Name:

ID:

CSCI 3104, Algorithms **Profs. Hoenigman & Agrawal**
Problem Set 6b (40 points) **Fall 2019, CU-Boulder**

Instructions for submitting your solution:

- The solutions **should be typed** and we cannot accept hand-written solutions. Here's a short intro to Latex.
- You should submit your work through **Gradescope** only.
- If you don't have an account on it, sign up for one using your CU email. You should have gotten an email to sign up. If your name based CU email doesn't work, try the identikey@colorado.edu version.
- Gradescope will only accept **.pdf** files (except for code files that should be submitted separately on Gradescope if a problem set has them) and **try to fit your work in the box provided**.
- You cannot submit a pdf which has less pages than what we provided you as Gradescope won't allow it.
- Verbal reasoning is typically insufficient for full credit. Instead, write a logical argument, in the style of a mathematical proof.
- For every problem in this class, you must justify your answer: show how you arrived at it and why it is correct. If there are assumptions you need to make along the way, state those clearly.
- You may work with other students. However, **all solutions must be written independently and in your own words.** Referencing solutions of any sort is strictly prohibited. You must explicitly cite any sources, as well as any collaborators.

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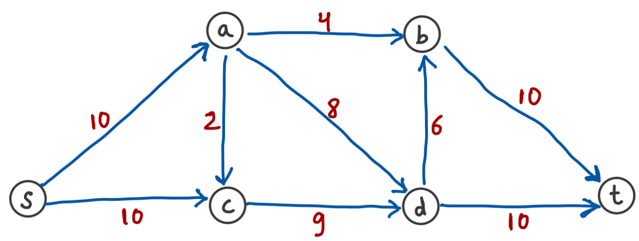
1. (19 pts) Based on the following network and the given edge capacities answer the following.



   (a) (12 pts) Suppose we start the Ford-Fulkerson algorithm and **select the path** $s- > a- > c- > d- > t$ **in the first iteration (Do not chose the first s-t path on your own).** Complete all the iterations of Ford-Fulkerson to find the Max-Flow (including the first round that is incomplete). Clearly show each round with

      i. The path that you are selecting in that round.
      ii. The bottleneck edge on this path.
      iii. The additional flow that you push from the source by augmenting (pushing maximum allowed flow along) this selected augmenting path.
      iv. The residual graph with the residual capacities (on both the forward and backward) edges.

   Also, report the Max-Flow after the algorithm terminates.

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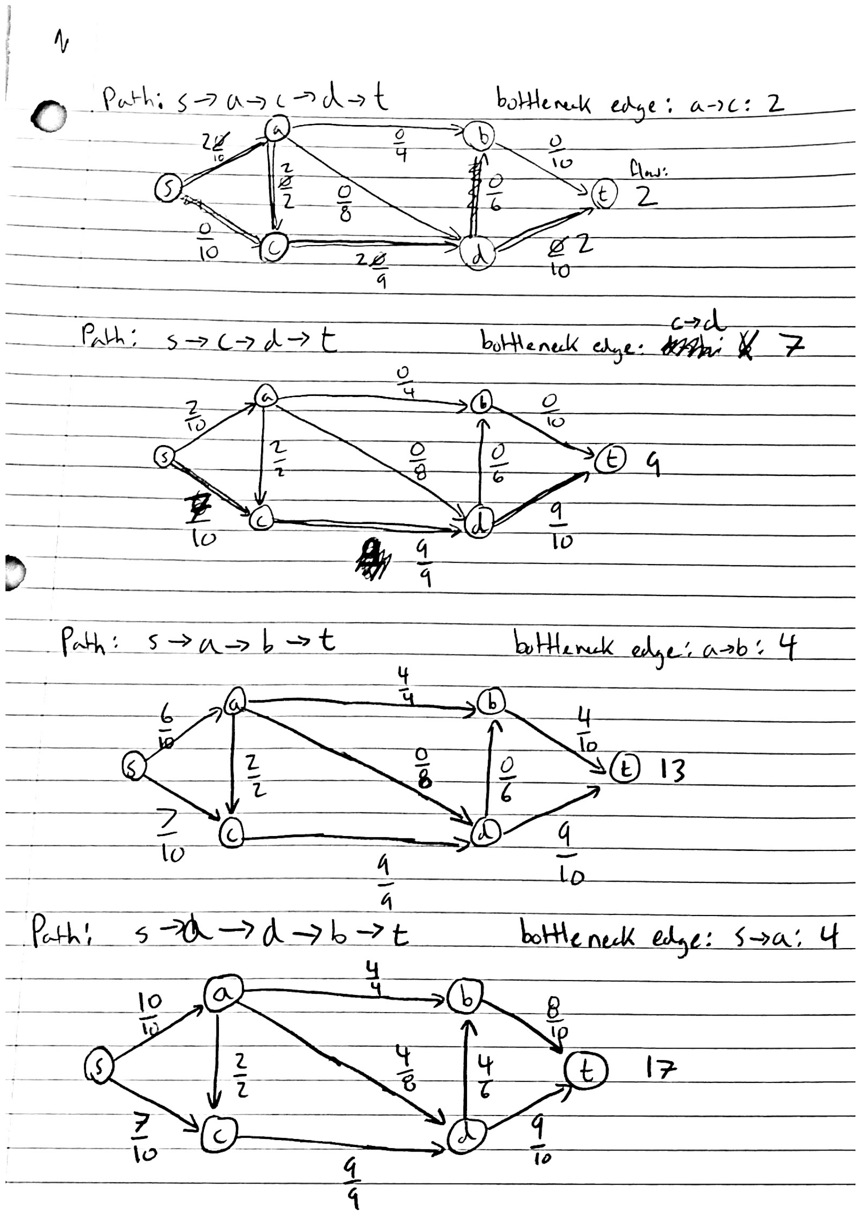
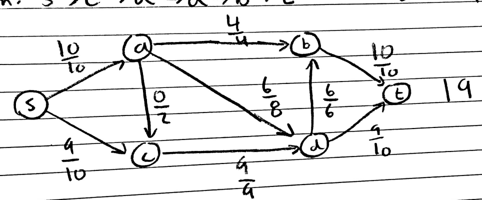
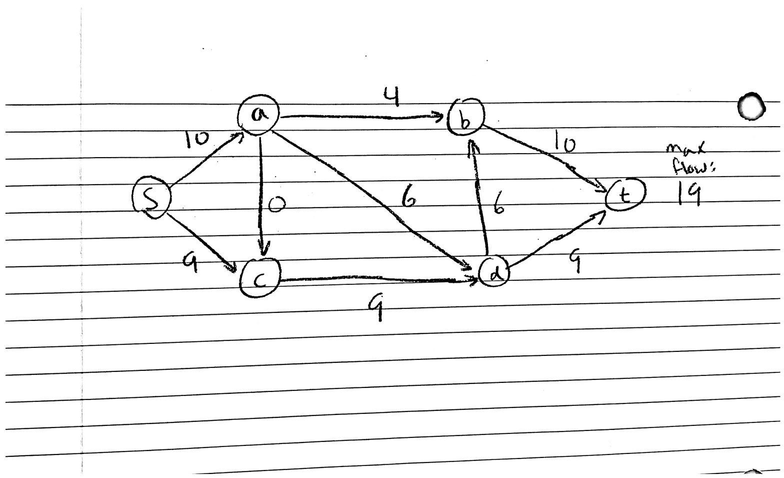


