that leads to max acc of load

no load torque

motor rotor inertia : $J_m$

load inertia : $J$

$$(2.13) : J_e \dot{\omega} = NT_0 - T_L$$

$$J_e = J + N^2 J_m$$

$$\Longrightarrow (J + N^2 J_m)\dot{\omega} = NT_0 - T_L$$

$$\dot{\omega} = \frac{NT_0 - T_L}{J + N^2 J_m} \qquad \frac{d}{dN}\left(\frac{\overset{a}{NT_0} - \overset{b}{T_L}}{\underset{c}{J} + \underset{d}{N^2 J_m}}\right) = 0$$

solving online yields

$$= \frac{T_0(J - J_m^2) + 2(T_L)(J_m)(N)}{(J + J_m N^2)^2} = 0$$

solving for $N^*$ gives

$$N^* = \frac{\sqrt{(J)(T_L) + (T_0)^2(J)(J_m)} + (T_L)(J_m)}{(T_0)(J_m)}$$

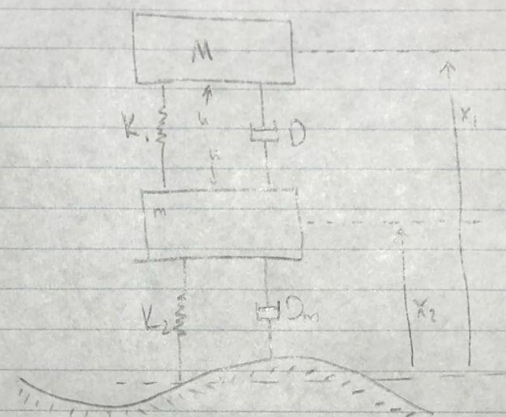
☆ after realizing the problem has $T_L = 0$ (oops) ☆

$$\frac{d}{dN}\left(\frac{NT_0}{J + N^2 J_m}\right) = 0 \Longrightarrow \frac{T_0(J - J_m N^2)}{(J + J_m N^2)^2} = 0$$

solving for $N^*$ yield

$$N^* = \sqrt{\frac{J}{J_M}}$$

3.) damper b/w m and ground, $D_m$




a) $g = 0$
rest lengths $= 0$

$u$ = input
$y_R(t)$ = disturbance input

output : $x_1$

$F_{spring} = k(x - x_0)$

$F_{K_1} = K_1(x_1 - x_2)$

$F_D = D(\dot{x}_1 - \dot{x}_2)$

$\dot{x}_1 = v_1$
$\dot{x}_2 = v_2$
$\dot{v}_1 =$
$\dot{v}_2 =$

states = $x_1, x_2, v_1, v_2$

$$M\frac{dv_1}{dt} = -K_1(x_1 - x_2) - D(v_1 - v_2) + u$$

$$m\frac{dv_2}{dt} = K_1(x_1 - x_2) + D(v_1 - v_2) - K_2(x_2 - y_R) - D_m(v_2 - \dot{y}_R) - u$$

