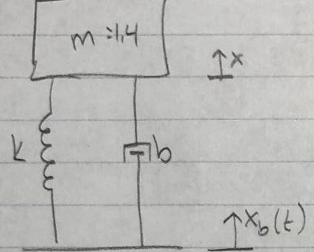
Luke Davidson
ME 5659
Hw #5

1.)



$x_b(t) = 0.5\sin(\omega t)$ cm

$\dot{x}_b(t) = 0.5\cos(\omega t)$

$X = x - x_b$

$\sum F_y = 0 = -m\ddot{x} - kX - b\dot{X}$

$= 0 = -m\ddot{x} - k(x-x_b) - b(\dot{x}-\dot{x}_b)$

$m\ddot{x} = -kx + kx_b - b\dot{x} + b\dot{x}_b$

X_{ss} = amplitude

$m\ddot{x} + kx + b\dot{x} = kx_b + b\dot{x}_b$

$\Rightarrow m\ddot{x} + kx + b\dot{x} = k(0.5\sin(\omega t)) + b(0.5\cos(\omega t))$ ①

Found online → for $mx'' + bx' + kx = B\cos\omega t$

mit.edu/jorloff/suppnotes

$$x_p = \frac{B}{\sqrt{(k-m\omega^2)^2 + b^2\omega^2}}\cos(\omega t - \phi) \qquad \phi = \tan^{-1}\left(\frac{b\omega}{k-m\omega^2}\right)$$

same for sin

apply that to ①

$$X_{ss} = \frac{k0.5}{\sqrt{(k-m\omega^2)^2 + b^2\omega^2}}\sin(\omega t - \phi) + \frac{b\,0.5}{\sqrt{(k-m\omega^2)^2 + b^2\omega^2}}\cos(\omega t - \phi)$$

$$\text{amp } |X_{ss}| = \sqrt{\sum x^2} = \sqrt{\left(\frac{0.5\,\omega b}{\sqrt{(k-m\omega^2)^2 + b^2\omega^2}}\right)^2 + \left(\frac{0.5k}{\sqrt{(k-m\omega^2)^2 + b^2\omega^2}}\right)^2}$$

$$X_{ss} = \left(\frac{\omega^2 b^2 + k^2}{(k - m\omega^2)^2 + b^2\omega^2} \right)^{1/2} 0.5$$

for $\omega = 30$, $X_{ss} = 0.7008$

① $$0.7008 = \left(\frac{(30)^2 b^2 + k^2}{(k - (1.4)(30)^2)^2 + b^2(30^2)} \right)^{1/2} 0.5$$

for $\omega = 5$, $X_{ss} = 0.5070$

② $$0.5070 = \left(\frac{5^2 b^2 + k^2}{(k - (1.4)(5^2))^2 + b^2(5^2)} \right)^{1/2} 0.5$$

$$\left(\frac{X_{ss}}{0.5}\right)^2 = \frac{\omega^2 b^2 + k^2}{(k - m\omega^2)^2 + b^2\omega^2} = (2x_{ss})^2 \left[(k - m\omega^2)^2 + b^2\omega^2\right] = \omega^2 b^2 + k^2$$

$$4x_{ss}^2 \left[k^2 - 2km\omega^2 + m^2\omega^4 + b^2\omega^2 \right] = \omega^2 b^2 + k^2$$

$$4x^2k^2 - 8x^2km\omega^2 + 4x^2m^2\omega^4 + 4x^2b^2\omega^2 = \omega^2 b^2 + k^2$$

$$\frac{4x^2k^2 - 8x^2km\omega^2 + 4x^2m^2\omega^4 - k^2}{\omega^2 - 4x^2\omega^2} = b^2$$

equate for ① $\omega = 30$, $x = .7008$

② $\omega = 5$, $x = .507$

$$\left(4(.7008)^2k^2 - 8(.7008)^2 k(1.4)(900) + 4(.7008)^2(1.4)^2(30)^4 - k^2\right) \frac{1}{900 - 4(.7008)^2(900)}$$

$-0.04\,k^2 + 102.1049\,k - 17868 =$

$-0.0011\,k^2 + 5.7031\,k - 35930$

$\Rightarrow k = 2532.8\ \text{N/m}$

$k = 2532.8 \longrightarrow b = 61.02\ \text{Ns/m}$

MatLAB attached

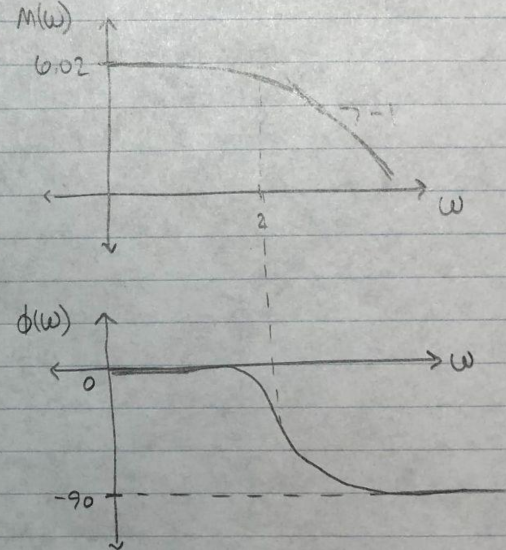
```
% Luke Davidson
% ME 5659
% HW 5

clc;
clear all;
close all;

x = 0.7008;
w = 30;
k = 2532.8;

denom = (w^2)-4*(x^2)*(w^2);
a = ((4*x^2)-1)*k^2;
b = (-(8*x^2)*(1.4)*(w^2))*k;
c = 4*(x^2)*(1.4)^2*(w^4);
a = a/denom;
b = b/denom;
c = c/denom;
bb = (a+b+c)^0.5;
disp(bb)


% Answer1= ((-b)+((b^2-4*a*c))^0.5)/(2*a);
% Answer2= ((-b)-((b^2-4*a*c))^0.5)/(2*a);
% disp(Answer1)
% disp(Answer2)
%
% a = -.04+.0011;
% b = 102.1049-5.7031;
% c = 1786.8+3593;
%
% Answer1= ((-b)+((b^2-4*a*c))^0.5)/(2*a);
% Answer2= ((-b)-((b^2-4*a*c))^0.5)/(2*a);
% disp(Answer1)
% disp(Answer2)
```

