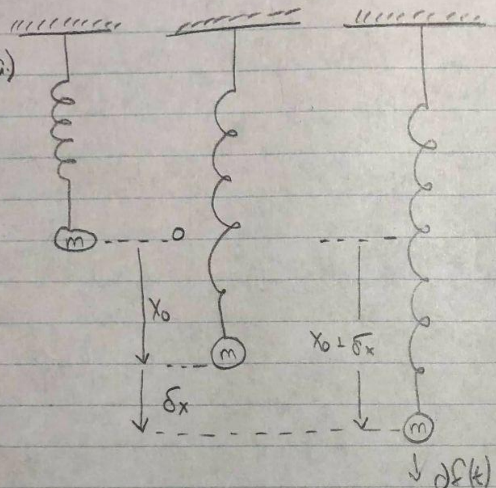
Luke Davidson
ME 5659
HW #2

1.)

a.)



$$\sum F = 0 = F_s - \delta f(t) - mg - m\ddot{y}$$

$$F_s = mg + m\ddot{\delta}_x + \delta f(t)$$

$$kx^3 = mg + m\ddot{\delta}_x + \delta f(t)$$

x_0 we have equilibrium

$$kx_0^3 = mg$$

$$x_0^3 = \frac{mg}{k} \Rightarrow x_0 = \sqrt[3]{\frac{mg}{k}}$$

$$\{x^3\} @ x_0 = x_0^3 + 3x_0^2 \delta x \qquad \left.\frac{d}{dx} f\right|_{x_0} = 3x_0^2$$

$$k(x_0^3 + 3x_0^2 \delta x) = mg + m\ddot{\delta}_x + \delta F(t)$$

$$\cancel{kx_0^3} + 3kx_0^2\delta x = \cancel{mg} + m\ddot{\delta}_x + \delta F(t)$$

$$3kx_0^2\delta x = m\ddot{\delta}_x + \delta F(t)$$

$$\delta F(t) = 3kx_0^2 \delta x + m\ddot{\delta}_x$$

$$\delta F(t) = \delta_x (3kx_0^2 + ms^2)$$

$$T(s) = \frac{\delta_x}{\delta F(t)} = \frac{1}{3kx_0^2 + ms^2} = T(s)$$

b.) linearize about $x = 2x_0$

$$\{x^3\} = (2x_0)^3 + 3(2x_0)^2\,\delta x$$

$$= 8x_0^3 + 12x_0^2\,\delta x$$

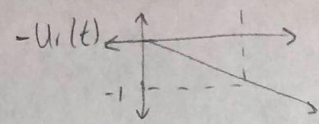
$$k(\cancel{8x_0^3} + 12x_0^2\,\delta x) = \cancel{mg} + m\ddot{\delta}_x + \cancel{F^*} + \delta F$$

$$\left.\begin{aligned} k(2x_0)^3 &= F^* + mg \\ k8x_0^3 &= F^* + mg \end{aligned}\right\}\text{@ equilibrium}$$

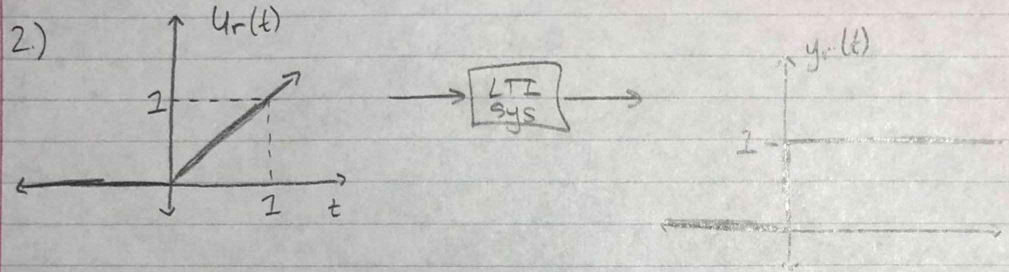
$$12kx_0^2\,\delta x = m\ddot{\delta x} + \delta F$$