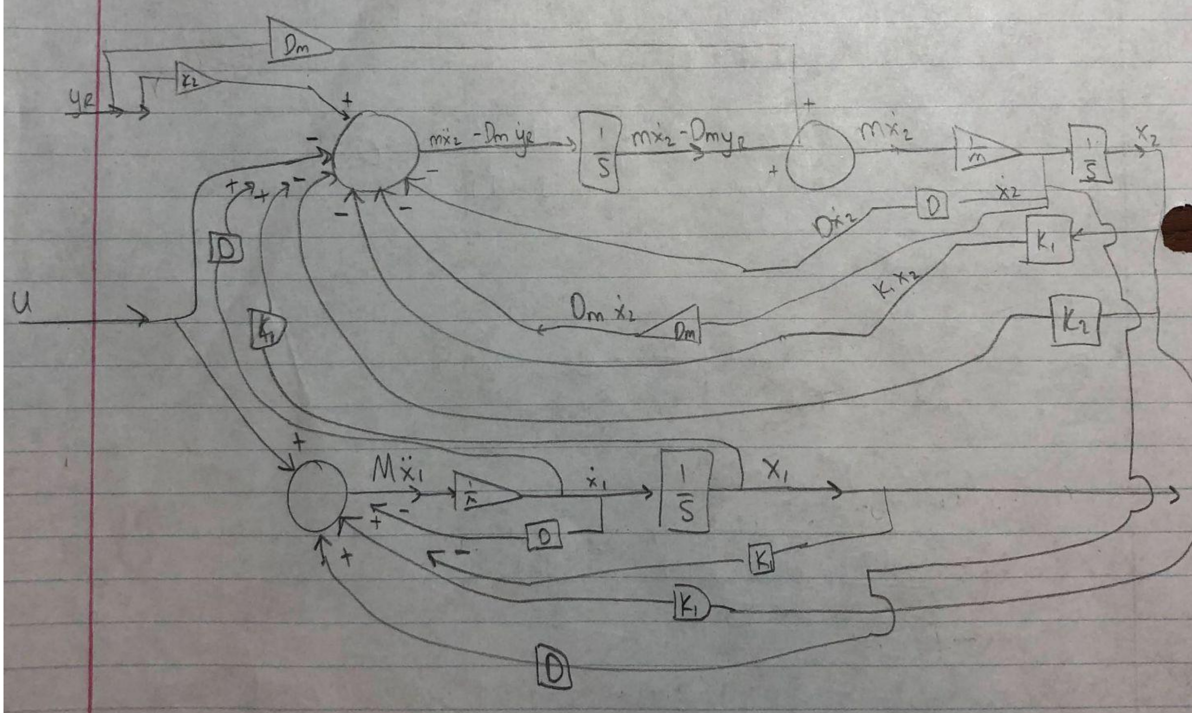
This is not possible because with the damper force from $D_m$ introduces a variable $\dot{y}_R$, which is a derivative of an exog. input and not a state or direct input. It cannot be represented in $y = Ax + Bu$.

3.) $M\ddot{x}_1 = -K_1(x_1 - x_2) - D(\dot{x}_1 - \dot{x}_2) + u$

$m\ddot{x}_2 = K_1(x_1 - x_2) + D(\dot{x}_1 - \dot{x}_2) - K_2(x_2 - y_R) - D_m(\dot{x}_2 - \dot{y}_R) - u$

$m\ddot{x}_2 - D_m\dot{y}_R = \underline{K_1x_1}\ \underline{-K_1x_2}\ \underline{+D\dot{x}_1}\ \underline{-D\dot{x}_2}\ \underline{-K_2x_2}\ \underline{+K_2y_R}\ \underline{-D_m\dot{x}_2}\ \underline{-u}$

$M\ddot{x}_1 = \underline{-K_1x_1}\ \underline{+K_1x_2}\ \underline{-D\dot{x}_1} + \underline{D\dot{x}_2}\ \underline{+u}$