2) $G_1(s) = \dfrac{4}{s+2}$ $\qquad G_2(s) = \dfrac{4}{(0.4s+1)(s+1)}$ $\qquad G_3(s) = \dfrac{8}{s(1.25s+1)(s+2)}$

a) $G_1(s) = \dfrac{4(1)}{2(\frac{1}{2}s+1)} = 2\left(\dfrac{1}{\frac{1}{2}s+1}\right) = 2\left(\dfrac{1}{\frac{s}{2}+1}\right)$

$20 \log(2) = \boxed{6.02}$

$\omega = 2$



$G_2(s) = \dfrac{4}{0.4s^2 + 1.4s + 1}$

$\dfrac{4}{0.4(s^2 + \frac{1.4}{0.4}s + \frac{1}{0.4})}$

$\dfrac{4}{0.4(s^2 + 3.5s + 2.5)}$

$\dfrac{10}{s^2 + 3.5s + 2.5}$ $\qquad \omega_n = \sqrt{2.5} = 1.58$ $\qquad \boxed{K=4}$ $\qquad$ graph on BACK

$2\zeta(1.58) = 3.5$

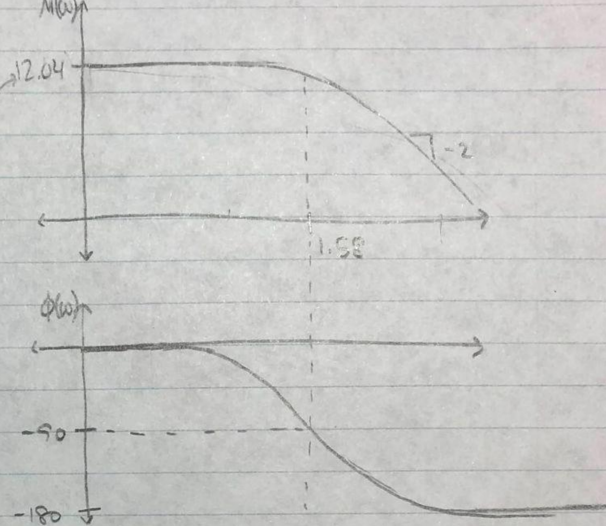
$4\left(\dfrac{2.5}{s^2 + 3.5s + 2.5}\right)$ $\qquad \zeta = 1.108$ which is $> \frac{\sqrt{2}}{2}$ so won't resonate

$G_2(s) = 4\left(\frac{2.5}{s^2 + 3.5s + 2.5}\right)$

$(s + 2.5)(s + 1)$

$20\log(4) = 12.04$

$\omega_n = \sqrt{2.5} = 1.58$



$G_3(s) = \frac{8}{s(1.25s+1)(s+2)}$

$8\left(\frac{1}{s}\right)\left(\frac{1}{1.25s+1}\right)\left(\frac{1}{s+2}\right)$

$\boxed{p = 0,\ -0.8,\ -2}$

$8\left(\frac{1}{s}\right)\left(\frac{1}{1.25s+1}\right)\left(\frac{0.5}{\frac{1}{2}s+1}\right)$

$\frac{8}{2(s)(1.25s+1)(\frac{1}{2}s+1)}$

$4\left(\frac{1}{s}\right)\left(\frac{1}{1.25s+1}\right)\left(\frac{1}{\frac{1}{2}s+1}\right)$

$\frac{4}{s(1.25s+1)(0.5s+1)}$

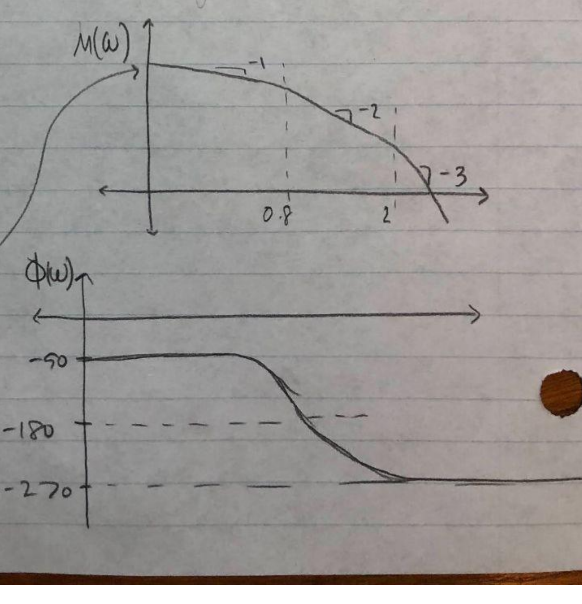
$\tau = 0.5$

$\omega_c = 2$

$\tau = 1.25$

$\omega =$

$k \to \infty$



b.) $G(s) = K \dfrac{(s+2)}{s(s+1)(s+10)}$ zero @ -2
pole @ $0, -1, -10$

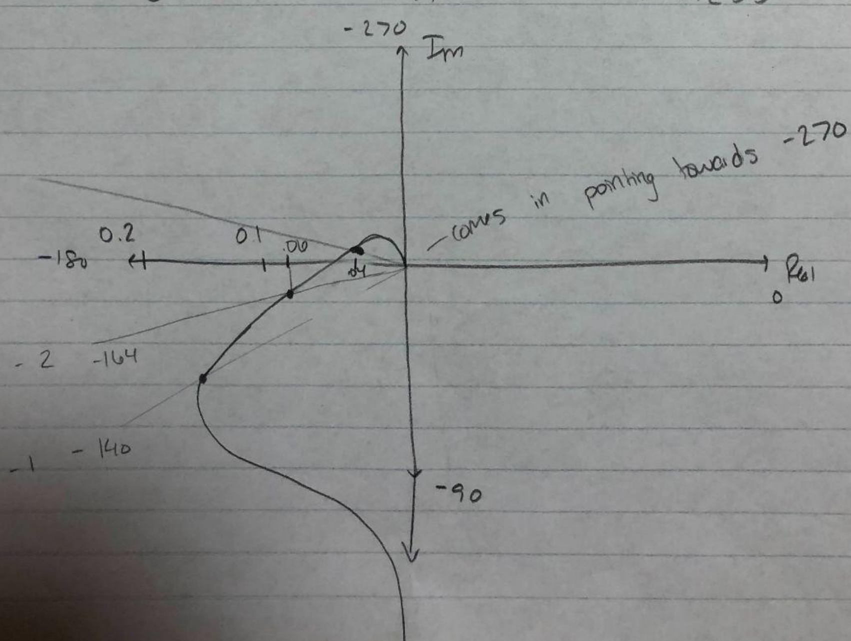
$$G(j\omega) = \frac{(j\omega + 2)}{j\omega(j\omega+1)(j\omega+10)}$$

