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CSCI 3104, Algorithms
              Profs. Hoenigman & Agrawal

Problem Set 10b (34 points)
              Fall 2019, CU-Boulder

Advice 1: For every problem in this class, you must justify your answer: show how you arrived at it and why it is correct. If there are assumptions you need to make along the way, state those clearly.

Advice 2: Verbal reasoning is typically insufficient for full credit. Instead, write a logical argument, in the style of a mathematical proof.

Instructions for submitting your solution:

- The solutions **should be typed** and we cannot accept hand-written solutions. Here's a short intro to Latex.

- You should submit your work through **Gradescope** only.

- If you don't have an account on it, sign up for one using your CU email. You should have gotten an email to sign up. If your name based CU email doesn't work, try the identikey@colorado.edu version.

- Gradescope will only accept **.pdf** files (except for code files that should be submitted separately on Gradescope if a problem set has them) and **try to fit your work in the box provided**.

- You cannot submit a pdf which has less pages than what we provided you as Gradescope won't allow it.

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Important: This assignment has 1 (Q1) coding question.

- You need to submit 1 python file.

- The .py file should run for you to get points and name the file as following -
  If Q1 asks for a python code, please submit it with the following naming convention -
  `Lastname-Firstname-PS10b-Q1.py`.

- You need to submit the code via Canvas but the table/plot/result should be on the
  main .pdf.

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1. (34 pts total) Recall that the *string alignment problem* takes as input two strings x and y, composed of symbols $x_i, y_j \in \Sigma$, for a fixed symbol set Σ, and returns a minimal-cost set of *edit* operations for transforming the string x into string y.

   Let x contain n_x symbols, let y contain n_y symbols, and let the set of edit operations be those defined in the lecture notes (substitution, insertion, and deletion).

   Let the cost of *insert* be c_{insert} and *delete* be c_{delete}, and the cost of *sub* be c_{sub}, except when $x_i = y_j$, which is a "no-op" and has cost 0.

   In this problem, you will implement and apply three functions.

   (i) `alignStrings(x,y, ` c_{insert}, c_{delete}, c_{sub}`)` takes as input two ASCII strings x and y, cost of the operations, and runs a dynamic programming algorithm to return the cost matrix S, which contains the optimal costs for all the subproblems for aligning these two strings.

   (ii) `extractAlignment(S,x,y, ` c_{insert}, c_{delete}, c_{sub}`)` takes as input an optimal cost matrix S, strings x, y, cost of the operations, and returns a vector a that represents an optimal sequence of edit operations to convert x into y. This optimal sequence is recovered by finding a path on the implicit DAG of decisions made by `alignStrings` to obtain the value $S[n_x, n_y]$, starting from $S[0,0]$.

   When storing the sequence of edit operations in a, use a special symbol to denote no-ops.

   (iii) `commonSubstrings(x,L,a)` which takes as input the ASCII string x, an integer $1 \le L \le n_x$, and an optimal sequence a of edits to x, which would transform x into y. This function returns each of the substrings of length at least L in x that aligns exactly, via a run of no-ops, to a substring in y.

   (a) (21 pts) From scratch, implement the functions `alignStrings`, `extractAlignment`, and `commonSubstrings`. You may not use any library functions that make their implementation trivial. Within your implementation of `extractAlignment`, ties must be broken uniformly at random.
       If you plan to create a version that saves the parent information in `alignStrings` itself, then you should break the ties randomly in `alignStrings` instead.
       Submit:
       - A brief paragraph for each function that explains how you implemented it (describe how it works and how it uses its data structures).
       - Your code implementation, with code (the code should be submitted on Canvas)

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- The cost matrix S that your code produces on the strings x=EXPONENTIAL and y=POLYNOMIAL with $c_{insert} = 2$, $c_{delete} = 1$, $c_{sub} = 2$

Solution.

