Chapter 1: Unsharpness as Indeterminacy

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Abstract. I introduce the central topic and thesis of the book: it's at least plausible that we have incomplete preferences and imprecise credences and that such 'unsharpness' is vagueness. I introduce the problem of value pumps faced by the unsharp, and state my view of the matter. That is Supersharp: given a 'tolerance-denying' theory of vagueness it's super-true that we have point-valued preferences and credences, but often vague what that point is. Finally, I preview the remaining chapters.

How many onions would you barter for 247 carrots, to be delivered on the last Thursday of next June? That question is purely academic for most of us, but what's your degree of belief (credence) that you'll be in a car accident in the next 28 days? If you've ever rented a car on holiday then you've almost certainly been offered some kind of supplementary insurance or waiver to cover such an accident. Whether the insurance is worth it depends in part on how likely an accident is.

These examples can be multiplied, and they illustrate that your preferences and degrees of belief *seem* not to be maximally complete and precise. You aren't maximally opinionated on every question of preference and belief, at least as far as you can tell. So maybe you lack maximally-detailed opinions, or maybe you have them but can't access them?

In this book, I'll argue that (if you are like most of us) many of your preferences and credences are vague so—unless perhaps you work in the food industry—it's vague how many onions you're willing to trade for the carrots, as it is vague how many grains of sand make a heap. Moreover, I'll argue that such vagueness is compatible with complete rationality and that it can resolve several puzzles about choice *without* abandoning what I'll call the 'standard model' of rational choice—standard decision theory.

1 The Standard Model

If you speak of 'maximising expected utility', you speak the language of standard decision theory. That theory is an enormous intellectual achievement.

The standard model of standard decision theory assumes that you are maximally opinionated. I'll work with the classic version due to Leonard Savage (1972).

For him, an *act* is a function from states of the world to consequences. In a very simple case, for example, there are two acts open to me. One is that I spend my last £1 on a lottery ticket, which will either have the consequence that I gain £1m (if the ticket wins) or that I am left with nothing (if the ticket loses). The other is that I keep my £1. Focusing only on these financial consequences, the two acts can be represented in a payoff table:

Table 1: Payoff table for buying a lottery ticket.

	Ticket wins	Ticket loses
Buy ticket	£1m	£0
Don't buy	£1	£1

There are two possible states or ways the world might be—the ticket I would have bought wins or it doesn't—and the various acts open to me (buy a ticket or don't) have different consequences in the different states.

To introduce a bit of space-saving notation, $\preccurlyeq$ is a relation among acts, where $f \preccurlyeq g$ means that you weakly prefer g to f: you either strictly/outright prefer g to f (written $f \prec g$), or you are indifferent between f and g (written $f \sim g$).

Unless there is something particularly odd going on, for most of us £5 $\prec$ £10, and where X and Y are two £10 notes, $X \sim Y$. We can see a useful terminological ambiguity here, in that this notation allows us to specify preferences between states (£5 $\prec$ £10), and between acts (pick up £5 $\prec$ pick up £10).[1]

The centrepiece of standard decision theory is the representation theorem. Given some assumptions about you, such a theorem says that your preferences can be represented by a pair of functions: a utility function u which takes each proposition to a real number representing how much you want that proposition to be true, and a credence function c which takes each proposition to a real number between 0 and 1 representing your degree of belief in that proposition.

Your credence $c(H)$ in a proposition H has natural end-points: 0 for absolute certainty that H is false, and 1 for absolute certainty that it is true.

But things are a touch more complicated for preferences. There are no natural end-points, and so your underlying *preference ordering* ($\preccurlyeq$ and the like) can be represented in many different ways. For example if the utility function $u(x)$ represents your preferences, then so does the function $v(x) = 6u(x) + 4$, and so simply saying that you attach a certain utility to a proposition means very little in isolation. But saying that you attach a certain credence to that proposition is meaningful. This complication I'll set aside, often using the common device of 'utilities linear in dollars' (or pounds), $u(x) = x$ where x is the number of dollars. The examples would have the same structure if you attach 6 utiles to each dollar, starting with 4 utiles at \$0—so $v(x)$—because such a function is a linear transformation of $u(x)$.

[1]This is because of the possibility of 'constant acts', which have the same consequence in every state. See Savage (1972), p. 25.

Given such a pair of functions, each act ϕ has an expected utility $E(\phi)$. This is the average utility of the act across all the ways the world might be, weighted according to how likely the world is to be that way. If there are n such ways $H_1, ..., H_n$ that the world might be, then your expected utility of doing ϕ is

Expected Utility. $E_{c,u}(\phi) = \sum_{i=1}^{n} c(H_i) \cdot u_{H_i}(\phi)$

For simplicity, let's assume that all you care about is money. Then we can stipulate that you attach 1 unit of utility (1 utile) to each pound as just discussed, at least if we also assume that you care about it linearly (you don't care more or less about each extra pound as you get richer).

Suppose again that you are offered the chance to buy a lottery ticket bearing your lucky numbers with your last £1. There are two possibilities for how the world will turn out:

- H_1: your lucky numbers come up in the lottery;
- H_2: your lucky numbers don't come up.

If H_1 is true and you buy the ticket then your utility is 1,000,000 because you have £1m; if H_2 is true and you buy the ticket then your utility is 0 because you have no money. If you don't buy a ticket then your utility is 1 either way because you still have your £1. This is what the payoff table represents.

Whether buying the ticket is a good idea or not depends on how likely you think it that your numbers will come up—your credence or degree of belief in H_1.

If your credence in H_1 is p then your credence in H_2 is $(1 - p)$, because H_1 and H_2 are the only two possibilities: either your numbers come up or they don't. That you are coherent in this way—that your credences sum to 1—is one of the assumptions of the representation theorem. The theorem works out the consequences of certain rationality axioms, including this one, and so if the axioms are not true of you then the theorem is not (proven to be) true of you, except insofar as the axioms are superfluous.

With credence p in H_1 and $(1-p)$ in H_2, your expected utility of buying the ticket is $[1000000p + 0(1 - p)] = 1000000p$ and your expected utility of not buying the ticket is $[1p+1(1-p) = 1]$. We are interested in which actions maximise expected utility:

Admissibility. An action is *admissible* or *sanctioned* according to a pair of functions (credence function c and utility function u) if and only if that action maximises expected utility according to that pair.

So here, buying the ticket is admissible if and only if the expected utility of buying the ticket is at least that of not buying the ticket, that is to say iff $1000000p \geq 1$. This is true iff $p \geq 1/1000000$, which is to say that you think the ticket has at least a one in a million chance of winning. So in expected utility terms, entering a lottery is often not a terrible idea.

I said that representation theorems are about working out the consequences of some assumptions. Savage's theorem says given some such assumptions, there are credence and utility functions which represent them. There's some expectation E such that:

Savage's Representation Theorem. $f \preccurlyeq g$ given B *iff* $p(B) = 0$ or $E(f - g|B) \leq 0$.[2]

Ignoring 'B' for a moment, this theorem says that you (weakly) prefer the act f to the act g if the expected utility of f is at least that of g. In other words, and notwithstanding the possibility of positive linear transformations, there is a *unique* pair of functions that represents your preferences via the expectation function.

The theorem says that anyone who obeys its assumptions will act so as to maximise expected utility. So if the assumptions of the theorem are normatively plausible—if it's plausible that agents *should* obey them—then it's plausible that agents *should* act so as to maximise expected utility. In that case, acts are permissible (they are what should be done) *iff* they are admissible.