$|G(j\omega)| \angle G(j\omega)$

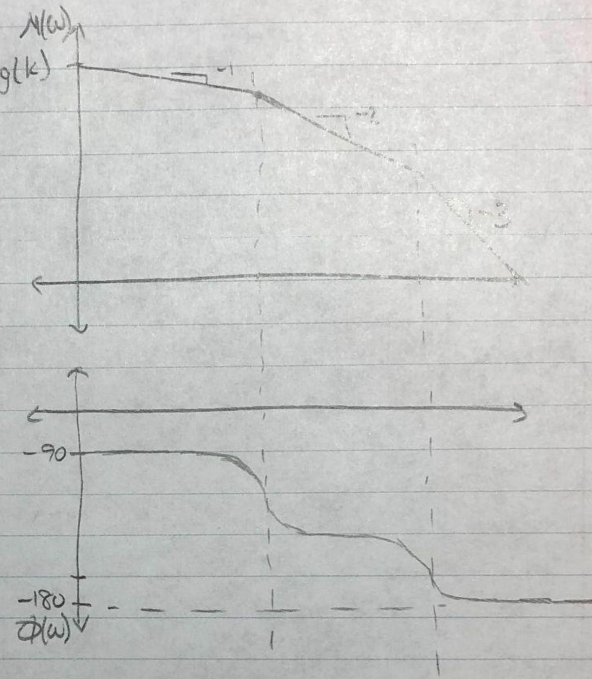
$$\frac{\sqrt{\omega^2+4}}{\omega\left(\sqrt{\omega^2+1}\right)\left(\sqrt{\omega^2+100}\right)}$$
$(10)(0)$

$$\phi = -90 - \tan^{-1}(\omega) - \tan^{-1}\left(\frac{\omega}{10}\right)$$

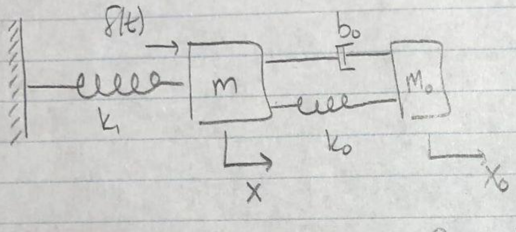
@ ω	$M(\omega)$	$\phi(\omega)$
0	∞	-90 or 270
∞	0	-270 or 90
1	0.1573	-140.7
2	0.062	-164.7
5	.047	-195.255



$$G(s) = \frac{K\,(s+2)}{s(s+1)(s+10)}$$



3.)

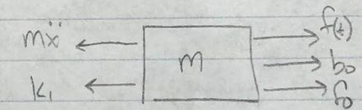


$G(s) = \frac{X(s)}{F(s)}$

a.) $m = 10$
$m_o = 1$
$k = 100$
$k_o = 10$

bode plots for $b_o = \{0.1, 1, 10\}$

super impose $k_o = 1000$
$b_o = 1000$



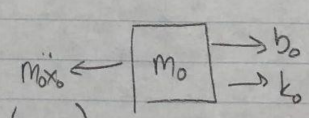
$$\Sigma F_{x_m} = m\ddot{x} + k_1(x) = f(t) + b_o(\dot{x}_o - \dot{x}) + k_o(x_o - x)$$

$$= m\ddot{x} + k_1 x = f(t) + b_o\dot{x}_o - b_o\dot{x} + k_o x_o - k_o x$$

$$m\ddot{x} + k_1 x + b_o\dot{x} + k_o x = f(t) + b_o\dot{x}_o + k_o x_o$$

$$X(s)\left(ms^2 + k_1 + b_o s + k_o\right) = F(s) + X_o(s)\left(b_o s + k_o\right)$$

$\Sigma F_{x\,m_o}$



$$\Sigma F_{x\,m_o} = m_o\ddot{x}_o = b_o(\dot{x} - \dot{x}_o) + k_o(x - x_o)$$

$$m_o\ddot{x}_o = b_o\dot{x} - b_o\dot{x}_o + k_o x - k_o x_o$$

$$m_o\ddot{x}_o + b_o\dot{x}_o + k_o x_o = b_o\dot{x} + k_o x$$

$$X_o(s)\left(m_o s^2 + b_o s + k_o\right) = X(s)\left(b_o s + k_o\right)$$

$$X_o(s) = X(s)\left(\frac{b_o s + k_o}{m_o s^2 + b_o s + k_o}\right)$$

substitute this in to get X(s) and F(s)

$$X(s)\left(ms^2+k_1+b_0s+k_0\right)=F(s)+X(s)\left(\frac{b_0s+k_0}{m_0s^2+b_0s+k_0}\right)\left(b_0s+k_0\right)$$

$$X(s)\left[ms^2+k_1+b_0s+k_0-\left(\frac{(b_0s+k_0)^2}{m_0s^2+b_0s+k_0}\right)\right]=F(s)$$

$$G(s)=\frac{X(s)}{F(s)} \Longrightarrow \frac{1}{\{\ \}}$$

simplified online using wolfram alpha

$$\frac{1}{\frac{(ms^2+b_0s+(k_1+k_0))(m_0s^2+b_0s+k_0)-(b_0s+k_0)^2}{m_0s^2+b_0s+k_0}}$$

$m=10,\ m_0=1$
$k_1=100,\ k_0=10$

num $=(10s^2+100+b_0s+10)(s^2+b_0s+10)-(b_0s+10)^2$

$=10s^4+10b_0s^3+100s^2+100s^2+100b_0s+1000+b_0s^3+\cancel{b_0^2s^2}+10b_0s+10s^2$

