Name: Luke Joyce

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**CSCI 3104, Algorithms** **Profs. Hoenigman & Agrawal**

**Problem Set 7a (14 points)** **Fall 2019, CU-Boulder**

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*Advice 1*: For every problem in this class, you must justify your answer: show how you arrived at it and why it is correct. If there are assumptions you need to make along the way, state those clearly.

*Advice 2*: Verbal reasoning is typically insufficient for full credit. Instead, write a logical argument, in the style of a mathematical proof.

**Instructions for submitting your solution**:

- The solutions **should be typed** and we cannot accept hand-written solutions. Here's a short intro to Latex.
- You should submit your work through **Gradescope** only.
- If you don't have an account on it, sign up for one using your CU email. You should have gotten an email to sign up. If your name based CU email doesn't work, try the identikey@colorado.edu version.
- Gradescope will only accept **.pdf** files (except for code files that should be submitted separately on Gradescope if a problem set has them) and **try to fit your work in the box provided**.
- You cannot submit a pdf which has less pages than what we provided you as Gradescope won't allow it.

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1. (1 pt) Provide a one-sentence description of each of the components of a divide and conquer algorithm.

   *Solution.* Divide and conquer algorithms involve recursion so that a big problem can be broken down into smaller parts (hence "divide") and then solved individually (hence "conquer") and then combined to have a final answer to the original problem.

2. (3 pts) Use the array $A = [2, 5, 1, 6, 7, 9, 3]$ for the following questions

   (a) (1 pt) What is the value of the pivot in the call $partition(A, 0, 6)$?
   *Solution.*
   The pivot in the call $partition(A, 0, 6)$ is 3, and index 6.

   (b) (1 pt) What is the index of that pivot value at the end of that call to $partition()$?
   *Solution.*
   The index of 3 after the call $partition(A, 0, 6)$ is 2.

   (c) (1 pt) On the next recursive call to Quicksort, what sub-array does $partition()$ evaluate?

   *Solution.* The sub-arrays created by partition in this case are [1, 2] and [5,6,7,9], and since in a quicksort algorithm the sub-array on the left is evaluated first, [1,2] will be the next evaluated sub-array.

3. (4 pts) Draw the tree of recursive calls that Quicksort makes to sort the list $E, X, A, M, P, L, E$ in alphabetical order. Use the last element in the sub-list in each recursive call as the pivot.

   *Solution.* Image for number 3 on second page

4. (6 pts) You are given a collection of $n$ bottles of different widths and $n$ lids of different widths and you need to find which lid goes with which bottle. You can compare a lid to a bottle, from which you can determine if the lid is larger than the bottle, smaller than the bottle, or the correct size. However, there is no way to compare the bottles or the lids directly to each other, i.e. you can't compare lids to lids or bottles to bottles. Design an algorithm for this problem with an average-case efficiency of $\Theta(nlgn)$

   *Solution.* If the psuedocode underneath is not correct, what I want it to do is choose the last element of Bottles to use as a pivot for caps. Once caps is reordered, choose the same pivot that was selected for bottles, in Caps, and use that to sort bottles. By the end all elements will be matching up accordingly.

```
QuickSortBC(B[], C[],j,n):
```

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```
#B = array of bottles
#C = array of caps
#n = length of both arrays
pivotCap = partition(C,j,n,B[n])
#Use the last index in caps as pivot for bottles
pivotBottle = partition(B,j,n,pivotCap)
#sort bottles based on pivotCap. this returns index for pivot
bottle for caps.
QuickSortBC(B[],C[],pivotCap, pivotBottle)
QuickSortBC(B[],C[],pivotBottle, pivotCap)
#Do a quicksort for bottles
based on pivot cap, and one
for caps based on pivot bottle
```

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