But are the assumptions normatively plausible? This might of course depend on what kind of normativity we have in mind; we might be restricting our attention to only instrumental normativity, for example. The focus of this book is Savage's assumption (P1):

Savage's (P1). $\preccurlyeq$ is a simple ordering—it is complete and transitive. (Savage 1972, 18)

This is a conjunctive assumption, so let's consider the conjuncts separately. Transitivity looks intuitively plausible both as a descriptive and a normative claim:

Transitivity of Preference. For any A, B, and C, if $A \preccurlyeq B$ and $B \preccurlyeq C$, then $A \preccurlyeq C$.

I think transitivity is obviously true, and will implicitly defend it throughout. I defend it explicitly in Chapter 5 against a tenacious challenge from the Puzzle of the Self-Torturer by Chrisoula Andreou and others. (Andreou 2023, 3) for example, contends that 'rationality does not invariably prohibit disorderly preferences'— where the disorderly includes the cyclic and the incomplete.

Completeness is a much more demanding standard, and lacks even the default credibility of Transitivity. Do you really have a preference between *any* two acts, no matter how obscure? (P1) assumes so:

Descriptive Preference Completeness. All agents have complete preferences. For any A and B, either $A \preccurlyeq B$ or $B \preccurlyeq A$.

But think again about the carrots. How many onions are you willing to trade for 247 of them, to be delivered on the last Thursday of next June? Descriptive Preference Completeness says that there is an answer to this question, because your preferences are complete: for each number of onions *o*, either you strictly prefer *o* onions to 247 carrots (to be delivered on the last Thursday of next June), or you strictly prefer 247 carrots to *o* onions (to be delivered...), or you are strictly indifferent between those offers.

Most of us have never thought about bartering carrots and onions, so doesn't that make completeness about such a choice false by default? Surprisingly, stan-

[2] Savage (1972), p. 82.

dard decision theory says No: our preferences are complete about every possible choice. If preferences are construed in terms of choices—for you to prefer A to B is to choose A over B under certain conditions—then whether you've thought about a choice has little relevance to its completeness. The idea is that since you *would* choose one way or another under the right circumstances, you do in fact have a preference. (In the next chapter I'll resist this 'Choice Argument'.)

Without Descriptive Preference Completeness there may not be a fact of the matter about which actions are admissible, because without a preference between two outcomes there may be no fact of the matter about which action has greater expected utility.

2 Preferential Incompleteness

So standard decision theory rests on an assumption—Descriptive Preference Completeness—that looks implausible. And it doesn't just look implausible about choices too obscure or unlikely to be worth thinking about. (Savage 1972, 21) himself noted that there seem to be 'introspective sensations of indecision or vacillation, which we may be reluctant to identify with indifference'.

Introspection often fails to reveal complete preferences, even after extensive consideration of an important choice:

> **Property Buying.** You are buying somewhere to live. Would you prefer the larger House in the suburbs with a garden, or the smaller Flat in town, walking distance to work and the shops but with no garden?
>
> **Job Hopper.** Hopper is an academic economist. He enjoys his job (Academic) for the intellectual stimulation and ability to take an afternoon nap. But he could easily find work with an investment bank. If he takes such a job (Banking) there will be no more naps, but his competitive spirit will be better satisfied and he'll have more money to spend on expensive meals and grooming.

Capitalised words such as 'Flat' and 'Banking' denote a *particular* place to live, job, or other good. '+' and '-' denote small improvements and detriments.

Many of us would have definite preferences in Property Buying and Job Hopper. But many of us wouldn't. The choices involve weighing many different competing factors against each other, and it's not surprising if Hopper lacks a precise number of hours at home that he'd trade for an extra £30,000 of salary. We often indeed seem to have Savage's sensations of indecision or vacillation.

I'll say that two outcomes A and B are *incomplete with each other* for you if you neither prefer A to B, nor prefer B to A, nor are indifferent between A and B. The terminology is perhaps a little ungainly, but it lacks the connotations of other obvious candidates— such as 'incomparable', 'incommensurate', 'incommensurable', and 'on a par'—in this literature. Unfortunately my terminology has an objective sound, so I wish to be clear that incompleteness is a feature *of the agent* and her preferences: if she is incomplete between A and B, then they fall into a ‘gap’ in her preferences. Someone else might well have a definite preference between them, and incompleteness is not (directly) a claim about objective values. My usage of ‘incomplete’ is analogous to how some authors might call A and B ‘indifferent’ to say that you are strictly indifferent between them.

As a descriptive claim, many of us seem to have incomplete preferences about many things. But wait! Why not claim that if we really don’t prefer one job to another, for example, then we must be strictly indifferent? That’s what Descriptive Preference Completeness says, with the implication that failing to identify our preferences in a particular case is a failure of introspection.

Small-Improvement Arguments are meant to show that in these sorts of cases, we are not indifferent. Suppose for the sake of argument that you *are* strictly indifferent between Flat and House, which both cost £300,000. Seeing you dither, the flat seller sweetens the deal to Flat+: the price is now £299,999. You care about money, so you clearly albeit slightly prefer Flat+ to Flat. If you really are indifferent between Flat and House, then by the transitivity of preference you clearly—albeit slightly—prefer Flat+ to House, and you should buy Flat+.

But here’s the heart of the argument: it’s often introspectively implausible that such small improvements (a £1 discount!) would tip the balance in this way. It’s implausible that such a small discount could make you prefer one property to another, so it’s implausible that you were strictly indifferent between Flat and House. This is not to say that you are *never* strictly indifferent in such cases, but that indifference doesn’t normally or always seem like an adequate description of your preferences in such large complex choices.

You sometimes see it claimed that Armstrong (1939) gave a small-improvement argument involving a pony and a bell, but I can find no such example in that paper. To my knowledge, the first example is due to Ronald de Sousa, and involves the famous but rather dubious example of a ‘fairly-virtuous wife’ who is unsure whether to take $1,000 to go for an adulterous weekend in California or to keep her ‘virtue’. Raising the ‘inducement’ to $1,500 to also leaves her hesitant. He argues that ‘connexivity’ (completeness) is not true of the fairly-virtuous wife.[3]

There are two core ideas behind small-improvement arguments. The first is that because of the transitivity of preference, strict indifference can only hold between (for example) House and *one* of Flat or Flat+. We can say that (1) House $\sim$ Flat or we can say that (2) House $\sim$ Flat+, but not both. (1) implies that House $\prec$ Flat+ which is incompatible with (2), whereas (2) implies that House $\succ$ Flat which is incompatible with (1). More generally, positing strict indifference at some point in the scale prevents us from positing it at some other point in the scale.

The second core idea is that some improvements are ‘too small to make a difference’, and the argument could just as easily begin at the other point in the scale. We could have begun property buying with a comparison between House and a flat that costs £299,999, then asking whether a small improvement to the latter so it costs £299,998 would have tipped the scales. And so positing strict indif-

[3] de Sousa (1974), p. 545.

ference simply shifts the implausibility, because it shifts the location at which a small improvement tips the scales.

Introspection bolstered by small-improvement arguments is enough to convince me that we have incomplete preferences, that Descriptive Preference Completeness is false. We all have incomplete preferences; as I'll put it, we are all incomplete.

But popularity needn't mean rationality. Most of us fall victim to the gambler's fallacy or the sunk-cost fallacy every so often, but doing so is irrational. Unlike in those fallacies, however, having incomplete preferences in the cases just surveyed does not feel irrational: it is not obviously a mistake. In that vein (de Sousa 1974, 545) also claims that though the wife in question lacks complete preferences, 'she is not irrational' to display this pattern of preferences. So he rejects

Normative Preference Completeness. All fully rational agents have complete preferences.