$+10b_0s\ \cancel{+100}\ \cancel{-b_0^2s^2}-20b_0s\ \cancel{-100}$

$=10s^4+(10b_0+b_0)s^3+(100+100+10)s^2+(100b_0+10b_0+10b_0-20b_0)s$
$+1000$

$$\Longrightarrow \frac{s^2+b_0s+10}{10s^4+11b_0s^3+210s^2+100b_0s+1000}=G(s)$$

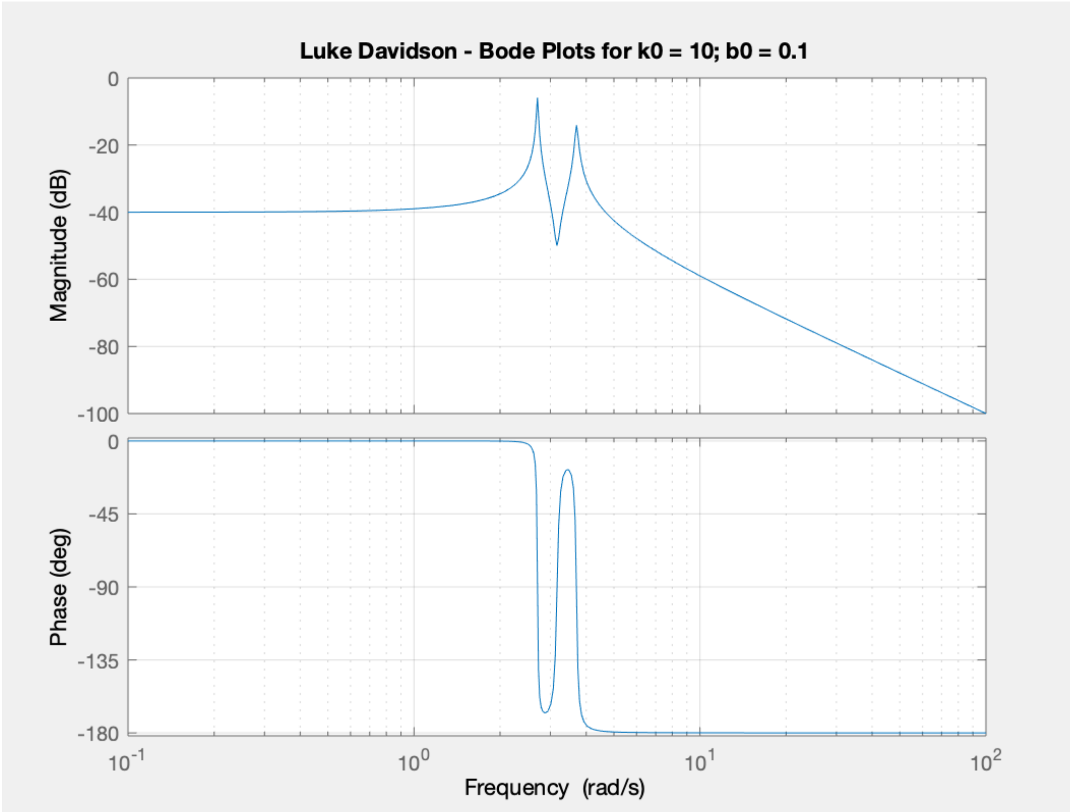
$b_0=1000,\ k_0=1000$ solved for online w/ wolfram alpha

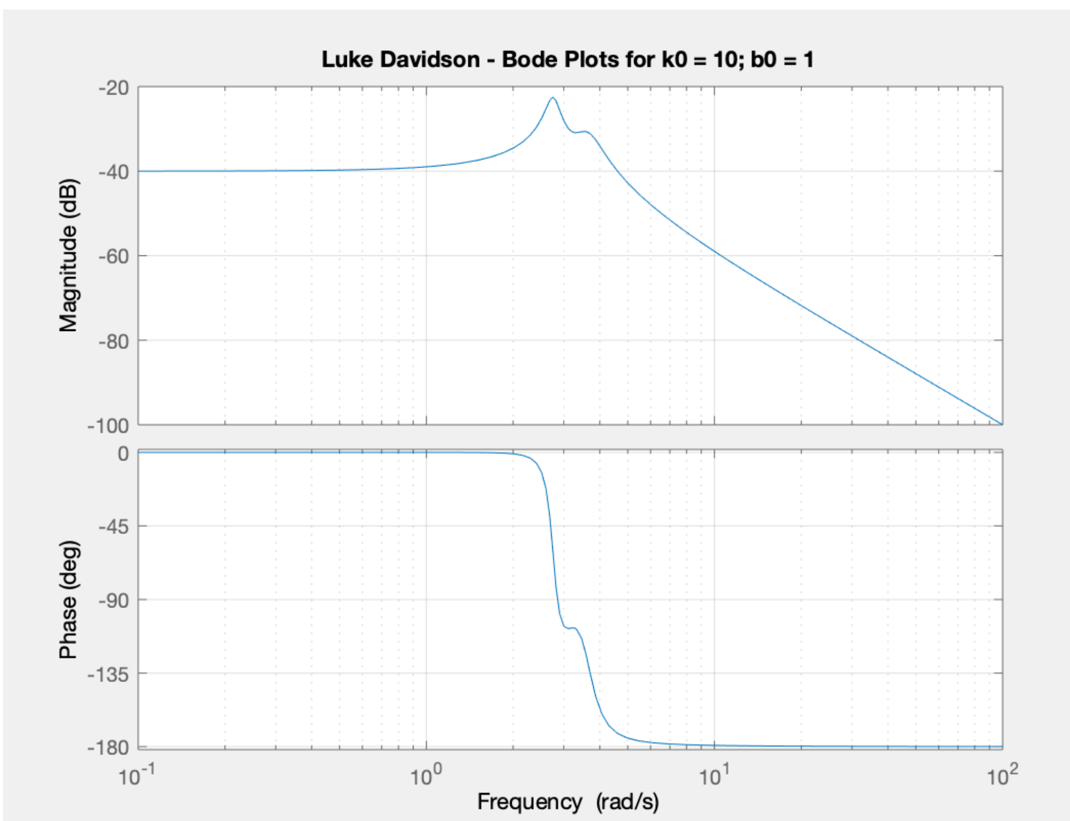
$$G(s)=\frac{s^2+1000s+10}{10s^4+11000s^3+11100s^2+100000s+100000}$$