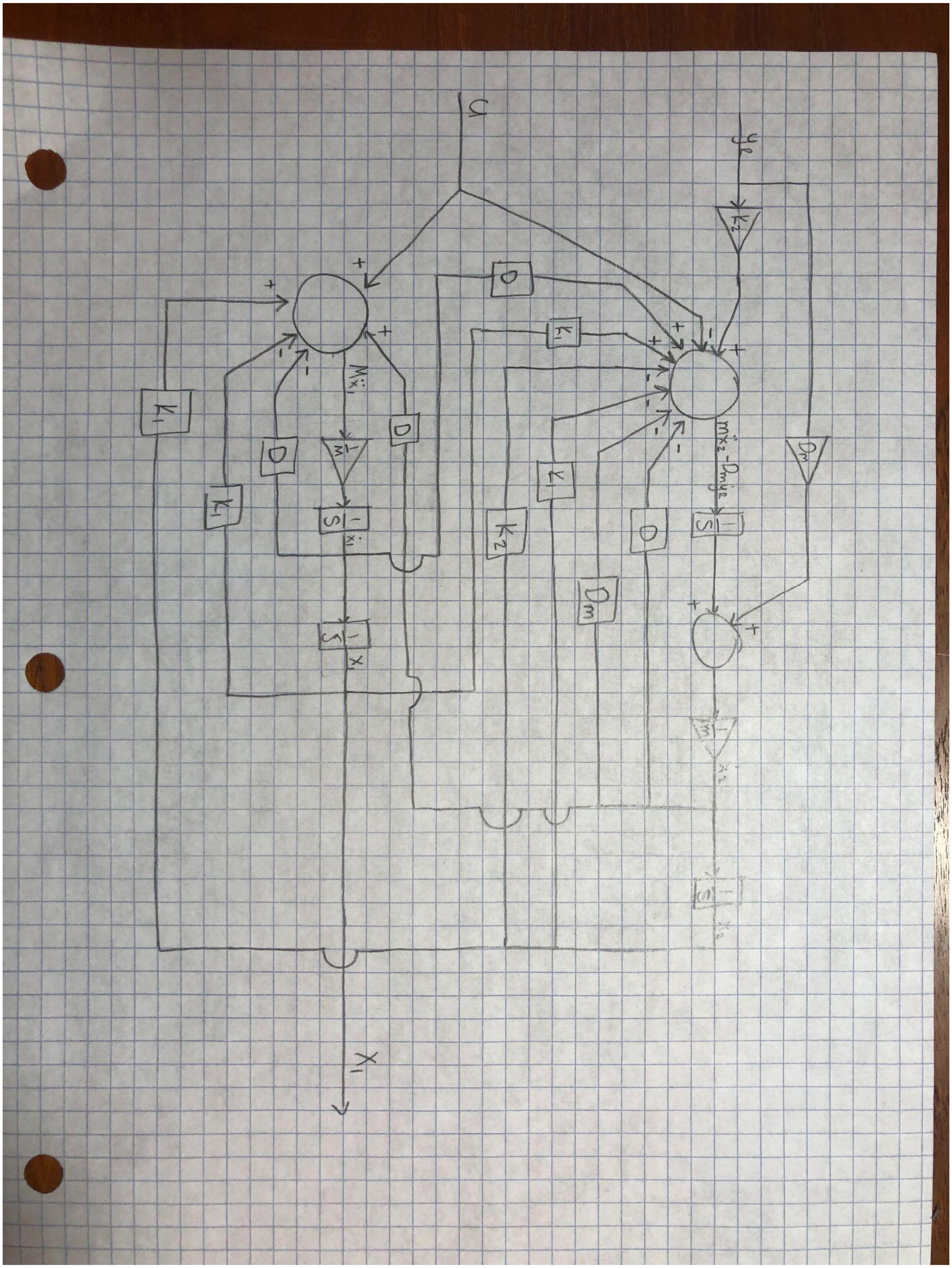



$m\ddot{x}_2 - \underline{D_m\dot{y}_R} = \underline{K_2y_R} - u + \underline{D\dot{x}_1}\ \underline{+K_1x_1}\ \underline{-K_2x_2}\ \underline{-K_1x_2}\ \underline{-D_m\dot{x}_2}\ \underline{-D\dot{x}_2}$