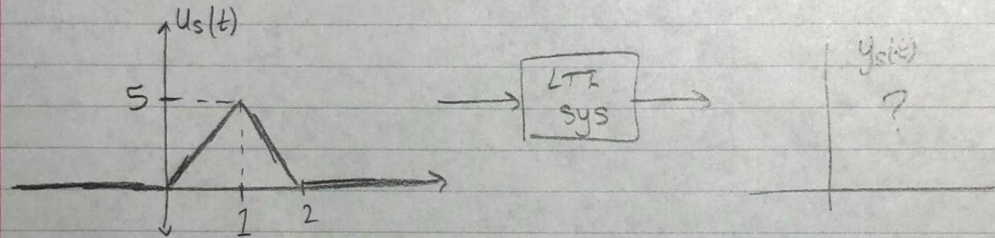
$$\delta F = 12kx_0^2\,\delta x - m\ddot{\delta x}$$

This is different than part (a) b/c if we treat $2x_0$ as the equilibrium point, we will have to substitute $2x_0$ in to our Taylor series, which will scale our $kx_0^2\delta x$ factor by a factor of 2^2.



2.)





a) $u_s = r_1 + r_2 + r_3$

$r_1 = 5\,u_r(t)$
$r_2 = -10\,u_r(t-1)$
$r_3 = 5\,u_r(t-2)$

from t $0 \to 1$, $u_s(t) = 5\,u_r(t)$

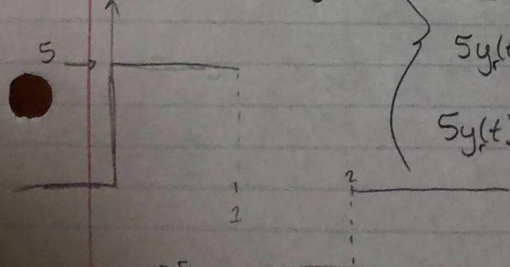
from t $1 \to 2$ total $= -5 = 5 + x$ $x = -10$

from $t \geq 2$ total $= 0 = -5 + x$ $x = 5$

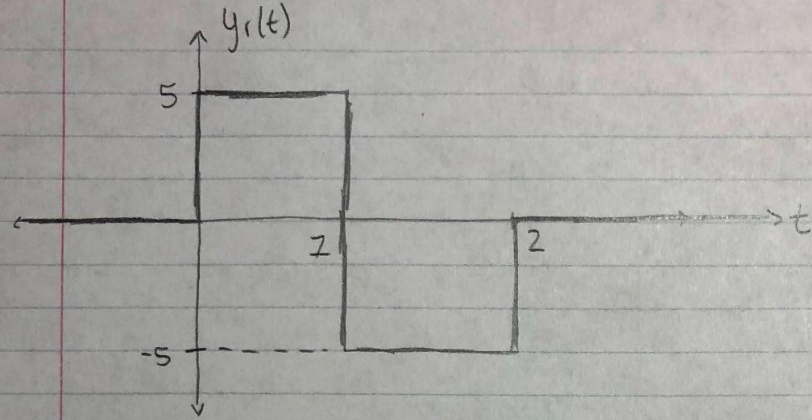
$u_s(t) = 5\,u_r(t) - 10\,u_r(t-1) + 5\,u_r(t-2)$

b.) $y_s(t) = y_r(t)\,5u_r(t) - 10\,y_r(t)\,u_r(t-1) + 5u_r(t-2)\,y_r(t)$

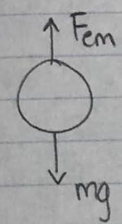
or $y_s(t) = \begin{cases} 5y_r(t) & 0 \leq t < 1 \\ 5y_r(t) - 10y_r(t-1) & 1 \leq t < 2 \\ 5y_r(t) - 10y_r(t-1) + 5y_r(t-2) & 2 \leq t \end{cases}$

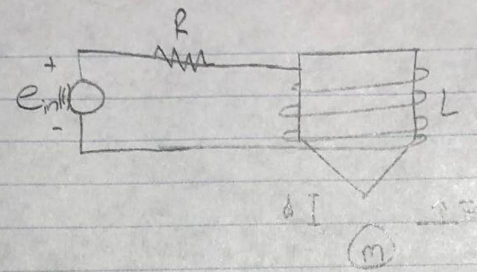


graph on BACK


$y_1(t)$
5
-5
1
2
t

3.)





