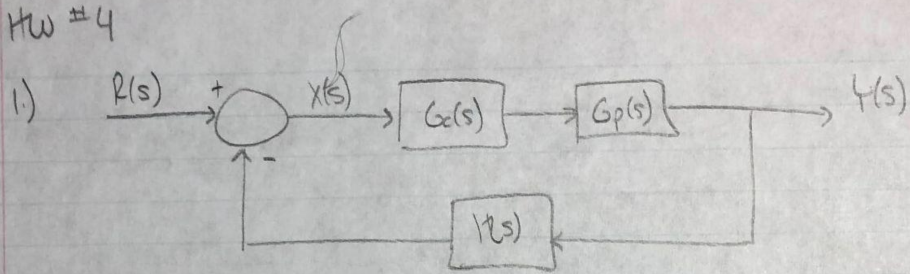
Luke Davidson
ME 5659
HW #4

1.)



a) $F(s) = G_c(s)\,G_p(s) = \dfrac{b(s)}{a(s)}$, $H(s) = 1$

$$G_{cl}(s) = \frac{Y(s)}{R(s)}$$

$$X(s) = R(s) - H(s)Y(s) \qquad Y(s) = X(s)G_c(s)G_p(s)$$

$$Y(s) = \big(R(s) - H(s)Y(s)\big)G_c(s)\,G_p(s)$$

$$Y(s) = R(s)\,G_c(s)\,G_p(s) - H(s)\,G_c(s)\,G_p(s)\,Y(s)$$

$$Y(s)\big[1 + G_c(s)\,G_p(s)\,H(s)\big] = R(s)\big[G_c(s)\,G_p(s)\big]$$

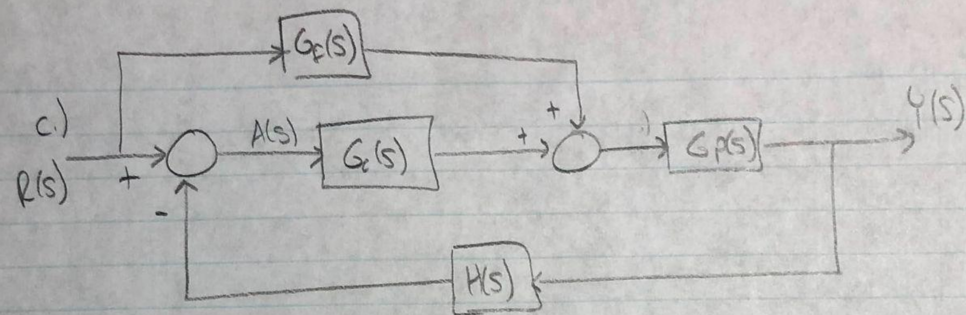
$$\frac{Y(s)}{R(s)} = \frac{G_c(s)\,G_p(s)}{1+G_c(s)\,G_p(s)\,H(s)} = \frac{\frac{b(s)}{a(s)}}{1 + \frac{b(s)}{a(s)}} = \frac{b(s)}{a(s) + b(s)} = G_{cl}(s)$$

b) $H(s) = \dfrac{p(s)}{q(s)}$

$$\frac{\frac{b(s)}{a(s)}}{1 + \frac{b(s)\,p(s)}{q(s)\,a(s)}} = \frac{b(s)}{a(s) + \frac{b(s)\,p(s)}{q(s)}}$$

$$G_{cl}(s) = \frac{b(s)\,q(s)}{a(s)\,q(s) + b(s)\,p(s)}$$

c.)



$$A(s) = R(s) - H(s)Y(s)$$

$$Y(s) = \left[A(s)\,G_c(s) + \overset{R(s)}{G_f(s)}\right] G_p(s)$$

$$y = \left[(r - hy)c + rf\right] p$$

$$y = (rc - hyc + rf)\, p$$

$$y = rcp - hycp + rfp$$

$$y(1 + hcp) = r(cp + fp)$$

$$\frac{y}{r} = \frac{cp + fp}{1 + hcp} \Rightarrow G_{cl}(s) = \frac{G_c(s)G_p(s) + G_f(s)G_p(s)}{1 + H(s)G_c(s)G_p(s)}$$