$M\ddot{x}_1 = D\dot{x}_2 - D\dot{x}_1 - K_1x_1 + K_1x_2 + u$

$M\ddot{x}_1 = D(\dot{x}_2 - \dot{x}_1) - K(x_1 - x_2) + u$ ✓

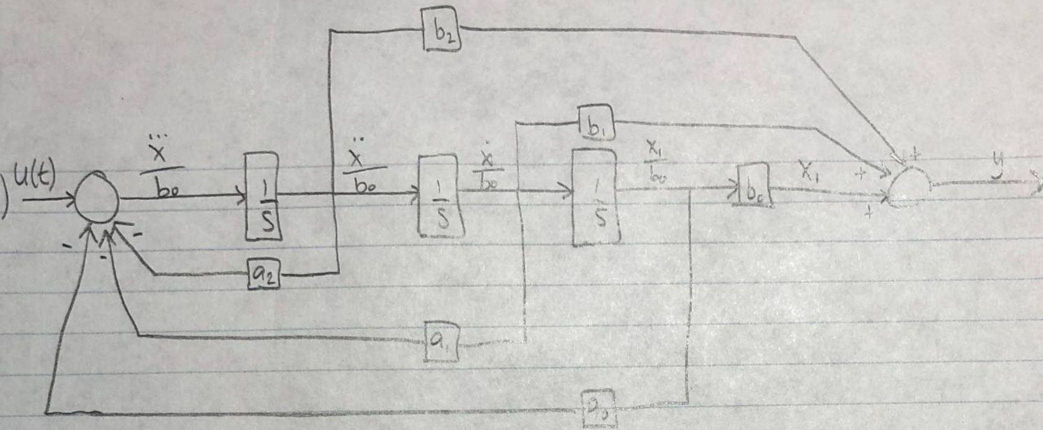
$K_2(y_R - x_2) + K_1(x_1 - x_2) + D(\dot{x}_1 - \dot{x}_2) + D_m(\dot{y}_R - \dot{x}_2) - u$


u
$y_e$
$K_1$
$K_2$
D
$D_m$
$x_1$

$$\text{states} = \left\{ \begin{matrix} x_1 \\ x_2 \\ \dot{x}_1 \\ m\dot{x}_2 - D_m y_R \end{matrix} \right\}$$

$$\frac{d}{dt}\begin{bmatrix} x_1 \\ x_2 \\ \dot{x}_1 \\ m\dot{x}_2 - D_m y_R \end{bmatrix} = \begin{bmatrix} \frac{-K_1}{D} & \frac{K_1 + K_2}{D} & \frac{D + D_m}{D} & \frac{1}{D} \\ \frac{K_1}{D + D_m} & \frac{-K_1 - K_2}{D + D_m} & 0 & \frac{-1}{D + D_m} \\ \frac{-K_1}{D} & \frac{K_1}{D} & 0 & 0 \\ K_1 & -K_1 - K_2 & D & 0 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ \dot{x}_1 \\ m\dot{x}_2 - D_m y_R \end{bmatrix} + \begin{bmatrix} 1 \\ -1 \\ 1 \\ 1 \end{bmatrix} u$$

4.)




label $y = x_1$. since $\dot{x} \longrightarrow \boxed{\frac{1}{s}} \longrightarrow x$, we can label the other main links (after dividing $b_0$)

$$y = x_1 + b_1\left(\frac{\dot{x}}{b_0}\right) + b_2\left(\frac{\ddot{x}}{b_0}\right)$$

$$\frac{\dddot{x}}{b_0} = -\left(\frac{\ddot{x}}{b_0}\right)a_2 - \left(\frac{\dot{x}}{b_0}\right)a_1 - \left(\frac{x}{b_0}\right)a_0 + u(t)$$

$$Y(s) = \left[1 + \frac{b_1}{b_0}s + \frac{b_2}{b_0}s^2\right]X = \left(\frac{b_0 + b_1 s + b_2 s^2}{b_0}\right)X$$

$$\Longrightarrow u(t) = \frac{\dddot{x}}{b_0} + \left(\frac{\ddot{x}}{b_0}\right)a_2 + \left(\frac{\dot{x}}{b_0}\right)a_1 + \left(\frac{x}{b_0}\right)a_0$$

