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ME 5659
HW #7

1.) $G(s) = \dfrac{1}{(s+8)(s^2-2s+2)}$

a.) $G_c(s) = K$

$G_{rl}(s) = \dfrac{K}{(s+8)(s^2-2s+2)}$

$(s+8) = 0$; $s^2-2s+2=0$

$\boxed{s=-8}$

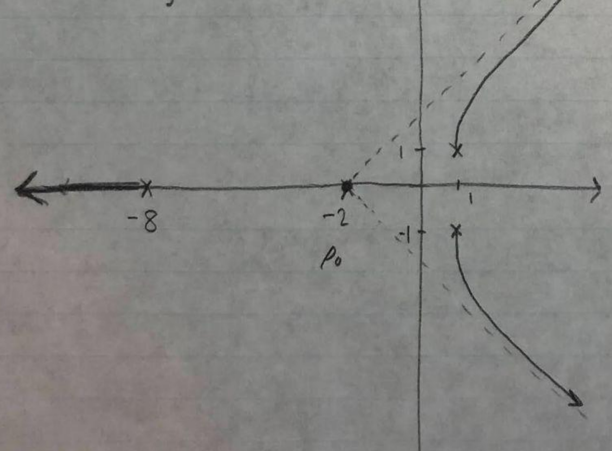
$s = \dfrac{2 \pm \sqrt{4-8}}{2} = \dfrac{2 \pm \sqrt{}}{}$ $\boxed{1 \pm i = s}$

$n-m = \boxed{3}$

$\rho_0 = \dfrac{\Sigma \text{ finite poles} - \Sigma \text{ finite zeros}}{n-m}$

$= \dfrac{(-8)+1+1}{3} = \boxed{-2 = \rho_0}$

3 = 



b.) $G_c(s) = \dfrac{K(s+1)(s+z)}{s}$ find z crit so $z_{crit} > z > 0$ is cond. stable

$$L(s) = G_c(s)G_p(s) = \frac{K(s+1)(s+z)}{s}\left(\frac{1}{(s+8)(s^2-2s+2)}\right)$$