2.) $G_{cl}(s) = \dfrac{G_c(s)\,G_p(s)}{1 + G_c(s)\,G_p(s)\,H(s)}$

a.)

$$= \frac{K_p \dfrac{1}{s^2+bs+c}}{1 + \dfrac{K_p}{s^2+bs+c}} = \frac{K_p}{s^2+bs+c+K_p} = G_{cl}(s)$$

b) $s^2 + bs + c + K_p$

$$\text{poles} = \frac{-b \pm \sqrt{b^2 - 4(c+K_p)}}{2} \qquad b^2 - 4(c+K_p)$$

$b > 0$ and $b^2 - 4(c+K_p) \leq 0$ or $b^2 - 4(c+K_p) < b$

c.)

$$(2\sqrt{c+K_p})(\xi) = b$$

$$(2\sqrt{c+K_p})(0.5) = b$$

$$c + K_p = b^2 \Longrightarrow K_p = b^2 - c$$

3.) $G_1(s) = \dfrac{K}{s^2 + cs + k}$

in form $G(s) = \dfrac{K\omega_n^2}{s^2 + 2\xi\omega_n s + \omega_n^2}$

a.)

$t_s = 0.25$

$0.25 = e^{\frac{-\pi\xi}{\sqrt{1-\xi^2}}}$

overshoot = 25%

$\ln(0.25) = \dfrac{-\pi\xi}{\sqrt{1-\xi^2}}$

$\sqrt{1-\xi^2}\,(\ln 0.25) = -\pi\xi$

$(1-\xi^2)(\ln(0.25))^2 = \pi^2\xi^2$

$(\ln(0.25))^2 = \pi^2\xi^2 + \xi^2(\ln(0.25))^2$

$\xi^2 = \dfrac{(\ln(0.25))^2}{\pi^2 + (\ln(0.25))^2}$

$\xi = \dfrac{\ln(0.25)}{\sqrt{\pi^2 + (\ln(0.25))^2}}$

$\xi = 0.404$

$t_s = 0.25 = \dfrac{4}{\xi\omega_n}$

$0.25(\xi)(\omega_n) = 4$

$0.25(0.404)\omega_n = 4$

$\omega_n = \dfrac{4}{(0.25)(0.404)} = \boxed{39.6 = \omega_n}$

comparing $G_1(s)$ w/ $G(s)$ canonical,

$c = 2\xi\omega_n$

$k = \omega_n^2 = 1568.16$

$c = 2(0.404)(39.6) = 32 = c$

$k = 1568.16$

$0.1 = \dfrac{K}{k} \Rightarrow K = 0.1k = 156.816 = K$

b.) $k \rightarrow 2k$ $\qquad G_1(s) = \frac{K}{s^2 + cs + 2k}$

$\omega_n^2 = 2(1568.16) =$

new $\boxed{\omega_n = 56}$ $\qquad K = 0.1(3136.32) = \boxed{313.632 = K}$

settling time would increase due to a doubled k,

overshoot would also increase, and DC Gain would

remain constant.

c.) output / MATLAB code attached.

we would expect gain to remain constant b/c
the $\lim_{s \to 0}$ will lead to the same result as the first.

The addition of a zero will cause overshoot to
decrease and rise time to increase due to a
higher damping ratio value when adding the zero.