Solution.

Path: s→c→a→d→b→t   bottleneck: c→a: 2



(b) (3 pts) Show the final flow f(e) for the edges of the original graph when the Ford-Fulkerson algorithm terminates.



Solution.

(c) (4 pts) Find the minimum capacity cut with respect to the capacities on the original graph. Is this minimum capacity equal to the Max-Flow that you earlier identified? Justify your answer in a sentence. Also, report the crossing edges in this cut that are saturated (can't carry any more flow).

Solution.

Saturated edges:

$s- > a$

$a- > b$

$c- > d$

$d- > b$

$b- > t$

The cut a-¿b-¿d-¿c, equals the max flow.

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2. (10 pts) Let (X, Y) be any s-t cut in the network G and a be any flow.

   (a) (5 pts) Prove that the value of the flow a equals the **net** flow that crosses the cut (X, Y).
   i.e. $value(a) = \sum_{\text{e out of X}} a(e) - \sum_{\text{e in to X}} a(e)$
   You should use the flow conservation property to complete the proof.
   Hint : Recollect the definition of a flow a.
   $value(a) = \sum_{\text{e out of s}} a(e) - \sum_{\text{e in to s}} a(e)$ where s is the source.

   Solution. The flow conservation theorem says that for any vertex $v \in s, t$, the flow in equals the flow out. If we consider the sum:
   $value(a) = \sum_{\text{v}}(\sum_{\text{e out of v}} a(e) - \sum_{\text{e in to v}} a(e))$. By rule of conservation, for each vertex v we have $\sum_{\text{e out of v}} a(e) - \sum_{\text{e in to v}} a(e) = 0$
   And so the total flow = a The sum of the flows always contains the outward and inward flow of each vertice, so they cancle out.
   $\therefore$ The value of flow a equals the net flow that crosses the cut (X,Y)

   (b) (5 pts) Use the above proof (from part Q3a) to prove that the value of the flow a $\leq$ Capacity of the cut (X, Y).

   Solution. We proved that the flow a equals the net flow that crosses (X,Y), and the flow through an edge can never exceed the capacity of an edge, there fore a is always less than or equal to the cpacity of cut (X,Y)

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3. (11 pts) CU is organising a spot job fair were many companies participate and take tests to select students. After their final round of interviews, they all find a preference list of candidates that they would like to hire. All the companies just want to hire one student (because recession). All the companies sat together and they realised that if they extend offers to the same students, only one of them would get the student so they decide to run an algorithm to hire the maximum number of students they can together.
Example - Following is one such preference of each company after the final round of interviews. If they all give the offer to just their first preference only 3 students will get hired. But a better offer is Apple - Alice, Google - Dave, Facebook - Carol, Amazon - Eliza, Uber - Frank, Netflix - Bob and this gets 6 students hired.

Help them come up with an algorithm to find an offer set that gets the maximum students the job using Ford-Fulkerson.

Preference	Apple	Google	Facebook	Amazon	Uber	Netflix
1	Alice	Alice	Alice	Carol	Carol	Bob
2	Bob	Bob	Carol	Eliza	Eliza	Eliza
3	Dave	Dave	Frank		Frank	Frank

(a) (6 pts) Draw a network G to represent this problem as a flow maximisation problem for the example given above. Clearly indicate the source, the edge directions, the sink and the capacities and label the vertices.

Solution. Its hard to see from the picture, but all edges are directed to the right (out of S, to companies, to students, into T)

(b) (5 pts) Assume that you have access to Fork-Fulkerson sub-routine called **Ford-Fulkerson(G)** that takes a network and gives out max-flow in terms of f(e) for all the edges. How will you use this sub-routine to find the offer set that employs the maximum number of students. Clearly explain your solution.

Solution. Each edge weight on the graph below are of weight 1, which means that once a student is given a company, there is no more capacity for other companies to "flow through" them. If a company A is a part of a path being checked, and all the students that they want are taken, an edge can be reversed so that a different company B gives up their student (reverses flow) to take another one, so the company A can get that student.

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(Space to solve Q3)