$$L(s) = K\left(\frac{(s+1)(s+z)}{s(s+8)(s^2-2s+2)}\right)$$

zeros : $-1, -z$
poles : $0, -8, 1 \pm i$

$n - m = 4 - 2 = \boxed{2} \longrightarrow \pm 90°$

$$\sigma = \frac{-8+2 - (-1-z)}{2}$$

$$L(s) + 1 = 0 = \frac{K(s+1)(s+z)}{s(s+8)(s^2-2s+2)} + 1 = 0$$

$$\boxed{\frac{-5+z}{2} = \sigma}$$

~~$K(s+1)(s+z) + s(s+8)(s^2-2s+2) = 0$~~

~~$K(s^2+sz+s+z) + s(s^3-2s^2+2s+8s^2-16s+16) = 0$~~

~~$Ks^2 + (z+K)s + zK + s^4 + 6s^3 - 14s^2 + 16s$~~

~~$s^4 + 6s^3 + s^2(K-14) + s(Kz+16+K) + zK = 0$~~

~~1~~	~~$K-14$~~	~~K~~
~~6~~	~~$Kz+K+16$~~	
~~$\square$~~		

~~$$\frac{6(K-14) - (Kz+K+16)}{6} = \frac{6K - 84 - Kz - K - 16}{6}$$~~

~~$$= \frac{K(6-1-z) - 100}{6}$$~~

~~$$= \frac{(5-z)(K) - 100}{6}$$~~

~~$(5-z)K - 100 > 0$~~

~~$(5-z)K > 100$~~

~~$5 - z > \frac{100}{K}$~~

~~$z_{crit} < 5 - \frac{100}{K}$~~

σ

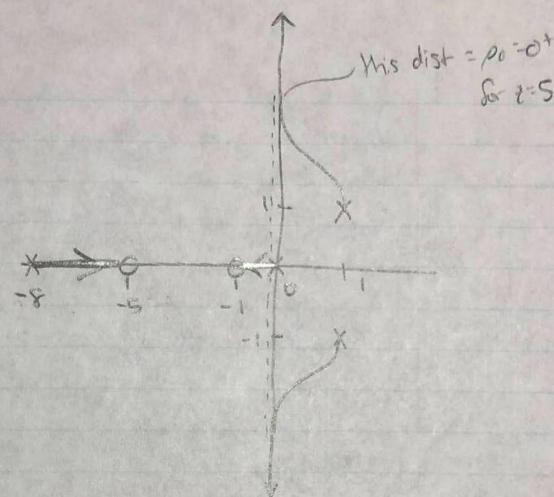
$\dfrac{-5+z}{2} < 0$

want approaching left half

$\boxed{z < 5}$

$z_{crit} = 5$

root locus sketch for $z=5$



$$G(s) = \frac{K(s+1)(s+5)}{s(s+8)(s^2-2s+2)}$$

$n-m = 4-2 = 2 = \pm 90$

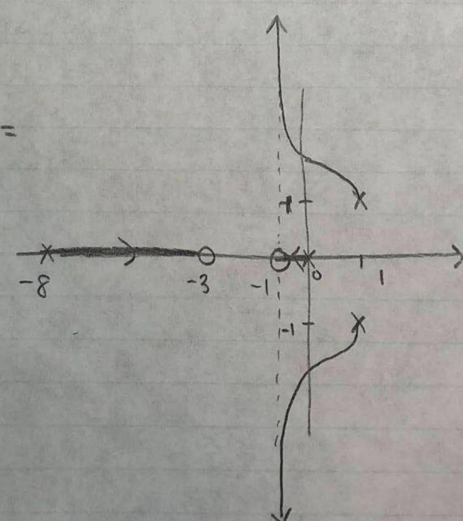
$\boxed{p_0 = 0^+}$

c.) dominant cl $= -1$ means

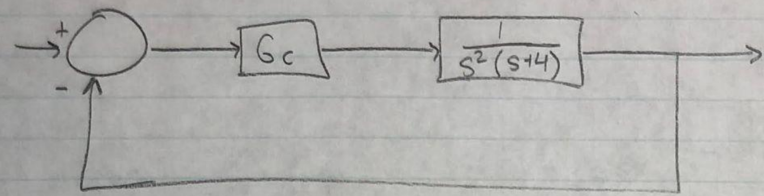
$\boxed{p_0 \longrightarrow -1}$ so $\frac{-5+z}{2} = -1$ $\boxed{z=3}$

sketch for $z=3$, or

$$G(s) = \frac{K(s+1)(s+3)}{s(s+8)(s^2-2s+2)} =$$



2.)



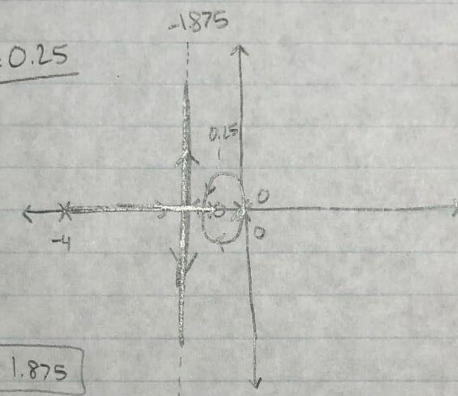
< 25% overshoot

$t_s < 5$
(2%)

$$\frac{K(s+b)}{s^2(s+4)}$$

$n-m = 3-1 = \boxed{2} = \pm 90°$

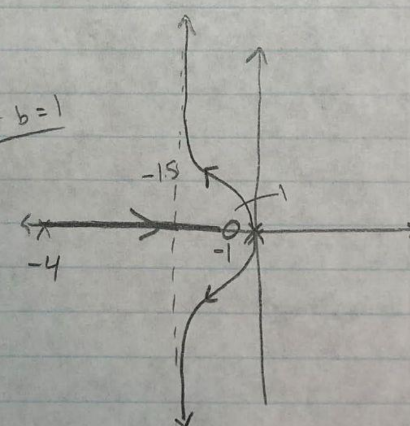
for b=0.25



b=0.25 $\quad \sigma_0 = \dfrac{-4-(-0.25)}{2} = \dfrac{-3.75}{2} = \boxed{-1.875}$