$$U(s) = \left(\frac{1}{b_0}s^3 + \frac{a_2}{b_0}s^2 + \frac{a_1}{b_0}s + \frac{a_0}{b_0}\right)X = \left(\frac{s^3 + a_2 s^2 + a_1 s + a_0}{b_0}\right)X$$

$$\frac{Y(s)\, b_0}{b_0 + b_1 s + b_2 s^2} = \frac{U(s)\, b_0}{s^3 + a_2 s^2 + a_1 s + a_0}$$

$$Y(s)\left[s^3 + a_2 s^2 + a_1 s + a_0\right] = U(s)\left(b_0 + b_1 s + b_2 s^2\right)$$

$$\Longrightarrow \dddot{y} + a_2 \ddot{y} + a_1 \dot{y} + a_0 y = u b_0 + b_1 \dot{u} + b_2 \ddot{u}$$

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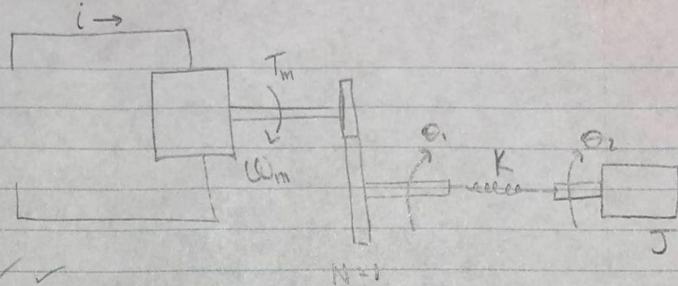


$T = k(\theta_1 - \theta_2)$

state variables = $\theta_2, \Delta, \omega_2, \Omega, i$

constants: $N, K, J_2$

input $v$

$\dot{\theta}_2 = \omega_2$

$\dot{i} = \dfrac{v - K_m N(\Omega + \omega_2) - Ri}{L}$

$\dot{\Delta} = \Omega$

$\dot{\omega}_2 = \dfrac{K\Delta}{J}$

$\dot{\Omega} = \dot{\omega}_1 - \dot{\omega}_2$

$= \dot{\omega}_1 - \dfrac{K\Delta}{J}$

$\boxed{\dot{\Omega} = \dfrac{K_m(i)}{J_m N} - \dfrac{K\Delta}{N^2 J_m} - \dfrac{K\Delta}{J}}$

$\Delta = \theta_1 - \theta_2$

$\dot{\Delta} = \dot{\theta}_1 - \dot{\theta}_2$

$\Omega = \dot{\theta}_1 - \dot{\theta}_2$

$\dot{\theta}_1 = \Omega + \dot{\theta}_2$

$\boxed{\dot{\theta}_1 = \Omega + \omega_2}$

$J\dot{\omega}_2 = k(\theta_1 - \theta_2) = K\Delta$

$J_m \dot{\omega}_m = T_m - \dfrac{K\Delta}{N}$

$\omega_m = N\omega_1$

$\boxed{\omega_m = N\dot{\theta}_1}$

$\dot{\omega}_2 = \dfrac{K\Delta}{J}$

$J_m \dot{\omega}_m = T_m - \dfrac{K\Delta}{N}$

$\dot{\omega}_m = \dfrac{T_m}{J_m} - \dfrac{K\Delta}{NJ_m}$

$\dot{\omega}_m = \dfrac{K_m i}{J_m} - \dfrac{K\Delta}{NJ_m}$

$\dot{\omega}_m = N\dot{\omega}_1$

$\dot{\omega}_1 = \dfrac{\dot{\omega}_m}{N}$

$L\dfrac{di}{dt} + Ri = v - K_m \omega_m$

$L\dot{i} + Ri = v - K_m N\dot{\theta}_1$

$\dot{i} = \dfrac{v - K_m N\dot{\theta}_1 - Ri}{L}$

$\dot{i} = \dfrac{v - K_m N(\Omega + \omega_2) - Ri}{L}$

$\dot{\omega}_1 = \dfrac{K_m(i)}{(J_m)N} - \dfrac{K\Delta}{N^2 J_m}$

$$\dot{\theta}_2 = \omega_2$$

$$\dot{\Delta} = \Omega$$

$$\dot{\omega}_2 = \frac{K\Delta}{J}$$

$$\dot{i} = \frac{1}{L}(v) - \frac{K_m(N)}{L}\Omega - \frac{K_m(N)}{L}\omega_2 - \frac{R}{L}(i)$$

