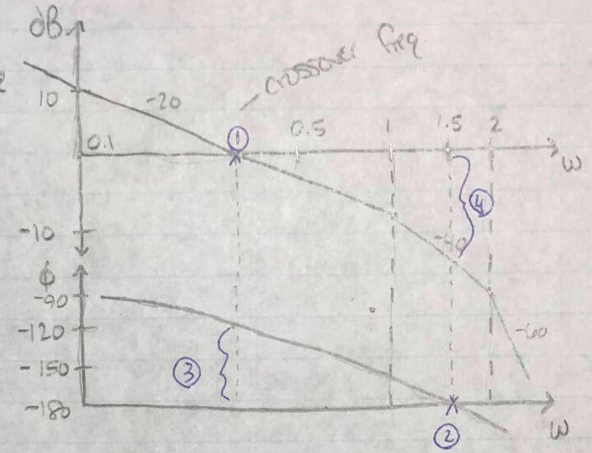
Luke Davidson
ME 5659
HW #6

1.) a.) ① = gain crossover freq
② = phase " " "
③ = phase margin
④ = gain margin



Both gain and phase margins are <u>positive</u>.

PM ≈ 60°
GM ≈ 12 dB

b.) open loop stable, no zeros, $\omega_c = 0.33$ rad/s

$$G(s) = \frac{K}{s(s+1)(s+2)}$$ 3 poles: s = 0, 1, 2

$$G(s) = \left(\frac{K}{2}\right)\frac{1}{s(s+1)(\frac{1}{2}s+1)} = \frac{\frac{K}{2}}{s(s+1)(\frac{1}{2}s+1)}$$

$$G(j\omega) = \frac{\frac{K}{2}}{(j\omega)(j\omega+1)(\frac{1}{2}j\omega+1)} \Rightarrow |G(j\omega)| = \frac{\frac{K}{2}}{(\omega)\sqrt{\omega^2+1}\sqrt{\frac{\omega^2}{4}+1}}$$

$$0 = 20\log\left(\frac{K}{2}\right) - 20\log(\omega) - 20\log\left(\sqrt{\omega^2+1}\right) - 20\log\sqrt{\frac{\omega^2}{4}+1}$$

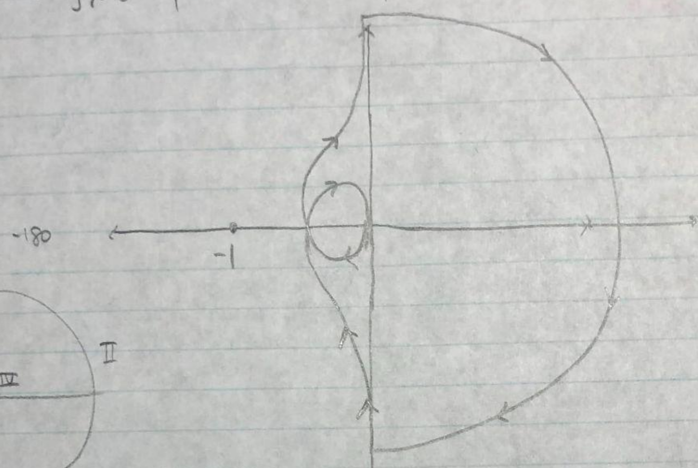
$$20\log\left(\frac{K}{2}\right) = -9.06414$$

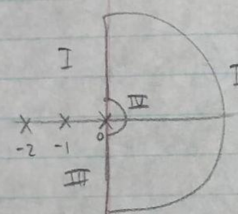
$\cancel{\frac{K}{2} =}$ $\boxed{K = 0.7044}$

$s^2 + s(\frac{1}{2}s+1$
$(\frac{1}{2}s^3 + s^2 + \frac{1}{2}s^2 + s)$
$\frac{1}{2}$ 1.5

$$G(s) = \frac{\frac{0.7044}{2}}{s(s+1)(\frac{1}{2}s+1)} = \frac{0.3522}{s(s+1)(\frac{1}{2}s+1)} = G(s)$$

c.) Nyquist plot from bode plots





$Z = N + P$

P = # RHP, unstable poles
N = # CW circlements of -1

in our case

$P = 0$
$N = 0$

if -1 was in the smaller circle, N would be equal to 2.