b=1 $\quad \sigma_0 = \dfrac{-4-(-1)}{2} = \dfrac{-3}{2} = \boxed{-1.5}$

for b=1



$$0.25 \lessgtr e^{\frac{-\xi\pi}{\sqrt{1-\xi^2}}}$$

$$\ln(0.25) \lessgtr \frac{-\xi\pi}{\sqrt{1-\xi^2}}$$

$$1.92 - 1.92\xi^2 \lessgtr \xi^2\pi^2$$

$$[\ln(0.25)]^2 \lessgtr \frac{\xi^2\pi^2}{1-\xi^2}$$

$$(\pi^2 + 1.92)(\xi^2) \geq 1.92$$

$t_s = 5 = \dfrac{4}{(.404)(\omega_n)} \Rightarrow \boxed{\omega_n = 1.98}$ or greater

$\boxed{\omega_n = 0.87}$

$t_s < 5$

$\dfrac{4}{.404(\omega_n)} < 5$ $\quad$ cos

$\boxed{\xi > 0.404}$ $\quad$ $\boxed{\xi > 0.404}$

$\cos(.404) = 66°$

$\cos(1) = 0$

$b=0.25$

$$1+G(s) = s^2(s+4) + K(s+0.25) = 0$$

$$s^3 + 4s^2 + Ks + K(0.25)$$

$$K = \frac{-s^2(s+4)}{s+0.25}$$

$$\frac{d}{ds}K = \frac{s(-2s^2 - 4.75s - 2)}{(s+0.25)^2} = 0$$

roots $= -1.828,\ -0.547,\ 0$

according to graph

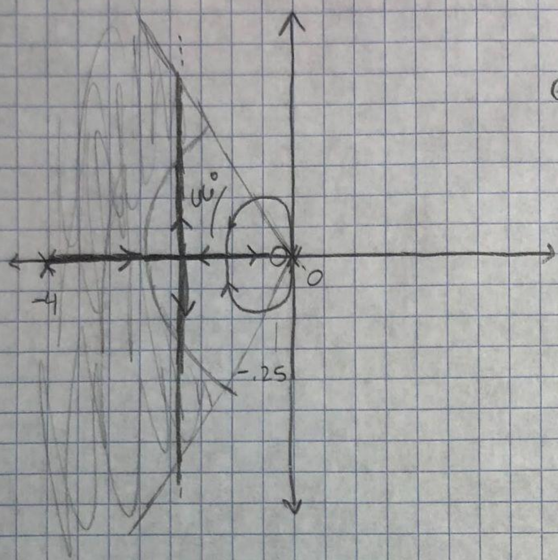
-0.547 = break in
$0, -1.828$ = break out

$b=1$

$$K = \frac{-s^2(s+4)}{s+1} \longrightarrow \frac{d}{ds}K = \frac{-s(2s^2+7s+8)}{(s+1)^2}$$

$=0 \rightarrow$ roots $=0$, so break out @ 0

Pros of placing the zero near the origin include creating a peak in the response of the system, although the overshoot is likely to be high and not easy to control.



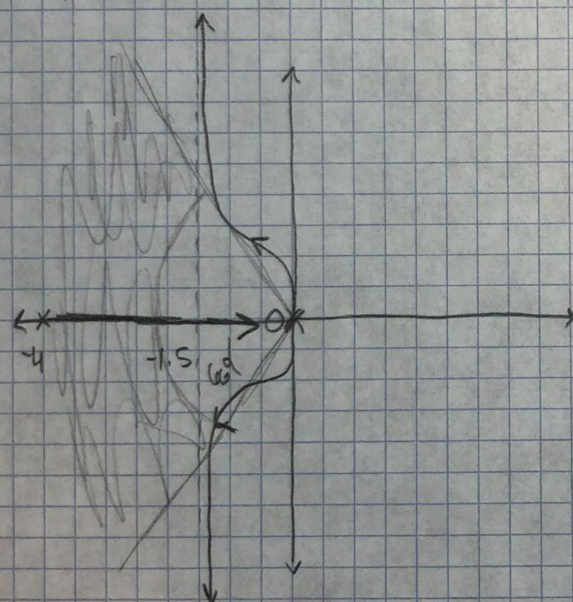
$$G(s) = \frac{K(s+0.25)}{s^2(s+4)}$$

$$1+G(s) = s^2(s+4) + K(s+0.25)$$

$\angle$ is between 0–66° @ magnitude

$W_n > 1.98$

(shaded area)