Similarly, it will take less time to settle.

```matlab
% Luke Davidson
% ME 5659
% HW4 Q3c

clc;
clear all;
close all;

sys1 = tf([15.6816 156.816],[1 32 1568.16]);
sys2 = tf([156.816],[1 32 1568.16]);

subplot(2,1,1);
step(sys1)
title('Luke Davidson - HW4 Q3 - Original TF');
grid on;

subplot(2,1,2);
step(sys2)
title('Luke Davidson - HW4 Q3 - Part C TF');
grid on;


S1 = stepinfo(sys1)
S2 = stepinfo(sys2)
```

```
S1 =

  struct with fields:

        RiseTime: 0.0055
    SettlingTime: 0.2512
     SettlingMin: 0.0508
     SettlingMax: 0.2969
       Overshoot: 196.9445
      Undershoot: 0
            Peak: 0.2969
        PeakTime: 0.0374


S2 =

  struct with fields:

        RiseTime: 0.0372
    SettlingTime: 0.2123
     SettlingMin: 0.0938
     SettlingMax: 0.1250
       Overshoot: 24.9636
      Undershoot: 0
            Peak: 0.1250
        PeakTime: 0.0863
```

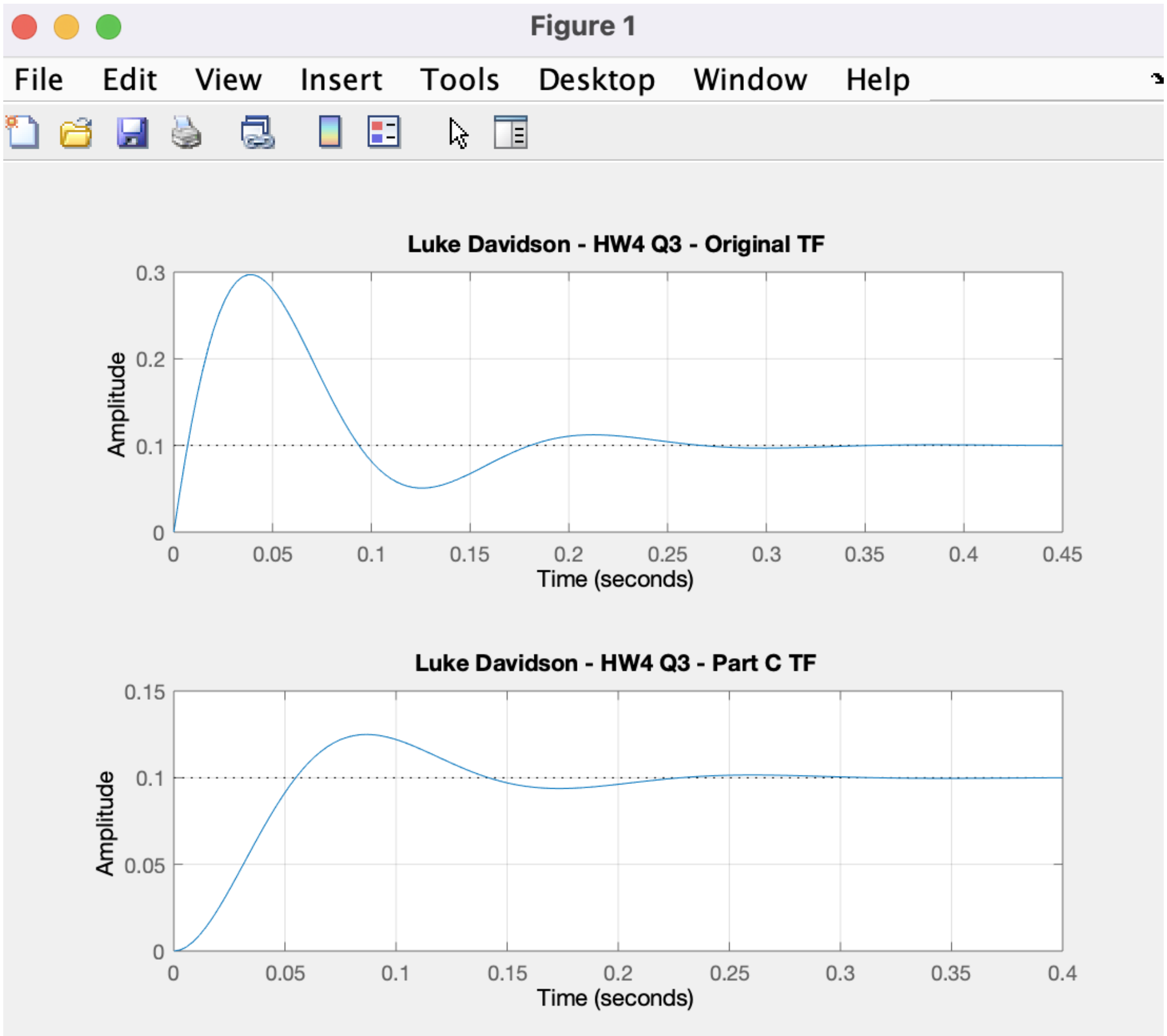


Figure 1
File Edit View Insert Tools Desktop Window Help
Luke Davidson - HW4 Q3 - Original TF
Amplitude
Time (seconds)
0 0.05 0.1 0.15 0.2 0.25 0.3 0.35 0.4 0.45
0 0.1 0.2 0.3
Luke Davidson - HW4 Q3 - Part C TF
Amplitude
Time (seconds)
0 0.05 0.1 0.15 0.2 0.25 0.3 0.35 0.4
0 0.05 0.1 0.15

4.) $\zeta > \frac{\sqrt{2}}{2}$

$\beta = \cos^{-1}\left(\frac{\sqrt{2}}{2}\right) = 45$

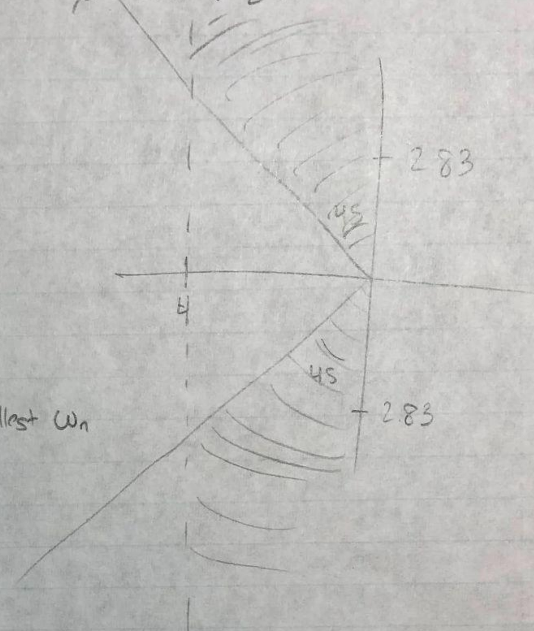
$\omega_n < 4 \text{ rad/s}$

$t_s < 4s$

$t_s < 4$

$t_s = \frac{4}{\zeta \omega_n}$

to find smallest ω_n



$\frac{4}{\frac{\sqrt{2}}{2}\omega_n} < 4$

$\frac{8}{4\sqrt{2}} < \omega_n$

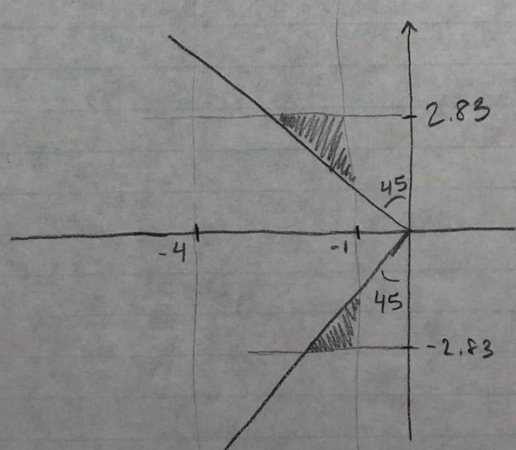
$\boxed{\omega_n > \frac{2}{\sqrt{2}}}$

$\left.\begin{array}{l} \frac{2}{\sqrt{2}} < \omega_n < 4 \\ \frac{\sqrt{2}}{2} < \zeta \le 1 \end{array}\right\}$ max $\omega_n \zeta = 4$ min $\omega_n \zeta = 1$