2.) B-7-5

$$G(s) = \frac{\omega_n^2}{s^2 + 2\zeta\omega_n s + \omega_n^2}$$

show $|G(j\omega_n)| = \frac{1}{2\zeta}$

$$G(j\omega_n) = \frac{\omega_n^2}{(j\omega_n)^2 + 2\zeta\omega_n(j\omega_n) + \omega_n^2}$$

$$\frac{\omega_n^2}{-\omega_n^2 + 2\zeta\omega_n^2 j + \omega_n^2} = \frac{\omega_n^2}{\omega_n^2(-1 + 2\zeta j + 1)} = \frac{1}{2\zeta j}$$

$$|G(j\omega_n)| = \left|\frac{1}{2\zeta j}\right| = \frac{1}{2\zeta} = |G(j\omega_n)|$$

$$\frac{\sqrt{1^2}}{\sqrt{4\zeta^2}} = \frac{1}{2\zeta}$$

3.) B7-6 unity feedback

$$G_{ol}(s) = \frac{s + 0.5}{s^3 + s^2 + 1}$$

open loop poles =

$s = -1.4656$

$s = 0.2328 \pm 0.7926j$

MATLAB attached

need to analyze $\angle G_{ol}(s)$ @ $\omega = 0$ and $\omega \to \infty$

$$G(\omega j) = \frac{j\omega + 0.5}{(j\omega)^3 + (j\omega)^2 + 1} = \frac{j\omega + 0.5}{-j\omega^3 - \omega^2 + 1} = G(j\omega)$$

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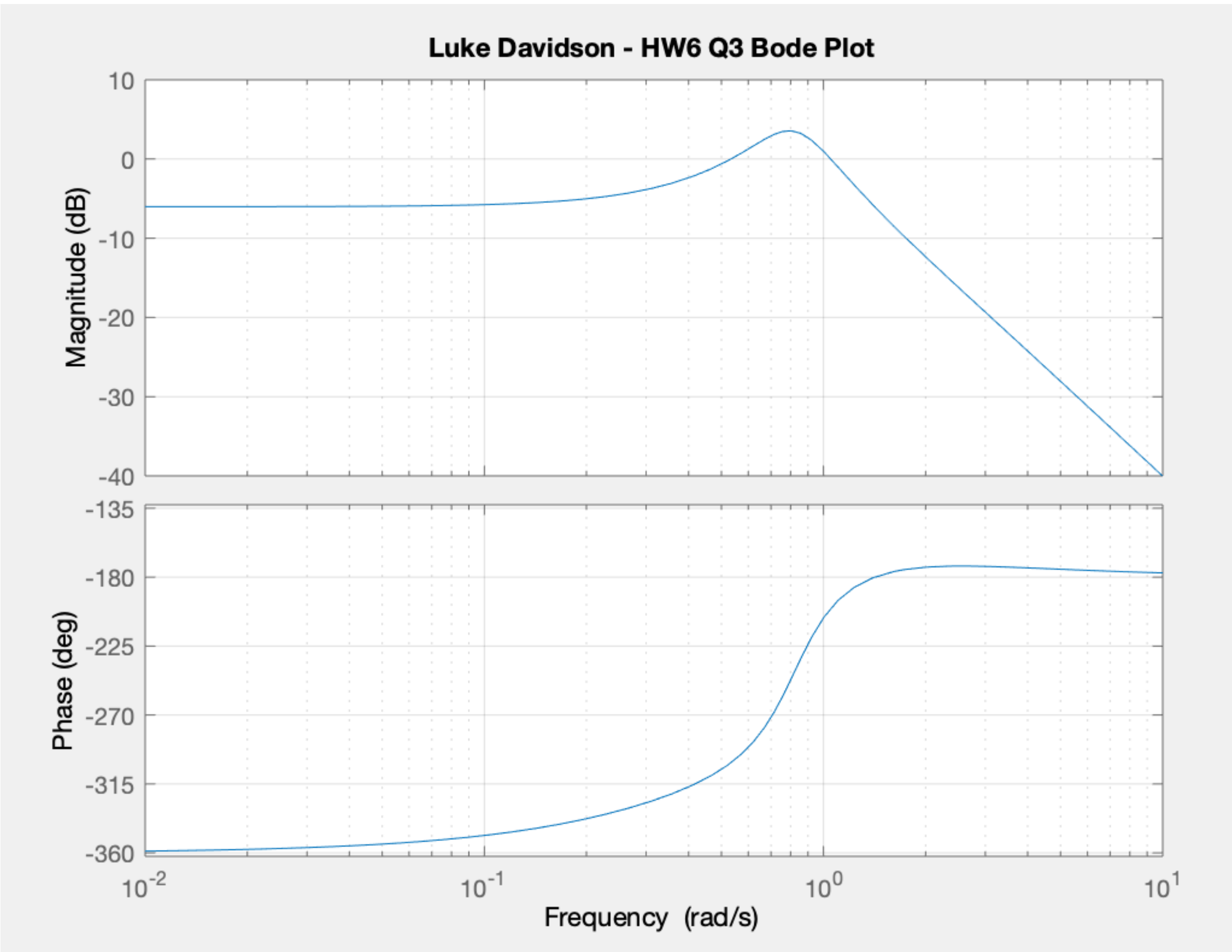
% Luke Davidson
% ME 5659
% HW 6 Q3

clc;
clear all;
close all;