Since incomplete preferences are not *obviously* irrational, we would need some argument to accept Normative Preference Completeness, lest it look unmotivated.

Unfortunately, there is such an argument, and it is that if you violate Descriptive Preference Completeness then you are a sucker. You can be *value-pumped*. To get the problem of value pumping on the table, we need to assume a decision theory for the sake of argument: expectation-maximising is not available because by definition the incomplete agent doesn't have complete expected utilities. We need a theory of which incomplete actions are permissible.

Here is one:

Incompleteness-Liberal. If X and Y are incomplete with each other, but preferred to all other options, then it is permissible to choose X and permissible to choose Y.

Incompleteness-Liberal is intuitively plausible. For example, in the case of House versus Flat, if you have narrowed your decision down to these final two properties and you still have no clear preference between them, but you prefer both to all other properties on the market and not buying at all, it seems clear that you should plump for one or the other. In everyday cases of incompleteness, the almost irresistible verdict is 'so you may choose either'. The fact that varying either property's price by £1 wouldn't tip the balance doesn't seem to detract from that verdict.

At a more theoretical level, if X is not strictly preferred to Y, how could it be impermissible to choose Y? And vice versa. We would expect that as your preferences become less 'opinionated', more actions will become permissible, and here by stipulation the agent is not at all opinionated—not even indifferent—between X and Y. As Simon Blackburn puts it about a somewhat related matter, 'both common sense and high theory tell us how to handle it. The agent has to *plump* for one alternative.'[4]

[4]Blackburn (2010), p. 50.

But here is the problem: if you are permitted to choose either of two incomplete options, we can keep offering you pairs of incomplete options that nevertheless lead you to a dispreferred outcome. This is a value pump.

Consider again Hopper, who is incomplete between Academic and Banking. He works at the latter, and—it is a very competitive field!—his boss offers him a move to Banking-, which is the same job with a slightly smaller salary. He should clearly reject this offer with a snarl, because switching from Banking to Banking- is all-else-equal impermissible. But Banking and Banking- are both incomplete with Academic, so according to Incompleteness-Liberal:

- it is permissible for Hopper to move from Banking to Academic.
- it is permissible for Hopper to move from Academic to Banking-.

The problem is that *together* these permissible acts amount to the impermissible move from Banking to Banking-. This is a value pump: two (apparently) permissible actions amount to an (apparently) impermissible one. Value pumping is grounded in a fundamental feature of incompleteness, which underlies Small-Improvement Arguments. This is the possibility of one-sided improvement, where we can improve one option slightly without 'tipping the balance': Banking and Banking- are both incomplete with Academic, but Banking is preferred to Banking-.

We could construct a similar example involving your property buying: voluntarily paying an extra £1 for the flat is (slightly but genuinely) irrational, all else equal, but Incompleteness-Liberal seems to permit you to switch from Flat+ to House and then from House to Flat, which amounts to the same thing.

Thus the combination of incomplete preferences and Incompleteness-Liberal seems to lead to disaster. Here it doesn't *force* us into disaster (it doesn't require you or Hopper to make the switches), but it does *permit* us to get into it. It's a permissive value-pump rather than a requiring one. But that too is a terrible outcome, because getting into a predictable disaster seems manifestly irrational. Thus we must either say that incomplete preferences are irrational (that is, accept Normative Preference Completeness) or that Incompleteness-Liberal is not the correct decision theory for such preferences.

As a last ditch, might we claim that the sequence is not a disaster, or that it need not be irrational to complete it? But would *you* respond to Hopper if he'd done this? We may imagine the pump continuing: Hopper takes progressively worse banking and academic jobs, until he works at an investment bank for minimum wage. This sequence of choices seems clearly irrational, the kind of thing decision theory should save us from. As we'll see in Chapter 4, not everybody agrees with this, but I find it almost impossible to deny.

So we must either accept Normative Preference Completeness or reject Incompleteness-Liberal. In some ways this choice frames the whole debate: can we make the former principle less demanding, so it seems less absurd that complete preferences might be a requirement of rationality alongside logical omniscience and the rest? Must I be so opinionated about obscure choices between onions and carrots?

The other option is to replace Incompleteness-Liberal with a decision rule that lets us accept incompleteness with a clear philosophical conscience. If we can't, then incompleteness leaves us naked against value pumps, a clear sign of irrationality. As we'll see very soon, this is effectively the argument of Elga (2010) about the credal cousin of incompleteness.

Defenders of Normative Preference Completeness can also correctly say that demandingness is not always decisive. A tragic view might hold that incompleteness *is* irrational but unavoidable for us, either because of our limitations or because it's mandated by other rational requirements. For example, if we think that our preferences should fit the objective evaluative facts as we perceive them, but we have only imprecise access to those facts, then it may be impossible to satisfy both requirements—for complete preferences and for preferences that fit the evaluative facts as we perceive them—simultaneously.

But in the end, I don't think accepting Normative Preference Completeness is correct, partly because it doesn't get us to the heart of the matter. Even if completeness is a rational norm, what if we are irrational sometimes? We want a decision rule to tell us how to cope in that situation. Even if incompleteness is irrational, it doesn't seem like it should leave us totally rationally adrift in the way that (say) believing both a proposition and its negation might. And there *does* seem to be a way to cope with incompleteness: don't fall victim to a value-pump.

That injunction—spelt out a little more—will effectively be my addition to Incompleteness-Liberal, and will serve twin purposes. It'll help those with incomplete preferences to avoid disastrous value pumps, and it'll thus undermine the 'incompleteness leads to disaster' argument for Normative Preference Completeness. By showing that incomplete preferences need not allow us to be irrational, we make it more plausible that such preferences are not themselves irrational. (Of course, there may be other motivations for Normative Preference Completeness, but value-pumping is the central one I'll contend with.)

Finding such a decision rule is one of the main tasks of this book. The rule I'll defend in chapters 3 and 4 is in the spirit of Incompleteness-Liberal, but it heavily qualifies the permissibility of choosing either of two incomplete options. In particular, I distinguish between (in Chapter 3) synchronic or one-shot choices where one may choose either, except in certain odd cases, and (in Chapter 4) synchronic or repeated choices where one must take account of previous choices to avoid a value pump. Hopper is not irrational to have incomplete preferences or even to choose either job, but he is irrational to be value pumped.

3 Imprecise Credences

Sometimes, like in the *Star Trek* episode 'Errand of Mercy', we put our credences—degrees of belief—in terms of odds:

> Captain Kirk: What would you say the odds are on our getting out of here?
>
> Mr. Spock: Difficult to be precise, Captain. I should say, approxi-

mately 7,824.7 to 1.

Spock's estimate of the odds is absurdly precise, despite his disclaimer. How *could* he have enough evidence about their likelihood of escaping the occupying Klingons on a planet they are visiting for the first time, sufficient to justify a credence that extends to five significant figures? It's intuitive that Spock's credence should be 'imprecise' in some way: *difficult to be precise, Captain. I should say it's rather unlikely.*

If we didn't take ourselves to have imprecise credences at least some of the time, then the *Star Trek* joke wouldn't be funny—and it is funny.