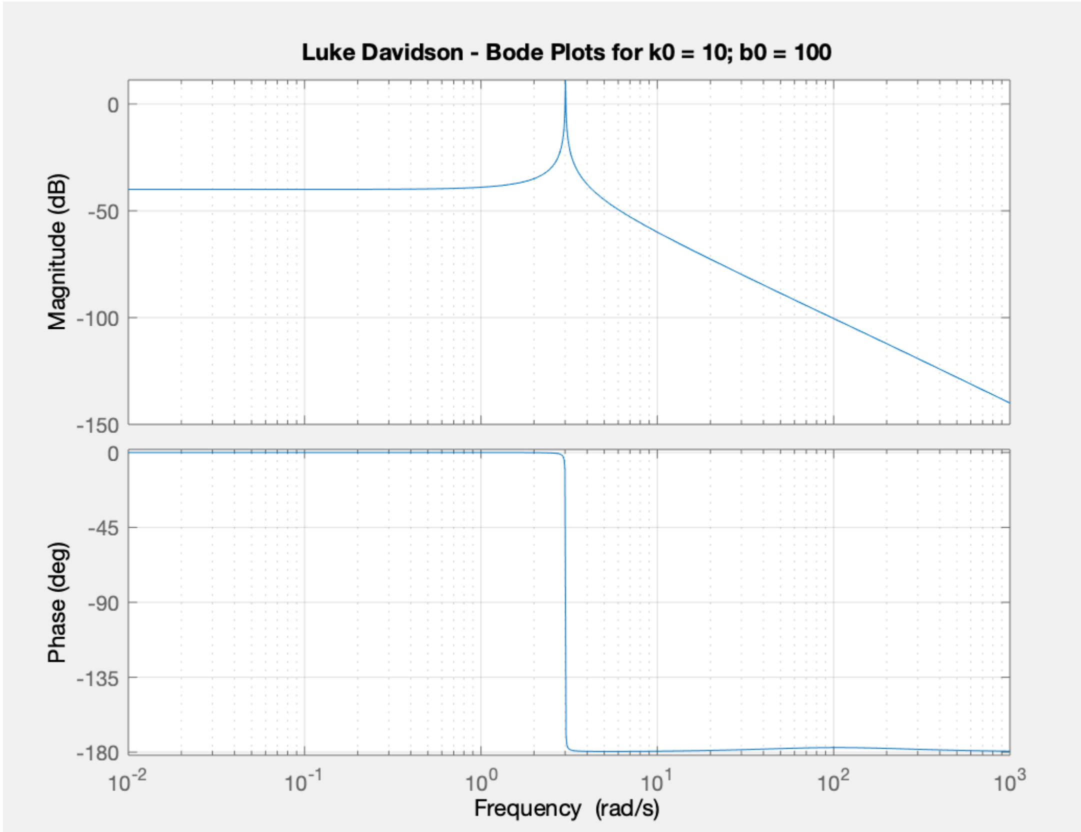
c.) $f(t) = \sin(t\sqrt{10})$ for $b_0 = 0.1$