b.) $G_c(s) = \frac{K(s+4)(s+b)}{(s+12)}$ $G(s) = \frac{1}{s^2(s+4)}$

$G_c \rightarrow s^2 + 8s + 32$

$$\boxed{G_c(s) \rightarrow \frac{K(s+4)(s+b)}{(s+12)(s^2+8s+32)}}$$

$\frac{1}{K_{re}} = 4$

$\sqrt{64} = \sqrt{64 - \underbrace{(4}_{128}}$

$4(a)(c) = 128$

$\frac{128}{4} =$

$s^2 + 8s + 32$

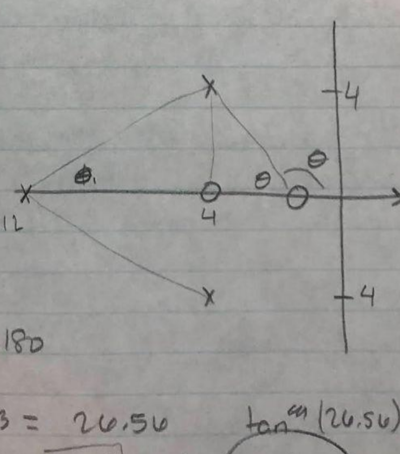
$\frac{-8 \pm \sqrt{64 - 4(32)}}{2}$

$\text{angle} = K_{re}\left(\frac{B_1 \ldots B_m}{A \cdot A_n}\right) e^{j(\theta_1 + \ldots + \theta_m) - (\phi_1 + \ldots + \phi_n))} = e^{j(\pm 180(2k+1))}$

$\theta_1 + \ldots + \theta_m - (\phi_1 + \ldots + \phi_n) = \pm 180(2k+1)$

$\alpha = n - m = 3 - 2 = \boxed{1}$

$\alpha = \pm 180$



$\theta_1 = \tan^{-1}(1/2) = 26.56$

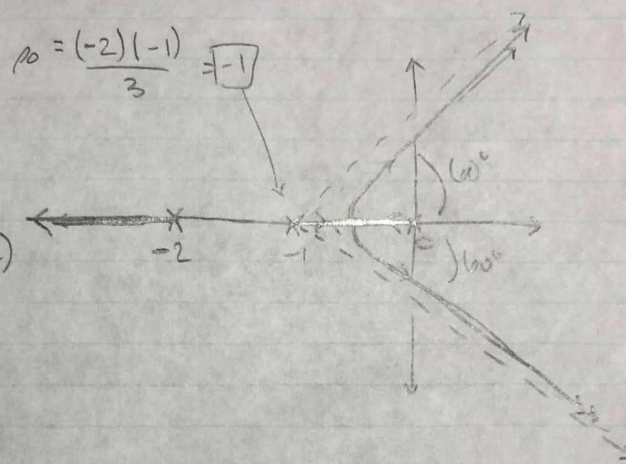
$= 26.56 +$
$90 + \theta - (26.56 + 90) = \pm 180$

$\theta = 153.43 \Rightarrow 180 - 153 = 26.56$ $\tan^{-1}(26.56) = \frac{4}{x}$

$\boxed{x = 8} \Rightarrow b = 4 \Rightarrow K_{re} = 0.25$

3.) $G(s) = \dfrac{K_p}{s(s+1+a)(s+2)}$

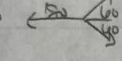
$\sigma_0 = \dfrac{(-2)(-1)}{3} = \boxed{-1}$



a.) $a = 0$

$G(s) = \dfrac{K_p}{s(s+1)(s+2)}$

$n - m = \boxed{3} \longrightarrow$



b.) $K_p = 1$

$G(s) = \dfrac{1}{s(s+1+a)(s+2)} \Longrightarrow \dfrac{L}{1+L} = \dfrac{1}{1 + s(s+1+a)(s+2)}$

$= \dfrac{1}{1 + s(s^2 + 2s + s + 2 + as + 2a)} = \dfrac{1}{1 + (s^3 + 3s^2 + as^2 + 2s + 2as)}$

