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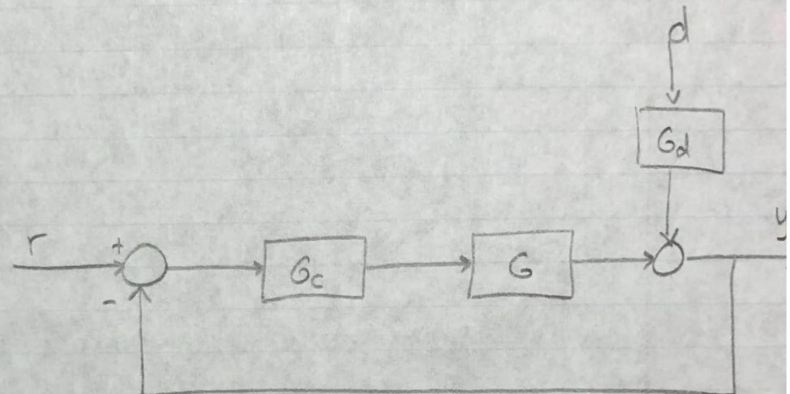
ME 5659
HW #9

1.) $G(s) = \dfrac{200}{(10s+1)(0.05s+1)^2}$ $\qquad G_d(s) = \dfrac{100}{10s+1}$

overshoot < 5%
$t_p \approx 0.3s$
$M_s < 1.5$
phase M > 45°

a) $K(s) = k_p$



$$Y(s) = \frac{G_c G}{1+G_c G}(R) + \frac{G_d}{1+G_c G} D(s)$$

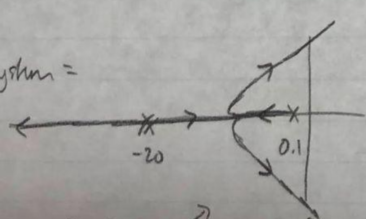
$$\boxed{Y(s) = \frac{KG}{1+KG} R(s) + \frac{G_d}{1+KG} D(s)}$$

$$\text{\% overshoot} = e^{\frac{-\pi \xi}{\sqrt{1-\xi^2}}}$$

$$\boxed{t_p = \frac{\pi}{\omega_d}}$$

$$\boxed{\omega_d = \omega_n \sqrt{1-\xi^2}}$$

system =



$n-m = 3 \Rightarrow$

$$0.3s = \frac{\pi}{\omega_n \sqrt{1-\xi^2}}$$

$$0.05 = e^{\frac{-\pi \xi}{\sqrt{1-\xi^2}}}$$

$$\ln(0.05)\left(\sqrt{1-\xi^2}\right) = -\pi \xi$$

$$[\ln(0.05)]^2 (1-\xi^2) = \pi^2 \xi^2$$

$$\left(\pi^2 + [\ln(0.05)]^2\right)\xi^2 = [\ln(0.05)]^2$$

$$\xi = \sqrt{\frac{(\ln 0.05)^2}{\pi^2 + [\ln(0.05)]^2}} = 0.69$$

$$\boxed{\xi = 0.69}$$

$$t_p = \frac{\pi}{\omega_n\sqrt{1-\zeta^2}} \qquad 0.35 = \frac{\pi}{\omega_n\sqrt{1-.69^2}}$$

$$\omega_n = \frac{\pi}{(0.35)(\sqrt{1-.69^2})} = \boxed{12.4 = \omega_n} \quad , \boxed{\omega_n < 12.4}$$

~~(10s +1)~~

$$\frac{200}{(10s+1)(0.05s+1)^2} K_p + 1$$

$$(10s+1)(0.05s+1)^2 + 200K_p$$

$$\boxed{K_p > 0}$$

$$(10s+1)(.0025s^2 + 0.1s + 1) + 200K_p$$

$$.025s^3 + s^2 + 10s + .0025s^2 + 0.1s + 1 + 200K_p$$

$$.025s^3 + 1.0025s^2 + 10.1s + 1 + 200K_p$$

<u>RH</u>

.025	10.1	200Kp
1.0025	1	
	$\boxed{200}K_p$	

$$\frac{10.12525 - .025}{1.0025} = \boxed{10.075} \qquad \frac{(1.0025)(200K_p) - }{1.0025} = 200K_p$$

$$\frac{(10.075)(1) - 1.0025(200K_p)}{10.075} > 0 =$$

$$1 - 19.9K_p > 0$$

$$K_p < \frac{1}{19.9} = \quad K_p < 0.05$$

requirements ① and ② contradict. peak time will increase as % overshoot decreases, and vice versa.