max $\omega_n\sqrt{1-\zeta^2} = 4(\sqrt{0.5}) = 2.83$

min " $= \frac{2}{\sqrt{2}} 0$

$\beta \geq 45$



5.) a) $\ddot{x}(t) + b\dot{x}(t) + kx(t) = F(t)$

$X(s)s^2 + bsX(s) + kX(s) = F(s)$

$X(s)(s^2 + bs + k) = F(s)$

$$\frac{X(s)}{F(s)} = \frac{1}{s^2 + bs + k}$$

$\omega_n^2 = k$ $\quad$ $\omega_n = \sqrt{k}$

$1 = Kk$

$2\zeta\omega_n = b$ $\quad$ $K = \frac{1}{k}$

$2\zeta\sqrt{k} = b$

$$\zeta = \frac{b}{2\sqrt{k}}$$

b) $t_{peak} = 0.7$

Over shoot $= 25\% = \frac{0.01}{0.04} =$

$0.25 = e^{\frac{-\pi\zeta}{\sqrt{1-\zeta^2}}}$ $\rightarrow$ from Q3 I know this is $\boxed{0.404 = \zeta}$

$t_p = 0.7 = \frac{\pi}{\omega_n\sqrt{1-\zeta^2}}$ $\quad$ $\Rightarrow k = 4.91^2$ $\quad$ $b = 3.96$

$\omega_n = \frac{\pi}{0.7\sqrt{1-0.404^2}} = \boxed{4.91 = \omega_n}$ $\quad$ $k = 24.1$

$b = 2\zeta\sqrt{k} = 2(.404)(4.91)$

or DC gain = $K = 0.04$

$K = \frac{1}{k} = 0.04$

$\Longrightarrow k = 25 \Longrightarrow \quad (.404) = \frac{b}{2\sqrt{25}} \Longrightarrow b = 4.04$

Both answers very close.

6.) a.) $G_a(s) = \dfrac{1}{(s+2)(s+10)}$

$k = \lim_{s \to 0} \dfrac{1}{(s+2)(s+10)} = \dfrac{1}{20}$

$s^2 + 12s + 20$

$s^2 + 12s + 20$

dom pole = -2

$\boxed{k = .05}$

$\dfrac{A}{s+2} + \dfrac{B}{s+10}$

right → $G_a(s) = \dfrac{0.1}{s+2}$

wrong → $G_a(s) = \dfrac{0.5}{s+10}$

plots attached,

top 2 plots are clearly closer.

b.) dom pole = -10

$\dfrac{1}{20}$

approx = $\dfrac{0.5}{s+10}$

wrong = $\dfrac{0.1}{s+2}$

plots attached,

same as above

c.) $G(s) = \dfrac{(\frac{5}{9}s + 1)}{(s+2)(s+10)}$

$\dfrac{A}{s+2} + \dfrac{B}{s+10} = \dfrac{\frac{5}{9}s + 1}{(s+2)(s+10)}$

$A = T(s) \cdot (s+2)\Big|_{s=-2} = \dfrac{\frac{5}{9}s + 1}{\cancel{(s+2)}(s+10)} \cancel{(s+2)} = \dfrac{\frac{5}{9}s + 1}{s+10}\Big|_{s=-2}$

$A = \dfrac{\frac{-10}{9} + \frac{9}{9}}{+8} = \dfrac{-1}{9} \cdot \dfrac{1}{+8} = \boxed{\dfrac{-1}{72} = A}$

