Name: Luke Joytce

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CSCI 3104, Algorithms **Profs. Hoenigman & Agrawal**

Problem Set 6a (10 points) **Fall 2019, CU-Boulder**

Instructions for submitting your solution:

- The solutions **should be typed** and we cannot accept hand-written solutions. Here's a short intro to Latex.
- You should submit your work through **Gradescope** only.
- If you don't have an account on it, sign up for one using your CU email. You should have gotten an email to sign up. If your name based CU email doesn't work, try the identikey@colorado.edu version.
- Gradescope will only accept **.pdf** files (except for code files that should be submitted separately on Gradescope if a problem set has them) and **try to fit your work in the box provided**.
- You cannot submit a pdf which has less pages than what we provided you as Gradescope won't allow it.
- Verbal reasoning is typically insufficient for full credit. Instead, write a logical argument, in the style of a mathematical proof.
- For every problem in this class, you must justify your answer: show how you arrived at it and why it is correct. If there are assumptions you need to make along the way, state those clearly.
- You may work with other students. However, **all solutions must be written independently and in your own words.** Referencing solutions of any sort is strictly prohibited. You must explicitly cite any sources, as well as any collaborators.

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1. (1 pt) What do the edge weights of a graph G in a maximum-flow network represent?

   Solution.
   The edge weights of a graph in a maximum flow network represent the greatest flow that can traverse through a each particular edge.

2. (2 pts) What are the two conditions that must be met for network flow?

   Solution.
   1. The flow exiting a node must be equal to the flow entering it.
   2. The maximum-flow must not exceed the flow entering the sink or exiting the source.

3. (2 pts) What do the edge weights in the residual graph G_f represent? Include both forward and backward edges.

   Solution. The edge weights on a residual graph represent the additional flow that the edge could take on. If the max flow of an edge is 5 and there is only 2 flow going through it on the max-flow network, then the forward edge weight would be 3. The backwards edges represent the flow that the reverse direction could hold. The backwards edge weight plus the forwards edge weight is the total capacity of that edge.

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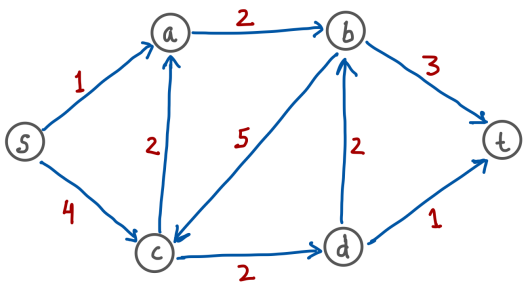
4. (5 pts) Based on the following network and the given edge capacities answer the following.



(a) (1 pts) Can the max flow be 5 (capacity(e_{sa}) + capacity(e_{sc}))? Justify your answer in one sentence.

Solution.
The max flow cannot be 5 because the flow entering the sink t is 5 (capacity(e_bt) + capacity(e_dt)), and the max flow cannot be greater than the amount entering the sink.

(b) (2 pts) For the graph, identify one simple $s-t$ path and the bottleneck edge value on that path. Also report the maximum allowed flow on this $s-t$ path.

Solution.
For path $S->C->D->T$, the minimum edge is 1, $D->T$, so that is the bottleneck edge value.

(c) (2 pts) Assuming all $f(e)$ are initially 0 where f represents flow, what are the residual capacities on the forward and backward edges of G_f after one iteration of the Ford-Fulkerson algorithm. Use the simple path you identified in Part b.

Solution.
Image of graph for residual capacities on the chose path is below.

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