$$F_{em} = \frac{K_f I^2}{(d-z)^2}$$

a) $$\frac{K_f I^2}{d^2} = mg$$

$z=0$ for nominal / equilibrium

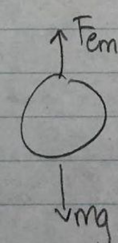
$$\frac{(2.6487 \times 10^{-5})(0.8)^2}{d^2} = 0.003(9.8)$$

$$d = \sqrt{\frac{(2.6487 \times 10^{-5})(0.8)^2}{0.003(9.8)}} = 0.024\,m = d$$

b.) $v = v^* + \partial v \qquad z = z^* + \partial z$

$V = 4 + \partial v$

$\boxed{mg = F_{em}}$



output = z

$V = 4 + \partial V$

$z = z^* + \partial z = .024 + \partial z$

$$\Rightarrow \frac{v^2}{(d-z)^2} = \frac{v^{*2}}{(d-z^*)^2} + \frac{\partial}{\partial v}\left(\frac{v^{*}}{(d-z^*)^2}\right) + \frac{\partial}{\partial z}\left(\frac{v^{*2}}{(d-z^*)^2}\right)$$

$$\frac{v^{*2}}{(d-z^*)^2} + \frac{2v^*}{(d-z^*)^2}\,\partial v - \frac{2v^{*2}}{(d-z)^3}\,\partial z$$

$$\Sigma F = ma_z + mg = F_{em}$$

$$m\ddot{dz} = \frac{K_f}{R^2}\left(\cancel{\frac{v^{*2}}{(d-z^*)^2}}^{\,0} + \frac{2v^*}{(d-z^*)^2}dv - \frac{2v^{*2}}{(d-z^*)^3}dz \right) - \cancel{mg}^{\,0}$$

$$m\ddot{dz} + \frac{2v^{*2}}{(d-z)^3}dz = \frac{2K_f v^*}{R^2(d-z)^2}dv$$

The mg will get cancelled b/c of the initial correlation $mg = F_{em}$. a constant (mg) will cause an ODE to not have homogeneity and therefore will normally get eliminated.

4.) $J\ddot{\theta} + b\dot{\theta} = K_t i$ input $u = V_s$

$L\dot{i} + Ri + K_b\dot{\theta} = V_s$ output $y = \theta$

a.) $\begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} \theta \\ \dot{\theta} \\ i \end{bmatrix}$ $x_2 = \dot{x}_1$

$$\begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \\ \dot{x}_3 \end{bmatrix} = \begin{bmatrix} \dot{\theta} \\ \ddot{\theta} \\ \dot{i} \end{bmatrix} = \overset{A}{\begin{bmatrix} 0 & 1 & 0 \\ 0 & -b/J & K_t/J \\ 0 & -K_b/L & -R/L \end{bmatrix}} \begin{bmatrix} \theta \\ \dot{\theta} \\ i \end{bmatrix} + \overset{B}{\begin{bmatrix} 0 \\ 0 \\ 1/L \end{bmatrix}} V_s$$

$$\ddot{\theta} = \frac{K_t i}{J} - \frac{b\dot{\theta}}{J}$$

θ

$$\dot{i} = \frac{V_s - K_b\dot{\theta} - Ri}{L}$$

b.)

$$\Theta(s)(Js^2 + bs) = K_t I(s)$$

$$I(s)(Ls + R) + \Theta(s)(K_b s) = V(s)$$

$$\frac{\Theta(s)(Js^2 + bs)}{K_t} = \frac{V(s) - \Theta(s)(K_b s)}{Ls + R}$$

$$\Theta(s)\frac{(Js^2 + bs)(Ls+R)}{K_t} = (V(s) - \Theta(s)(K_b s))$$

$$\Theta(s)\left(\frac{(Js^2 + bs)(Ls+R)}{K_t} + K_b(s)\right) = V(s)$$

$$V_s = \frac{(J\ddot{\theta} + b\dot{\theta})(L\dot{\theta} + R)}{K_t} + K_b\dot{\theta} = \frac{LJ\dddot{\theta} + RJ\ddot{\theta} + Lb\ddot{\theta} + b\dot{\theta}R}{K_t} + K_b\dot{\theta}$$

$$\Rightarrow V_s = \frac{LJ}{K_t}\dddot{\theta} + \left(\frac{RJ+Lb}{K_t}\right)\ddot{\theta} + \left(\frac{Rb}{K_t} + K_b\right)\dot{\theta}$$