A precise credence is given by a real number between 0 and 1, but imprecise credences are somehow not representable by such a number. Imprecise credences are often informally given by a range or pair of numbers ('between 0.1 and 0.2'). Formally, they are usually typically represented by a set of functions, called a 'representor' or 'credal set'. In Spock's position, for example, if p is the proposition that we will get out of here, his stated credence $c(p)$ is very precise and very low. We can imagine that Kirk is less pessimistic and less precise, so his credal set contains functions with different verdicts on p, ranging from $c(p) = 0.1$ at the bottom end to $c(p) = 0.4$ at the top.

As with incomplete preferences, we might ask whether our credences are ever imprecise, and whether they ought not to be. There are two credal analogues to the principles about preferences:

Descriptive Credal Precision. All agents have precise credences.
Normative Credal Precision. All fully-rational agents have precise credences.

(Elga 2010, 3) puts the latter as "perfect rationality requires one to have sharp degrees of belief".

Introspection tells against Descriptive Credal Precision. We're not likely to be in Spock's position, but how confident are you that you won't have a car accident in the next week, in the car you are renting from the airport? An accident is fairly unlikely, but driving an unfamiliar car in a new area, it's far from impossible. Any assignment of a real number to the proposition that you will have an accident in the next week seems arbitrary. Or to use a common kind of example, how confident are you that it will rain in London on July 4th?[5] There are many competing considerations on both sides: the UK is generally very rainy, but London is in the rain shadow of Wales; the date is in July, but due to climate change the weather in July may be more unpredictable. It rained on n days in London last year, and so on.

As in the preferential case, we may mount a small-improvement argument against the claim that you should have a precise credence in this proposition, demonstrating what Schoenfield (2012) calls 'insensitivity to mild evidential sweetening'. Consider again the proposition that it'll rain in London on the 4th of July, or Schoenfield's example that on average, 24 men in Bulgaria stand on their heads on Sundays. It seems permissible and natural to be neither more

[5]See, for example, Mahtani (2018). She presents a version of a small-improvement argument in endnote 2.

nor less confident in the claim that it will rain than that it won't rain. So in a parallel to the preferential case, perhaps we are 'strictly indifferent' in the sense that following the Principle of Indifference, we have credence 0.5 in this proposition? In the argument to follow, 0.5 is really a stand-in for any precise credence we might have for whatever reason.

First, add a little evidential sugar to one side. For example, suppose we learn that in the London borough in question it rained on (n+1) days last year, not n. This is a little bit more evidence that it'll rain there this July 4th, and it should make us a little more confident in that proposition, as the financial sugar made us prefer Flat+ to Flat. But just as the small-improvement in that case didn't seem enough to make us prefer Flat+ to House, so the evidential sweetening here doesn't seem enough to make us more confident that it will rain on July 4th than that it *won't*—that is, it doesn't seem enough to raise our credence above 0.5.[6]

I mentioned that one might reject Normative Preference Completeness by arguing that our preferences ought to be tied to the objective evaluative facts as we see them and that sometimes we don't see them very clearly. The credal analogue of this argument (against Normative Credal Precision) is much more convincing: our credences ought reflect our evidence, but that evidence often seems not to support precision—how could Spock possibly have landed on his estimate, for example? Think how many pieces of conflicting evidence there are about your driving over the next week. I won't engage with these epistemological issues about whether our credences *ought* to be imprecise because my focus is decision-theoretic, not epistemic.[7]

I'm interested in whether imprecision leads to disaster. The simple fact is that many of us do seem to be unsharp (a term I use to encompass incompleteness and imprecision, as I explain below), and so there are two questions that interest me on the credal side: if we are imprecise are we irrational in so being, and if so how should we act in our fallen state? The most influential defence of Normative Credal Precision is that there is no good answer to the latter question, so you best not have imprecise credences. This is a decision-theoretic argument against imprecision, and it is structurally similar to the value-pump argument against incomplete preferences.

As before, it begins with an example. Returning to the rental counter, you must bet on the outcome that you'll have an accident in the next week, when you decide whether or not to take the supplementary insurance or damage waiver at a certain price. The most intuitively plausible decision rule is an analogue of Incomplete-Liberal.

It's time to introduce a little more terminology, due to Isaac Levi. As I mentioned, the standard model of imprecise credences is a set of credence functions—a representor or credal set. E-admissible actions are those which are sanctioned by at least one of those functions:

[6]See Schoenfield (2012), especially pp. 200–202, for a particularly careful presentation of the argument.

[7]A classic here is Joyce (2010). See Carr (2020) for recent criticism of the argument from evidence, and Rinard (2014) and the references there on the Principle of Indifference.

E-admissibility. An action is E-admissible if and only if that action maximises utility according to (is sanctioned by) at least *one* sharpening in the agent's representor.[8]

Credal-Liberal says that if an action is admissible according to at least one credence function in an agent's credal set, then it is permissible for that agent:

Credal-Liberal. If an act ϕ is E-admissible, then ϕ is permissible.

For example, if taking the insurance is admissible according to one function and rejecting it is admissible according to others, then you may permissibly do either. Your imprecise credences leave it rationally up to you.

But with this rule in hand, imprecise credences also engender predictably irrational sequences of action. Here's a particularly clear example due to Paul Weirich:

> ... some may object to applying [Credal-Liberal] to a series of decisions. For example, if the probability of rain tomorrow ranges from 0.4 to 0.6, then, under standard simplifying assumptions about the utility of money, the rule authorizes paying $0.6 for a gamble that pays $1 if it rains, and paying $0.6 for a gamble that pays $1 if it does not rain. But if one makes both bets, one is sure to lose money.[9]

The idea is that both bets are E-admissible and so Credal-Liberal permits them both. But if you do make both bets you pay $1.20 and lose money whatever happens: if it rains, you'll receive $1 from the first bet and nothing from the second; if it doesn't rain, you'll receive nothing from the first bet and $1 from the second. Either way, you'll receive $1, which is less than $1.20. Taking both bets means a sure loss, and looks grossly irrational. This is a familiar story now. Imprecise credences together with Credal-Liberal allow us to complete a sequence of actions that seems clearly impermissible, here taking both bets and definitely losing money.

As above, we have three broad options. Argue that it's not in fact irrational to take both bets. Argue that the imprecision is what got you into the mess and so Normative Credal Precision is true (this is the line famously taken by Adam Elga, and responding to his argument will be the main focus of Chapter 4). Or reject Normative Credal Precision and find a better decision rule than Credal-Liberal.

4 Hard Unsharpness

I've tried to show that preferential incompleteness and credal imprecision are motivated in some of the same ways (through introspection and small-improvements) and that they engender some of the same decision-theoretic issues, especially value pumping.

The theoretical space for each phenomenon is also much the same. There's an epistemic option, that Savage's (P1) is true. Descriptive Preference Complete-

[8]Levi (1974), pp. 409-410.

[9]Weirich (2001), pp. 439-440.

ness is true, but we are often ignorant of our own preferences. Of course such ignorance happens sometimes—nobody is immune to failed introspection or self-deception—but Descriptive Preference Completeness says that if we judge ourselves incomplete between two options or lacking precise credences, then we are *always* mistaken. We may or may not add the claim that only one point-valued credence is justified by the evidence.

If we don't take the epistemic option, then we must reject or heavily qualify (P1). When your preferences are incomplete and/or your credences imprecise, then you depart from having a real-valued utility function and credence function.

But there are many ways to *not* have a function. As we've seen, a natural option is to talk in terms of sets. The incomplete agent has (in some sense of 'has') a non-singleton set of preference orderings rather than just one ordering. If she is incomplete between X and Y, her 'preferential set' as concerns those two is $\{X \preccurlyeq Y, Y \preccurlyeq X\}$. If she follows Incomplete-Liberal in making a one-shot choice between X and Y—and thus chooses X or chooses Y—then these are the two orderings compatible with her behaviour.