G = tf([1 0.5],[1 1 0 1]);

figure(1);
bode(G);
grid on;
title('Luke Davidson - HW6 Q3 Bode Plot');
```



$$\angle G(j\omega) = \tan^{-1}\left(\frac{j}{R}\right)$$

$$\angle G(j\omega) = \tan^{-1}\left(\frac{\omega}{0.5}\right) - \tan^{-1}\left(\frac{-\omega^3}{-\omega^2+1}\right)$$

$$\angle G(0) = \tan^{-1}(0) - \tan^{-1}(0)$$

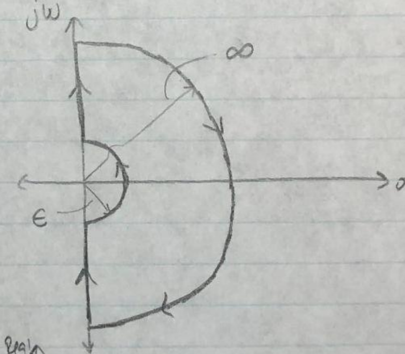
$\boxed{\angle G(0) = 0^\circ}$ ← starts @ 0° (or -360° in the attached MATLAB) ✓

$$\angle G(\infty) = \tan^{-1}\left(\frac{\infty}{0.5}\right) - \tan^{-1}\left(\frac{-\infty^3}{-\infty^2+1}\right)$$

$$= \tan^{-1}(\infty) - \tan^{-1}(-\infty)$$

$= \frac{\pi}{2} - \left(-\frac{\pi}{2}\right) = \pi = \boxed{180^\circ}$ ← should approach 180° ✓

4.) B7-15

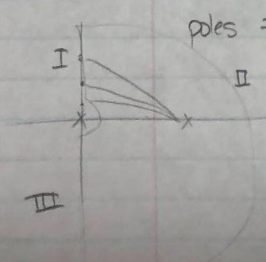


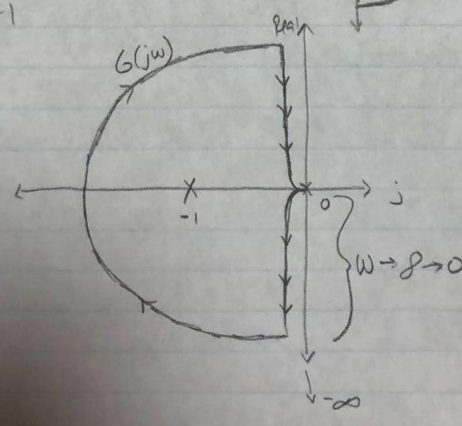
$$G(s) = \frac{1}{s(s-1)}$$

starts @

length decreases from ∞ as you go up I

poles = 0, +1





$$G(j\omega) = \frac{1}{j\omega(j\omega-1)}$$

$$G(j\omega) = \frac{1}{-\omega^2 - j\omega}$$

$$G(j\omega) = -\frac{1}{\omega^2 + j\omega}$$

$Z = N + P$

\# of CI poles (zeros of $1+L(s)$)

\# CW circles of -1

\# RHP (unstable poles) of $L(s)$

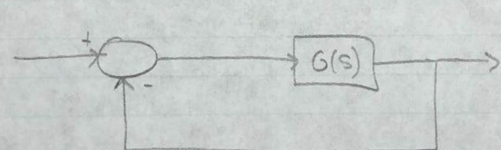
according to our plot

$\left.\begin{array}{l} N = 1 \\ P = 1 \end{array}\right\} \quad Z = 1+1 \implies \boxed{Z = 2}$

2 zeros of $1+L(s)$

so $\boxed{\text{unstable}}$

5.) B7-16



a.) $P=0$, $N =$ CW circles of -1

$N = -1+1 = 0$ so $Z = N+P = \boxed{0 = Z}$

so <u>stable</u>

b.) $P=0$, $N=2$

$Z = N+P = \boxed{2 = Z}$

so <u>unstable</u>

6.) B7-26

$$G(s) = \frac{K}{s(s^2+s+4)}$$

a.) K such that phase margin = 50°

$$\phi = -(180-50) = -130°$$

$$G(j\omega) = \frac{K}{j\omega((j\omega)^2 + j\omega + 4)}$$

$$\frac{K}{j\omega(-\omega^2 + j\omega + 4)} =$$

$$\angle G(j\omega) = \underbrace{\tan^{-1}(0)}_{K} - \underbrace{\tan^{-1}\left(\frac{\omega}{4-\omega^2}\right)}_{-\omega^2+j\omega+4} - \underbrace{90}_{j\omega}$$

$$-130 = -90 - \tan^{-1}\left(\frac{\omega}{4-\omega^2}\right)$$

$$40 = \tan^{-1}\left(\frac{\omega}{4-\omega^2}\right) \Rightarrow \quad 0.8391(4-\omega^2) = \omega$$

$$3.3564 - 0.8391\omega^2 = \omega \Rightarrow 0.8391\omega^2 + \omega - 3.3564$$

$$\omega = 1.491 \text{ rad/s}$$