2) $\dfrac{Y(s)}{U(s)} = \dfrac{10}{(s+1)(s+2)(s+3)}$ $\quad$ $x_1 = y$, $\quad x_2 = \dot{x}_1 = \dot{y}$, $\quad x_3 = \dot{x}_2 = \ddot{x}_1 = \ddot{y}$

$$Y(s)\left[(s^2+3s+2)(s+3)\right] = 10(U(s))$$

$$Y(s)\left[s^3+3s^2+3s^2+6s+2s+6\right] = 10(U(s))$$

$$\dddot{y} + 6\ddot{y} + 11\dot{y} + 6y = 10u$$

$$\begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \\ \dot{x}_3 \end{bmatrix} = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ -6 & -11 & -6 \end{bmatrix}\begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} + \begin{bmatrix} 0 \\ 0 \\ 10 \end{bmatrix} u \qquad y = \begin{bmatrix} 1 & 0 & 0 \end{bmatrix}\begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} + \underline{[0]}\, D$$

$$\dot{x}_3 = \dddot{y} = -6\ddot{y} - 11\dot{y} - 6y + 10u$$

$$\dddot{y} = -6x_3 - 11x_2 - 6x_1 + 10u$$

$$A = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ -6 & -11 & -6 \end{bmatrix}; \quad B = \begin{bmatrix} 0 \\ 0 \\ 10 \end{bmatrix}; \quad C = \begin{bmatrix} 1 & 0 & 0 \end{bmatrix}; \quad D = \underline{[0]}$$

$$A_{cl} = A - BK = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ -6 & -11 & -6 \end{bmatrix} - \begin{bmatrix} 0 \\ 0 \\ 10 \end{bmatrix}\begin{bmatrix} k_1 & k_2 & k_3 \end{bmatrix}$$

$$A_{cl} = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ -6-10k_1 & -11-10k_2 & -6-10k_3 \end{bmatrix}$$

$$|\lambda I - A_{cl}| = \left|\begin{bmatrix} \lambda & 0 & 0 \\ 0 & \lambda & 0 \\ 0 & 0 & \lambda \end{bmatrix} - \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ -6-10k_1 & -11-10k_2 & -6-10k_3 \end{bmatrix}\right|$$

$$= \begin{vmatrix} \lambda & -1 & 0 \\ 0 & \lambda & -1 \\ 6+10k_1 & 11+10k_2 & \lambda+6+10k_3 \end{vmatrix} = 0$$

$$\lambda \begin{vmatrix} \lambda & -1 \\ 11+10k_2 & \lambda+6+10k_3 \end{vmatrix} + \begin{vmatrix} 0 & -1 \\ 6+10k_1 & \lambda+6+10k_3 \end{vmatrix}$$

$$\lambda\Big(\lambda(\lambda+6+10k_3) + 11+10k_2\Big) + 6+10k_1 = 0$$

$$\lambda\left(\lambda^2 + 6\lambda + 10\lambda k_3 + 11 + 10k_2\right) + 6 + 10k_1 = 0$$

$$\lambda^3 + 6\lambda^2 + 10\lambda^2 k_3 + 11\lambda + 10\lambda k_2 + 6 + 10k_1 = 0$$

$$(\lambda^3) + (\lambda^2)(6+10k_3) + (\lambda)\left[11 + 10k_2\right] + (6+10k_1) = 0$$

$$\frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

let $a = 0.5$
$b = 2$

$b^2 - 4ac = -12$
$4 - 4(0.5)c = -12$
$4 - 2c = -12$
$-2c = -16$
$\boxed{c = 8}$

$0.5x^2 + 2x + 8$
$\boxed{x^2 + 4x + 16}$

$$(\lambda+10)(\lambda^2 + 4\lambda + 16) = \lambda^3 + 4\lambda^2 + 16\lambda + 10\lambda^2 + 40\lambda + 160 =$$

$$\lambda^3 + 14\lambda^2 + 56\lambda + 160 =$$

$6 + 10k_3 = 14$ | $11 + 10k_2 = 56$ | $6 + 10k_1 = 160$
$k_3 = 0.8$ | $k_2 = 4.5$ | $k_1 = 15.4$

$$K = \begin{bmatrix} 15.4 & 4.5 & 0.8 \end{bmatrix}$$