Similarly, when your credences are imprecise, your credal set C contains two or more credence functions. But precisely what is it to have a non-singleton credal set? It's not as if there's a credence function or set thereof in my brain to be examined by neurosurgeons. Credence functions and credal sets are mathematical representations of my beliefs. And these formal descriptions—a set instead of a single function—are compatible with many interpretations.

I think it's at least theoretically fruitful to examine the possibility that incompleteness and imprecision are both manifestations of the same phenomenon—*unsharpness*—and so my terminology is regimented:

Table 2: regimented terminology.

Incomplete	Preferences
Imprecise	Credences
Unsharp	Both preferences and credences
Incommensurate	The evaluative

Aside from some passing references, the evaluative isn't the focus until Chapter 7 when I turn to 'spectrum arguments' about betterness.

I think that incompleteness and imprecision are two sides of the same coin. Imagine again that you have utility linear in pounds, and it's indeterminate whether the maximum price you are willing to pay for a lottery ticket is £10 or £20. This suggests that it's indeterminate whether your expected payoff from the lottery is £10 or £20. The expected payoff is simply c(victory) times u(victory), so we can model this unsharpness in several ways:

- as determinate utility of victory (20 utiles) but imprecision in your credence that the ticket will win (either 1 or 0.5), or

- as a precise credence that the ticket will win (0.5) but incomplete preferences (20 or 40 utiles for winning), or
- as some combination of both phenomena.

Offering you further choices will narrow down the range of ways in which we can model your decisions. But the point is that much the same phenomena can arise because of *either* preferential incompleteness or credal imprecision.

Now it could be that the similarities are shallow ones, simply caused by moving in both cases from a single function to a set of functions. Then beyond this formal similarity, credence and preference are simply too different to receive a unified theoretical treatment. I reject this view, and will respond to it by pushing a unified treatment as far as I can. I'm also cheating: on the Savage-style construal of credences as willingness to take bets at certain odds, and to grossly overstate things, credences simply are preferences.

So how to interpret a non-singleton representor? To adapt some useful terminology that I first encountered about the evaluative in Broome (1997), we must distinguish between 'hard' and 'soft' interpretations.

In genuine or 'hard' unsharpness, unsharp preferences and credences are *not* given by real numbers (or functions that output such numbers). The representation is fundamentally different, perhaps a non-singleton set of numbers or functions, and your state is different in kind from a sharp one. Your credence could be given by a set of precise credence functions, a set of sets of such functions, a range of numbers ('between 0.6 and 0.8'), or a pair of numbers (a buying price and a selling price).[10] What makes these views hard is that if your credence in some proposition p is unsharp, then for every real number x it's determinately false that x is your credence in p.

Hard views say that if Spock lacks a precise credence that they will get off the planet, then his credal state is not represented by a number but by a set or a range or something else. But without a number, there is no expected utility, hence no expected-utility maximisation, hence no standard decision theory, at least not without hard work to extend it to sets. There are many hard views and so it's very difficult to say anything beyond the sketch just given without being false or misleading about one of them. So I'll consider two example views.

Walley (1991) is a classic of the genre, not merely due to its doorstop size. If you have a precise credence in a proposition H, then you will be indifferent between buying and not buying, selling and not selling, a bet on H at a 'fair' price determined by that credence. For example, if some lottery ticket costs £1 has an expected payoff of £1 (and your utilities are once again linear in pounds), then you are indifferent between selling or buying the ticket or not. These all have the same expected utility.

Consider

The Bet. The buyer pays the seller the price of £1; the seller gives the buyer nothing if H is false and £10 if p is true.

[10] Mahtani (2020) considers a 'set of sets' view.

Table 3 shows the payoff table for The Bet as a potential buyer. If you don't buy then you keep your £1. If you do buy then you get £10 with likelihood $c(H)$ and lose everything with credence $1 - c(H)$.

Table 3: Payoff table in pounds for buying The Bet

	H true	H false	Expected payoff
Buy	10	0	$10c(H) + 0(1 - c(H)) = 10c(H)$
Don't buy	1	1	1

Thus we can see that if your credence in H is 0.1 then you should be indifferent between buying The Bet and not buying it, because in that case $10c(H) = 1$. The expected payoffs of buying and not buying are equal.

Now let's suppose you are the bookmaker, and you are considering selling The Bet on the same terms. Table 4 shows your payoff table.

Table 4: Payoff table in pounds as bookmaker

	H true	H false	Expected payoff
Sell	-9	1	$-9c(H) + 1(1 - c(H)) = 1 - 10c(H)$
Don't sell	0	0	0

Again if your credence in H is 0.1 then your expected payoff for selling is $1 = 10 * 0.1 = 0$, and you are indifferent between selling and not selling the bet. Thus if $c(H) = 0.1$ you regard £1 as a *fair* price for The Bet because you are indifferent between buying and not buying or (as the bookie) selling and not selling.

For (Walley 1991, 52ff), imprecise credences occur when these buying and selling prices diverge. It might be, for example, that you would buy The Bet for 80p or less and sell it for £1.20 or more. (Such 'spreads' are common in gambling and currency trading, but that's because the bookie or currency dealer needs to make a living; here we're assuming that the bookie is happy to break even.)

So what if you are offered a chance to buy The Bet for 90p? Walley considers several models of unsharpness, but most relevant here is the *exhaustive interpretation* where you have no disposition either way about buying or selling The Bet at prices between 80p and £1.20, because the facts in the previous paragraph exhaust the truths about your credences. On this interpetation, imprecision is due to 'real indeterminacy in beliefs' (Walley 1991, 104).

We can see here the possibility of permissive value-pumping. If I buy The Bet now for 90p and sell it in an hour for 85p without having learnt anything new about p, then this looks clearly irrational: I am now 5p worse off. Under the exhaustive interpretation, my dispositions do not *force* me to make this series of bets and so my imprecise credences do not impose a sure loss on me in that sense, but they do seem to *permit* taking the sure loss.

I mentioned that not everyone agrees that such sequences are irrational. Sarah Moss's *Time-Slice Epistemology* construes imprecise credences as sets of precise credence functions (and incomplete preferences as sets of complete preferences). But there is a distinctive notion of agency at play because in choosing, a rational agent also 'identifies with some precise mental state for purposes of action' (Moss 2015, 673). Even when such an agent has an unsharp representor, she identifies with a single sharpening therein. The agent must act as if this privileged member of her representor were her sharp representor—only acts sanctioned by that sharp pair are choiceworthy. This is a purely synchronic norm and it can permit being value-pumped: if an agent identifies now with a sharpening that sanctions buying The Bet for 90p and later with another that sanctions selling The Bet for 85p, then she may be value pumped without ever violating the synchronic norm.

Hard unsharpness tends naturally to credal incomparability—where your credence in p is neither higher than nor lower than nor equal to your credence in q—because there seems to be no non-arbitrary way to compare credences that 'overlap' in set membership or similar. It is thus a fairly radical departure from the orthodox picture, because your credences are not 'point-valued': you do not have a credence that is representable by a single real number.

For the rest of this book, I will not be considering hard-unsharpness except in the occasional mention. My goal is to defend a *soft* unsharpness, according to which it is vagueness, partly with a view to fleshing out that theory so we don't need to go hard.

5 Supersharp

I will defend a more specific version of the following view:

Unsharpness-as-Vagueness. When an agent has an unsharp representor, it is vague which precise credence function and complete utility function is hers.