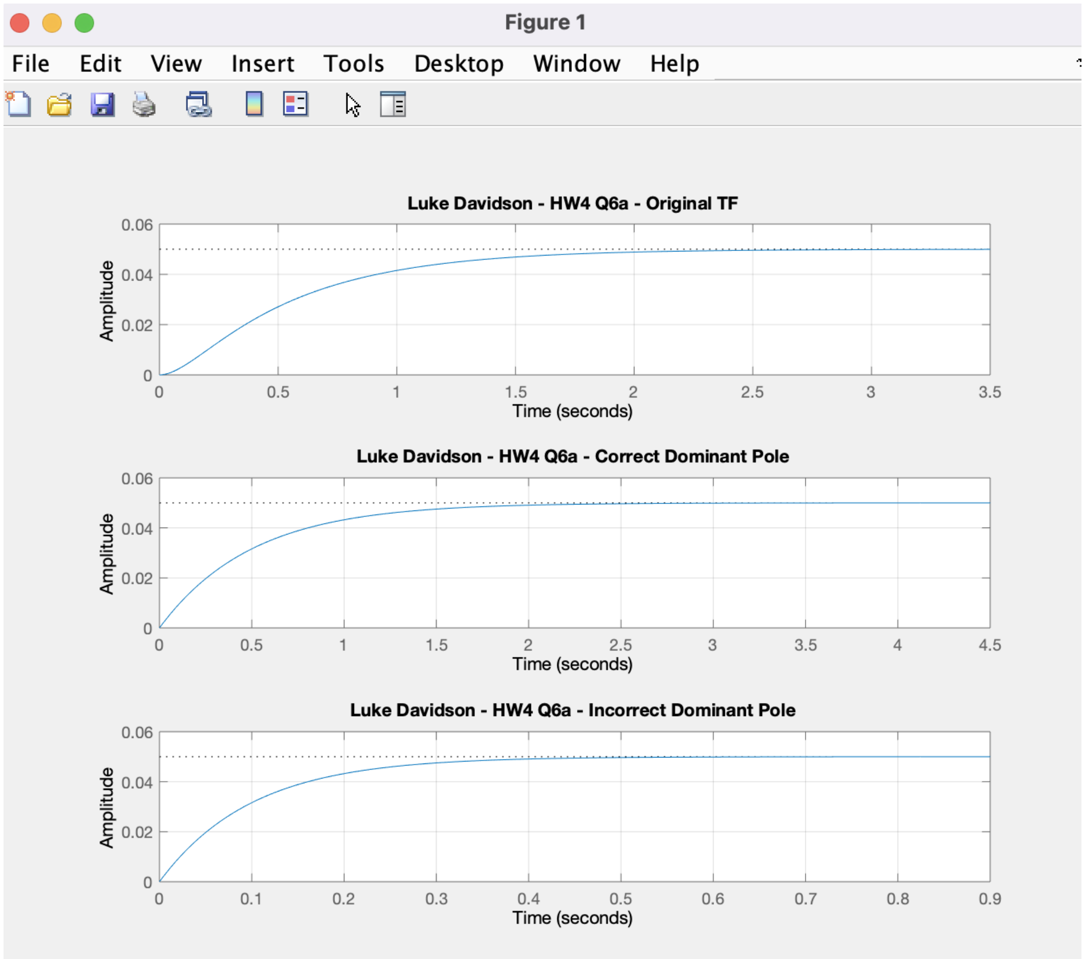
$$B = T(s)(s+10)\Big|_{s=-10} = \frac{\frac{5}{9}s+1}{(s+2)\cancel{(s+10)}}\cancel{(s+10)} = \frac{\frac{5}{9}s+1}{s+2}\Bigg|_{s=-10}$$

$$= \frac{\frac{-50}{9}+\frac{9}{9}}{-8} = \frac{-41}{9}\cdot\frac{1}{-8} = \frac{+41}{72} \qquad \boxed{B = \frac{41}{72}}$$

$$\frac{41}{72}\left(\frac{1}{s+10}\right) - \frac{1}{72}\left(\frac{1}{s+2}\right) = \frac{\frac{5}{9}s+1}{(s+10)(s+2)}$$