For the alignStrings function: 2-D array S is initiated with the first row being $(0, len(x))$ and the first column being $(0, len(y))$ for the initial DP-table before operations are done. The next nested for-loop (lines 10 - 15) iterate through the DP table and, for each cell, checks which operation is most cost efficient to make the previous cell have two equal characters.

For the extractAlignment function, the DP table input as S is read to tell what the best sequence is. This is done by comparing the delete, substitute, and insert operations to see which one is the most cost efficient.

For the commonSubstring function, it iterates string x and performs the operations given in array a to form a substring until the operation is not "/", and once a substring is produced from no operations that is longer than L, it is added to the list of substrings. This function does not use the sub and delete operations because is only detecting verbatim substrings.

```
        P  O  L  Y  N  O  M  I  A   L
   [0,  1, 2, 3, 4, 5, 6, 7, 8, 9, 10]
E  [1,  2, 3, 4, 5, 6, 7, 8, 9, 10, 11]
X  [2,  3, 4, 5, 6, 7, 8, 9, 10, 11, 12]
P  [3,  2, 4, 6, 7, 8, 9, 10, 11, 12, 13]
O  [4,  3, 2, 4, 6, 8, 8, 10, 12, 13, 14]
N  [5,  4, 3, 4, 6, 6, 8, 10, 12, 14, 15]
E  [6,  5, 4, 5, 6, 7, 8, 10, 12, 14, 16]
N  [7,  6, 5, 6, 7, 6, 8, 10, 12, 14, 16]
T  [8,  7, 6, 7, 8, 7, 8, 10, 12, 14, 16]
I  [9,  8, 7, 8, 9, 8, 9, 10, 10, 12, 14]
A  [10, 9, 8, 9, 10, 9, 10, 11, 11, 10, 12]
L  [11, 10, 9, 8, 10, 10, 11, 12, 12, 11, 10]
```

(b) (7 pts) Using asymptotic analysis, determine the running time of the call
commonSubstrings(x, L, extractAlignment(alignStrings(x,y,c_{insert},c_{delete},c_{sub}),
x,y,c_{insert},c_{delete},c_{sub}))
Justify your answer.

Solution. For the alignStrings portion, initiating the DP table takes $O(n_x + n_y)$, and iterating through the DP table takes $O(n_x n_y)$, which comes out to $O(n_x n_y)$ for the whole function since the first part is negligible. For the extractAlignment

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function, we have $O(n)$ since the single while loop iterates until we reach index 0 for either i or j, and each operation inside the while loop decrements at least one of them. for commonSubstrings we have another single for-loop which gives us $O(n)$. Since none of the functions are called within another function, we simply add the runtimes of each one and get $O(n_x n_y + n + n) = O(n_x n_y)$

(c) (6 pts) **Plagiarism detector** - String alignment algorithms can be used to detect changes between different versions of the same document (as in version control systems) or to detect verbatim copying between different documents (as in plagiarism detection systems).

The two song lyrics files for PS10b (see class Canvas) contain lyrics of two different songs in text format. Use your functions from (1a) with $c_{insert} = 1$, $c_{delete} = 1$, $c_{sub} = 1$ to align the text of these two documents. Utilize your **commonSubstrings** function for plagiarism detection. Present the results of your analysis, including a reporting of all the substrings in x of length $L = 10$ or more that could have been taken from y in two columns with the first being the length of the substring and the second being the actual common substring obtained via continuous 'no-op' run.

Briefly comment on whether these songs could be reasonably considered original works, under CU's academic integrity policy.

Solution.

```
Length: | Substring:
    18 | I hear the train a
    12 |  it's rollin
    28 | round the bend  And I ain't
    26 |  since I don't know when
    42 |     When I was just a baby my mama told me
    32 | When I hear that whistle blowin'
    20 |  I hang my head and
    22 |  rich folks eatin' in
    36 |  fancy dining car  They're probably
    44 |  if that railroad train was mine  I bet I'd
    43 | n a little farther down the line  Far from
    62 |  to stay  And I'd let that lonesome whistle blow my blues away
```

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