$= \dfrac{1}{s^3 + s^2(3+a) + s(2+2a) + 1}$

$\phi(s) = s^3 + 3s^2 + as^2 + 2s + 2as + 1 = 0$

$s^3 + 3s^2 + 2s + 1 + a(s^2 + 2s) = 0$

$\Longrightarrow \dfrac{a(s^2+2s)}{s^3+3s^2+2s+1} + 1 = 0$

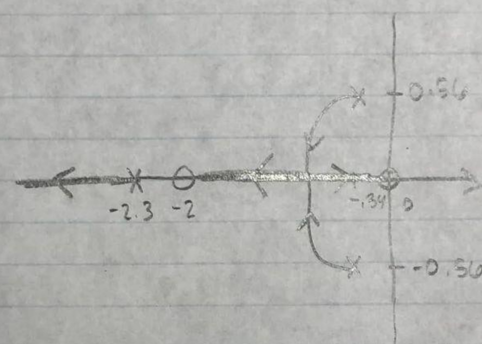
$= \dfrac{-a(s^2+2s)}{s^3+3s^2+2s+1}$

$G_{rl}(s) = \boxed{\dfrac{as(s+2)}{s^3+3s^2+2s+1}} = -1$

c.) $G(s) = \dfrac{a\ s(s+2)}{s^3+3s^2+2s+1}$

$\hookrightarrow$ $-2.3 \quad -0.34 \pm 0.56$

$n-m = 3-2 = \boxed{1}$ 180



a positive "a" value will increase stability margin as it will allow the root locus graph/branches to converge towards the left hand half of the plot. With a negative "a", or root locus gain, the root locus branches will shift towards the right hand side of the graph, causing the system to be unstable.

4.) a.) (i) $G(s) = \dfrac{k\;(s-2)}{s^2+1.6s+0.8}$

$k(s-2) + s^2 + 1.6s + 0.8 = 0$

$ks - 2k + s^2 + 1.6s + 0.8 = 0$

$s^2 + s(1.6+k) + (0.8-2k) = 0$

$0.8 - 2k > 0$

$2k < 0.8$

$\boxed{k < 0.4}$

$$\begin{array}{cc} 1 & (0.8-2k) \\ 1.6+k & 0 \end{array} \Longrightarrow \frac{(1.6+k)(0.8-2k)}{1.6+k} > 0$$