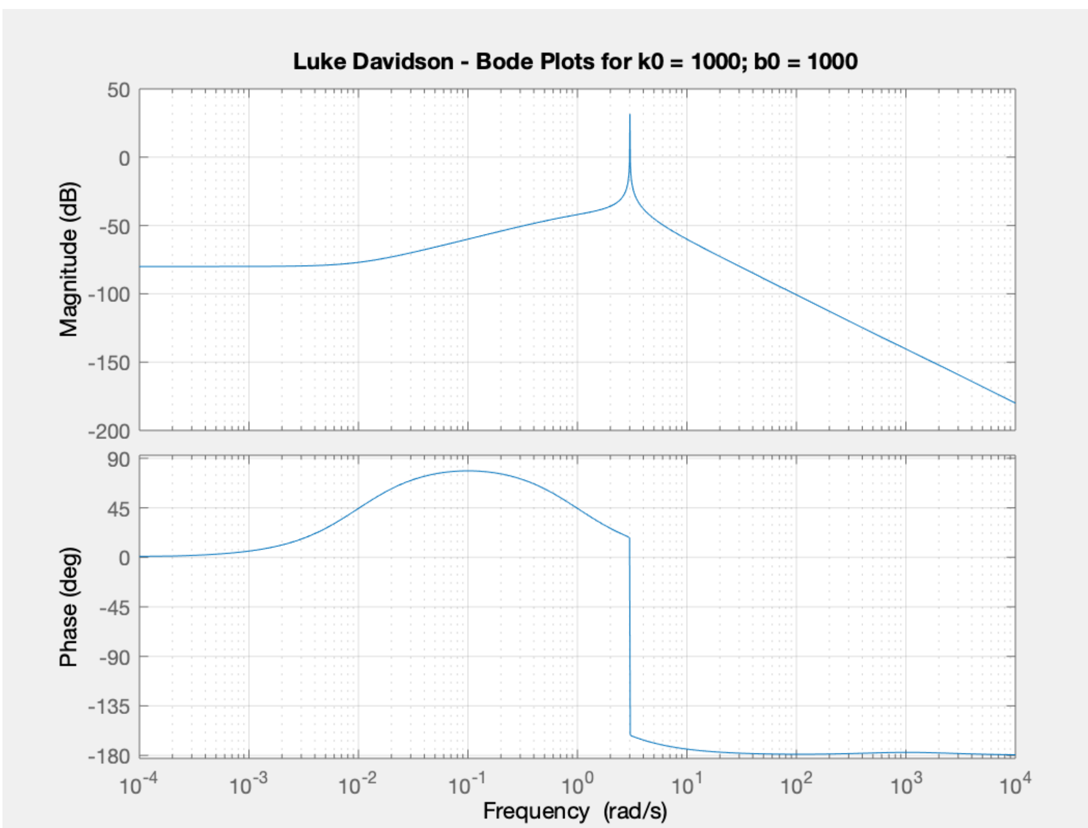
$$F(s) = \frac{\sqrt{10}}{s^2 + 10}$$

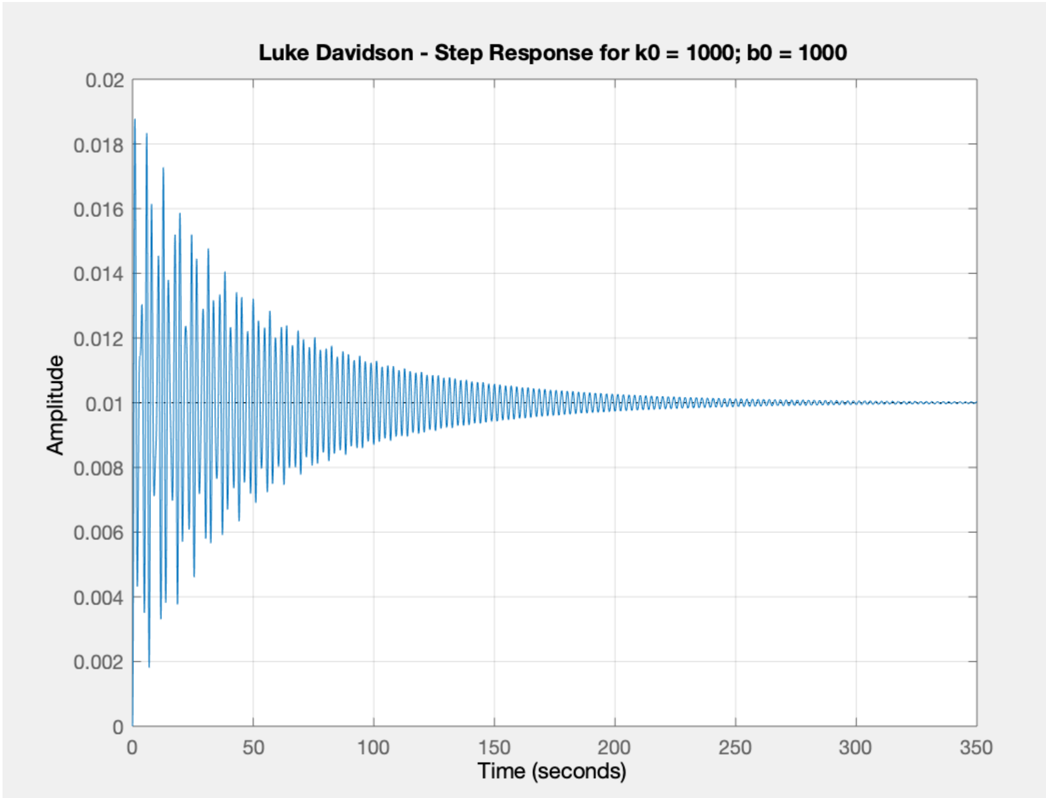
plots attached

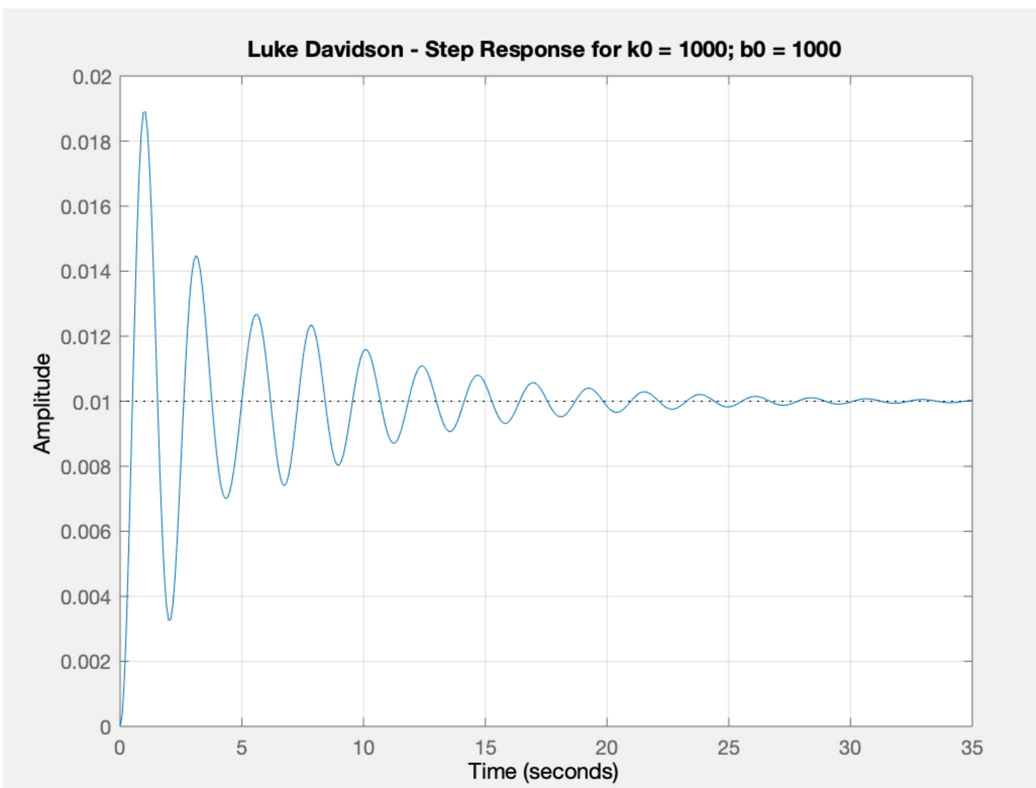

Luke Davidson - Bode Plots for k0 = 10; b0 = 0.1
Magnitude (dB)
Phase (deg)
Frequency (rad/s)
0
-20
-40
-60
-80
-100
-45
-90
-135
-180
10^{-1}
10^{0}
10^{1}
10^{2}

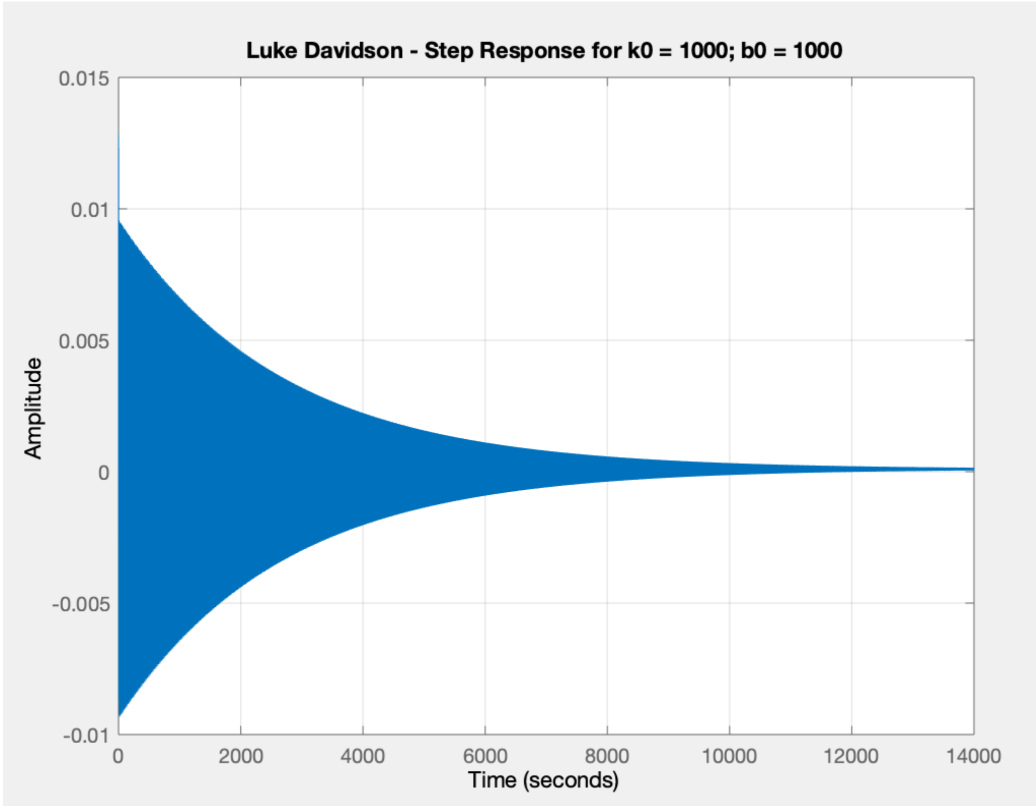

Luke Davidson - Bode Plots for k0 = 10; b0 = 1
Magnitude (dB)
Phase (deg)
Frequency (rad/s)
-20
-40
-60
-80
-100
0
-45
-90
-135
-180
10^{-1}
10^{0}
10^{1}
10^{2}