Vagueness is mostly closely associated with the paradox of the sorites, or heap. In that paradox, it is compelling that removing one grain from a heap of sand leaves a non-heap: that if n grains form a heap, then $(n-1)$ grains form a heap. But if 10,000 grains form a heap, this implies (via repeated modus ponens) that five grains form a heap, even in contexts where this is clearly false. Such reasoning is a sorites on 'is a heap'. It is vague where we tip from a heap into a non-heap, and this is what allows the puzzle to take hold.

Sorites hang on what *tolerance* principles such as 'one grain isn't the difference between a heap and a non-heap'. Tolerance principles are compelling: even given a theoretical commitment to its falsity, 'growing a short man by 1mm won't stop him being short' is almost undeniable.[11]

There are obviously many accounts of vagueness, and I will assume that the correct account is *tolerance-denying*: tolerance principles are false. So, for exam-

[11]See Wright ([1976] 1997), p. 156.

ple, a tolerance-denying theory of vagueness says that it's false that growing a short man by 1mm won't stop him being short. In particular, I'll rely on *penumbral connections* or *super-truths*, including the falsity of such tolerance principles. Different tolerance-denying theories disagree about what makes such principles false, and about the truth or falsity of *other* claims.

Consider supervaluationism. It says that there are many ways in which 'is tall' can be sharpened consistent with our verdicts about clear cases:[12]

- claims which are false on all sharpenings ('a 50cm tall man is tall') are false;
- claims which are true on all sharpenings ('a 250cm tall man is tall') are true;
- claims which are true on some sharpenings and false on others—'Timothy Williamson is thin', to use an example from Williamson and Simons (1992)—are indeterminate or borderline.

Supervaluationism is tolerance-denying because tolerance principles are false on every sharpening (and so 'super-false'), but false 'in different ways' so to speak: every sharpening draws the line between the tall and the not-tall, the thin and the not-thin in different places. But they *do* all draw a line.

So, for example, supervaluationism says that it's determinately true (super-true) that there is a 1mm threshold between being short and not-short, but for no particular number of millimetres is it super-true that *that* is the threshold number. For some heights (where people are clearly short or not-short) it is super-false and for others (where people are borderline-short) it is indeterminate.

I said my view was more specific. I will defend the conjunction of Unsharpness-as-Vagueness and a tolerance-denying account of vagueness. The combined view *Supersharp* assumes a tolerance-denying account of vagueness. Supervaluationists and other tolerance-denying indeterminists claim that there is a minimum number of grains required for a heap but it's indeterminate what that number is, perhaps because meaning depends on use but our use hasn't fixed a precise threshold. Supervaluationism is my working theory, but of course epistemicism about vagueness is also tolerance-denying. It says that some number is determinately the minimum but we don't and likely can't know what it is. Vagueness is ignorance.

Terminologically, I reserve 'indeterminate' for *unsettledness* or there being no fact of a matter. I my terms, epistemicism is a denial that vagueness is indeterminacy (as opposed to epistemicism being an account *of* indeterminacy).

As a matter of philosophical temperament I lean towards indeterminism but because this book explores the consequences of Supersharp—Unsharpness-as-Vagueness plus tolerance-denying about vagueness—most of what I say will be compatible with epistemicism. As we'll see in the next chapter, whether my argument supports the claim that our preferences (and credences) are indeterminate or determinate-but-unknown hangs on some details about possible worlds.

[12] For classic presentations of supervaluationism, see Fine (1975). An excellent book-length defence is Keefe (2000).

No matter what our theory of vagueness, tolerance principles are overwhelmingly plausible: it's very plausible that if you take a short man and add 1mm to his height, you have a short man. The sorites' paradoxical status is testament to their plausibility. But in assuming a tolerance-denying account of vagueness I assume that they are false. It is true, and determinately so, that adding 1mm to a short man can get you a not-short man. In the background is a basically psychological explanation of the plausibility of tolerance principles: because it is indeterminate or unknowable how they are true (where the threshold number of mm is, for example), we find them very hard to deny.

Given supervaluationism, Supersharp says that every agent has a full preference ordering—I use 'full' to avoid confusion, as part of terminological regimentation—but it can be indeterminate what it is, just as it is indeterminate how many grains of sand are required to form a heap. Thus you *do* have a number of onions that you'd trade for 247 carrots, but it's indeterminate what that number is. To be incomplete between X and Y is for it to be indeterminate which you prefer, in a way that's insensitive to sweetening. In the simplest case, $\{X \succ Y, Y \succ X, X \sim Y\}$ is the set of sharpenings and it's vague which of those orderings is yours. (In the next chapter I'll argue that $\sim$ can be dispensed with.)

Descriptive Preference Completeness has the same status as 'there is a minimum number of grains of sand required to form a heap': super-true, but indeterminate how. Similarly, imprecise credences are vague credences, so you have a point-valued credence in every proposition, but it's sometimes vague what that credence is.

Why did I say every agent having a complete preference ordering is potentially misleading? Because according to Supersharp our preferences (and credences) are all point-valued, but may nevertheless be incomplete or imprecise. The way to resolve this apparent contradiction is that when our credences are imprecise, it's vague what our point-valued credence is... but determinate and knowable that we *do* have one. Similarly, when we have incomplete preferences we have point-valued utility functions, and the incompleteness lurks in the vagueness. In some vagueness jargon: 'complete'/'precise'/'sharp' and their negations are terms in the meta-language; 'point-valued'/'full' are terms in the object language:

Supersharp. There are agents whose utility and credence functions are vague (and thus soft-unsharp), nevertheless it is determinately true that they have full, point-valued credences and preferences.

Each agent has a point-valued credence function, but it can be indeterminate what that credence is. When she has a non-singleton credal credal set $C = \{c_1, ..., c_n\}$ of credence functions, it is super-true that she has a point-valued credence function, but indeterminate whether it is c_1, or ..., or c_n. Unsharpness-as-Vagueness interprets C as containing the admissible sharpenings of her beliefs. Given a tolerance-denying account of vagueness, an object-language version of Descriptive Credal Precision is super-true. A similar story is true of her preferential set U of utility functions.

Her *joint representor* (or just 'representor') R is a subset of the Cartesian product

$C \times U$, the set of pairs of credence functions and utility functions. If both U and C are singleton sets, then she is sharp; if they are not, then she is unsharp.

If she is unsharp, R will be a proper subset of $C \times U$ if certain members of C pair only with certain members of U. Suppose it's indeterminate whether our agent's credence-preference function pair is (c_1, u_1) or (c_2, u_2). Then her credal set is $\{c_1, c_2\}$ and her utility set is $\{u_1, u_2\}$. (c_1, u_2) is a member of the Cartesian product of the two sets, but it's not a member of her joint representor $R = \{(c_1, u_1), (c_2, u_2)\}$. Supersharp says that it's determinately true that some member of R is her pair of functions.

I won't defend the assumption of tolerance-denying accounts, but rather explore the consequences of such a choice. In taking this anti-tolerance stance I'm *not* being ecumenical between all theories of vagueness. Supersharp is just one species of Unsharpness-as-Vagueness, other theories of vagueness reject supertruths or are otherwise not tolerance-denying. For example, Braun and Sider argue that supertruths are one of the more objectionable features of supervaluationism and according to their Semantic Nihilism, claims such as 'some member of R is your credence-utility pair' are merely approximately true. More recently, Andrew Bacon has turned to propositional vagueness, and argues that one shouldn't intrinsically care about vague propositions; I consider Bacon's argument in Chapter 3.[13]

It seems clear that we sometimes have sharp credences and preferences. How clear?[14] At the very least, the sharp case seems to be relatively unproblematic. If we are *never* truly sharp, then my task of providing a theory of unsharpness becomes more pressing. The vagueness view allows us to stay in the realm of comparing real numbers against each other. This is not just a convenience point—though calling it 'theoretical tractability' somehow sounds more respectable—but it points towards the main advantage of the theory.