$$\dot{\Omega} = \frac{K_m}{J_m N} i - \left(\frac{K}{N^2 J_m} + \frac{K}{J}\right)\Delta$$

$$\frac{d}{dt}\begin{bmatrix}\theta_2\\ \Delta\\ \omega_2\\ \Omega\\ i\end{bmatrix} = \begin{bmatrix}0 & 0 & 1 & 0 & 0\\ 0 & 0 & 0 & 1 & 0\\ 0 & (K/J) & 0 & 0 & 0\\ 0 & \left(\frac{-K}{N^2 J_m} - \frac{K}{J}\right) & 0 & 0 & \left(\frac{K_m}{J_m N}\right)\\ 0 & 0 & \left(\frac{-K_m N}{L}\right) & \left(\frac{-K_m N}{L}\right) & \left(-\frac{R}{L}\right)\end{bmatrix}\begin{bmatrix}\theta_2\\ \Delta\\ \omega_2\\ \Omega\\ i\end{bmatrix} + \begin{bmatrix}0\\ 0\\ 0\\ 0\\ \frac{1}{L}\end{bmatrix} V$$

$$y = \begin{bmatrix}\theta_2\\ \omega_2\end{bmatrix} = \begin{bmatrix}1 & 0 & 0 & 0 & 0\\ 0 & 0 & 1 & 0 & 0\end{bmatrix}\begin{bmatrix}\theta_2\\ \Delta\\ \omega_2\\ \Omega\\ i\end{bmatrix}$$

b.)

$$\dot{\theta}_2 = \omega_2$$

$$\dot{\Delta} = \Omega$$

$$\dot{\omega}_2 = 25000\Delta$$

$$\dot{i} = 20v - 12\Omega - 12\omega_2 - 24i$$

$$\dot{\Omega} = 0.02844i - 10023.7\Delta$$

## 5c.)

```
%Luke Davidson
%ME 5659
%HW1 Q5

clc;
clear all;
close all;

%initialize params
N = 12;
Jm = 8e-4;
J = .02;
Km = 0.05;
K = 500;
R = 1.2;
L = 0.05;

%build state matrices
A = [0 0 1 0 0; 0 0 0 1 0; 0 K/J 0 0 0; 0 -K/((N.^2)*Jm)-K/J 0 0 Km/(Jm*N); 0 0 -Km*N/L -Km*N/L -R/L]
B = [0; 0; 0; 0; 1/L]
C = [1 0 0 0 0; 0 0 1 0 0];
D = 0;
[By,Bx] = size(B);

%lsim matrices
t = 0:0.1:4;
[ty,tx] = size(t);
U = zeros(Bx,tx);
U(1,1:22) = 3;
U(1,21:end) = -3;

%lsim
sys = ss(A,B,C,D);
[Y_data,T,X_data] = lsim(sys,U,t);
% plot(T,Y_data,T,X_data)
plot(T,X_data)
title('Luke Davidson - HW1 Q5c');
legend('Theta (rad)','Delta (rad)', 'omega (rad/s)','Omega(rad/s^2)','Current (A)');
grid on;
grid minor;
```