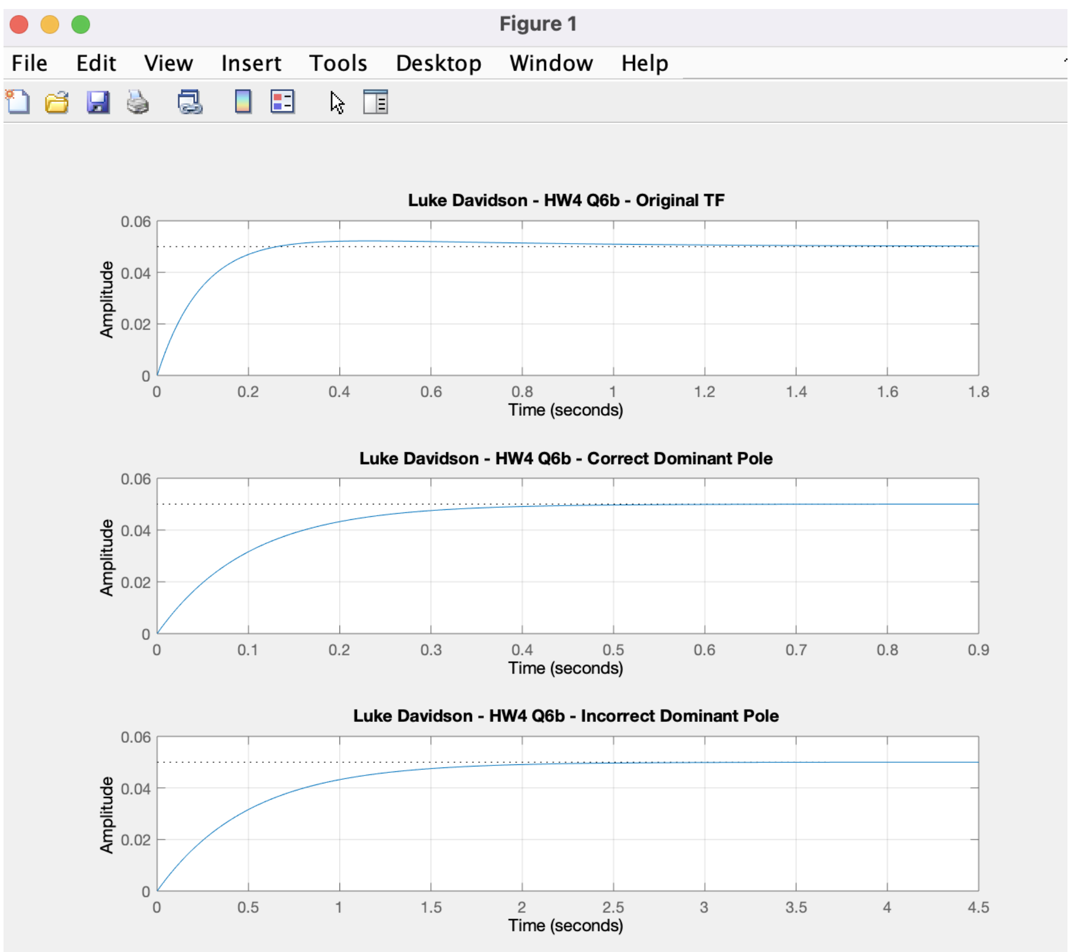
It is evident that the pole @ -2 has much less of an effect on the output due to its coefficient of $\frac{-1}{72}$ compared to pole @ -10's coeff. of $\frac{41}{72}$.

```
% Luke Davidson
% ME 5659
% HW4 Q3c

clc;
clear all;
close all;

sys1 = tf([1],[1 12 20]);
sys2 = tf([0.1],[1 2]);
sys3 = tf([0.5],[1 10]);

subplot(3,1,1);
step(sys1)
title('Luke Davidson - HW4 Q6a - Original TF')
grid on;

subplot(3,1,2);
step(sys2)
title('Luke Davidson - HW4 Q6a - Correct Dominant Pole')
grid on;

subplot(3,1,3);
step(sys3)
title('Luke Davidson - HW4 Q6a - Incorrect Dominant Pole')
grid on;
```