$(1.6+k)(0.8-2k) > 0$

$1.28 - 3.2k + 0.8k - 2k^2 > 0$

$-2k^2 - 2.4k + 1.28 > 0$

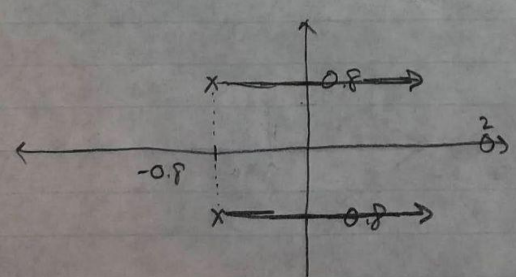
$k^2 + 1.2k - 0.64 \not> 0$

$$\frac{-1.2 \pm \sqrt{(1.2)^2 - 4(-0.64)}}{2} = \frac{-1.2 \pm \sqrt{4}}{2}$$

$= \dfrac{0.8}{2} = \boxed{0.4} \quad \boxed{-1.6} \qquad -1.6 < k < 0.4$

(ii) $n - m = 2 - 1 = \boxed{1} \rightarrow \pm 180^\circ$

poles $= \dfrac{-1.6 \pm \sqrt{1.6^2 - 4(0.8)}}{2} = \boxed{-0.8 \pm 0.8i}$



$\rho_0 =$

$\phi(s) = s^2 + s(1.6 + k) + (0.8 - 2k) = 0$

$\phi(jw) = (jw)^2 + (jw)(1.6 + k) + 0.8 - 2k = 0 + 0j$

$-w + jw(1.6 + k) + 0.8 - 2k = 0 + 0j$

$\underline{j}$

$w(1.6 + k) = 0$

$\boxed{k = -1.6}$

$\underline{R}$

$-w + 0.8 - 2k = 0$

$-w + 0.8 - 2(-1.6))$

$-w + 0.8 + 3.2 = 0$

$w = 4$

b) (i) $G(s) = \dfrac{k}{(s+1)^2 (s-0.5)}$ $+1 = 0$

$\phi(s) = (s+1)^2 (s-0.5) + k = 0$

$(s^2 + 2s + 1)(s - 0.5) + k = 0$

$s^3 - 0.5s^2 + 2s^2 - s + s - 0.5 + k = 0$

$k > 0$

$s^3 + 1.5s^2 - 0.5 + k$

1	$\cancel{-0.5 + k}$ 0	
1.5	$-0.5 + k$	

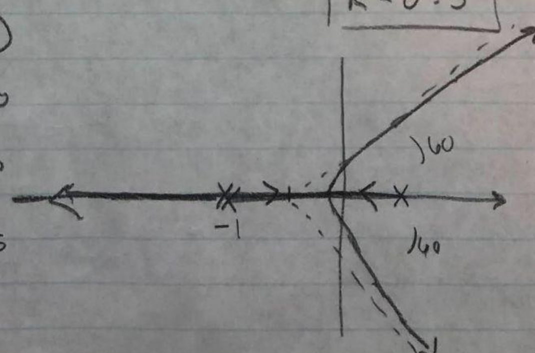
$= \dfrac{-(-0.5 + k)}{1.5} = \dfrac{0.5 - k}{1.5} > 0$