The advantage is that it dodges a difficult choice. Orthodox rational choice theory with its commitment to expected-utility maximiation has been incredibly successful and fruitful. But insofar as it implies descriptive sharpness then it looks implausible since—as even its defenders admit—such sharpness is at odds with our subjective experience. If instead sharpness is merely a normative claim, then I've argued that this is even more unsatisfying because the norm seems unmotiaved and we still would like some guidance for how to cope when we violate that norm.

The other option is to go unorthodox, by moving to hard unsharpness or other constructions. As we've seen, the problem with value-pumping is that on the face of it, a series of permissible choices amounts to an impermissible one. Unorthodox views try to avoid this by denying that being value-pumped really is impermissible, or by saying that one of the choices in the series was not permissible after all, or by denying that the permissibility of a series of choices is determined by the permissibility of each one.

Like me, many such unorthodox accounts appeal to vague preferences or

[13] Braun and Sider (2007), especially p. 145; Bacon (2018), especially p. 195.
[14] I must thank an anonymous reader here for this question.

projects. Typically, they will argue that we must go unorthodox to cope with them because vagueness induces incomplete or intransitive rational preferences. We'll see in later chapters exactly how this is supposed to go, but it often hangs on Warren Quinn's notorious *Puzzle of the Self-Torturer*. In that puzzle, you are repeatedly offered $10,000 (1990 dollars!) for a very small—but permanent—increase in the amount of pain you suffer. To many people, it seems clear that each particular instance of the deal would be irrational to decline. But you can see that if you keep taking it you will end up very rich but in agony and you'll regret taking the deal so many times.[15] This puzzle is rather niche, but it has been extremely influential: if it can be defanged and explained in orthodox terms (as I try to do) much of the motivation for going unorthodox is undermined.

Tenenbaum and Raffman (2012), for example, argue that to avoid disaster that to avoid disaster we must at some point act *against* our rational preferences between adjacent stages in the series. More recently, Tenenbaum (2020) relies in large part on this case to argue that orthodox decision theory can't cope with vague ends and must forbid them. But 'the puzzle vindicates the idea that such restriction is indefensible', because the puzzle is supposed to both require such ends and also reflect a very common plight. In Chapter 6, for example, I'll argue that we are in a similar position when it comes to climate change.

On this basis, Tenenbaum embraces the decidedly unorthodox

Non-Supervenience Thesis. The rationality of an agent through a time interval t_1 to t_n does not supervene on the rationality of the agent at each moment between t_1 and t_n. (Tenenbaum 2020, 47)

Tenenbaum's rich theory is to some extent in a different, more Kantian tradition—think ends not preferences—and so I won't really take it on its own terms. But I will argue against it by arguing that the orthodox Humean view can cope with cases like the Self-Torturer and so that it need not forbid vague ends or preferences.

Closer to orthodoxy is Chrisoula Andreou, who argues that intransitive preference cycles can be rational, again in cases like the Self-Torturer. Distinguishing between 'relational' appraisal responses (preferences) and 'categorial' ones (which classify options as, for example, fantasic or disastrous) she argues that it can be rational to have cyclic preferences but irrational to *act* on them.[16] Acting against our rational preferences is how we avoid following them around a cycle into a disastrous outcome.

Despite the centrality of the Self-Torturer, both authors are clear that it is meant to be a particularly tenacious and troublesome representative of the vagueness or indeterminacy that we frequently face. Given the welshcake crumbs on the page as I write this, I'll mention also the example of wanting to eat more crisps but to not overindulge. This example exhibits the central role of vagueness, in this case the vagueness of of *overindulge*: 'your goal of not overindulging is vague in the sense that there is no sharp crossover point in the process of eating chips

[15] Quinn (1990).
[16] See, for example, Andreou (2023), p. 67.

one at a time that takes one from not overindulging to overindulging' (Andreou 2023, 166).

So, the choice seems to be to stick with demanding orthodoxy or to embrace the intransitive and non-supervenient. I reject this choice: Supersharp can escape this dilemma. In cases like the Self-Torturer, vagueness engenders the illusion of cyclic preferences: because tolerance principles are false, there *is* a crossover point into overindulging, but it is vague what that point is. Supersharp promises a tractable middle-ground. The presence of full preference orders, for example—for any A and B, either $A \preccurlyeq B$ or $B \preccurlyeq A$—is a supertruth but it can be indeterminate which disjunct is true (in which case you have full but incomplete preferences). We neither have hard unsharpness nor complete *determinate and knowable* preferences.

Since it's (super-)true that we have point-valued preferences and credences, we can exploit the machinery of standard decision theory to that extent, forbidding value-pumping without appealing to an unacceptable decision rule. But because it's vague what those preferences and credences are, we can accommodate a realistic degree of incompleteness and imprecision. However, our decision rule must be augmented to cope with the vagueness around us, and that's what I'll do in chapters 3 and 4.

When I sat down to write this book as a book during lockdown, I thought that I must be reinventing the wheel: aren't decision theory and vagueness both too well-trodden to say anything new? I can't deny that I may have failed to notice a particular precursor to Supersharp, but I've been struck by how *recent* the literature in this area is.[17] And the sorites have only (relatively) recently come back into focus: in his discussion of vagueness and the free-rider problem that'll inform much of Chapter 6, Richard Tuck (1979) describes the paradox of the sorites as something that is only recently attracting the attention of logicians and philosophers of language again. So perhaps there is room to say more.

But has Supersharp been defended before? It can be difficult to attribute views on this matter—terminology such as 'vague', 'unsharp', 'indeterminate', and 'imprecise' is not always used in consistent ways—but there are some clear cases. Bas van Fraassen provides technical discussions of how imprecise credences can be represented with a set of functions, and interprets this set supervaluationally; Susanna Rinard relies much more explicitly on features of supervaluationism to defend a decision theory for imprecise credences ('Moderate', which I'll reject in Chapter 3) and so comes very close to Supersharp, even recognising that we have a precise-but-indeterminate credence in each proposition. The clearest case of a view akin to Supersharp is Anna Mahtani's account where the term 'credence' is vague and given a supervaluational treatment, and I return to her position in the next chapter.[18]

Despite the terminological overlap, the 'vague credences' of Aidan Lyon (2017) address a different question: not an interpretation of the credal set in terms of

[17] See Bradley (2018), especially p. 16.

[18] van Fraassen (1990); van Fraassen (2006); Rinard (2015), p. 2 fn. 1; Hájek (2003), especially pp. 277-8; Mahtani (2019).

vagueness, but the point that—as he plausibly argues—the credal set *itself* will very often have imprecise boundaries. On my picture, this phenomenon would correspond to second-order vagueness.