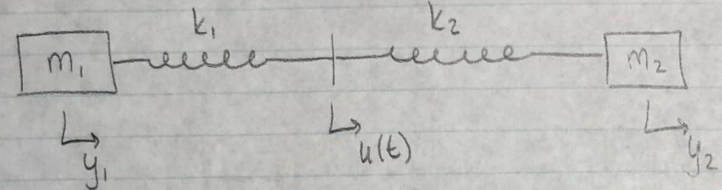



```
% Luke Davidson
```

```
% ME 5659
% HW4 Q3c

clc;
clear all;
close all;

sys1 = tf([5/9 1],[1 12 20]);
sys2 = tf([0.5],[1 10]);
sys3 = tf([0.1],[1 2]);
subplot(3,1,1);
step(sys1)
title('Luke Davidson - HW4 Q6b - Original TF')
grid on;

subplot(3,1,2);
step(sys2)
title('Luke Davidson - HW4 Q6b - Correct Dominant Pole')
grid on;

subplot(3,1,3);
step(sys3)
title('Luke Davidson - HW4 Q6b - Incorrect Dominant Pole')
grid on;
```



7.)



states $= y_1, y_2, \dot y_1, \dot y_2$

$$\left(\frac{m_2\ddot y_2 + b_2\dot y_2 + k_2 y_2 - k_2 u}{b_2}\right) b_1 + k_1 u = m\ddot y_1 + b_1\dot y_1 + k_1 y_1$$

$$\boxed{\begin{bmatrix} \dot y_1 \\ \ddot y_1 \\ \dot y_2 \\ \ddot y_2 \end{bmatrix} = \begin{bmatrix} 0 & 1 & 0 & 0 \\ -\frac{k_1}{m_1} & -\frac{b_1}{m_1} & \frac{b_1k_2}{m_1b_2} & \frac{b_1}{m_1} \\ 0 & 0 & 0 & 1 \\ \frac{k_1b_2}{b_1m_2} & \frac{b_2}{m_2} & -\frac{k_2}{m_2} & -\frac{b_2}{m_2} \end{bmatrix} \begin{bmatrix} y_1 \\ \dot y_1 \\ y_2 \\ \dot y_2 \end{bmatrix} + \begin{bmatrix} 0 \\ -\frac{b_1k_2}{b_2m_1} + \frac{k_1}{m_1} \\ 0 \\ -\frac{k_1b_2}{b_1m_2} + \frac{k_2}{m_2} \end{bmatrix} u}$$

$$\frac{1}{m_1}\left(b_1\left(\frac{m_2}{b_2}\right)\ddot y_2 + b_1\dot y_2 + \frac{b_1k_2}{b_2} y_2 - \frac{b_1k_2}{b_2} u + k_1 u - b_1\dot y_1 - k_1 y_1\right) = \ddot y_1$$

$$\frac{m_1b_2}{b_1m_2}\ddot y_1 + \underbrace{\frac{\cancel{b_1}b_2}{\cancel{b_1}m_2}}_{\frac{b_2}{m_2}}\dot y_1 + \frac{k_1b_2}{b_1m_2}y_1 - \frac{k_1b_2}{b_1m_2}u + \frac{k_2}{m_2}u - \frac{k_2}{m_2}y_2 - \frac{b_2}{m_2}\dot y_2 = \ddot y_2$$

$b_1 = b_2 = 0$

$$\Longrightarrow A = \begin{bmatrix} 0 & 1 & 0 & 0 \\ -\frac{k_1}{m_1} & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & -\frac{k_2}{m_2} & 0 \end{bmatrix}, \quad B = \begin{bmatrix} 0 \\ k_1/m_1 \\ 0 \\ k_2/m_2 \end{bmatrix}$$

$$\ddot{y}_1 = \frac{-k_1}{m_1} y_1 + \frac{k_1}{m_1} u$$

$$\ddot{y}_2 = \frac{-k_2}{m_2} y_2 + \frac{k_2}{m_2} u$$

This makes sense because it is the balance of forces w/o effects of the damper. Both sides/equations are the same due to the symmetry of the problem.