$\boxed{k < 0.5}$

$k < 0.5$

(ii) $n - m = \boxed{3}$ = 180, 60, -60

$\sigma = \dfrac{(-1)(-1) + 0.5}{3} = \dfrac{-1.5}{3} = -.5$



(iii) $\phi(s) = s^3 + 1.5s^2 - 0.5 + k$

$\phi(j\omega) = (j\omega)^3 + 1.5(j\omega)^2 - 0.5 + k = 0 + 0j$

$-\omega^3 j - 1.5\omega^2 - 0.5 + k = 0 + 0j$

$\underline{j}$

$-\omega^3 = 0$

$\omega = 0$

$\underline{\mathbb{R}}$

$-1.5\omega^2 - 0.5 + k = 0$

$k = 0.5$

c.) $G(s) = \dfrac{k(s+1)}{(s^2+1)(s+0.5)}$

$\phi(s) = (s^2+1)(s+0.5) + k(s+1) = 0$

$s^3 + 0.5s^2 + s + 0.5 + ks + k$

$s^3 + 0.5s^2 + s(1+k) + 0.5 + k = 0$

$k > 0.5$

$k + 1 > 0$

$\boxed{k > -1}$

1	$1+k$
0.5	$0.5+k$

$(0.5+k) - (1+k)(0.5)$

$(0.5)(1+k) - (0.5+k)$

$\cancel{0.5} + 0.5k - \cancel{0.5} - k > 0$

$-0.5k > 0$

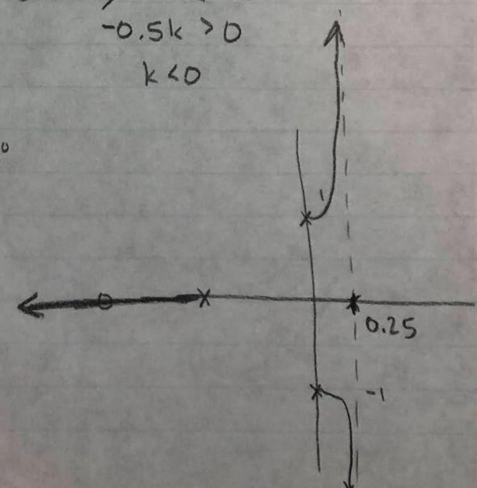
$k < 0$

$\boxed{k > 0.5}$

$-0.5 < k < 0$

(ii) $n - m = 3 - 1 = \boxed{2}$ $\quad \pm 90°$

$\rho_0 = \dfrac{-0.5 - (-1)}{2} = \dfrac{0.5}{2} = 0.25$



(iii) $s^3 + 0.5s^2 + s + 0.5 + ks + k = 0$

$(s) \to$

$(j\omega) = -j\omega^3 - \omega^2 0.5 + j\omega + 0.5 + kj\omega + k = 0 + 0j$

$\underline{j}$

$-\omega^3 + \omega + k\omega = 0$

$\omega(1 + k - \omega^2)$

$\omega = \sqrt{1+k}$

$\omega = 1$

$\underline{\Re}$

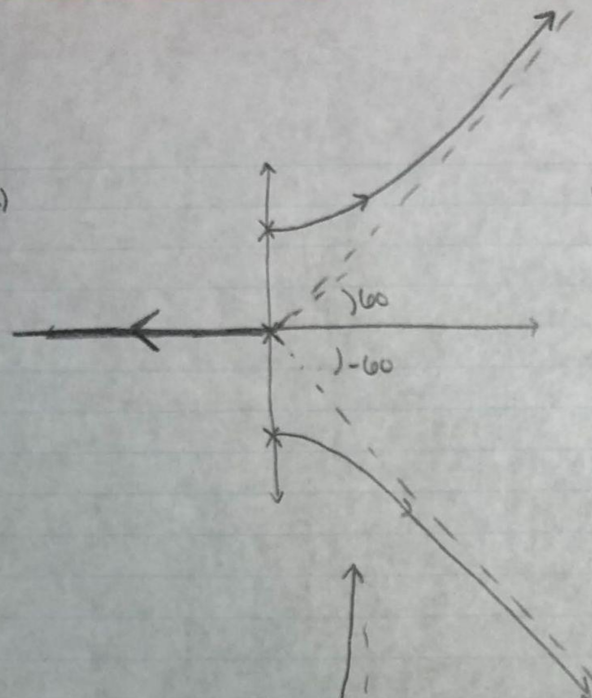
$-0.5\omega^2 + 0.5 + k = 0$

$-0.5(1+k) + 0.5 + k = 0$

$\cancel{-0.5} - 0.5k + \cancel{0.5} + k = 0$

$k = 0$

5.) a)

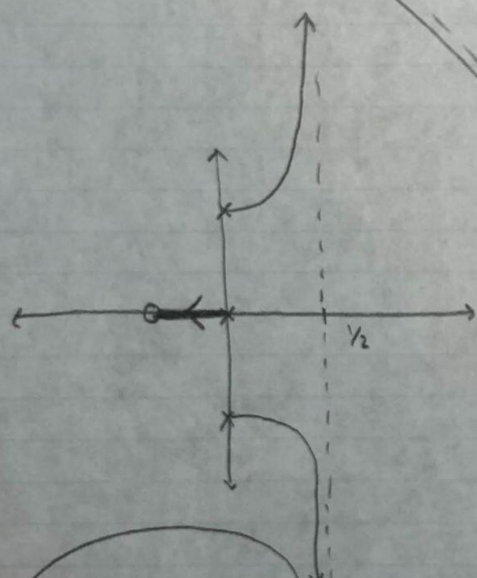


$n-m=3$

$\theta = 180, \pm 60$

$\sigma = 0$

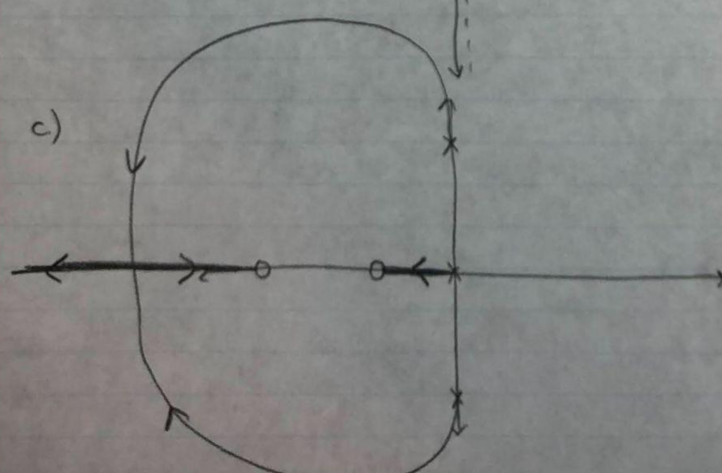
b.)



$n-m = 2 = \pm 90$

$\sigma = \dfrac{0-(-1)}{2} = \dfrac{1}{2}$

c.)



$n-m = 1 = 180$