$$\left|\frac{K}{j(1.491)(-(1.491)^2 + j(1.491) + 4)}\right| = 1$$

$$-3.3146j - 2.223 + 5.964j \Rightarrow \frac{K}{2.649j - 2.223} = 1$$

$$\frac{K}{\sqrt{(2.649)^2 + (2.223)^2}} = 1 \Rightarrow K = 3.459$$

range of K for which system is stable using Routh-Horwitz

$$s^3 + s^2 + 4s + K$$

$$0 < K < 4$$

$$\begin{array}{c|cc} s^3 & 1 & 4 \\ s^2 & 1 & K \\ s^1 & 4-K & \\ s^0 & K & \end{array}$$

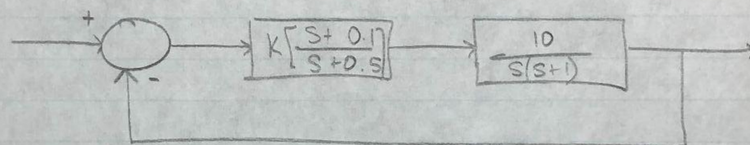
$K > 0$

$4 - K > 0$

$K < 4$

7.) B7-27

Bode Diagram



$$G(s) = K\frac{(s+0.1)(10)}{(s+0.5)(s)(s+1)} = \frac{10s+1}{s(s+1)(2s+1)}(2K)$$

MATLAB attached of Bode

phase margin = 50

$$\angle G(j\omega) = -(180-50) = -130$$

$$G(j\omega) = \frac{K(20j\omega + 2)}{(j\omega)(j\omega+1)(2j\omega+1)}$$

$$\angle G(j\omega) = \tan^{-1}\left(\frac{20\omega}{2}\right) - 90 - \tan^{-1}(\omega) - \tan^{-1}(2\omega)$$

$$-130 = \tan^{-1}(10\omega) - 90 - \tan^{-1}(\omega) - \tan^{-1}(2\omega)$$

$$-40 = \tan^{-1}\left(\frac{10\omega - \omega}{1+10\omega^2}\right) - \tan^{-1}(2\omega)$$

$$\frac{\frac{(10\omega - \omega)}{1+10\omega^2} - 2\omega}{1 + \frac{10\omega - \omega}{1+10\omega^2}(2\omega)} = \tan(-40)$$

$$\frac{9\omega - 2\omega(1+10\omega^2)}{1+10\omega^2} \cdot \frac{1+10\omega^2}{1+28\omega^2}$$

$$\frac{9\omega - 2\omega(1+10\omega^2)}{1+28\omega^2} \qquad \frac{9\omega - 2\omega - 20\omega^3}{1+28\omega^2}$$

$$\frac{7\omega - 20\omega^3}{1+28\omega^2} = \tan^{-1}(-40)$$

solving online yields $\boxed{\omega = 1.438 \text{ rad/s}}$

$$|G(j\omega)| = \frac{(20s+2)(K)}{s(s+1)(2s+1)} = \frac{(20j\omega+2)(K)}{j\omega(j\omega+1)(2j\omega+1)}$$

$$\frac{\sqrt{(20\omega)^2+4}}{\omega\sqrt{\omega^2+1}\sqrt{4\omega^2+1}} = \left.\frac{\sqrt{400\omega^2+4}}{\omega(\sqrt{\omega^2+1})(\sqrt{4\omega^2+1})}\right|_{\omega=1.438}$$

$$|G(j\omega)| = \frac{28.8295}{(1.438)(1.7515)(3.0449)} = 3.759$$

$$20\log(3.759) = \boxed{11.502}$$

$$20\log K = -11.502$$

$$K = 0.266$$

gain margin with $K=0.266$

$$= \infty$$

shown in attached MATLAB

% Luke Davidson

```
% ME 5659
% HW 6 Q7

clc;
clear all;
close all;

G = tf([20 2],[2 3 1 0]);

figure(1);
bode(G);
grid on;
title('Luke Davidson - HW6 Q7 Bode Plot');

G2 = tf([20*0.266 2*0.266],[2 3 1 0]);

figure(2);
margin(G2);
grid on;
```