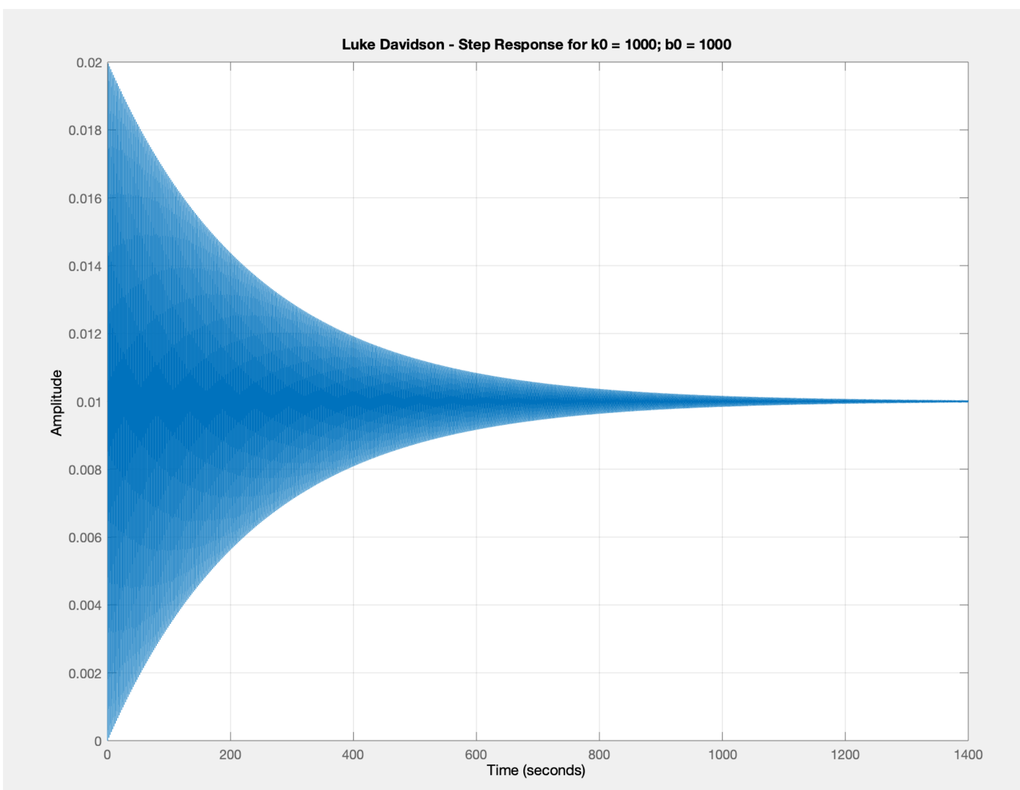

Luke Davidson - Bode Plots for k0 = 10; b0 = 100
Magnitude (dB)
Phase (deg)
Frequency (rad/s)

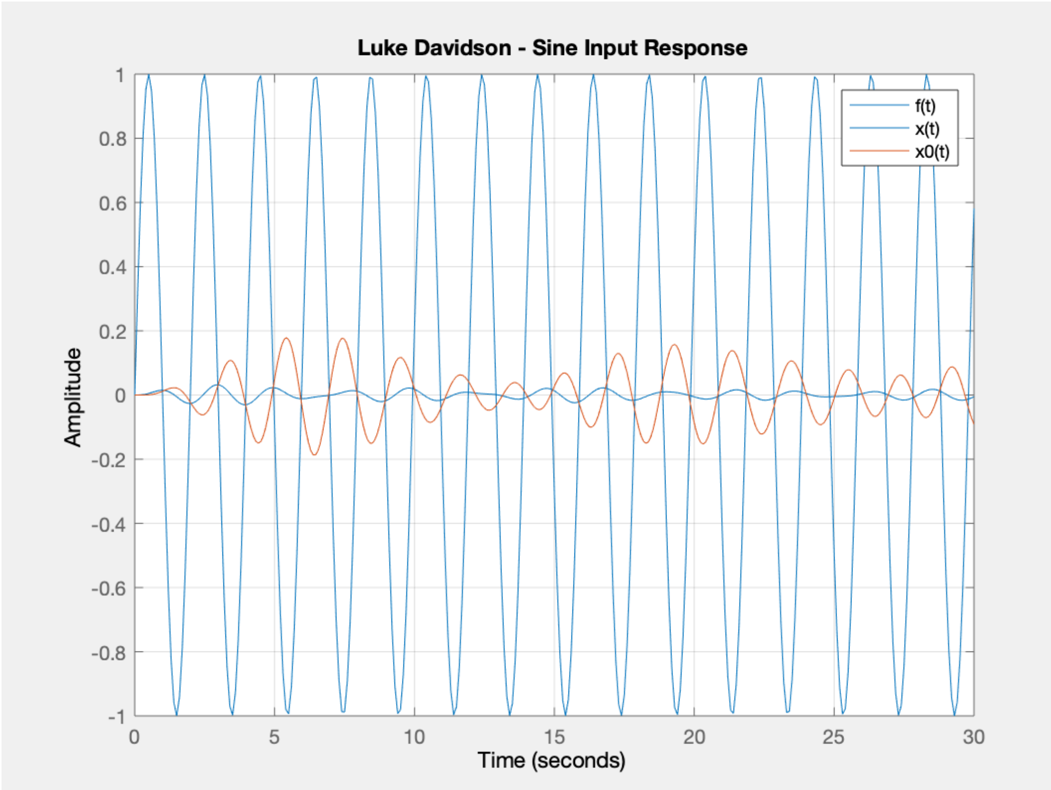

Luke Davidson - Bode Plots for k0 = 1000; b0 = 1000
Magnitude (dB)
Phase (deg)
Frequency (rad/s)