$$\begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} \theta \\ \dot{\theta} \\ \ddot{\theta} \end{bmatrix}$$

$$\begin{bmatrix} \dot{\theta} \\ \ddot{\theta} \\ \dddot{\theta} \end{bmatrix} = \overbrace{\begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 0 & -\left(\frac{-Rb}{K_t} + K_b\right) & -\left(\frac{RJ+Lb}{K_t}\right) \end{bmatrix}}^{A} \begin{bmatrix} \theta \\ \dot{\theta} \\ \ddot{\theta} \end{bmatrix} + \overbrace{\begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix}}^{B} V_s$$

$$\dddot{\theta} = V_s - \left(\frac{RJ+Lb}{K_t}\right)\ddot{\theta} - \left(\frac{Rb}{K_t} + K_b\right)\dot{\theta}$$

d.) Both have zeroes in first column because that column corresponds to θ, which is not present in either of the equations.

To add a θ, we could introduce an outside <u>torque</u>.

e.) attached code.

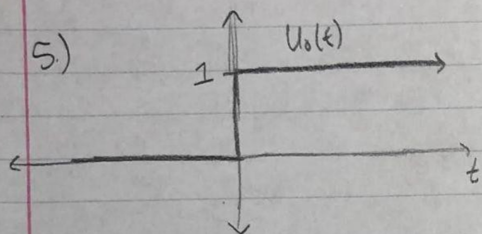
4e.) MATLAB code and output

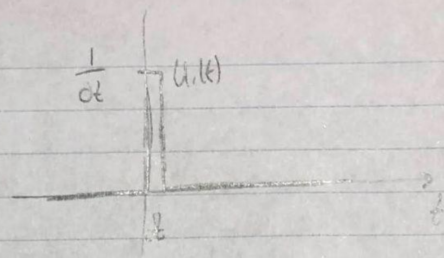
```
% Luke Davidson
% ME 5659
% HW2 Q4e

Aa = [0 1 0; 0 -1 1; 0 -1 -1];

eig(Aa)

Ac = [0 1 0; 0 0 1; 0 -2 -2];

eig(Ac)
```

Output:

```
>> ME5659_HW2_Q4e

ans =

   0.0000 + 0.0000i
  -1.0000 + 1.0000i
  -1.0000 - 1.0000i


ans =

   0.0000 + 0.0000i
  -1.0000 + 1.0000i
  -1.0000 - 1.0000i
```

5.)





$y_0(t) = T(s)\,U_0(t)$

$y_1(t) = T(s)\,U_1(t)$

a.)

$$U_1(t) = \frac{1}{\delta t} U_0(t) - \frac{1}{\delta t} U_0(t - \partial t)$$

$$y_1(t) = y_0(t)\left[(1/\delta t)U_0(t) - (1/\delta t)U_0(t - \delta t)\right]$$

$$y_1(t) = \frac{1}{\delta t} y_0(t) - \frac{1}{\delta t} y_0(t - \delta t)$$

b.) $y_0(t) = 1 - e^{-t} = 1 - \frac{1}{e^t}$

$1 - \frac{1}{e^t} \longrightarrow \boxed{1}$

what is $y_1(t)$ for $\partial t \to 0$

$$y_1(t) = \frac{1 - e^{-t}}{\delta t} - \frac{1 - e^{-(t - \partial t)}}{\partial t} =$$

$$\frac{1}{\partial t}\left(1 - e^{-t} - \left(1 - e^{-t} e^{\partial t}\right)\right) = \frac{1}{\partial t}\left(1 - e^{-t} - 1 + e^{-t} e^{\partial t}\right)$$

$$= \frac{1}{\partial t}\left(-e^{-t} + e^{-t} e^{\partial t}\right) = \frac{e^{-t}}{\partial t}\left(e^{\partial t} - 1\right)$$

$$\lim_{\partial t \to 0} \frac{\partial}{\cancel{\partial t}}\left(\frac{e^{-t}}{\partial t}\left(e^{\partial t} - 1\right)\right)$$

$$\Longrightarrow \lim_{\partial t \to 0} \left(e^{-t}\right)\left(\frac{1 + \partial t + \partial t/2 \ldots}{\partial t}\right) \longrightarrow e^{-t}$$