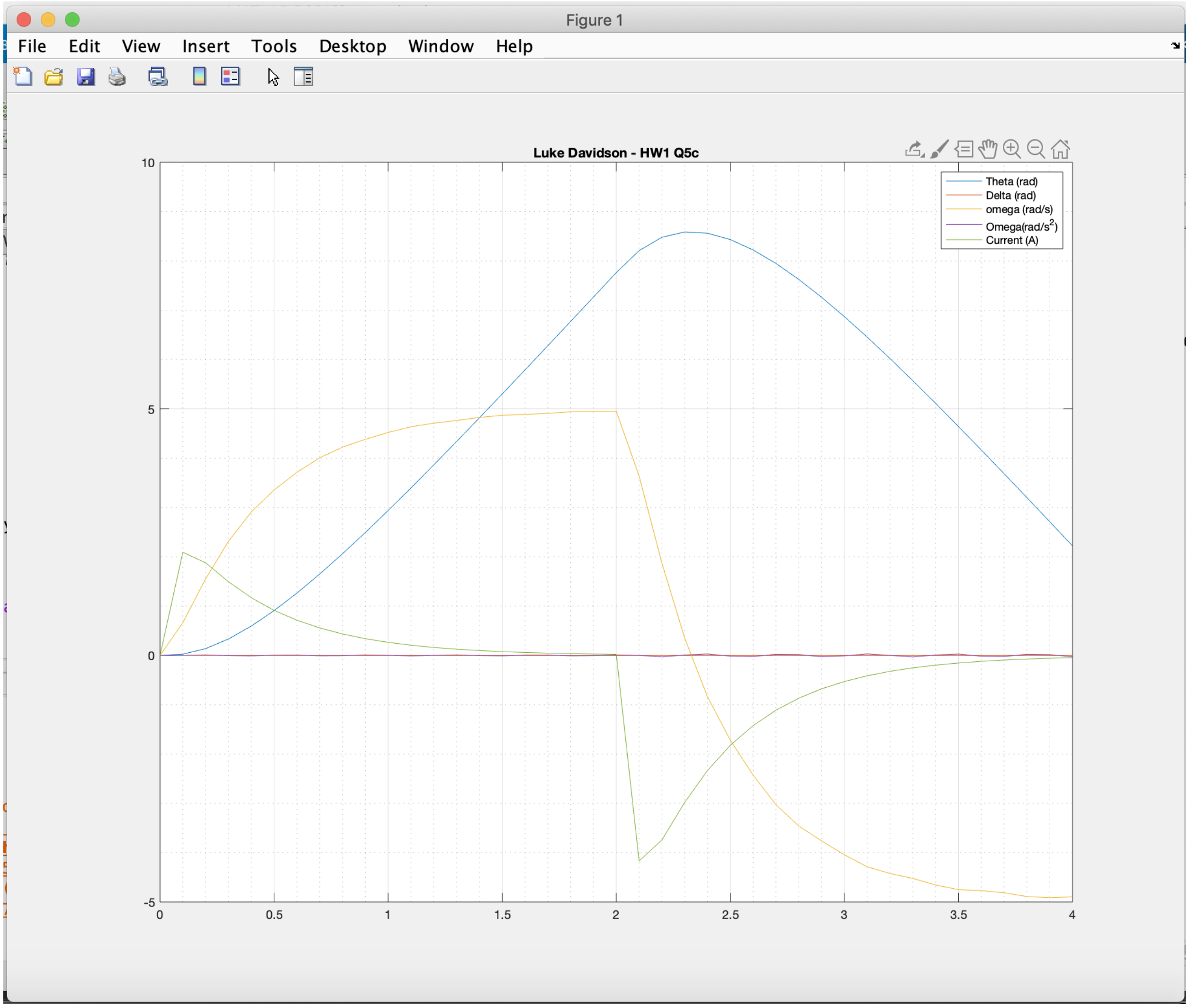
Figure 1
File Edit View Insert Tools Desktop Window Help
Luke Davidson - HW1 Q5c
Theta (rad)
Delta (rad)
omega (rad/s)
Omega(rad/s$^2$)
Current (A)
10
5
0
-5
0
0.5
1
1.5
2
2.5
3
3.5
4