Luke Davidson - Step Response for k0 = 1000; b0 = 1000
Amplitude
Time (seconds)
0.02
0.018
0.016
0.014
0.012
0.01
0.008
0.006
0.004
0.002
0
0
50
100
150
200
250
300
350


Luke Davidson - Step Response for k0 = 1000; b0 = 1000
Amplitude
Time (seconds)
0.02
0.018
0.016
0.014
0.012
0.01
0.008
0.006
0.004
0.002
0
0
5
10
15
20
25
30
35


Luke Davidson - Step Response for k0 = 1000; b0 = 1000
Amplitude
Time (seconds)
0.015
0.01
0.005
0
-0.005
-0.01
0
2000
4000
6000
8000
10000
12000
14000


Luke Davidson - Step Response for k0 = 1000; b0 = 1000
Amplitude
Time (seconds)
0.02
0.018
0.016
0.014
0.012
0.01
0.008
0.006
0.004
0.002
0
0
200
400
600
800
1000
1200
1400




```
% Luke Davidson
% ME 5659
% HW 5

clc;
clear all;
close all;

% b = {0.1, 1, 100}
b1 = 0.1;
b2 = 1;
b3 = 100;

G1 = tf([1 b1 10],[10 11*b1 210 100*b1 1000]);
G2 = tf([1 b2 10],[10 11*b2 210 100*b2 1000]);
G3 = tf([1 b3 10],[10 11*b3 210 100*b3 1000]);
G4 = tf([1 1000 10],[10 11000 11100 100000 100000]);

figure(1);
bode(G1);
grid on;
title('Luke Davidson - Bode Plots for k0 = 10; b0 = 0.1');

figure(2);
bode(G2);
grid on;
title('Luke Davidson - Bode Plots for k0 = 10; b0 = 1');

figure(3);
bode(G3);
grid on;
title('Luke Davidson - Bode Plots for k0 = 10; b0 = 100');

figure(4)
```

```
bode(G4);
grid on;
title('Luke Davidson - Bode Plots for k0 = 1000; b0 = 1000');

figure(5)
step(G1);
grid on;
title('Luke Davidson - Step Response for k0 = 1000; b0 = 1000');

figure(6)
step(G2);
grid on;
title('Luke Davidson - Step Response for k0 = 1000; b0 = 1000');

figure(7)
step(G3);
grid on;
title('Luke Davidson - Step Response for k0 = 1000; b0 = 1000');

figure(8)
step(G4);
grid on;
title('Luke Davidson - Step Response for k0 = 1000; b0 = 1000');


figure(9)
G = tf([1 0.1 10],[10 1.1 210 10 1000]);
H = tf([0.1 10],[1 0.1 10]);
t = 0:0.1:30;
plot(t,sin(sqrt(10)*t))
hold on
impulse(G*(tf([sqrt(10)],[1 0 10])),30)
impulse(H*G*(tf([sqrt(10)],[1 0 10])),30)
grid on
title('Luke Davidson - Sine Input Response')
legend('f(t)','x(t)', 'x0(t)')
```

4.) $u(t) = 100 \sin(\omega t)$

a) flapping angle   DC limit $= \omega \to 0$

$K = 20$ ~~$\log K = 1$~~ $\boxed{K = 10^\circ}$ $\log K = 1$

$K = 10$

flap $\angle = 10 \times \frac{\pi}{180^\circ} =$

$K = 20$ flap $\angle = 0.1745$ rad

b) flap $\angle = 100^\circ$

can we meet w/ 100V input

$V = 100$ so flap $\angle = \left(10^{\frac{33}{20}}\right) = 44.67^\circ$ so no

$\omega_n = 10$ rad/s

cannot reach 100°.

$K = 33$ at ω_n

min voltage? $\Longrightarrow$ $\frac{100\,v}{44.67} = \frac{?\,v}{100}$

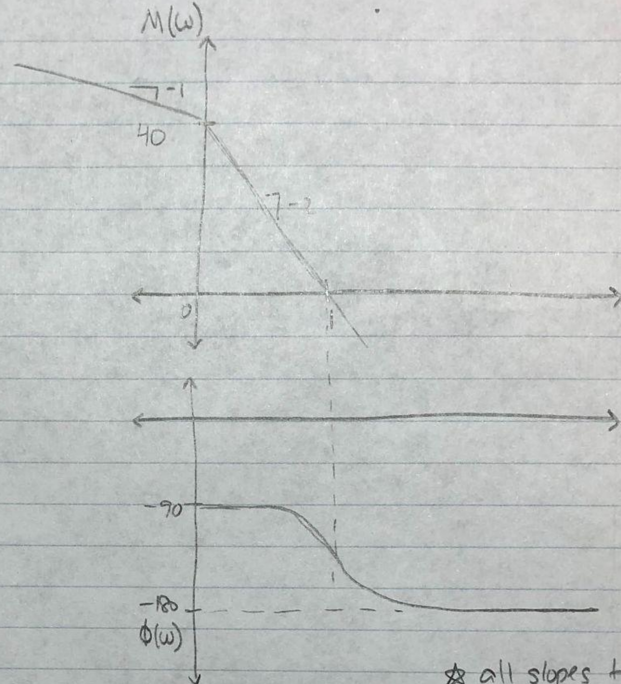
$V = 223.87\,v$

5.) $G_a(s) = \dfrac{100}{s(s+1)}$

$20\log(100) = 40$

$\boxed{\omega_c = 1}$

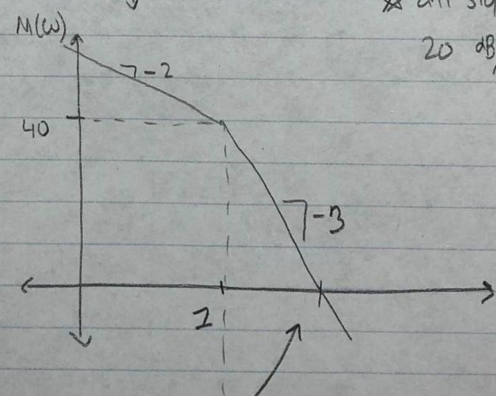
cornering = crossover



b.) $\dfrac{100}{s^2(s+1)}$

double pole @ 0

$s = -1$ ↳ cornering



★ all slopes times 20 dB/dec

$\omega_{crossover} = 0 = 2 - 3\log(\omega)$

$\log(\omega) = \dfrac{2}{3}$

$\omega = 10^{2/3} = 4.64 \text{ rad/s} = \omega_{cross}$