Where should we situate Supersharp? I've said that it's a middle ground that avoids the hard choice above. It says that incomplete preferences and imprecise credences are *vague* preferences and credences. But if it's supertrue that agents have point-valued preferences and credences, then are they 'really' unsharp? There are two ways to go, here. We might say that there is no unsharpness, merely vagueness. Alternatively, and in common with my previous work on evaluative incommensurability, I'll say that there is unsharpness but it is soft: unsharpness *is* vagueness.[19]

Here the differences between Supersharp's indeterminist and epistemicist sects emerge. Given indeterminacy, my credences are imprecise because indeterminate: there's no number x such that 'my credence that it will rain in London on July 4th is x' is determinately true. It looks hard to deny that this is a form of unsharpness. The supertruth that I have point-valued preferences and credences (a weaker analogue of Descriptive Preference Completeness and the like) is a useful technical artefact, but it remains indeterminate *how* such principles are made true. As supertruths involving 'tall' don't dissolve the puzzles of vagueness about height, so many puzzles of rational choice under vagueness remain. Vagueness as decision-theoretic phenomenon involving indeterminate preferences and credences will occupy most of the rest of this book.

But for the epistemicist Supersharper we have determinate but sometimes-unknown preferences and credences. There *is* a number x where it's determinately true that my credence that it'll rain is x. But we don't know it, and perhaps this ignorance is of the same sort (and maybe has the same source) as our ignorance of the extension of 'tall'. Either way, it is recalcitrant in much the same way: it's unclear how we could find out what x is, and x may be highly unstable, shifting from moment to moment.

I concede that this could well be considered a version of sharpism, and will have many of the same practical upshots: rational choice with unknown preferences and credences is on the face of it a rather more tractable problem. But that tractability allows us to make progress: we can explain why our preferences *seem* incomplete (or at least point to other cases of vagueness as a companion in guilt) and apply standard decision rules to cases such as self-torture.

That's another reason—besides philosophical temperament—that I talk in indeterminist terms: it carves out a distinctive philosophical space, of orthodox rational choice theory but with indeterminate preferences and credences.

6 Preview and Caveats

In this chapter I've introduced incompleteness in preferences and imprecision in credences—which together come under the heading of *unsharpness*—and mo-

[19]Especially Elson (2017). See the citations there for others who defend a similar view about incommensurability.

tivated them. The bulk of the book defends and explores the claim that such unsharpness is vagueness (unsharpness-as-vagueness), in particular given a tolerance-denying account of vagueness, which gets us Supersharp: it's determinately true that we have point-valued preferences credences but often indeterminate or unknown what they are.

In **Chapter 2**, I'll explain how such vagueness in our preferences and credences could come about. In particular, I'll explain how it could come about given a choice-based construal of preference (a 'revealed preference' view), where to prefer X to Y is to choose X over Y when nothing else is at stake. It's often assumed that this construal automatically entails completeness (after all, you *would* make a choice one way or the other), but I'll argue that this entailment need not hold.

Chapters 3 and 4 turn to a decision theory for Supersharp. In **Chapter 3**, I provide a *synchronic* decision rule for actions under unsharpness. Its exact statement requires some terminology, so here is the main idea:

Hierarchical Liberal (rough). Any action that borderline-maximises expected utility is permissible, *unless* there's an alternative that determinately maximises expected utility.

I'll argue that this rule settles a number of puzzles concerning action under vagueness, for example explaining away the plausibility of the argument for John Broome's 'collapsing principle'. More speculatively, it also explains how Joseph Raz's 'basic belief'—that we often have many rational options—could be true, and how principles of structural rationality can be vindicated as mere shadows of substantive claims about our reasons.

Chapter 4 extends that decision theory to *diachronic* puzzles involving sequences of action, to cope with value-pumping:

Compound with Necessary Detachment (rough). Do not complete impermissible sequences of action. If (i) $\phi_1\&\psi_2$ is permissible, (ii) $\phi_1\&\phi_2$ is impermissible, and (iii) you have done ϕ_1, then ϕ_2 is impermissible.

I turn in **Chapter 5** to Warren Quinn's Puzzle of the Self-Torturer and other vague projects. As I explained above, unorthodox decision theories are motivated by appeal to them, and since I'm trying to take standard decision theory as far as it can go towards the vague, it's crucial for me that the Self-Torturer can be accommodated within the extended standard framework. In Chapter 5 I argue that the Self-Torturer doesn't really have intransitive or cyclic preferences. Intransitivity is an illusion based in vagueness, along the lines of a Sorites tolerance principle. Standard decision theory must be supplemented to cope with vague cases where one doesn't have a determinate preference either way, but it is never rational to go *against* one's determinate rational preferences.

With some crucial exceptions, philosophical discussions of incommensurability and incompleteness (especially involving vague projects) have focused on individual action. In **Chapter 6** I turn to a group vague project, which engenders something like the free-rider problem or tragedy of the commons. My focus is on the real-life case of climate change, and my view is pessimistic: especially given the distinctive features of carbon emissions, it may be very hard to escape that value-pump.

Chapter 7 marks a slight change of topic, from preferences to the evaluative. When two items are evaluatively incommensurate (or 'incomparable') neither is better than the other, but they are also not equally good: none of these 'trichotomous' comparisons holds between them. It has been argued, most famously by John Broome but also by me and others, that such incommensurability is vagueness.[20]

Incommensurability clearly has a lot in common with preferential incompleteness. Both are motivated by small-improvement arguments: would one extra pony in the Brecon Beacons make it a better landscape than the Gower? The 'trichotomy thesis' (that of any two such items, one must be better or they must be equally good, if they are comparable at all) is also a clear cousin of Completeness, and value-pumping is a major feature in the incommensurability debate.

Incommensurability drags in (at least potentially) questions about the metaphysics of value. It is open to an evaluative nihilist to deny the reality of incommensurability because, they claim, no evaluative comparisons hold between anything. But it would be much harder to claim that we don't have preferences. More generally, running together preferences and credences on one side and values on the other risks a muddle.

Nevertheless *spectrum arguments* for the intransitivity of 'better than' look an awful lot like the other examples in this book. As with the Self-Torturer, I argue that the apparent intransitivity at work in such cases is really an artefact of the Sorites, and so rational preferences and 'better than' are both neatly transitive. In summary, my defence of orthodoxy concludes with the good.

Chapter 8 recapitulates.

I'll close this introduction with an apology. There are at least four literatures relevant to my topic: incommensurability, vagueness, and spectrum arguments in the broadly evaluative literature; decision puzzles surrounding intransitivity and value-pumping in rational choice; imprecise credences and the like in formal epistemology; and models of rational choice and the constraints thereon in mathematics and economics. As we go down this list, discussions tend to the more formal and technical, more concerned with the mathematics than with how it is to be interpreted. My home is in the first two entries. We might also add applications of these issues to machine learning, but that is a topic for future work.

These debates clearly do interact, but large tracts also run in parallel. I've tried to cast a wide net, but to make some progress I've had to risk overlooking classic contributions and making basic mistakes or reinventing the wheel. This is a risk worth taking, but I nevertheless apologise for my mistakes and oversights.

Since my discussion is largely about how unsharpness is to be understood or interpreted, I've not used much by way of serious formalism. But when discussing

[20]See Broome (1997). See also Sugden (2009), Qizilbash (2014), Andersson (2017), Elson (2017), and Broome (2021).

vagueness one has to be *careful* to avoid ambiguities. 'There is a minimum number of grains required for a heap', for example, could be understood either as 'determinately, there is a minimum...' or as 'there is a determinate minimum...' which is one reason I've regimented my terminology (distinguishing 'precise' from 'point-valued' credences, for example). I've tried to emulate the work I've found most helpful in this area, where technical details inform but don't overwhelm the central philosophical questions. The worst case scenario is that what I've written is neither technically rigorous nor substantive and readable, but I hope I've skirted that.

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